The b-quark running mass in QCD and SM

A. Bednyakov

BLTP, JINR

18.07.2016

Quantum Field Theory at the Limits:
from Strong Fields to Heavy Quarks, 2016

Outline

Quark Mass Definitions

- In QCD, the mass of the bottom quark m_b is a fundamental parameter of the Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \bar{q}_j \left(i\hat{D} - m_j \right) q_j, \quad q_i = \{u, d, c, s, b, t\}$$

NB: Confinement:

- difficult to determine experimentally;
- No "physical mass" like in the case of electron.

Quark Mass Definitions

- In QCD, the mass of the bottom quark m_b is a fundamental parameter of the Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \bar{q}_j \left(i\hat{D} - m_j \right) q_j, \quad q_i = \{u, d, c, s, b, t\}$$

  NB: Confinement:
  - difficult to determine experimentally;
  - No “physical mass” like in the case of electron.
- There are several definitions on the market [El-Khadra and Luke, 2002]

Quark Mass Definitions

- In QCD, the mass of the bottom quark m_b is a fundamental parameter of the Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \bar{q}_j \left(i\hat{D} - m_j \right) q_j, \quad q_i = \{u, d, c, s, b, t\}$$

NB: Confinement:

  - difficult to determine experimentally;
  - No "physical mass" like in the case of electron.

- There are several definitions on the market [El-Khadra and Luke, 2002]
  - The pole mass M_b

Quark Mass Definitions

- In QCD, the mass of the bottom quark m_b is a fundamental parameter of the Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \bar{q}_j \left(i\hat{D} - m_j \right) q_j, \quad q_i = \{u, d, c, s, b, t\}$$

  NB: Confinement:
  - difficult to determine experimentally;
  - No "physical mass" like in the case of electron.
- There are several definitions on the market [El-Khadra and Luke, 2002]
  - The pole mass M_b
  - The running mass in the $\overline{\mathrm{MS}}$-scheme $m_b(\mu)$

Quark Mass Definitions

- In QCD, the mass of the bottom quark m_b is a fundamental parameter of the Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \bar{q}_j \left(i\hat{D} - m_j \right) q_j, \quad q_i = \{u, d, c, s, b, t\}$$

NB: Confinement:

  - difficult to determine experimentally;
  - No "physical mass" like in the case of electron.

- There are several definitions on the market [El-Khadra and Luke, 2002]
  - The pole mass M_b
  - The running mass in the $\overline{\rm MS}$-scheme $m_b(\mu)$
  - "Threshold" masses
    - ★ Potential subtracted (PS) mass $m_b^{\rm PS}(\mu_f)$ [Beneke, 1998]
    - ★ 1S mass $m_b^{\rm 1S}$ [Hoang et al., 1999]
    - ★ Renormalon subtracted (RS) mass $m_b^{\rm RS}(\mu_f)$ [Pineda, 2001]
    - ★ Kinetic mass $m_b^{\rm kin}(\mu_f)$ [Bigi et al., 1997]

Quark Mass Definitions

- In QCD, the mass of the bottom quark m_b is a fundamental parameter of the Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \bar{q}_j \left(i\hat{D} - m_j \right) q_j, \quad q_i = \{u, d, c, s, b, t\}$$

NB: Confinement:

  - difficult to determine experimentally;
  - No "physical mass" like in the case of electron.

- There are several definitions on the market [El-Khadra and Luke, 2002]
  - The pole mass M_b
  - The running mass in the $\overline{\text{MS}}$-scheme $m_b(\mu)$
  - "Threshold" masses
    - ★ Potential subtracted (PS) mass $m_b^{\text{PS}}(\mu_f)$ [Beneke, 1998]
    - ★ 1S mass $m_b^{1\text{S}}$ [Hoang et al., 1999]
    - ★ Renormalon subtracted (RS) mass $m_b^{\text{RS}}(\mu_f)$ [Pineda, 2001]
    - ★ Kinetic mass $m_b^{\text{kin}}(\mu_f)$ [Bigi et al., 1997]

See also lectures on B-physics at this School.

Pole mass of a fermions

Fermion resummed propagator

$$i\left(\hat{p} - m - \Sigma(\hat{p}, m_i)\right)^{-1}$$

Fermion self-energy

$$\Sigma(\hat{p}, m_i) = \hat{p}\Sigma_V(p^2, m_i^2) + \hat{p}\gamma_5\Sigma_A(p^2, m_i^2) \\ + m\Sigma_S(p^2, m_i)$$

The pole mass $M^2 = \mathrm{Re}(s)$ satisfies the following equation:

$$\left(\left(1 - \Sigma_V(s, m_i^2)\right)^2 - \Sigma_A^2(s, m_i^2)\right) s \\ - m^2\left(1 + \Sigma_S(s, m_i)\right)^2 = 0$$

Here, m and m_i are mass parameters (bare or renormalized).

Quark pole mass

- The pole mass of a quark is a well-defined, IR-finite, and gauge-independent quantity in a finite order of PT [Tarrach, 1981].
- However, due to confinement the precision of experimental determination of the quark pole mass is limited by the ratio Λ_{QCD}/M_q [Bigi et al., 1994], [Beneke and Braun, 1994].
- For the b-quark the uncertainty is significant $\sim 10\%$.
- Nevertheless, it can be used as a bridge between different definitions!

Quark running mass in $\overline{\text{MS}}$ scheme

- A "short-distance" mass parameter
  - renormalized at a scale μ
  - insensitive to physics at distances large $1/\mu \to$ no IR problem.
- Based on convenient dimensional regularization $D = 4 \to D = 4 - 2\epsilon$
- Modified minimal subtractions ($\overline{\text{MS}}$) scheme is used to define $m_b(\mu)$

The pole mass M_b can be expressed in terms of $m_b(\mu)$:

- In QCD we have (see [Marquard et al., 2015] for recent 4-loop result):

$$M_b = m_b \left(1 + \underbrace{0.4244\alpha_s}_{9.6\%} + \underbrace{0.9401\alpha_s^2}_{4.8\%} + \underbrace{3.045\alpha_s^3}_{3.5\%} + \underbrace{(12.57 \pm 0.38)\alpha_s^4}_{3.3\pm0.1\%} \right),$$

  where $\mu = m_b$ is chosen, and $\alpha_s \equiv {\alpha_s}^{(5)}(m_b) = 0.2268$.
- In SM, electroweak corrections are taken into account at two-loop (see [Kniehl and Veretin, 2014]). But there are subtleties: m_b in the SM?

Motivations to consider m_b

- Extraction of V_{ub}

$$\Gamma(B \to X_u l\bar{\nu}) \sim G_F^2 m_b^5 |V_{ub}|^2$$

- Extraction of V_{cb}

$$\Gamma(B \to X_c l\bar{\nu}) \sim G_F^2 m_b^5 f(m_c^2/m_b^2) |V_{cb}|^2$$

- Higgs decay (dominant decay mode for $M_h = 125$ GeV)

$$\Gamma(H \to b\bar{b}) \sim \frac{G_F M_H}{4\sqrt{2}\pi} m_b^2$$

- ...

Up to now, the only source of information on the Higgs coupling y_b to b-quarks in the SM.

Motivations to consider m_b

- Extraction of V_{ub}

$$\Gamma(B \to X_u l\bar{\nu}) \sim G_F^2 m_b^5 |V_{ub}|^2$$

- Extraction of V_{cb}

$$\Gamma(B \to X_c l\bar{\nu}) \sim G_F^2 m_b^5 f(m_c^2/m_b^2) |V_{cb}|^2$$

- Higgs decay (dominant decay mode for $M_h = 125$ GeV)

$$\Gamma(H \to b\bar{b}) \sim \frac{G_F M_H}{4\sqrt{2}\pi} m_b^2(M_H)$$

- ...

Up to now, the only source of information on the Higgs coupling y_b to b-quarks in the SM.

QCD embedded in the SM

$$\mathcal{L}_{\rm SM} = \mathcal{L}^{\rm gauge}_{\rm QCD} + \mathcal{L}^{\rm gauge}_{\rm SU(2)\times U(1)} + \mathcal{L}_{\rm Yukawa} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm g.f.} + \mathcal{L}_{\rm ghosts}$$

- In the QCD embedded in the SM, quark mass terms are generated via Yukawa interactions with the Higgs vacuum expectation value $v^2 = -m^2/\lambda$:

$$m_q = \frac{y_q v}{\sqrt{2}} \Rightarrow \text{Main motivation: } m_b \to y_b!$$

- Due to spontaneous symmetry breaking (SSB) all other SM masses are also proportional to v

$$M_W^2 = \frac{g_2^2 v^2}{4}, \qquad M_Z^2 = \frac{g_1^2 + g_2^2}{4} v^2, \qquad M_h^2 = 2\lambda v^2$$

QCD embedded in the SM

$$\mathcal{L}_{\rm SM} = \mathcal{L}^{\sf gauge}_{\sf QCD} + \mathcal{L}^{\sf gauge}_{\sf SU(2)\times U(1)} + \mathcal{L}_{\sf Yukawa} + \mathcal{L}_{\sf Higgs} + \mathcal{L}_{\sf g.f.} + \mathcal{L}_{\sf ghosts}$$

- Introducing fine-structure constant α and Weinberg angle θ_W

$$(4\pi)\alpha = \frac{g_1^2 g_2^2}{g_1^2 + g_2^2} = g_2^2 \sin^2\theta_W = g_1^2 \cos^2\theta_W$$

- Parametrization:

$$y_q^2 = \frac{4\pi\alpha}{\sin^2\theta_W}\frac{m_q^2}{M_W^2}, \qquad \lambda = \frac{4\pi\alpha}{8\sin^2\theta_W}\frac{M_h^2}{M_W^2}$$

  - All the parameters here are bare (or $\overline{\rm MS}$ renormalizied) ones.
  - NB: In the formal limit $v \to \infty$ the mass ratios are finite.

Parameter values and the choice of renormalization scheme

- The values of the SM parameters are not predicted by the theory but extracted from an experiment.

- Sometimes, it is convenient to use $\overline{\mathrm{MS}}$ renormalization scheme.
- In order to determine the value,e.g., of $\alpha_s(\mu)$, an observable $\mathcal{O}$ is matched to the corresponding theoretical prediction

$$\mathcal{O} = \alpha_s^k(\mu) \left[c_0(\mu) + c_1(\mu)\alpha_s(\mu) + c_2(\mu)\alpha_s^2(\mu) + ... \right],$$

  so that $\alpha_s(\mu_0)$ at some matching μ_0 is extracted.
- To avoid large logarithms the scale μ_0 is usually chosen around the typical scale involved in the measurement of $\mathcal{O}$.
- NB: in $\overline{\mathrm{MS}}$ additional effort is required if a theory involves different mass scales (apparent violation of the [Appelquist and Carazzone, 1975] decoupling theorem)

Running SM parameters: taking loops into account

- Predictions of particle pole masses ("observables") in terms of running parameters:

$$2^{1/2}M_f = y_f v(1+\bar{\delta}_f), \quad 4M_W^2 = g_2^2 v^2(1+\bar{\delta}_W),$$
$$4M_Z^2 = \left(g_1^2+g_2^2\right) v(1+\bar{\delta}_Z), \quad M_h^2 = \lambda v^2(1+\bar{\delta}_h),$$

where all $\bar{\delta}$ are loop corrections (series in α and α_s).

- Two more equations are needed to "close" the system:

$$2^{1/2}G_F = v^{-2}(1+\bar{\delta}r), \quad (4\pi)^2\alpha_s^{(5)}(\mu) = g_s^2(1+\bar{\delta}\alpha_s),$$

They come from considering effective theories (see below).

Running SM parameters: taking loops into account

- Predictions of particle pole masses ("observables") in terms of running parameters:

$$2^{1/2} M_f = y_f v (1 + \bar{\delta}_f), \quad 4 M_W^2 = g_2^2 v^2 (1 + \bar{\delta}_W),$$
$$4 M_Z^2 = \left(g_1^2 + g_2^2\right) v (1 + \bar{\delta}_Z), \quad M_h^2 = \lambda v^2 (1 + \bar{\delta}_h),$$

  where all $\bar{\delta}$ are loop corrections (series in α and α_s).

- Two more equations are needed to "close" the system:

$$2^{1/2} G_F = v^{-2} (1 + \bar{\delta} r), \quad (4\pi)^2 \alpha_s^{(5)}(\mu) = g_s^2 (1 + \bar{\delta}\alpha_s),$$

  They come from considering effective theories (see below).

The running parameters can be obtained by solving the system (see [Bednyakov et al., 2015] for recent two-loop analysis in the SM).

The b-quark Yukawa coupling: electroweak corrections

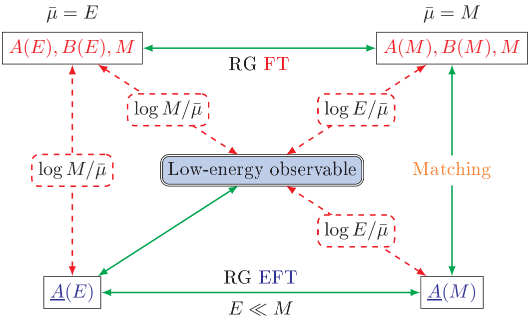
- The relation between the $\overline{\rm MS}$ Yukawa coupling at μ and physical(=pole) masses [Hempfling and Kniehl, 1995]:

$$y_b(\mu) = 2^{3/4} G_F^{1/2} M_b [1 + \delta_b(\underbrace{M_b}_{4\ \mathrm{GeV}}, \underbrace{M_t, M_W, M_Z, M_h}_{100-200\ \mathrm{GeV}}, \mu)],$$

involve more than one scale: $M_b \ll M_Z$:

$\Rightarrow$ potentially large logarithms $\log M_b/M_Z$

- It is better to re-summ large logs by the effective theory approach. See, e.g, Lectures by Andrey Grozin at this and previous [Grozin, 2009, Grozin, 2014] Schools.



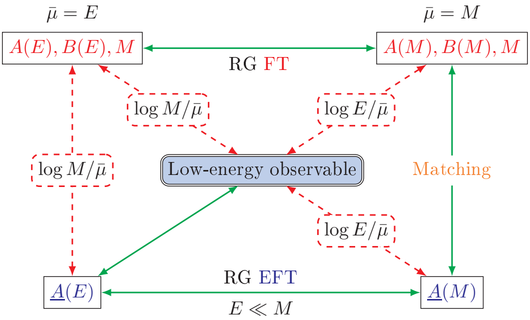
A well-known example:

- $A(\bar{\mu}) = \alpha_s^{(6)}(\bar{\mu})$,
- $M = M_t$,
- $\underline{A}(\bar{\mu}) = \alpha_s^{(5)}(\bar{\mu})$

Matching 6-flavor QCD with 5-flavor QCD without top quark.

- Effective theory (ET) describes the interactions of light fields at low energies $E \ll M$ and parametrized by running $\underline{A}(\bar{\mu})$ coupling .
- The latter can be expressed via matching in terms of (running) parameters of "full" theory (FT) - $A(\bar{\mu})$,$B(\bar{\mu})$ and a heavy mass M.
- Large $\log E/M$ are re-summed by solving renormalization group (RG) equations in the effective theory with initial conditions at $\bar{\mu} = M$.

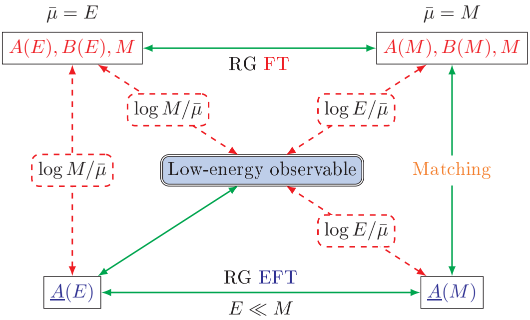
Re-summation and effective theories (in a nutshell)



Can be used to find $A(\bar{\mu})$ given $\underline{A}(\bar{\mu})$, $B(\bar{\mu})$ and M.

- Effective theory (ET) describes the interactions of light fields at low energies $E \ll M$ and parametrized by running $\underline{A}(\bar{\mu})$ coupling .
- The latter can be expressed via matching in terms of (running) parameters of "full" theory (FT) - $A(\bar{\mu})$,$B(\bar{\mu})$ and a heavy mass M.
- Large $\log E/M$ are re-summed by solving renormalization group (RG) equations in the effective theory with initial conditions at $\bar{\mu} = M$.

Re-summation and effective theories (in a nutshell)



Can be used to find $A(\bar{\mu})$ given $\underline{A}(\bar{\mu})$, $B(\bar{\mu})$ and M.

This is how $\alpha_s^{(6)}(\bar{\mu})$ is found from the value $\alpha_s^{(5)}(M_Z) = 0.1185 \pm 0.0006$, quoted in PDG'14 [Olive et al., 2014]

- Effective theory (ET) describes the interactions of light fields at low energies $E \ll M$ and parametrized by running $\underline{A}(\bar{\mu})$ coupling .
- The latter can be expressed via matching in terms of (running) parameters of "full" theory (FT) - $A(\bar{\mu})$,$B(\bar{\mu})$ and a heavy mass M.
- Large $\log E/M$ are re-summed by solving renormalization group (RG) equations in the effective theory with initial conditions at $\bar{\mu} = M$.

QED x QCD as an effective low-energy theory

- As a "low-energy" effective theory for the SM we consider a (toy) QCD x QED theory describing strong and electromagnetic interactions of five quarks (u, d, c, s, b) and leptons.

$$\mathcal{L}_{SM}\left(\alpha_s^{SM}, g_1, g_2, y_t, \lambda, ...\right) \Rightarrow \mathcal{L}_{QCD\times QED}^{(n_f=5)}\left(\alpha_s^{(5)}, \alpha_{EM}, m_b\right)$$

- Similar to the QCD case we "integrate out" top quark, electroweak gauge bosons and Higgs fields. We also neglect Fermi-like non-renormalizable interactions "$G_F\bar\psi\psi\bar\psi\psi$" with $G_F \propto \frac{g_s^2}{M_W^2}$.

- Formally, we consider the limit $v \to \infty$, which is different from that $y_t, g_2, \lambda \to \infty, v = fixed$ usually implied in the discussions of "non-decoupling" feature of the models with SSB (see [Pich, 1998]).

QED x QCD as an effective low-energy theory

- As a “low-energy” effective theory for the SM we consider a (toy) QCD x QED theory describing strong and electromagnetic interactions of five quarks (u, d, c, s, b) and leptons.

$$\mathcal{L}_{SM}\left(\alpha_s^{SM}, g_1, g_2, y_t, \lambda, ...\right) \Rightarrow \mathcal{L}_{QCD\times QED}^{(n_f=5)}\left(\alpha_s^{(5)}, \alpha_{EM}, m_b\right)$$

- Similar to the QCD case we “integrate out” top quark, electroweak gauge bosons and Higgs fields. We also neglect Fermi-like non-renormalizable interactions "$G_F\bar{\psi}\psi\bar{\psi}\psi$" with $G_F \propto \frac{g_s^2}{M_W^2}$.
- Formally, we consider the limit $v \to \infty$, which is different from that $y_t, g_2, \lambda \to \infty, v = fixed$ usually implied in the discussions of “non-decoupling” feature of the models with SSB (see [Pich, 1998]).
- From the phenomelogical point of view we miss a lot of electroweak physics, goverened at low energies by the Fermi constant G_F!

QED x QCD as an effective low-energy theory

- As a "low-energy" effective theory for the SM we consider a (toy) QCD x QED theory describing strong and electromagnetic interactions of five quarks (u, d, c, s, b) and leptons.

$$\mathcal{L}_{SM}\left(\alpha_s^{SM}, g_1, g_2, y_t, \lambda, ...\right) \Rightarrow \mathcal{L}_{QCD\times QED}^{(n_f=5)}\left(\alpha_s^{(5)}, \alpha_{EM}, m_b\right)$$

- Similar to the QCD case we "integrate out" top quark, electroweak gauge bosons and Higgs fields. We also neglect Fermi-like non-renormalizable interactions "$G_F\bar{\psi}\psi\bar{\psi}\psi$" with $G_F \propto \frac{g_s^2}{M_W^2}$.

- Due to the chosen $\overline{\mathrm{MS}}$ scheme, the result is also valid in the effective QED×QCD×Fermi theory!

Matching via "pseudo"-observables

$$M_b = m_b(\mu)\left(1 + \sum_{i+j=1}^{2} \alpha_s^i \cdot \alpha^j \cdot \sigma_{ij}(M_b, M, \mu) + \cdots\right)_{\text{SM}}$$
$$= \underline{m}_b(\mu)\left(1 + \sum_{i+j=1}^{2} \underline{\alpha}_s^i \cdot \underline{\alpha}^j \cdot \underline{\sigma}_{ij}(M_b, \mu) + \cdots\right)_{\text{QEDxQCD}} + \mathcal{O}\left(\frac{m_b}{M}\right)$$

Matching via pseudo-observable - the pole mass M_b - calculated either in the full SM or in $n_f = 5$ QCDxQED with $\underline{m}_b(\mu) \equiv m_b^{(5)}(\mu)$, etc.

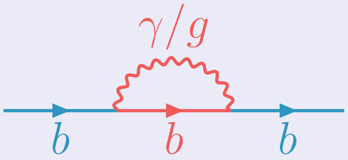
- Both σ_{ij} and $\underline{\sigma}_{ij}$ are extracted from the b-quark self-energies. Only photon and gluon exchange contribute to $\underline{\sigma}_{ij}$, while σ_{ij} also involve exchange of heavy virtual particles with mass $M = \{M_t, M_W, M_Z, M_H\}$

Matching via "pseudo"-observables

$$
\begin{aligned}
M_b &= m_b(\mu)\left(1+\sum_{i+j=1}^{2}\alpha_s^i\cdot\alpha^j\cdot\sigma_{ij}(M_b,M,\mu)+\cdots\right)_{\rm SM}\\
&= \underline{m}_b(\mu)\left(1+\sum_{i+j=1}^{2}\underline{\alpha}_s^i\cdot\underline{\alpha}^j\cdot\underline{\sigma}_{ij}(M_b,\mu)+\cdots\right)_{\rm QEDxQCD} + \mathcal{O}\left(\frac{m_b}{M}\right)
\end{aligned}
$$

Matching via pseudo-observable - the pole mass M_b - calculated either in the full SM or in $n_f = 5$ QCDxQED with $\underline{m}_b(\mu) \equiv m_b^{(5)}(\mu)$, etc.

$\underline{m}_b(\mu)$, $\underline{\alpha}(\mu)$, $\underline{\alpha}_s(\mu)$ are related to their counterparts in the SM by means



of decoupling constants $\zeta = 1 + \delta\zeta$:

Matching via "pseudo"-observables

$$M_b = m_b(\mu)\left(1 + \sum_{i+j=1}^{2} \alpha_s^i \cdot \alpha^j \cdot \sigma_{ij}(M_b, M, \mu) + \cdots\right)_{\text{SM}}$$
$$= \zeta_b \cdot m_b(\mu)\left(1 + \sum_{i+j=1}^{2} \zeta_{\alpha_s}^i \zeta_\alpha^j \alpha_s^i \cdot \alpha^j \cdot \underline{\sigma}_{ij}(M_b, \mu) + \cdots\right)_{\text{QEDxQCD}} + \cdots$$

Matching via pseudo-observable - the pole mass M_b - calculated either in the full SM or in $n_f = 5$ QCDxQED with $\underline{m}_b(\mu) \equiv m_b^{(5)}(\mu)$, etc.

But what is $m_b(\mu)$ in the SM formula for M_b?

Running m_b in the SM

- Running $\overline{\text{MS}}$ mass is not a fundamental SM parameter and is expressed in terms of y_b and the Higgs field v.e.v

$$m_b(\mu) = y_b(\mu) v/\sqrt{2}$$

- What is v (beyond the tree level approximation)?

Running m_b in the SM

- Running $\overline{\text{MS}}$ mass is not a fundamental SM parameter and is expressed in terms of y_b and the Higgs field v.e.v

$$m_b(\mu) = y_b(\mu) v/\sqrt{2}$$

- What is v (beyond the tree level approximation)?
  - $v \stackrel{?}{\equiv} v(\mu) \equiv \sqrt{\frac{-m^2(\mu)}{\lambda(\mu)}}$ - minimizes tree-level potential expressed in $\overline{\text{MS}}$ parameters (gauge-independent),$\gamma_{m_b} \neq \beta_{y_b}$

- $\frac{d}{d\ln\mu^2} m_b = \gamma_b m_b, \qquad \frac{d}{d\ln\mu^2} y_b = \beta_{y_b} y_b$

Running m_b in the SM

- Running $\overline{\text{MS}}$ mass is not a fundamental SM parameter and is expressed in terms of y_b and the Higgs field v.e.v

$$m_b(\mu) = y_b(\mu) v/\sqrt{2}$$

- What is v (beyond the tree level approximation)?
  - $v \overset{?}{\equiv} v(\mu) \equiv \sqrt{\frac{-m^2(\mu)}{\lambda(\mu)}}$ - minimizes tree-level potential expressed in $\overline{\text{MS}}$ parameters (gauge-independent),$\gamma_{m_b} \neq \beta_{y_b}$
  - $v \overset{?}{\equiv} \tilde{v}(\mu)$ - minimizes Higgs effective-potential expressed in terms $\overline{\text{MS}}$ parameters (*gauge-dependent*), $\gamma_{m_b} \neq \beta_{y_b}$

- $\frac{d}{d\ln\mu^2} m_b = \gamma_b m_b, \qquad \frac{d}{d\ln\mu^2} y_b = \beta_{y_b} y_b$

Running m_b in the SM

- Running $\overline{\text{MS}}$ mass is not a fundamental SM parameter and is expressed in terms of y_b and the Higgs field v.e.v

$$m_b(\mu) = y_b(\mu) v/\sqrt{2}$$

- What is v (beyond the tree level approximation)?
  - $v \stackrel{?}{\equiv} v(\mu) \equiv \sqrt{\frac{-m^2(\mu)}{\lambda(\mu)}}$ - minimizes tree-level potential expressed in $\overline{\text{MS}}$ parameters (gauge-independent),$\gamma_{m_b} \neq \beta_{y_b}$
  - $v \stackrel{?}{\equiv} \tilde{v}(\mu)$ - minimizes Higgs effective-potential expressed in terms $\overline{\text{MS}}$ parameters (*gauge-dependent*), $\gamma_{m_b} \neq \beta_{y_b}$
  - $v^2 = \frac{1}{\sqrt{2} G_F}$ - gauge-independent vev from the tree-level matching to the Fermi-theory, $\gamma_{m_b} = \beta_{y_b}$.
- $\frac{d}{d \ln \mu^2} m_b = \gamma_b m_b, \qquad \frac{d}{d \ln \mu^2} y_b = \beta_{y_b} y_b$

Running m_b in the SM

- Reorganize series for M_b in the SM, use [Hempfling and Kniehl, 1995, Kniehl and Veretin, 2014, Kniehl et al., 2015]

$$\underline{m}_b(\mu) \to m_{b,Y}(\mu) = y_b(\mu) 2^{-3/4} G_f^{-1/2}$$

  - via matching to Fermi theory - $v^2(\mu) = \frac{m^2(\mu)}{\lambda(\mu)} = \frac{1}{\sqrt{2} G_f (1 + \bar{\delta} r(\mu))}$ with $\delta r(\mu)$ being an appropriate variant of Sirlin's Δr parameter
- Advantage - the relation between M_b and $m_{b,Y}(\mu)$ does not involve numerically "dangerous" tadpole terms, scaling as $M_t^4/(M_W^2 M_h^2)$.

Running m_b in the SM

- Reorganize series for M_b in the SM, use [Hempfling and Kniehl, 1995, Kniehl and Veretin, 2014, Kniehl et al., 2015]

$$\underline{m}_b(\mu) \to m_{b,Y}(\mu) = y_b(\mu) 2^{-3/4} G_f^{-1/2}$$

  - via matching to Fermi theory - $v^2(\mu) = \frac{m^2(\mu)}{\lambda(\mu)} = \frac{1}{\sqrt{2} G_f (1 + \bar{\delta} r(\mu))}$ with $\delta r(\mu)$ being an appropriate variant of Sirlin's Δr parameter
- Advantage - the relation between M_b and $m_{b,Y}(\mu)$ does not involve numerically "dangerous" tadpole terms, scaling as $M_t^4/(M_W^2 M_h^2)$.
- Crucial check - independence of ξ_{m_b} on "soft" scale M_b

$$\underline{m}_b(\mu) = m_{b,Y}(\mu) \cdot \zeta_{m_b}(\alpha_s, \alpha, M, \cancel{M_b}, \mu) = m_{b,Y}(\mu) \cdot (1 + \delta\zeta_{m_b})$$

Running m_b in the SM

- Reorganize series for M_b in the SM, use [Hempfling and Kniehl, 1995, Kniehl and Veretin, 2014, Kniehl et al., 2015]

$$\underline{m}_b(\mu) \to m_{b,Y}(\mu) = y_b(\mu) 2^{-3/4} G_f^{-1/2}$$

  - via matching to Fermi theory - $v^2(\mu) = \frac{m^2(\mu)}{\lambda(\mu)} = \frac{1}{\sqrt{2} G_f (1 + \bar{\delta} r(\mu))}$ with $\delta r(\mu)$ being an appropriate variant of Sirlin's Δr parameter
- Advantage - the relation between M_b and $m_{b,Y}(\mu)$ does not involve numerically "dangerous" tadpole terms, scaling as $M_t^4/(M_W^2 M_h^2)$.
- Crucial check - independence of ξ_{m_b} on "soft" scale M_b

$$\underline{m}_b(\mu) = m_{b,Y}(\mu) \cdot \zeta_{m_b}(\alpha_s, \alpha, M, \not{M_b}, \mu) = m_{b,Y}(\mu) \cdot (1 + \delta\zeta_{m_b})$$

- The relation is used to find the boundary value of the Yukawa coupling from the $n_f = 5$ QCDxQED b-quark running mass

$$y_b(\mu) = 2^{3/4} G_f^{1/2} \cdot \underline{m}_b(\mu) \cdot \zeta_{m_b}^{-1}(\mu)$$

Results

- The results of matching after proper re-expansion can be casted, e.g., into the following form

$$\alpha_s = {\alpha_s}^{(5)} \left(1 + \frac{{\alpha_s}^{(5)}}{4\pi} \delta\zeta^{(1)}_{\alpha_s'} + \frac{\left({\alpha_s}^{(5)}\right)^2}{(4\pi)^2} \delta\zeta^{(2)}_{\alpha_s'} + \frac{{\alpha_s}^{(5)}\alpha_F}{(4\pi)^2} \delta\zeta^{(2)}_{\alpha_s'\alpha} + \cdots \right.$$

$$m_{b,Y} = \underline{m}_b \left(1 + \cancel{\frac{{\alpha_s}^{(5)}}{4\pi} \delta\zeta^{(1)}_{\alpha_s'}} + \frac{\alpha_F}{4\pi} \delta\zeta^{(1)}_{\alpha_F} + \frac{\left({\alpha_s}^{(5)}\right)^2}{(4\pi)^2} \delta\zeta^{(2)}_{\alpha_s'} \right.$$
$$\left. + \frac{{\alpha_s}^{(5)}\alpha_F}{(4\pi)^2} \delta\zeta^{(2)}_{\alpha_s'\alpha} + \frac{(\alpha_F)^2}{(4\pi)^2} \delta\zeta^{(2)}_{\alpha} \cdots \right)$$

- $\alpha_F \equiv \frac{\sqrt{2} G_F M_W^2}{\pi} \left(1 - \frac{M_W^2}{M_Z^2}\right) = 132.233$ (for PDG'14)
- The required expressions for the strong coupling is available from [Bednyakov, 2014] and the results for the b-quark mass are extracted from full two-loop electroweak corrections considered in the paper by [Kniehl et al., 2015].

Numerical estimates of the EW corrections to m_b

- At Z - boson mass scale (with RunDec [Chetyrkin et al., 2000]):

$$y_b(M_Z) = 2^{3/4} G_f^{1/2} \cdot \underline{m}_b(M_Z) \cdot \left[1 - \underbrace{0.00838}_{\alpha} - \underbrace{0.00074}_{\alpha_s^2} - \underbrace{0.00023}_{\alpha_s^3} + \underbrace{0.00068}_{\alpha_s \alpha} - \underbrace{0.00005}_{\alpha^2} + \cdots \right]$$

in comparision with

$$y_b(M_Z) = 2^{3/4} G_f^{1/2} \cdot M_b \left[1 - \underbrace{0.010}_{\alpha} - \underbrace{0.270}_{\alpha_s} - \underbrace{0.0784}_{\alpha_s^2} + \underbrace{0.0032}_{\alpha_s \alpha} + \underbrace{0.0003}_{\alpha^2} + \cdots \right]$$

Conclusions

- A well-known decoupling procedure in QCD is extendend to include electroweak corrections due to heavy degrees of freedom in the SM.
- Two-loop electroweak decoupling corrections for $y_b(\mu)$ are found by considering the b-quark pole mass in the SM and effective QCDxQED.
- The obtained result is gauge-indepdent and is free from soft scales, thus, allowing to use RGE for resummation of $\log m_b/M_t$.
- The convergence of PT series for threshold corrections to $y_b(\mu)$ is much better if expressed in terms of running $\underline{m}_b(\mu)$ in the five-flavour QCD.

Thank you for your attention!

And special thanks to B. Kniehl, A. Pikelner, and O. Veretin
for sharing analytic expresions
from [Kniehl and Veretin, 2014] and [Kniehl et al., 2015]!

References I

Appelquist, T. and Carazzone, J. (1975).
Infrared Singularities and Massive Fields.
Phys.Rev., D11:2856.

Baikov, P. A., Chetyrkin, K. G., and Khn, J. H. (2014).
Quark Mass and Field Anomalous Dimensions to $\mathcal{O}(\alpha_s^5)$.
JHEP, 10:076.

Baikov, P. A., Chetyrkin, K. G., and Khn, J. H. (2016).
Five-Loop Running of the QCD coupling constant.

Bednyakov, A. (2014).
On the electroweak contribution to the matching of the strong coupling constant in the SM.

References II

Bednyakov, A. V., Kniehl, B. A., Pikelner, A. F., and Veretin, O. L. (2015).
Stability of the Electroweak Vacuum: Gauge Independence and Advanced Precision.
Phys. Rev. Lett., 115(20):201802.

Beneke, M. (1998).
A Quark mass definition adequate for threshold problems.
Phys. Lett., B434:115–125.

Beneke, M. and Braun, V. M. (1994).
Heavy quark effective theory beyond perturbation theory: Renormalons, the pole mass and the residual mass term.
Nucl. Phys., B426:301–343.

References III

Bigi, I. I. Y., Shifman, M. A., and Uraltsev, N. (1997).
Aspects of heavy quark theory.
Ann. Rev. Nucl. Part. Sci., 47:591–661.

Bigi, I. I. Y., Shifman, M. A., Uraltsev, N. G., and Vainshtein, A. I. (1994).
The Pole mass of the heavy quark. Perturbation theory and beyond.
Phys. Rev., D50:2234–2246.

Chetyrkin, K., Kniehl, B. A., and Steinhauser, M. (1998).
Decoupling relations to O (alpha-s**3) and their connection to low-energy theorems.
Nucl.Phys., B510:61–87.

References IV

Chetyrkin, K., Kuhn, J. H., and Steinhauser, M. (2000).
RunDec: A Mathematica package for running and decoupling of the strong coupling and quark masses.
Comput.Phys.Commun., 133:43–65.

El-Khadra, A. X. and Luke, M. (2002).
The Mass of the b quark.
Ann. Rev. Nucl. Part. Sci., 52:201–251.

Grozin, A. (2014).
Effective weak Lagrangians in the Standard Model and B decays.
In *Proceedings, Helmholtz International Summer School on Physics of Heavy Quarks and Hadrons (HQ 2013)*, pages 78–98.
[,78(2014)].

References V

Grozin, A. G. (2009).
Introduction to effective field theories. 1. Heisenberg-Euler effective theory, decoupling of heavy flavours.
In *Helmholtz International School - Workshop on Calculations for Modern and Future Colliders (CALC 2009) Dubna, Russia, July 10-20, 2009*.

Hempfling, R. and Kniehl, B. A. (1995).
On the relation between the fermion pole mass and MS Yukawa coupling in the standard model.
Phys.Rev., D51:1386–1394.

Hoang, A. H., Ligeti, Z., and Manohar, A. V. (1999).
B decay and the Upsilon mass.
Phys. Rev. Lett., 82:277–280.

References VI

Kniehl, B. A., Pikelner, A. F., and Veretin, O. L. (2015).
Two-loop electroweak threshold corrections in the Standard Model.

Kniehl, B. A. and Veretin, O. L. (2014).
Two-loop electroweak threshold corrections to the bottom and top Yukawa couplings.
Nucl.Phys., B885:459.

Marquard, P., Smirnov, A. V., Smirnov, V. A., and Steinhauser, M. (2015).
Quark Mass Relations to Four-Loop Order in Perturbative QCD.
Phys. Rev. Lett., 114(14):142002.

Mihaila, L. (2013).
Three-loop gauge beta function in non-simple gauge groups.
PoS, RADCOR2013:060.

References VII

Olive, K. et al. (2014).
Review of Particle Physics.
Chin.Phys., C38:090001.

Pich, A. (1998).
Effective field theory: Course.
pages 949–1049.

Pineda, A. (2001).
Determination of the bottom quark mass from the Upsilon(1S) system.
JHEP, 06:022.

Tarrach, R. (1981).
The Pole Mass in Perturbative QCD.
Nucl. Phys., B183:384–396.

Backup: One-loop ζ's

- Decouple the top quark and W-boson at one-loop ($N_c = 3$, $T_f = 1/2$, $Q_u = 2/3$):

$$\underline{\alpha}_s(\mu) = \alpha_s(\mu)\zeta_{\alpha_s}, \quad \zeta_{\alpha_s} = 1 + \frac{4}{3}T_f\frac{\alpha_s}{4\pi}\log\frac{M_t^2}{\mu^2} + \dots$$

$$\underline{\alpha}(\mu) = \alpha(\mu)\zeta_\alpha, \quad \zeta_\alpha = 1 + \frac{\alpha}{4\pi}\left(\frac{2}{3} - 7\log\frac{M_W^2}{\mu^2} + \frac{4}{3}N_c Q_u^2\log\frac{M_t^2}{\mu^2}\right) + \dots$$

- Decouple t,H,Z and W ($L_X \equiv \log M_X^2/\mu^2$, $L_{XY} \equiv \log M_X^2/M_Y^2$):

$$\delta\zeta_{m_b}(\mu) = \sum_{i+j=1}^{\infty}\left(\frac{\alpha(\mu)}{4\pi}\right)^i\left(\frac{\alpha_s(\mu)}{4\pi}\right)^j\delta\zeta_{ij}^{(b)}(M,\mu), \quad \zeta_{01}^{(b)} = 0.$$

$$\begin{aligned}
\delta\zeta_{10}^{(b)} &= -\frac{5}{18} + \frac{1}{3}L_Z + \frac{1}{\sin^2\theta_W}\left[\frac{41}{36} - \frac{3}{4}L_W - \frac{1}{6}L_Z\right] - \frac{3}{8}\frac{L_{WZ}}{\sin^4\theta_W} \\
&+ \frac{1}{\sin^2 2\theta_W}\left[\frac{13}{9} - \frac{1}{4M_Z^2}(M_t^2 + M_H^2) - \frac{5}{6}L_Z + \frac{3M_t^2}{2M_Z^2}L_t\right] \\
&+ \frac{3}{8\sin^2\theta_W}\left[\frac{M_H^2}{M_H^2 - M_W^2}L_{WH} - \frac{M_t^2}{M_t^2 - M_W^2} - \frac{M_t^4}{(M_t^2 - M_W^2)^2}L_{Wt}\right]
\end{aligned}$$

Backup: two-loop b-quark pole mass in QCD x QED

$$
\begin{aligned}
M_b &= \underline{m}_b(\mu)\Big\{1 + \frac{\alpha_s}{4\pi} C_F\,(4+3L_b) + \frac{\alpha}{4\pi} Q_d^2\,(4+3L_b) \\
&+ 2\frac{\alpha_s\alpha}{(4\pi)^2} C_F Q_d^2 \left[\frac{121}{8} + 30\zeta_2 + 8I_3 + \frac{27}{2}L_b + \frac{9}{2}L_b^2\right] \\
&+ \frac{\alpha^2}{(4\pi)^2} Q_d^2 (Q_e^2 + 2Q_u^2)\left(-\frac{71}{2} - 24\zeta_2 - 26L_b - 6L_b^2\right) \\
&+ \frac{\alpha^2}{(4\pi)^2} Q_d^4 \left(-\frac{1019}{8} + 30\zeta_2 + 8I_3 - \frac{129}{2}L_b - \frac{27}{2}L_b^2\right) \\
&+ \frac{\alpha_s^2}{(4\pi)^2} C_F \Bigg[C_F\left(\frac{121}{8} + 30\zeta_2 + 8I_3\right) + C_A\left(\frac{1111}{24} - 8\zeta_2 - 4I_3\right) \\
&\qquad - T_f\left(\left[\frac{71}{6} + 8\zeta_2\right] n_f + 12(1 - 2\zeta_2)\right) \\
&\qquad + L_b\left(\frac{27}{2}C_F + \frac{185}{6}C_A - \frac{26}{3}n_f T_f\right) \qquad C_F = 4/3, C_A = 3 \\
&\qquad + L_b^2\left(\frac{9}{2}C_F + \frac{11}{2}C_A - 2n_f T_f\right)\Bigg]\Big\}, \qquad T_f = 1/2, n_f = 5
\end{aligned}
$$

with $L_b = \ln(\mu^2/M_b^2)$, $I_3 = 3/2\zeta_3 - 6\zeta_2 \log 2$, $Q_d = -1/3$, $Q_u = 2/3$, $Q_e = -1$.

Backup: RGEs in QCDxQED at 3 loops

Can be deduced, e.g., from [Mihaila, 2013] (QCD - 5-loop [Baikov et al., 2016]!):

$$\mu^2 \frac{d\underline{\alpha}_i}{d\mu^2} = \underline{\alpha}_i \beta^i, \quad \underline{\alpha}_i = \{\underline{\alpha}, \underline{\alpha}_s\}, \quad \beta_i = \sum_{k,l=1}^{3} \beta^i_{kl} \left(\frac{\alpha}{4\pi}\right)^k \left(\frac{\alpha_s}{4\pi}\right)^l + \ldots,$$

$n_f = N_u + N_d$, number of active quark flavors, $N_l = 3$ number of charged leptons

$$\beta^{\alpha}_{10} = \frac{4}{3}\left[N_l Q_e^2 + N_c\left(N_d Q_d^2 + N_u Q_u^2\right)\right], \qquad \beta^{\alpha}_{0j} \equiv 0, \quad j = 1, \ldots$$

$$\beta^{\alpha}_{11} = 4 C_F N_c \left[N_d Q_d^2 + N_d Q_u^2\right],$$

$$\beta^{\alpha}_{20} = 4\left[N_l Q_e^4 + N_c\left(N_d Q_d^4 + N_u Q_u^4\right)\right],$$

$$\begin{aligned}\beta^{\alpha}_{30} = &-\frac{44}{9} N_c^2 \left[N_d^2 Q_d^6 + N_d^2 Q_u^6 + N_u N_d \left(Q_u^2 + Q_d^2\right) Q_u^2 Q_d^2\right] \\ &- \frac{44}{9} N_l Q_e^2 \left[N_c \left[Q_e^2 \left(N_u Q_u^2 + N_d Q_d^2\right) + N_u Q_u^4 + N_d Q_d^4\right] + N_l Q_e^4\right] \\ &- 2\left[N_c (N_u Q_u^6 + N_d Q_d^6) + N_l Q_e^6\right],\end{aligned}$$

$$\beta^{\alpha}_{21} = -4 C_F N_c \left[N_u Q_u^4 + N_d Q_d^2\right],$$

$$\beta^{\alpha}_{12} = C_F N_c \left(N_u Q_u^2 + N_d Q_d^2\right)\left[\frac{133}{9} C_A - 2 C_F - \frac{44}{9} T_f (N_u + N_d)\right],$$

Backup: RGEs in QCDxQED at 3 loops

Can be deduced, e.g., from [Mihaila, 2013] (QCD - 5-loop [Baikov et al., 2016]!):

$$\mu^2 \frac{d\underline{\alpha}_i}{d\mu^2} = \underline{\alpha}_i \beta^i, \quad \underline{\alpha}_i = \{\underline{\alpha}, \underline{\alpha}_s\}, \quad \beta_i = \sum_{k,l=1}^{3} \beta^i_{kl} \left(\frac{\underline{\alpha}}{4\pi}\right)^k \left(\frac{\underline{\alpha}_s}{4\pi}\right)^l + \ldots,$$

$n_f = N_u + N_d$, number of active quark flavors, $N_l = 3$ number of charged leptons

$$\begin{aligned}
\beta^{\alpha_s}_{01} &= -\frac{11}{3} C_A + \frac{4}{3} T_f n_f, \qquad \beta^{\alpha_s}_{j0} \equiv 0, \quad j = 1, \ldots \\
\beta^{\alpha_s}_{11} &= 4 T_F \left[N_u Q_u^2 + N_d Q_d^2 \right], \\
\beta^{\alpha_s}_{02} &= -\frac{34}{3} C_A^2 + T_f n_f \left[4 C_F + \frac{20}{3} C_A \right], \\
\beta^{\alpha_s}_{03} &= -\frac{2857}{54} C_A^3 + \frac{1415}{27} C_A^2 T_f n_f + \frac{205}{9} C_A C_F T_f n_f - \frac{158}{27} C_A T_f^2 n_f^2 \\
&\quad - 2 C_F^2 T_f n_f - \frac{44}{9} C_F T_f^2 n_f^2, \\
\beta^{\alpha_s}_{12} &= -\frac{44}{9} T_f \left[N_c (N_u Q_u^2 + N_d Q_d^2)^2 + N_l Q_e^2 (N_u Q_u^2 + N_d Q_d^2) \right] \\
&\quad - 2 T_f (N_u Q_u^4 + N_d Q_d^4)
\end{aligned}$$

Backup: RGEs in QCDxQED at 3 loops

In QCD, γ_m^q is known up to 5-loop [Baikov et al., 2014]

$$\mu^2 \frac{d m_b}{d\mu^2} = \gamma_m^b m_b, \qquad \gamma_m^b = -\sum_{i+j=1}^{3} \gamma_{ij}^b \left(\frac{\alpha}{4\pi}\right)^i \left(\frac{\alpha_s}{4\pi}\right)^j + \ldots,$$

$$\gamma_{01}^b = 3C_F, \qquad \gamma_{10}^b = 3Q_d^2, \qquad \gamma_{11}^b = 3C_f Q_d^2,$$

$$\gamma_{02}^b = \frac{3}{2}C_F^2 + \frac{97}{6}C_F C_A - \frac{10}{3}C_F T_f n_f,$$

$$\gamma_{20}^b = \frac{3}{2}Q_d^4 - \frac{10}{3}Q_d^2 \left[N_c\left(N_u Q_u^2 + N_d Q_d^2\right) + N_l Q_e^2\right]$$

$$\gamma_{03}^b = -\frac{129}{4}C_F^2 C_A + \frac{11413}{108}C_F C_A^2 + C_F C_A T_f n_f\left(-\frac{556}{27} - 48\zeta_3\right) + \frac{129}{2}C_F^3 - \frac{140}{27}C_F T_f^2 n_f^2 + C_F^2 T_f n_f(-45 + 48\zeta_3) - C_F^2 T_f n_f,$$

$$\gamma_{12}^b = -\frac{129}{4}C_F C_A Q_d^2 + 3\cdot\frac{129}{2}C_F^2 Q_d^2 - C_F T_f (N_u + N_d) Q_d^2 + C_F T_f(-45 + 48\zeta_3)(N_d Q_d^2 + N_u Q_u^2)$$

$$\gamma_{21}^b = 3\cdot\frac{219}{2}C_F Q_d^4 - C_F Q_d^2\left[N_e Q_e^2 + N_c(N_u Q_u^2 + N_d Q_d^2)\right] + C_F Q_d^2 N_c(-45 + 48\zeta_3)(N_d Q_d^2 + N_u Q_u^2)$$

$$\gamma_{30}^b = \frac{129}{2}Q_d^6 - \frac{140}{27}Q_d^2\left[N_e Q_e^2 + N_c(N_u Q_u^2 + N_d Q_d^2)\right]^2 - Q_d^4\left[N_e Q_e^4 + N_c(N_u Q_u^4 + N_d Q_d^4)\right]$$

See [Chetyrkin et al., 1998] for details...

$$\alpha_{s,0}^{(5)} = \zeta_{\alpha_s,0}[\alpha_{s,0}, \alpha_0, M_0] \times \alpha_{s,0}$$

Due to SU(3) gauge invariance, the bare decoupling constant $\xi_{\alpha_{s,0}}$ can be found in a number of ways:

$$\zeta_{\alpha_s,0} = \zeta_{cGc,0}^{2}\zeta_{c,0}^{-2}\zeta_{G,0}^{-1} = \zeta_{qGq,0}^{2}\zeta_{q,0}^{-2}\zeta_{G,0}^{-1} = \ldots$$

in which different ζs are found by considering three- and two-point 1PI Green functions in the SM so that

- $\zeta_{cGc,0}$and $\zeta_{qGq,0}$ correspond to the leading terms in Taylor expansion of the integrand of the ghost-gluon and (light)-quark-gluon vertices in small masses and momenta, respectively.
- $\zeta_{c,0}, \zeta_{G,0}, \zeta_{q,0}$ involve only $\ln M/\mu$ terms coming from ghost, gluon and quark propagators.

Backup: Matching at the bare level

See [Chetyrkin et al., 1998] for details. . .

$$\alpha_{s,0}^{(5)} = \zeta_{\alpha_s,0}[\alpha_{s,0}, \alpha_0, M_0] \times \alpha_{s,0}$$

Due to SU(3) gauge invariance, the bare decoupling constant $\xi_{\alpha_s,0}$ can be found in a number of ways:

$$\zeta_{\alpha_s,0} = \zeta_{cGc,0}^2 \zeta_{c,0}^{-2} \zeta_{G,0}^{-1} = \zeta_{qGq,0}^2 \zeta_{q,0}^{-2} \zeta_{G,0}^{-1} = \ldots$$

in which different ζs are found by considering three- and two-point 1PI Green functions in the SM

Taylor expansion can produce spurious IR-divergent $\frac{1}{(q^2)^2}$ terms, which, upon integration, lead to additional IR poles in $\epsilon = (4-d)/2$ in bare ζs.

Backup: Matching at the bare level

See [Chetyrkin et al., 1998] for details. . .

$$\alpha_s^{(5)}(\mu) = \frac{Z_{\alpha_s}[\alpha_s, \alpha, M]}{Z_{\alpha_{s(5)}}\left[\alpha_s^{(5)}\right]} \zeta_{\alpha_s,0}\left[Z_{\alpha_s}\alpha_s, Z_\alpha \alpha, Z_M M\right] \times \alpha_s(\mu)$$

Due to SU(3) gauge invariance, the bare decoupling constant $\xi_{\alpha_s,0}$ can be found in a number of ways:

$$\zeta_{\alpha_s,0} = \zeta_{cGc,0}^2 \zeta_{c,0}^{-2} \zeta_{G,0}^{-1} = \zeta_{qGq,0}^2 \zeta_{q,0}^{-2} \zeta_{G,0}^{-1} = \ldots$$

in which different ζs are found by considering three- and two-point 1PI Green functions in the SM

But the spurious IR poles are canceled in the matching relation for the running couplings *after* renormalization.