Implications from $B \to K^* \ell\ell$ observables using LHCb data

Rahul Sinha

The Institute of Mathematical Sciences

Chennai India

Helmholtz International Summer School

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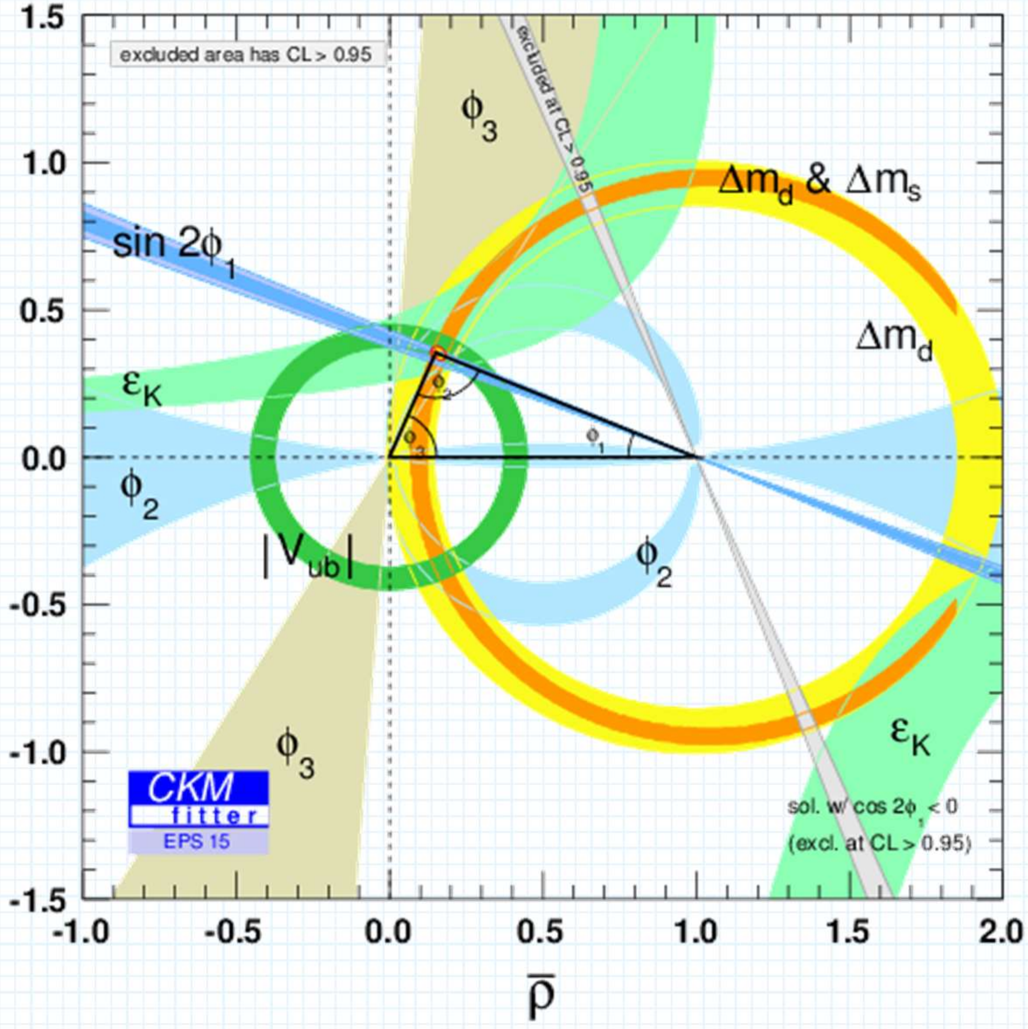
Outline

1. Why search New Physics through heavy flavor.
2. Complications in studying heavy flavor decays. Operator product expansion and effective Hamiltonian.
3. $B \to VV$ and introduction to angular analysis.
4. Why $B \to K^* \ell^+ \ell^-$?
5. Amplitude for $B \to K^* \ell^+ \ell^-$, non-local contributions and hadronic matrix elements.
6. Angular analysis in $B \to K^* \ell^+ \ell^-$.
7. Observables in $B \to K^* \ell^+ \ell^-$.
8. Effect of non-local contributions.
9. Techniques for low q^2 and high q^2.
10. Various approaches to clean observables $P_i^{(\prime)}$.
11. Our own approach to clean observables…

My apologies for the patchy referencing. Happy to add citations. Hopefully update these lectures in near future. Correct typos and add references.

Why study B decays?



SM is a gauge theory capable in principle of explaining interactions between the observed particles. All ingredients completed by 1974. Soon there were attempts to go beyond. In fact even before SM could be tested attempts were made to extend it. Many arguments were given to go beyond

- *Too many parameters-No fundamental understanding of masses, mixing & CP violation*
- *Existence of a fundamental scalar:* *Naturalness Problem: Higgs mass is unstable to radiative corrections. One-loop correction to Higgs boson mass due to quantum fluctuations of a size characterised by the scale Λ_N are $\delta M_H^2 \approx \alpha\Lambda_N^2$ If we require $\delta M_H^2 \lesssim M_H^2 \Rightarrow \Lambda_N \simeq 1\text{TeV}$.*
- *Strong reason to believe that there exists dark matter and dark energy. Observable Universe only 5%.*
- *Gravity is not included …*

Many model and attempts have been made to extend SM

- *Technicolor*
- *Grand Unified Theories*
- *Supersymmetry*
- *Extra dimensions*
- *Little Higgs, …*

However, no conclusive signal of physics beyond the SM has been seen in last 40+ years. How does one see a signal of New Physics?

New Physics can be discovered either by

- *direct production of new particles at high energies*
- *indirect searches at high luminosity facilities. New physics can contribute virtually to loop processes.*

1. Collider Signals
2. Precision tests and rare decays

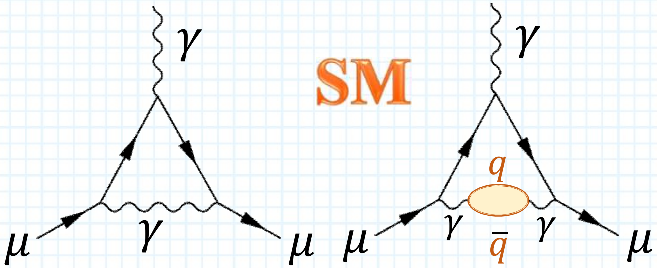
μ magnetic moment

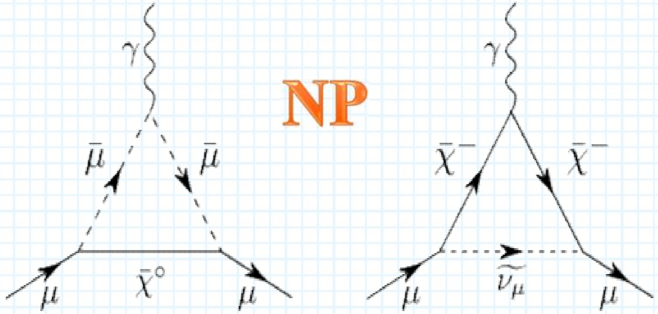
At $\left(\frac{\alpha}{\pi}\right)^4$ 891 Feynman diagrams

a_μ=116 592 089 (63) × 10^{-11} (Exp)

a_μ=116 591 840 (59) × 10^{-11} (SM)

a_μ= 6949.1(37.2)(21) × 10^{-11}(hadronic)





New Physics: flavour physics perspective

- ***Why are there 3 generations of quarks and leptons? Flavour problem***
- ***The amount of CP violation observed is too small for Baryogenesis.***

  $$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 10^{-20} \textit{in SM, but} \sim 6 \times 10^{-10} \textit{is needed.}$$
- ***CP violation is observed phenomenon the origin of which have to be better understood.***
- ***To explain the CKM elements, understand what makes quarks and lepton elements so different.***
- ***New Physics must satisfy FCNC constraints.***

New physics modifies low energy effective Hamiltonian:

- *Cause new contributions to the SM operators.*
- *Generate new operators.*
- *Lead to new CP violating phases.*

NP cannot show up everywhere.

Where should one, then, look for NP? The obvious rules are to look for observables that are:

- *Easy to measure, so that there is no large systematics or backgrounds to deal with.*
- *Have a small SM (theory) uncertainty.*
- *Sensitive to NP, so that NP signals are seen with the least numbers of B or D mesons.*

Loop level processes that are easy to calculate or we should smartly be able to remove SM uncertainty.

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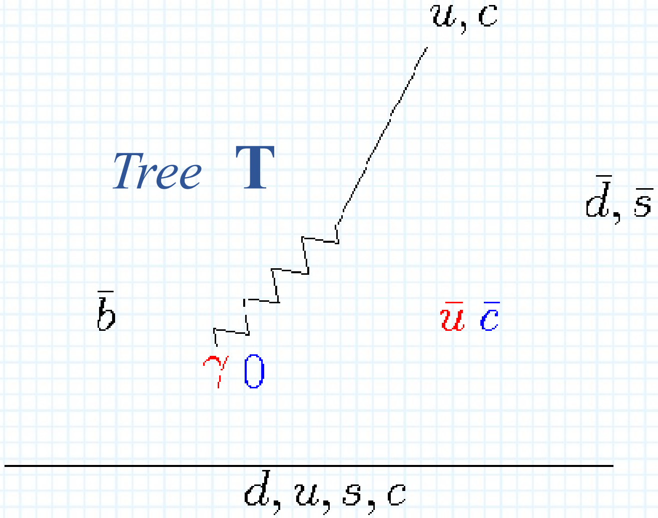
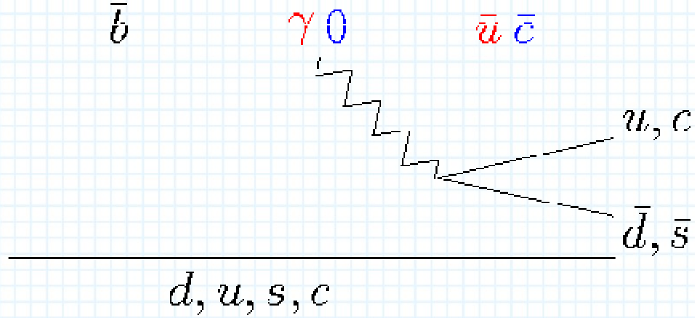
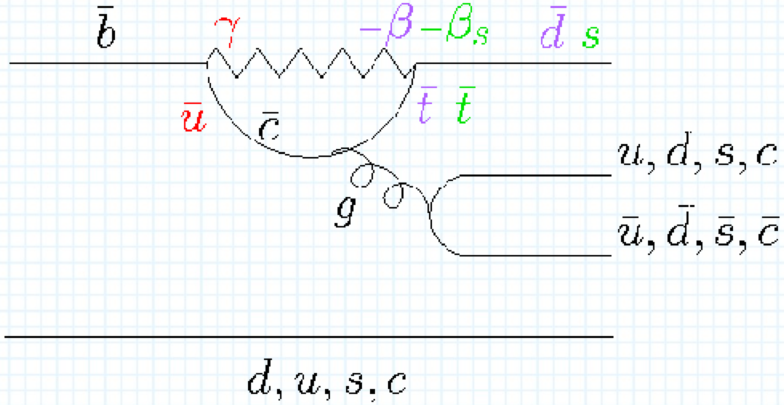
Decays of the B mesons

Tree **T**

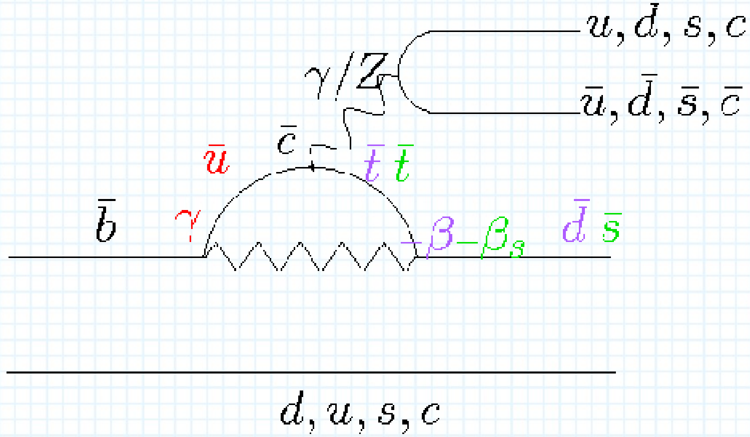


Color Suppressed Tree **C**





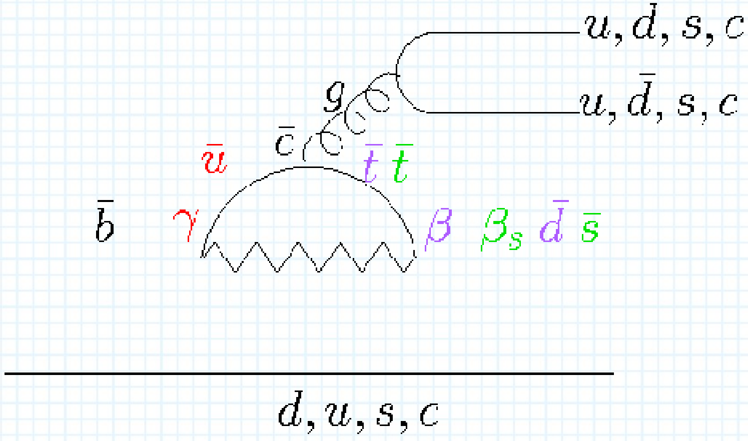
Penguin **P**



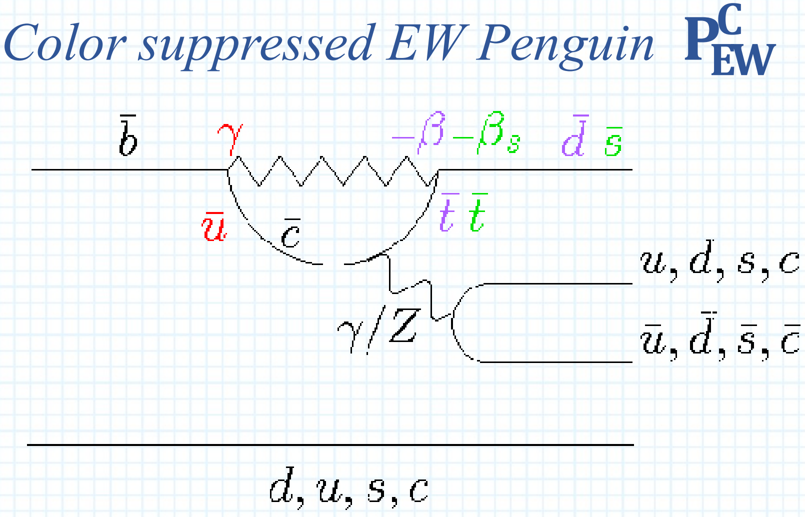
Electroweak Penguin $\mathbf{P_{EW}}$

Weak decays of hadrons are actually weak interaction of their constituent quarks.

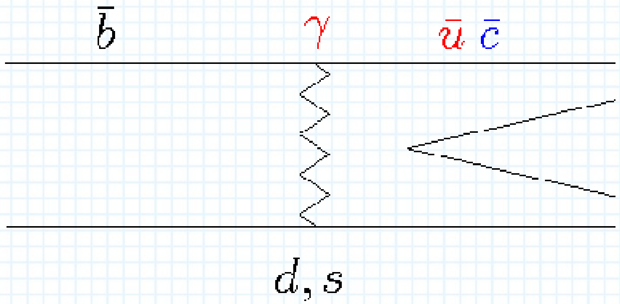
OZI suppressed Penguin $\mathbf{P_{OZI}}$



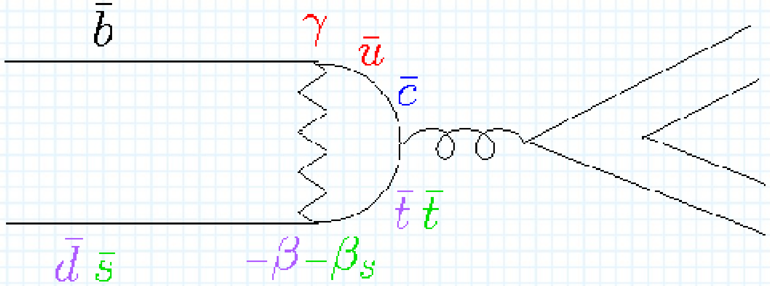
Color suppressed EW Penguin $\mathbf{P_{EW}^{C}}$



Annihilation **A**



Exchange **E**



Penguin Annihilation **PA**

\+ More smaller contributions

Operator product expansion

a) ***Hadrons are complicated superposition of an infinite no of short lived quark and gluon configuration.***

b) ***The weak interaction time is considerably shorter than the typical lifetime of these fluctuations because of the large mass of the gauge boson W.***

c) ***These strong interaction processes can be divided in long and short distance phenomena by the momentum scales involved. The long distance effect happen at distances of usual hadrons size which include bound state wave functions, soft gluon radiation and final state interactions. On the other hand, short distance effects originates in hard gluon interaction.***

d) ***All long distance effects are absorbed in the initial and final hadronic wave functions whereas short distance corrections are associated with the effective weak Hamiltonian*** $\mathcal{H}_{\text{eff}}$.

e) ***Correspondingly, the weak amplitudes are given by matrix elements of*** $\mathcal{H}_{\text{eff}}$ ***between asymptotic states***

Hierarchy of scales

$$\underbrace{\begin{matrix}\Lambda_{\mathrm{QCD}} \\ \mathbf{10^{-4}}\boldsymbol{TeV}\end{matrix}}_{\textit{Long distances}} \ll \underbrace{\begin{matrix}\Lambda_{\mathrm{EW}} \\ \mathbf{10^{-1}}\boldsymbol{TeV}\end{matrix}}_{\textit{Short distances}} \ll \underbrace{\begin{matrix}\Lambda_{\mathrm{NP}} \\ > \mathbf{10^{0}}\boldsymbol{TeV}\end{matrix}}_{\textit{Very short distances}}$$

Lot of effort put into understanding these decays. The technique utilized to address this is Effective Field Theory. It allows one to look at the physics of the shortest distance/time scales ignoring the longer ones, and then move sequentially to longer distances/times.

- ***Heavy degrees of freedom (new physics particles, top, Z, W) are integrated out from appearing explicitly→short-distance loop functions. Renormalization group allows summation of large*** $\log(\mu_{SD}/\mu_{LD})$.
- ***Long distance effects involve calculation of matrix elements of local quark bilinear operators. Form factors…***

In SM the effective Hamiltonian for $b \to s\ell^+\ell^-$ is given by

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\left(\sum_{i=1}^{2}(\lambda_u C_i \mathcal{O}_i^u + \lambda_c C_i \mathcal{O}_i^c) - \lambda_t \sum_{i=3}^{10} C_i \mathcal{O}_i\right)$$

Unitarity $\lambda_u + \lambda_c + \lambda_t = 0$ $\qquad \lambda_i \equiv V_{is}^* V_{ib}$

$$\mathcal{O}_1^q = (\bar{s}_i q_j)_{V-A}(\bar{q}_j b_i)_{V-A} \qquad \mathcal{O}_2^q = (\bar{s}q)_{V-A}(\bar{q}b)_{V-A}$$

$$\mathcal{O}_3^q = (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V-A} \qquad \mathcal{O}_4^q = (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V-A}$$

$$\mathcal{O}_5^q = (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V+A} \qquad \mathcal{O}_6^q = (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V+A}$$

$$\mathcal{O}_7^q = -\frac{em_b}{8\pi^2}\bar{s}_i\sigma_{\mu\nu}(1+\gamma_5)b_i F^{\mu\nu} \qquad \mathcal{O}_9^q = \frac{e^2}{8\pi^2}(\bar{s}b)_{V-A}(\bar{\ell}\ell)_V$$

$$\mathcal{O}_8^q = -\frac{gm_b}{8\pi^2}\bar{s}_i\sigma_{\mu\nu}(1+\gamma_5)T_{ij}^a b_j G^{\mu\nu} \qquad \mathcal{O}_{10}^q = \frac{e^2}{8\pi^2}(\bar{s}b)_{V-A}(\bar{\ell}\ell)_A$$

i and j are color indices

$g(e)$ strong (e.m) couplings

m_b running b-quark mass in $\overline{MS}$ scheme

$$(\bar{s}b)_{V\pm A} = \bar{s}\gamma_\mu(1\pm\gamma_5)b$$

G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125.

$B \to V_1V_2$ and angular analysis

- ***B decays to two vector mesons special due to the presence of 3 helicity amplitudes***

Spin J=0 $B \to V_1V_2$ V_1 and V_2 both have Spin 1 $\Rightarrow$ L=0, 1, 2 or S, P, D waves

- ***In the rest frame of B, the momenta of V_1 and V_2 are equal and opposite hence the helicities of both the vector mesons are same .***

- ***CP of a final state depends on the partial wave: $(-1)^L \Rightarrow$ final state admixture of CP-odd and CP-even. Since asymmetry has opposite sign for the two CP states one has cancellation or dilution of overall asymmetry. Separation of CP-even and CP-odd components needed.***

POLARIZATION IN B^0 DECAY

Γ_L/Γ in $B^0 \to J/\psi(1S)K^*(892)^0$	0.570 ± 0.008
$\Gamma_\perp/\Gamma$ in $B^0 \to J/\psi K^{*0}$	0.219 ± 0.010 (S = 1.2)
$\phi_\parallel$ in $B^0 \to J/\psi K^{*0}$	-2.86 ± 0.11 rad (S = 1.5)
$\phi_\perp$ in $B^0 \to J/\psi K^{*0}$	3.01 ± 0.14 rad (S = 2.9)
Γ_L/Γ in $B^0 \to \psi(2S)K^*(892)^0$	0.46 ± 0.04
$\Gamma_\perp/\Gamma$ in $B^0 \to \psi(2S)K^{*0}$	0.30 ± 0.06
$\phi_\parallel$ in $B^0 \to \psi(2S)K^{*0}$	-2.8 ± 0.4 rad
$\phi_\perp$ in $B^0 \to \psi(2S)K^{*0}$	2.8 ± 0.32 rad
Γ_L/Γ in $B^0 \to \chi_{c1}K^*(892)^0$	$0.83\ ^{+0.06}_{-0.08}$ (S = 1.3)
$\Gamma_\perp/\Gamma$ in $B^0 \to \chi_{c1}K^*(892)^0$	0.03 ± 0.04
$\phi_\parallel$ in $B^0 \to \chi_{c1}K^*(892)^0$	0.0 ± 0.32 rad
Γ_L/Γ in $B^0 \to D_s^{*+}D^{*-}$	0.52 ± 0.05
Γ_L/Γ in $B^0 \to D^{*-}\rho^+$	0.885 ± 0.020
Γ_L/Γ in $B^0 \to D_s^{*+}\rho^-$	0.84 ± 0.30
Γ_L/Γ in $B^0 \to D_s^{*+}K^{*-}$	$0.92\ ^{+0.40}_{-0.32}$
Γ_L/Γ in $B^0 \to D^{*+}D^{*-}$	0.624 ± 0.031
$\Gamma_\perp/\Gamma$ in $B^0 \to D^{*+}D^{*-}$	0.147 ± 0.019
Γ_L/Γ in $B^0 \to \overline{D}^{*0}\omega$	0.67 ± 0.05
Γ_L/Γ in $B^0 \to D^{*-}\omega\pi^+$	0.65 ± 0.04
Γ_L/Γ in $B^0 \to \omega K^{*0}$	0.69 ± 0.13
Γ_L/Γ in $B^0 \to \omega K_2^*(1430)^0$	0.45 ± 0.12
Γ_L/Γ in $B^0 \to K^{*0}\overline{K}^{*0}$	$0.80\ ^{+0.12}_{-0.13}$
Γ_L/Γ in $B^0 \to \phi K^*(892)^0$	0.480 ± 0.030
$\Gamma_\perp/\Gamma$ in $B^0 \to \phi K^{*0}$	0.24 ± 0.05 (S = 1.5)
$\phi_\parallel$ in $B^0 \to \phi K^{*0}$	2.40 ± 0.13 rad
$\phi_\perp$ in $B^0 \to \phi K^{*0}$	2.39 ± 0.13 rad
$\delta_0(B^0 \to \phi K^{*0})$	2.82 ± 0.17 rad

Experimental status
Polarization measurements in B decays

A_{CP}^0 in $B^0 \to \phi K^{*0}$	0.04 ± 0.06
$A_{CP}^\perp$ in $B^0 \to \phi K^{*0}$	-0.11 ± 0.12
$\Delta\phi_\parallel$ in $B^0 \to \phi K^{*0}$	0.11 ± 0.22 rad (S = 1.7)
$\Delta\phi_\perp$ in $B^0 \to \phi K^{*0}$	0.08 ± 0.22 rad (S = 1.7)
$\Delta\delta_0(B^0 \to \phi K^{*0})$	0.27 ± 0.16 rad
$\Delta\phi_{00}(B^0 \to \phi K_0^*(1430)^0)$	0.3 ± 0.4 rad
Γ_L/Γ in $B^0 \to \phi K_2^*(1430)^0$	$0.90\ ^{+0.06}_{-0.07}$
$\Gamma_\perp/\Gamma$ in $B^0 \to \phi K_2^*(1430)^0$	$0.002\ ^{+0.040}_{-0.031}$
$\phi_\parallel$ in $B^0 \to \phi K_2^*(1430)^0$	4.0 ± 0.4 rad
$\phi_\perp$ in $B^0 \to \phi K_2^*(1430)^0$	
$\delta_0(B^0 \to \phi K_2^*(1430)^0)$	3.41 ± 0.18 rad
A_{CP}^0 in $B^0 \to \phi K_2^*(1430)^0$	-0.05 ± 0.06
$\Delta\phi_\parallel(B^0 \to \phi K_2^*(1430)^0)$	-1.0 ± 0.4 rad
$\Delta\delta_0$ in $B^0 \to \phi K_2^*(1430)^0$	0.11 ± 0.14 rad
Γ_L/Γ in $B^0 \to K^*(892)^0\rho^0$	0.40 ± 0.14
Γ_L/Γ in $B^0 \to K^{*+}\rho^-$	0.38 ± 0.13
Γ_L/Γ in $B^0 \to \rho^+\rho^-$	$0.977\ ^{+0.028}_{-0.024}$
Γ_L/Γ in $B^0 \to \rho^0\rho^0$	$0.75\ ^{+0.12}_{-0.15}$
Γ_L/Γ in $B^0 \to a_1(1260)^+a_1(1260)^-$	0.31 ± 0.24
Γ_L/Γ in $B^0 \to p\bar{p}K^*(892)^0$	1.01 ± 0.13
Γ_L/Γ in $B^0 \to \Lambda\bar{\Lambda}K^*(892)^0$	0.60 ± 0.23

- *Need to perform angular analysis to obtain the helicity amplitudes from expt. data. Already been done for several $B \to V_1 V_2$ decay modes.*
- *Three different angular momentum projections that are used*
  - *Helicity basis*
  - *Transversity basis*
  - *Partial wave decomposition*
- *Theses bases are equivalent; different basis states have different physical interpretation giving different physical insights about the underlying processes.*
- *The most general covariant amplitude for a $B \to VV$ e.g. $B \to D^* \rho$*

$$A(B(p) \to V_1(k, \epsilon_1) V_2(q, \epsilon_2)) = \epsilon_1^{\mu} \epsilon_2^{\nu} \left(a\, g_{\mu\nu} + \frac{b}{m_1 m_2} p_\mu p_\nu + i\, \frac{c}{m_1 m_2} \epsilon_{\mu\nu\alpha\beta}\, k^\alpha q^\beta \right)$$

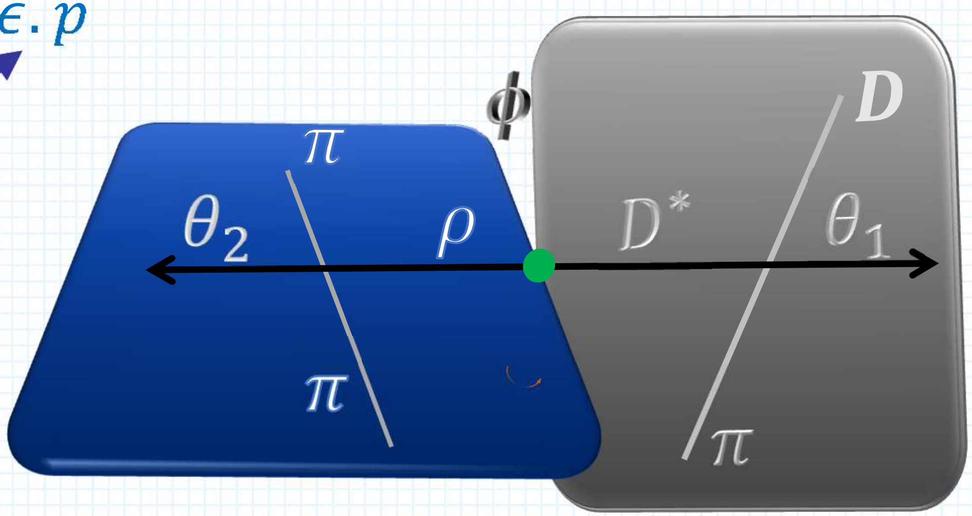
S & D wave admixture *P wave*

Consider decay of vector $V(\epsilon_\mu)$ to two pseudoscalars. In the rest frame this is described by

$$\epsilon = (0, \vec{\epsilon}),\ p_1 = \left(\sqrt{m_1^2 + \vec{p}^2}, \vec{p}\right), p_2 = \left(\sqrt{m_2^2 + \vec{p}^2}, -\vec{p}\right)$$

$$\mathcal{M}\left(V\left(\epsilon_\mu\right) \to P_1(p_1)P_2(p_2)\right) \propto \epsilon_\mu(p_1 - p_2) \propto \vec{\epsilon}.\vec{p}$$

Momentum of pseudoscalar mesons is peaked along the polarization direction



$$A_L = -x\ a\ - (x^2 - 1)b$$
$$A_\parallel = \sqrt{2}a$$
$$A_\perp = \sqrt{2}(x^2 - 1)c$$

$$x = \frac{k.q}{m_1 m_2}$$

$$A(B \to V_1 V_2) \propto \left(A_L \cos\theta_1 \cos\theta_2 + \frac{A_\parallel}{\sqrt{2}} \sin\theta_1 \sin\theta_2 \cos\phi\ + i\ \frac{A_\perp}{\sqrt{2}} \sin\theta_1 \sin\theta_2 \sin\phi\ \right)$$

- $A_0, A_\parallel$ *and* $A_\perp$ *referred to as the transersity amplitudes*
- *Helicity amplitudes are defined as* H_0, H_+, H_-, *where*

$$H_\pm = \frac{1}{\sqrt{2}}(A_\parallel \pm A_\perp), H_0 = A_0$$

- *The partial wave amplitudes are described in terms of transversity amplitudes as*

$$S = \frac{1}{\sqrt{3}}\left(\sqrt{2}A_\parallel - A_0\right), \qquad P = A_\perp, \qquad D = \frac{1}{\sqrt{3}}\left(A_\parallel + \sqrt{2}A_0\right)$$

Transversity basis is convenient for separation of CP-even and CP-odd components of the amplitude.

The partial wave amplitudes S, P, D ***and the helicty amplitudes*** H_0, H_+, H_- ***are not very useful for our purpose.***

Note $A_0 \equiv A_L$

Hence, the partial decay rate for $B \to D^{*}\rho \to (D\pi)(\pi\pi)$ can be written as

$$\frac{d\Gamma}{d\cos\theta_1\, d\cos\theta_2\, d\phi}$$

$$\propto \left(|A_L|^2\ \cos^2\theta_1 \cos^2\theta_2 + \frac{|A_\perp|^2}{2}\sin^2\theta_1 \sin^2\theta_2 \sin^2\phi\ + \frac{|\boldsymbol{A}_\parallel|^2}{2}\sin^2\theta_1 \sin^2\theta_2 \cos^2\phi \right.$$

$$\left. + \frac{\mathrm{Re}(A_L A_\parallel^*)}{2\sqrt{2}}\sin 2\theta_1\ \sin 2\theta_2 \cos\phi - \frac{\mathrm{Im}(A_\perp A^*_L)}{2\sqrt{2}}\sin 2\theta_1\ \sin 2\theta_2 \sin\phi - \frac{\mathrm{Im}(A_\perp A_\parallel^*)}{2}\sin^2\theta_1\ \sin^2\theta_2 \sin 2\phi \right)$$

Blue terms are P and T even

Red terms are P and T odd

However if one considers the decay $B \to K^{*} J/\psi \to (K\pi)(ll)$ the rate is given by

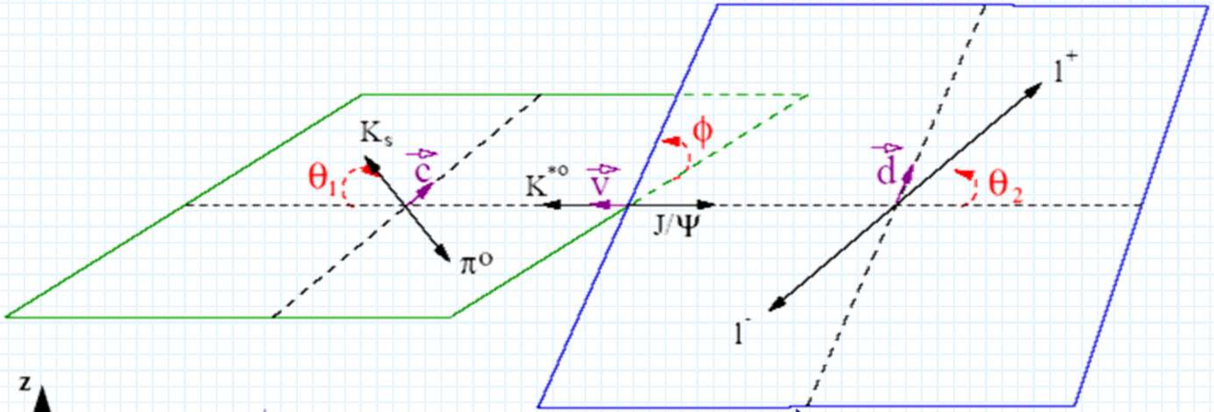
$$\propto \left(|A_L|^2\ \cos^2\theta_1 \sin^2\theta_2 + \frac{|A_\perp|^2}{2}\sin^2\theta_1 (\cos^2\theta_2 \sin^2\phi\ + \cos^2\phi) + \frac{|\boldsymbol{A}_\parallel|^2}{2}\sin^2\theta_1 (\cos^2\theta_2 \cos^2\phi + \sin^2\phi) \right.$$

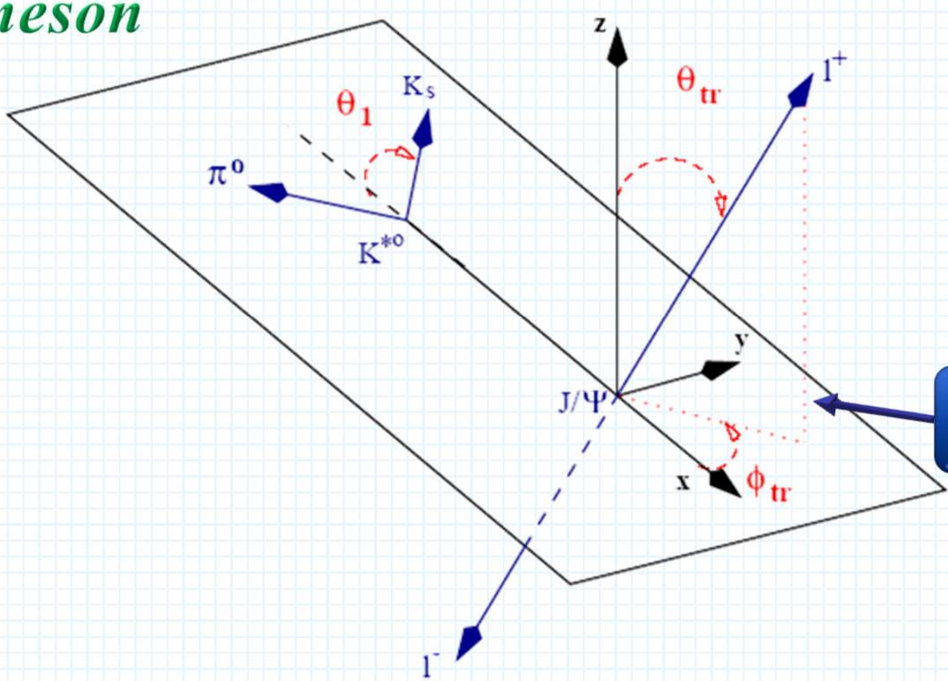
$$\left. + \frac{\mathrm{Re}(A_L A_\parallel^*)}{2\sqrt{2}}\sin 2\theta_1\ \sin 2\theta_2 \cos\phi - \frac{\mathrm{Im}(A_\perp A_L^*)}{2\sqrt{2}}\sin 2\theta_1\ \sin 2\theta_2 \sin\phi - \frac{\mathrm{Im}(A_\perp A_\parallel^*)}{2}\sin^2\theta_1\ \sin^2\theta_2 \sin 2\phi \right)$$

Expressions depend on decay mode !

Confusion??? !!! e.g. consider $B \to K^* J/\psi \to K\pi\, l^+ l^-$

A common choice is to assume that J/ψ at rest and not the B meson





Helicity frame B at rest

Transversity frame J/ψ at rest

See BaBar Physics Book

$$\cos\theta_2 = \sin\theta_{tr}\cos\theta_{tr}$$
$$\sin\theta_2 \sin\phi = \cos\theta_{tr}$$
$$\sin\theta_2 \cos\phi = \sin\theta_{tr}\sin\phi_{tr}$$

Transversity frame popular among experimentalists

In the transversity frame the partial decay rate in terms of transversity amplitudes is written as

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_1 d\cos\theta_{tr}\, d\,\phi_{tr}}$$

$$= \frac{9}{32}\frac{1}{|A_L|^2+|A_\parallel|^2+|A_\perp|^2}\Big(|A_L|^2\, 2\, \cos^2\theta_1\,(1-\sin^2\theta_{tr}\cos^2\phi_{tr}) + |A_\parallel|^2\cos^2\theta_1\,(1-\sin^2\theta_{tr}\sin^2\phi_{tr})$$

$$+|A_\perp|^2\sin^2\theta_1\sin^2\theta_{tr} - \frac{\mathrm{Re}(A_L A_\parallel^*)}{\sqrt{2}}\sin 2\theta_1\,\sin^2\theta_{tr}\sin 2\phi_{tr} + \frac{\mathrm{Im}(A_\perp A_L^*)}{\sqrt{2}}\sin 2\theta_1\,\sin 2\theta_{tr}\cos 2\phi_{tr}$$

$$+\frac{\mathrm{Im}(A_\perp A_\parallel^*)}{2}\sin^2\theta_1\,\sin 2\theta_{tr}\sin\phi_{tr}\Big)$$

We define the longitudinal polarization ratios as

$$F_L = \frac{\Gamma_L}{\Gamma} = \frac{|A_L|^2}{|A_L|^2+|A_\parallel|^2+|A_\perp|^2}$$

F_L easilyobtained by a one parameter fit to the θ_1, θ_2 distribution

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_1 d\cos\theta_2} = \frac{9}{32}(1-F_L)\sin^2\theta_1\,(1+\cos^2\theta_2) + \frac{9}{8}F_L\cos^2\theta_1\sin^2\theta_2$$

$F_\perp$ is similarly defined as $F_\perp = \frac{\Gamma_\perp}{\Gamma} = \frac{|A_\perp|^2}{|A_L|^2+|A_\parallel|^2+|A_\perp|^2}$

$F_\perp$ is easily obtained by a one parameter fit in transversity basis

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_{tr}} = \frac{3}{8}(1-F_\perp)(1+\cos^2\theta_{tr}) + \frac{3}{4}F_\perp\sin^2\theta_{tr}$$

The above relations are for $B \to K^* J/\psi$. For $B \to D^*\rho$ the analogous relations are

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_1 d\cos\theta_2} = \frac{9}{16}(1-F_L)\sin^2\theta_1\sin^2\theta_2 + \frac{9}{4}F_L\cos^2\theta_1\cos^2\theta_2$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_{tr}} = \frac{3}{4}(1-F_\perp)\sin^2\theta_{tr} + \frac{3}{2}F_\perp\cos^2\theta_{tr}$$

Important:* *The choice of frame may help in simplifying the problem we are interested in.

One is however interested in all the information that angular analysis can provide and not just in one component.

The best choice is to use the transversity basis in the Helicity frame, especially to study interesting CP violating signals that do not need flavour or time tagging.

We learnt that the differential decay rate for B → VV decays may be written as

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_1\, d\cos\theta_2\, d\phi} = \frac{9}{16\pi}\left(|F_L|^2 f_{LL} + |F_\perp|^2 f_{\perp\perp} + \left|F_\parallel\right|^2 f_{\parallel\parallel} + \mathrm{Re}\left(F_L F_\parallel^*\right) f_{L\parallel} - \mathrm{Im}\left(F_\perp F_L^*\right) f_{\perp L} - \mathrm{Im}\left(F_\perp F_\parallel^*\right) f_{\perp\parallel}\right)$$

where $|F_L|^2 + \left|F_\parallel\right|^2 + |F_\perp|^2 = 1$ ***and*** $f_{\lambda\sigma}$ ***are the coefficients of the helicity amplitudes that depend purely on the angles describing the kinematics.***

Consider the decay corresponding to the conjugate process; its amplitude may be written using CPT invariance as

$$A(B(p) \to V_1(k,\epsilon_1)V_2(q,\epsilon_2) = \epsilon_1^\mu \epsilon_2^\nu \left(a\, g_{\mu\nu} + \frac{b}{m_1 m_2} p_\mu p_\nu + i\, \frac{c}{m_1 m_2} \epsilon_{\mu\nu\alpha\beta}\, k^\alpha q^\beta\right)$$

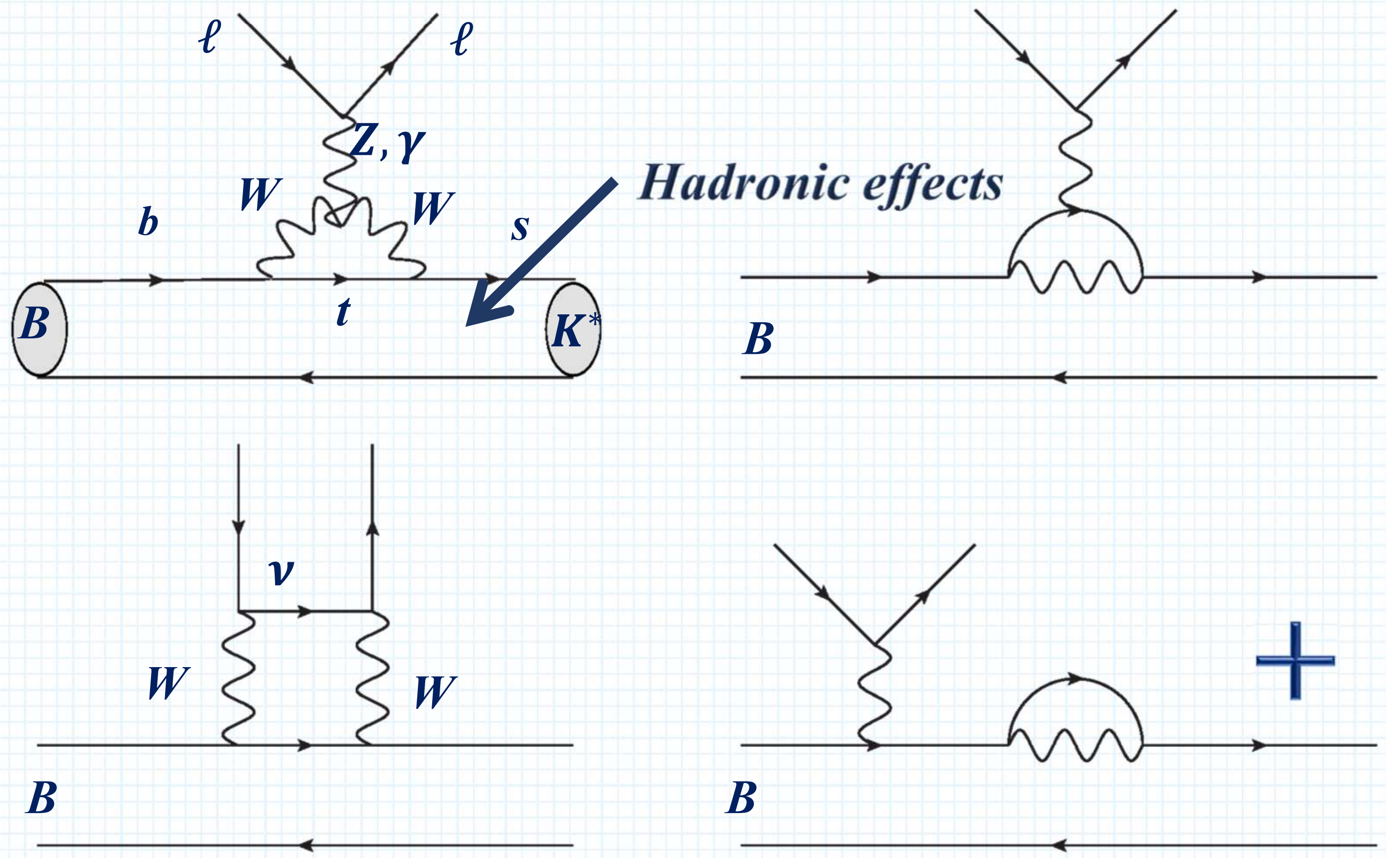
Note the switch of sign of the P-wave term in the conjugate process when compared with the amplitude for the process.

$$A(\bar{B}(p) \to \bar{V}_1(k,\epsilon_1)\bar{V}_2(q,\epsilon_2) = \epsilon_1^\mu \epsilon_2^\nu \left(\bar{a}\, g_{\mu\nu} + \frac{\bar{b}}{m_1 m_2} p_\mu p_\nu - i\, \frac{\bar{c}}{m_1 m_2} \epsilon_{\mu\nu\alpha\beta}\, k^\alpha q^\beta\right)$$

Why $B \to K^* \ell^+ \ell^-$?

1. *Penguin process. Rare FCNC decay. Good place to look for NP.*
2. *One has a large number of related observables each measured as a function of the dilepton invariant mass. This mode that get contribution from variety of operators i.e. various new particles in the loop. It is therefore more likely that NP contributes to this mode than say $B_s \to \mu^+ \mu^-$.*
3. *Clean mode. Can be studied in a manner where there is almost none or reduced hadronic uncertainty.*
4. *Several asymmetries in addition to A_{FB} can be measured which are sensitive to NP via interference as linear effects.*

Semileptonic FCNC process… a cleaner route



Effective Hamiltonian for b→ sℓℓ

The decay $B(p) \to K^*(k)\ell^-(q_1)\ell^+(q_2)$ at the quark level can be described by effective weak Hamiltonian $\mathcal{H}_{\text{eff}}$ for $b \to s\ell^+\ell^-$ as

$$\mathcal{H}_{eff} = \frac{G_F\alpha}{\sqrt{2}\pi}V_{tb}V_{ts}^*[\, C_9^{eff}(\bar{s}\gamma^\mu P_L b)\bar{l}\gamma^\mu l \; + \; C_{10}(\bar{s}\gamma^\mu P_L b)\bar{l}\gamma^\mu\gamma^5 l$$

$$- \; \frac{2C_7^{eff}}{q^2}(\bar{s}i\sigma_{\mu\nu}q^\nu m_b P_R b)\bar{l}\gamma^\mu l]$$

dominant terms only

$q = q_1 + q_2$ $P_{L,R} = \frac{(1\mp\gamma_5)}{2}$

Neglected s quark mass but it can be included

$C_7^{\text{eff}} = -0.304$ $\quad C_9^{\text{eff}} = 4.211 + Y(q^2)$ $\quad C_{10} = -4.103$

$$= C_7 - \frac{1}{3}C_3 - \frac{4}{9}C_4 - \frac{20}{3}C_5 - \frac{80}{9}C_6$$

$\alpha_s(m_W) = 0.120$, $\alpha_s(m_b) = 0.214$, $m_t(m_t) = 162.3$ GeV, $m_W = 80.4$GeV and $\sin^2 2\theta_W = 0.23$.

$$Y(q^2) = \frac{4}{3}C_3 + \frac{64}{9}C_5 + \frac{64}{27}C_6 + h(q^2, m_c)\left(\frac{4}{3}C_1 + C_2 + 6\,C_3 + 60C_5\right)$$

$$-\frac{1}{2}h(q^2, m_b)\left(7\,C_3 + \frac{4}{3}C_4 + 76C_5 + \frac{64}{3}C_6\right) - \frac{1}{2}h(q^2, 0)\left(C_3 + \frac{4}{3}C_4 + 16C_5 + \frac{64}{3}C_6\right)$$

$$h(q^2, m_q) = -\frac{4}{9}\left(\ln\frac{m_q^2}{\mu^2} - \frac{2}{3} - y\right) - \frac{4}{9}(2+y)\sqrt{|y-1|} \times \begin{cases} \tan^{-1}\dfrac{1}{\sqrt{y-1}} & y > 1 \\ \ln\dfrac{1+\sqrt{1-y}}{\sqrt{y}} - i\dfrac{\pi}{2} & y \le 1 \end{cases}$$

$$y = \frac{4m_q^2}{q^2}$$

Matrix element for $B \to K^* \ell^+ \ell^-$

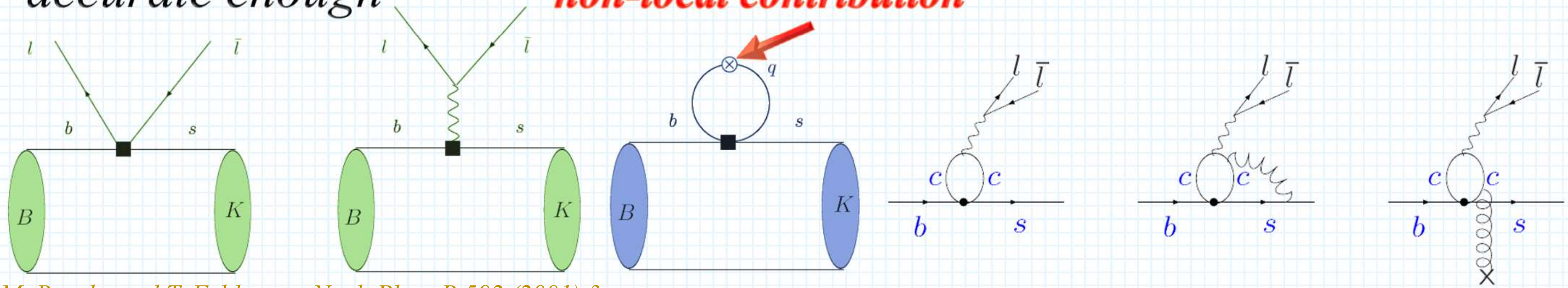
The matrix element for the decay mode $B(p) \to K^*(k)\ell^-(q_1)\ell^+(q_2)$

$$\mathcal{M} = \frac{G_F\alpha}{\sqrt{2}\pi} V_{tb}V_{ts}^*\Big\{C_9 \langle K^*|\bar{s}\gamma^\mu P_L b|\bar{B}\rangle\, \bar{\ell}\gamma_\mu \ell + C_{10}\langle K^*|\bar{s}\gamma^\mu P_L b|\bar{B}\rangle \bar{\ell}\gamma_\mu\gamma_5 \ell$$
$$- \frac{2m_b}{q^2} C_7 \underbrace{\langle K^*|\bar{s}i\sigma^{\mu\nu}q_\nu P_R b|\bar{B}\rangle}_{\text{Hadronic matrix element}} \bar{\ell}\gamma_\mu \ell\Big\}$$

challenge to reliably calculate. Estimated in various theories: LCSR, Lattice QCD, HQET, LEET … tremendous effort in literature

Unfortunately simple picture of decay presented above is not accurate enough

non-local contribution



M. Beneke and T. Feldmann, Nucl. Phys. B 592 (2001) 3
A. Khodjamirian, T. Mannel, A. A. Pivovarov and Y.-M. Wang, JHEP 1009, 089 (2010);A. Khodjamirian arXiv:1312.6480

Non-local contributions

Since the matrix element of the semi-leptonic operator $\mathcal{O}_{9,10}$ can be expressed through $B \to K^*$ form factors, the non-factorizable corrections contribute to the decay amplitude only through the production of a virtual photon, which then decays into lepton pair.

Exist additional non-factorizable and long-distance contributions
Electromagnetic corrections of purely hadronic operators
$\Rightarrow$ Complete Hamiltonian

$$A(B(p) \to K^*(k)\ell^+\ell^-)$$
$$= \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \Bigg[\Bigg\{ C_9 \langle K^* | \bar{s}\gamma^\mu P_L b | \bar{B} \rangle - \frac{2C_7}{q^2} \langle K^* | \bar{s} i \sigma^{\mu\nu} q_\nu (m_b P_R + m_s P_L) b | \bar{B} \rangle$$
$$- \frac{16\pi^2}{q^2} \sum_{i=\{1-6,8\}} C_i \mathcal{H}_i^\mu \Bigg\} \bar{\ell}\gamma_\mu \ell + C_{10} \langle K^* | \bar{s}\gamma^\mu P_L b | \bar{B} \rangle \bar{\ell}\gamma_\mu \gamma_5 \ell \Bigg]$$

nonlocal hadronic matrix elements

$$\mathcal{H}_i^\mu = \left\langle K^*(k) \right| i \int d^4x \, e^{iq.x} T\{ j_{em}^\mu(x), \mathcal{O}_i(0) \} \left| \bar{B}(p) \right\rangle$$

Hadronic matrix elements

Lorentz invariance to write the most general form of the Matrix element

$$\langle K^*(\epsilon^*,k)|\bar{s}\gamma^\mu P_L b|B(p)\rangle$$

$$= \epsilon_\nu^*\left(\mathcal{X}_0 q^\mu q^\nu + \mathcal{X}_1\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \mathcal{X}_2\left(k^\mu - \frac{k.q}{q^2}q^\mu\right)q^\nu + i\mathcal{X}_3\epsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma\right)$$

Vector current conserved and only the $\mathcal{X}_0$ term in divergence of axial part survives. q^μ term vanishes in the limit of massless leptons.

$$\left\langle K^*(\epsilon^*,k)\middle|i\bar{s}\sigma^{\mu\nu}q_\nu P_{R,L}b\middle|B(p)\right\rangle$$

$$= \epsilon_\nu^*\left(\pm\mathcal{Y}_1\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) \pm \mathcal{Y}_2\left(k^\mu - \frac{k.q}{q^2}q^\mu\right)q^\nu + i\mathcal{Y}_3\epsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma\right)$$

Ensure that $q_\mu\left\langle K^*(\epsilon^*,k)\middle|i\bar{s}\sigma^{\mu\nu}q_\nu P_{R,L}b\middle|B(p)\right\rangle = 0$

Nonlocal hadronic matrix elements

$$\mathcal{H}_i^\mu = \left\langle K^*(\epsilon^*,k)\middle|i\int d^4x\, e^{iq.x}\, T\{j_{em}^\mu(x),\mathcal{O}_i(0)\}\middle|B(p)\right\rangle$$

$$= \epsilon_\nu^*\left(\mathcal{Z}_1^i\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \mathcal{Z}_2^i\left(k^\mu - \frac{k.q}{q^2}q^\mu\right)q^\nu + i\mathcal{Z}_3^i\,\epsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma\right)$$

Ignore nonlocal contributions for the moment. Get back to them later…

In terms of q^2 dependent well known from factors $V, A_{1,2}, T_{1,2,3}$

$$\langle K^*(\epsilon^*, k)|\bar{s}\gamma^\mu P_L b|B(p)\rangle$$

$$= -i\epsilon^*_\mu (m_B + m_{K^*})A_1(q^2) + p_\mu\,(\epsilon^*.q)\frac{2\,A_2(q^2)}{m_B + m_{K^*}} + i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho k^\sigma \frac{2\,V(q^2)}{m_B + m_{K^*}}$$

$$\left\langle K^*(\epsilon^*, k)\middle| i\bar{s}\sigma^{\mu\nu}q_\nu P_{R,L} b\middle| B(p)\right\rangle$$

$$= i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho k^\sigma 2\,T_1(q^2) \pm T_2(q^2)\left[\epsilon^*_\mu\left(m_B^2 - m_{K^*}^2\right) - 2(\epsilon^*.q)p_\mu\right]$$

$$\mp(\epsilon^*.q)q^2\frac{2\,T_3(q^2)}{m_B^2 - m_{K^*}^2}p^\mu$$

$\mathcal{X}_i$'s and $\mathcal{Y}_i$'s can be related to form factors $V, A_{0,1,2}$ and $T_{1,2,3}$

$$\mathcal{X}_1 = -\frac{1}{2}(m_B + m_{K^*})A_1(q^2), \qquad \mathcal{Y}_1 = \frac{1}{2}(m_B^2 - m_{K^*}^2)T_2(q^2),$$

$$\mathcal{X}_2 = \frac{A_2(q^2)}{m_B + m_{K^*}}, \qquad \mathcal{Y}_2 = -T_2(q^2) - \frac{q^2}{m_B^2 - m_{K^*}^2}T_3(q^2),$$

$$\mathcal{X}_3 = \frac{V(q^2)}{m_B + m_{K^*}}, \qquad \mathcal{Y}_3 = -T_1(q^2).$$

What we really want is the final state where $K^*(\epsilon^*_\mu, k)$ has decayed to $K(k_1)\pi(k_2)$. This is achieved by replacing $\epsilon^*_\mu \to D_{K^*}\left(k^2\right)W_\mu$, where $W_\mu = K_\mu - \xi k_\mu$, $k = k_1 + k_2$, $K = k_1 - k_2$ and $\xi = \frac{m_K^2 - m_\pi^2}{k^2}$.

$$|D_{K^*}(k^2)|^2 = \frac{g^2_{K^*K\pi}}{\left(k^2 - m^2_{K^*}\right)^2 + (m_{K^*}\Gamma_{K^*})^2} \xrightarrow{\Gamma_{K^*} \ll m_{K^*}} \frac{\pi g^2_{K^*K\pi}}{m_{K^*}\Gamma_{K^*}}\delta(k^2 - m^2_{K^*})$$

$$A(B(p) \to k(k_1)\pi(k_2)\ell^+\ell^-) = \frac{G_F\alpha}{\sqrt{2}\pi} V_{tb}V^*_{ts} D_{K^*}(k^2)$$
$$\times \left\{\left[W_\mu \mathcal{V}_1^L + W.q\; k_\mu \mathcal{V}_2^L + i\epsilon_{\mu\nu\rho\sigma} K^{*\nu} k^\rho q^\sigma \mathcal{V}_3^L\right]\bar{\ell}\gamma^\mu P_L \ell + L \to R\right\}$$

$$\mathcal{V}_i^{L,R} = C_{L,R}\mathcal{X}_i - \frac{2m_b}{q^2} C_7 \mathcal{Y}_i$$

Remember that in the massless lepton limit L and R terms do not interfere

$$C_{L,R} = C_9 \mp C_{10}$$

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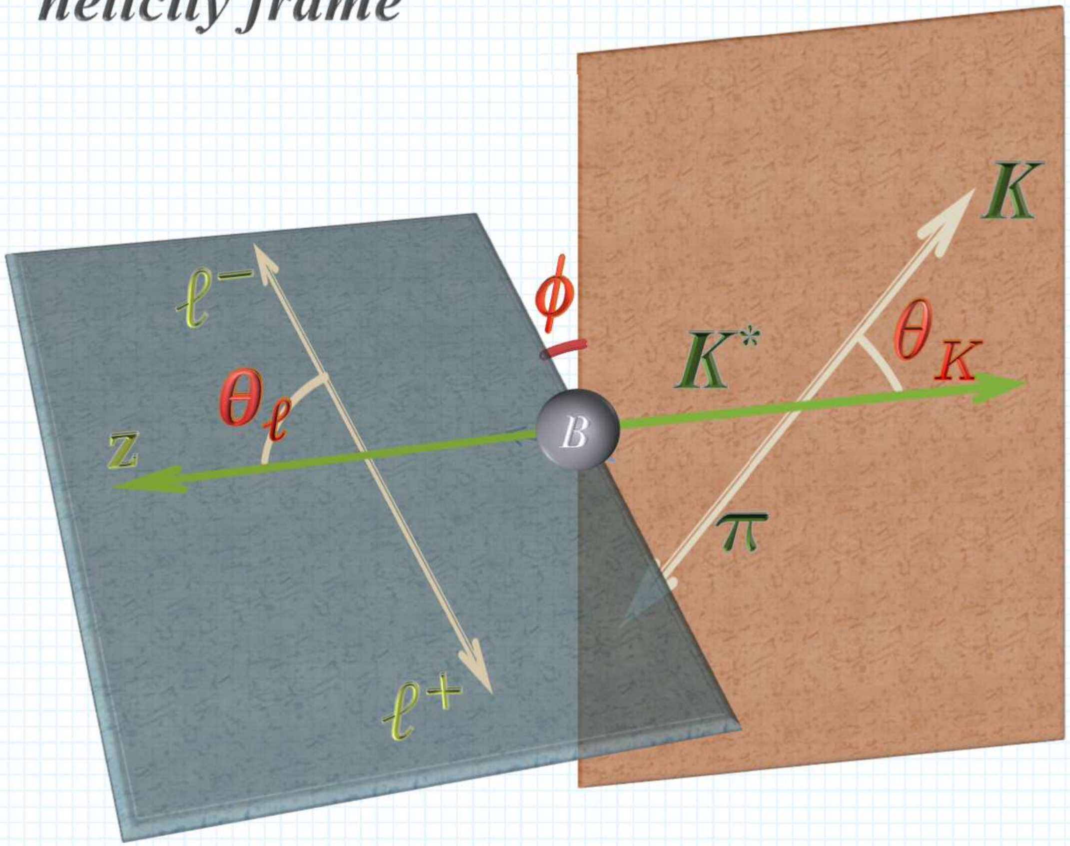
R. S., hep-ph:/9608341 ***Full angular study***
*F. Kruger, L. M. Sehgal, N. Sinha, R.S., Phys. Rev. D61,114028 (2000) [hep-ph/9907386]****Full angular study***
A. Ali, P. Ball, L. T. Handoko and G. Hiller, Phys. Rev. D61, 074024 (2000) [hep-ph/9910221]. ***A_{FB} zero crossing***
A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner and V. E. Lyubovitskij, Eur. Phys. J. C4, 18 (2002) ***lepton mass effect***

The decay $B(p) \to K^*\big(\to K(k_1)\pi(k_2)\big)\ell^+(q_2)\ell^-(q_1)$ in the helicity frame



$$X = \frac{1}{2}\lambda^{1/2}(m_B^2, q^2, m_{K^*}^2)$$

$$\beta_{K^*} = \frac{\lambda^{1/2}(m_{K^*}^2, m_K^2, m_\pi^2)}{m_{K^*}^2}$$

$$q.K = \xi(k.q) + \beta_{K^*} X \cos\theta_K$$

$$k.K = \xi m_{K^*}^2$$

$$k.Q = X\cos\theta_\ell$$

$$K.Q = \xi k.Q + \beta_{K^*}\left[k.q\ \cos\theta_\ell \cos\theta_K - \sqrt{q^2 m_{K^*}^2}\sin\theta_\ell \sin\theta_K \cos\phi\right]$$

$$\epsilon_{\mu\nu\alpha\beta} k^\mu K^\nu q^\alpha Q^\beta = -\sqrt{q^2 m_{K^*}^2}\beta_{K^*} X \sin\theta_\ell \sin\theta_K \sin\phi$$

It is easily seen that

$$\frac{d^4\Gamma(B \to (K\pi)\ell^+\ell^-)}{dq^2 d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = I(q^2, \theta_\ell, \theta_K, \phi)$$

$$= \frac{9}{32\pi}\left[I_1^s \sin^2\theta_K + I_1^c \cos^2\theta_K + (I_2^s \sin^2\theta_K + I_2^c \cos^2\theta_K)\cos 2\theta_\ell\right.$$

$$+I_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi + I_5 \sin 2\theta_K \sin\theta_\ell \cos\phi$$

$$+I_6 \sin^2\theta_K \cos\theta_\ell + I_7 \sin 2\theta_K \sin\theta_\ell \sin\phi + I_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi$$

$$\left.+I_9 \sin^2\theta_K \sin^2\theta_\ell\ \sin 2\phi\right]$$

Define transversity amplitudes

$$\mathcal{A}_t = -\frac{N}{m_K^*}\sqrt{q^2}\lambda^{1/2}(m_B^2, m_{K^*}^2, q^2)\hat{C}_{10}\mathcal{X}_0,$$

$$\mathcal{A}_\parallel^{L,R} = 2\sqrt{2}\, N\, \mathcal{V}_1^{L,R}$$

$$N = V_{tb}V_{ts}^*\left[\frac{G_F^2\alpha^2}{3\times 2^{10}\pi^5 m_B^3}q^2\sqrt{\lambda(m_B^2, m_{K^*}^2, q^2)}\beta\right]^{1/2}.$$

$$\mathcal{A}_0^{L,R} = \frac{N}{2\, m_{K^*}\sqrt{q^2}}\left[4k.q\mathcal{V}_1^{L,R} + \lambda(m_B^2, m_{K^*}^2, q^2)\mathcal{V}_2^{L,R}\right]$$

$$\mathcal{A}_\perp^{L,R} = \sqrt{2}\, N\lambda^{1/2}(m_B^2, m_{K^*}^2, q^2)\mathcal{V}_3^{L,R}$$

Where we have the amplitudes in terms of conventional form-factors:

$$\mathcal{A}_\perp^{L,R} = N\sqrt{2}\sqrt{\lambda(m_B^2, m_{K^*}^2, q^2)}\left[\left[(C_9^{\rm eff} \mp C_{10}^{\rm eff})\right]\frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2}C_7^{\rm eff}T_1(q^2)\right],$$

$$\mathcal{A}_\parallel^{L,R} = -N\sqrt{2}(m_B^2 - m_{K^*}^2)\left[\left[(C_9^{\rm eff} \mp C_{10}^{\rm eff})\right]\frac{A_1(q^2)}{m_B - m_{K^*}} + \frac{2m_b}{q^2}C_7^{\rm eff}T_2(q^2)\right],$$

$$\mathcal{A}_0^{L,R} = -\frac{N}{2m_{K^*}\sqrt{q^2}}\Bigg(\left[(C_9^{\rm eff} \mp C_{10}^{\rm eff})\right] \times \left[(m_B^2 - m_{K^*}^2 - q^2)(m_B + m_{K^*})A_1(q^2) - \lambda(m_B^2, m_{K^*}^2, q^2)\frac{A_2(q^2)}{m_B + m_{K^*}}\right] + 2m_bC_7^{\rm eff}\left[(m_B^2 + 3m_{K^*}^2 - q^2)T_2(q^2) - \frac{\lambda(m_B^2, m_{K^*}^2, q^2)}{m_B^2 - m_{K^*}^2}T_3(q^2)\right]\Bigg)$$

implicit dependence on q^2

$$I_1^s = \frac{(2+\beta^2)}{4}\left[|\mathcal{A}_\perp^L|^2 + |\mathcal{A}_\parallel^L|^2 + (L \to R)\right] + \frac{4m^2}{q^2}\mathrm{Re}(\mathcal{A}_\perp^L\mathcal{A}_\perp^{R*} + \mathcal{A}_\parallel^L\mathcal{A}_\parallel^{R*}),$$

$$I_1^c = |\mathcal{A}_0^L|^2 + |\mathcal{A}_0^R|^2 + \frac{4m^2}{q^2}\left[|\mathcal{A}_t|^2 + 2\mathrm{Re}(\mathcal{A}_0^L\mathcal{A}_0^{R*})\right],$$

$$I_2^s = \frac{\beta^2}{4}\left[|\mathcal{A}_\perp^L|^2 + |\mathcal{A}_\parallel^L|^2 + (L \to R)\right],$$

$$I_2^c = -\beta^2\left[|\mathcal{A}_0^L|^2 + (L \to R)\right],$$

$$\beta = \sqrt{1 - \frac{4m^2}{q^2}}$$

$$I_3 = \frac{\beta^2}{2}\left[|\mathcal{A}_\perp^L|^2 - |\mathcal{A}_\parallel^L|^2 + (L \to R)\right],$$

$$I_4 = \frac{\beta^2}{\sqrt{2}}\left[\mathrm{Re}(\mathcal{A}_0^L\mathcal{A}_\parallel^{L*}) + (L \to R)\right],$$

$$\Rightarrow I_5 = \sqrt{2}\beta\left[\mathrm{Re}(\mathcal{A}_0^L\mathcal{A}_\perp^{L*}) - (L \to R)\right],$$

$$\Rightarrow I_6^s = 2\beta\left[\mathrm{Re}(\mathcal{A}_\parallel^L\mathcal{A}_\perp^{L*}) - (L \to R)\right],$$

$$I_7 = \sqrt{2}\beta\left[\mathrm{Im}(\mathcal{A}_0^L\mathcal{A}_\parallel^{L*}) - (L \to R)\right],$$

$$\Rightarrow I_8 = \frac{1}{\sqrt{2}}\beta^2\left[\mathrm{Im}(\mathcal{A}_0^L\mathcal{A}_\perp^{L*}) + (L \to R)\right],$$

$$\Rightarrow I_9 = \beta^2\left[\mathrm{Im}(\mathcal{A}_\parallel^{L*}\mathcal{A}_\perp^L) + (L \to R)\right],$$

In SM for this mode CP violation is very small. Ignoring the small CP violation we have for the conjugate mode $\bar{B} \to (\bar{K}\,\bar{\pi})\ell^+\ell^-$

$$\frac{d^4\Gamma(\bar{B} \to (\bar{K}\,\bar{\pi})\ell^+\ell^-)}{dq^2 d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = \bar{I}(q^2, \theta_\ell, \theta_K, \phi)$$

$$= \frac{9}{32\pi}\big[I_1^s \sin^2\theta_K + I_1^c \cos^2\theta_K + (I_2^s \sin^2\theta_K + I_2^c \cos^2\theta_K)\cos 2\theta_\ell$$

$$+I_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi - I_5 \sin 2\theta_K \sin\theta_\ell \cos\phi$$

$$-I_6 \sin^2\theta_K \cos\theta_\ell + I_7 \sin 2\theta_K \sin\theta_\ell \sin\phi - I_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi$$

$$-I_9 \sin^2\theta_K \sin^2\theta_\ell\, \sin 2\phi\big]$$

Note that the terms linearly proportional to $\mathcal{A}_\perp$ ***have switched sign when going from mode to conjugate mode. If*** $K \equiv K_s$ ***and*** $\pi \equiv \pi^0$ ***adding partial decay rates for*** B^0 ***and*** $\overline{B^0}$ ***will result in vanishing contribution from*** $I_{5,6,8,9}$. ***These rates have t be subtracted to obtain a CP conserving partial rate***

Observables in $B \to K^* \ell^+ \ell^-$

$$F_L = \frac{|\mathcal{A}_0^L|^2 + |\mathcal{A}_0^R|^2}{\Gamma_f} \qquad \frac{d\Gamma}{dq^2} = \sum_{\lambda=0,||,\perp} \left(|\mathcal{A}_\lambda^L|^2 + |\mathcal{A}_\lambda^R|^2\right)$$

λ is K^* polarization

L, R chirality of ℓ^-

$$F_{||} = \frac{|\mathcal{A}_{||}^L|^2 + |\mathcal{A}_{||}^R|^2}{\Gamma_f} \qquad F_L + F_{||} + F_\perp = 1$$

3 observables $F_L, F_\perp, \cancel{\Gamma_f}$

$$F_\perp = \frac{|\mathcal{A}_\perp^L|^2 + |\mathcal{A}_\perp^R|^2}{\Gamma_f}$$

$$\int_D \equiv \int_0^1 - \int_{-1}^0$$

$$\int_{D_{LR}} \equiv \int_{-\pi/2}^{\pi/2} - \int_{\pi/2}^{3\pi/2}$$

$$A_4 = \frac{1}{\Gamma_f} \int_{D_{LR}} d\phi \int_D d\cos\theta_K \int_D d\cos\theta_\ell \frac{d^4(\Gamma + \bar{\Gamma})}{dq^2 d^3\Omega}$$

$$A_5 = \frac{1}{\Gamma_f} \int_{-1}^{1} d\cos\theta_\ell \int_{D_{LR}} d\phi \int_D d\cos\theta_K \frac{d^4(\Gamma - \bar{\Gamma})}{dq^2 d^3\Omega}$$

$$A_{FB} = \frac{1}{\Gamma_f} \int_{-1}^{1} d\cos\theta_K \int_0^{2\pi} d\phi \int_D d\cos\theta_\ell \frac{d^4(\Gamma - \bar{\Gamma})}{dq^2 d^3\Omega}$$

3 asymmetry observables $\propto Re(A_\lambda A_\sigma^*)$

A_7, A_8, A_9 are 3 more asymmetry observables $\propto Im(A_\lambda A_\sigma^*)$ /Require imaginary amplitudes

Total 9 CP conserving observables

For the massless lepton case

Relations between observables and Amplitudes

$$A_{FB} = \frac{3}{2}\frac{\mathrm{Re}(\mathcal{A}_\parallel^L \mathcal{A}_\perp^{L^*} - \mathcal{A}_\parallel^R \mathcal{A}_\perp^{R^*})}{\Gamma_f},$$

$$A_4 = \frac{\sqrt{2}}{\pi}\frac{\mathrm{Re}(\mathcal{A}_0^L \mathcal{A}_\parallel^{L^*} + \mathcal{A}_0^R \mathcal{A}_\parallel^{R^*})}{\Gamma_f},$$

$$A_5 = \frac{3}{2\sqrt{2}}\frac{\mathrm{Re}(\mathcal{A}_0^L \mathcal{A}_\perp^{L^*} - \mathcal{A}_0^R \mathcal{A}_\perp^{R^*})}{\Gamma_f},$$

$$A_7 = \frac{3}{2\sqrt{2}}\frac{\mathrm{Im}(\mathcal{A}_0^L \mathcal{A}_\parallel^{L*} - \mathcal{A}_0^R \mathcal{A}_\parallel^{R*})}{\Gamma_f},$$

$$A_8 = \frac{\sqrt{2}}{\pi}\frac{\mathrm{Im}(\mathcal{A}_0^L \mathcal{A}_\perp^{L\,*} + \mathcal{A}_0^R \mathcal{A}_\perp^{R\,*})}{\Gamma_f},$$

$$A_9 = \frac{3}{2\pi}\frac{\mathrm{Im}(\mathcal{A}_\parallel^{L*} \mathcal{A}_\perp^L + \mathcal{A}_\parallel^{R*} \mathcal{A}_\perp^R)}{\Gamma_f}.$$

J. Gratrex, M. Hopfer and R. Zwicky, 1506.03970

The observables A_4, A_5, A_{FB}, A_7, A_8 and A_9 are related to the CP averaged observables S_4 , S_5 , A_{FB}^{LHCb} , S_7, S_8 and S_9 measured by LHCb

$$A_4 = -\frac{2}{\pi}S_4,\ \ A_5 = \frac{3}{4}S_5,$$

$$A_{FB} = -A_{FB}^{\mathrm{LHCb}},\ A_7 = \frac{3}{4}S_7,$$

$$A_8 = -\frac{2}{\pi}S_8,\ \ A_9 = \frac{3}{2\pi}S_9,$$

Observables absent in $B \to \psi K^*$ mode. However, they will appear if lepton helicity measured

Effects of non-factorizable contributions

It is easy to see that the effect of non-local contributions can be straight forwardly incorporated into C_9^{eff}. We simply make the replacement $C_9 \mathcal{X}_j \rightarrow C_9^j \mathcal{X}_j$

$$-\frac{16\,\pi^2}{q^2} \sum_{i=\{1-6,8\}} C_i \frac{\mathcal{Z}_j^i}{\mathcal{X}_j} = \Delta C_9^{(\text{fac})}(q^2) + \Delta C_9^{j,(\text{non}-\text{fac})}(q^2) \quad j = 1,2,3$$

$$C_9 \rightarrow C_9^j = C_9 + \Delta C_9^{(\text{fac})}(q^2) + \Delta C_9^{j,(\text{non}-\text{fac})}(q^2)$$

$$\frac{2(m_b + m_s)}{q^2} C_7 \mathcal{Y}_j \rightarrow \tilde{\mathcal{Y}}_j = \frac{2(m_b + m_s)}{q^2} C_7 \mathcal{Y}_j + \cdots$$

factorizable & non factorizable contributions

Not to be confused with $C_{9,\perp}, C_{9,\parallel}$ defined in Beneke, Feldmann and Seidel

Rusa Mandal, R.S. and Diganta Das Phys. Rev. D90 096006 (2014)
Diganta Das and R.S. Phys. Rev. D 86, 056006(2012)

Effective helicity index due to non-factorizable corrections in C_9:

$$C_9^{\perp} \equiv C_9^{(3)}, C_9^{\parallel} \equiv C_9^{(1)}, C_9^{0} \equiv C_9^{(2)}\kappa$$

$$\kappa = 1 + \frac{C_9^{(1)} - C_9^{(2)}}{C_9^{(2)}} \frac{4\,k.q\mathcal{X}_1}{4\,k.q\mathcal{X}_1 + \lambda(m_B^2, m_{K^*}^2, q^2)\mathcal{X}_2}$$

The seven amplitudes can be written in terms of the form-factors $\mathcal{X}_{0,1,2,3}$ and $\mathcal{Y}_{1,2,3}$

$$\mathcal{A}_{\perp}^{L,R} = \sqrt{2}\,N\sqrt{\lambda(m_B^2, m_{K^*}^2, q^2)}\left[(C_9^{\perp} \mp C_{10})\mathcal{X}_3 - \widetilde{\mathcal{Y}}_3\right]$$

$$\mathcal{A}_{\parallel}^{L,R} = 2\sqrt{2}\,N\left[(C_9^{\parallel} \mp C_{10})\mathcal{X}_1 - \zeta_0\widetilde{\mathcal{Y}}_1\right]$$

$$\mathcal{A}_0^{L,R} = \frac{N}{2m_{K^*}\sqrt{q^2}}\left[\begin{array}{c}(C_9^0\kappa \mp C_{10})(4\,k.q\mathcal{X}_1 + \lambda(m_B^2, m_{K^*}^2, q^2)\mathcal{X}_2) \\ -\zeta_0(4\,k.q\widetilde{\mathcal{Y}}_1 + \lambda(m_B^2, m_{K^*}^2, q^2)\widetilde{\mathcal{Y}}_2)\end{array}\right]$$

$$\mathcal{A}_t = -\frac{N}{m_{K^*}}\sqrt{q^2}\sqrt{\lambda(m_B^2, m_{K^*}^2, q^2)}\,C_{10}\mathcal{X}_0 \qquad \zeta_0 = \frac{m_b - m_s}{m_b + m_s}$$

Note the amplitude $\mathcal{A}^{L,R}_{0,\parallel,\perp}$ have the form:

$$\mathcal{A}^{L,R}_{\lambda} = \boldsymbol{C^{\lambda}_{L,R}\mathcal{F}_{\lambda} - \widetilde{\mathcal{G}}_{\lambda} = \left(C^{\lambda}_{9} \mp C_{10}\right)\mathcal{F}_{\lambda} - \widetilde{\mathcal{G}}_{\lambda}}$$

Where, $\mathcal{F}_{\lambda}$, $\widetilde{\mathcal{G}}_{\lambda}$ new form factors that can be related to conventional form factors at a given order

$$\mathcal{F}_{\perp} = \sqrt{2}\, N \sqrt{\lambda(m_B^2, m_{K^*}^2, q^2)}\mathcal{X}_3 \qquad \mathcal{F}_{\parallel} = 2\sqrt{2}\, N \mathcal{X}_1$$

$$\mathcal{F}_0 = \frac{N}{2m_{K^*}\sqrt{q^2}}\left(4\, k.q\mathcal{X}_1 + \lambda\left(m_B^2, m_{K^*}^2, q^2\right)\mathcal{X}_2\right)$$

$$\tilde{\mathcal{G}}_0 = \sqrt{2}\, N \sqrt{\lambda(m_B^2, m_{K^*}^2, q^2)}\,\frac{2\,(m_b - m_s)}{q^2}\hat{C}_7\mathcal{Y}_3 + \cdots$$

$$\tilde{\mathcal{G}}_{\parallel} = 2\,\sqrt{2}\, N \frac{2\,(m_b - m_s)}{q^2}\hat{C}_7\mathcal{Y}_1 + \cdots$$

$$\tilde{\mathcal{G}}_0 = \frac{N}{2m_{K^*}\sqrt{q^2}}\frac{2\,(m_b - m_s)}{q^2}\hat{C}_7\left(4\, k.q\mathcal{Y}_1 + \lambda\left(m_B^2, m_{K^*}^2, q^2\right)\mathcal{Y}_2\right) + \cdots$$

We have the amplitudes (massless limit):

Simple to define amplitudes in terms of some new form factors as

$$\mathcal{A}_{\lambda}^{L,R} = C_{L,R}^{\lambda}\mathcal{F}_{\lambda} - \widetilde{\mathcal{G}}_{\lambda} = \left(C_9^{\lambda} \mp C_{10}\right)\mathcal{F}_{\lambda} - \widetilde{\mathcal{G}}_{\lambda} \leftarrow \textit{includes } C_7$$

implicit dependence on $\mathbf{q^2}$

$\mathcal{F}_{\lambda}, \tilde{\mathcal{G}}_{\lambda}$ new form factors that can be related to conventional form factors at a given order

An important step is to separate the real and imaginary parts of the amplitude. Three observables are non-zero only if the amplitude has an imaginary part

$$\mathcal{A}_{\lambda}^{L,R} = \left(C_9^{\lambda} \mp C_{10}\right)\mathcal{F}_{\lambda} - \widetilde{\mathcal{G}}_{\lambda} = (\mp C_{10} - r_{\lambda})\mathcal{F}_{\lambda} + i\varepsilon_{\lambda}$$

$$r_{\lambda} = \frac{\mathrm{Re}\,(\widetilde{\mathcal{G}}_{\lambda})}{\mathcal{F}_{\lambda}} - \mathrm{Re}\left(C_9^{\lambda}\right)$$

$$\varepsilon_{\lambda} = \mathrm{Im}\left(C_9^{\lambda}\right)\mathcal{F}_{\lambda} - \mathrm{Im}(\widetilde{\mathcal{G}}_{\lambda})$$

9 observables in terms of 10 parameters

$$F_L\Gamma_f = 2\mathcal{F}_0^2(r_0^2 + C_{10}^2) + 2\,\varepsilon_0^2$$

$$F_\parallel\Gamma_f = 2\mathcal{F}_\parallel^2\left(r_\parallel^2 + C_{10}^2\right) + 2\,\varepsilon_\parallel^2$$

$$F_\perp\Gamma_f = 2\mathcal{F}_\perp^2(r_\perp^2 + C_{10}^2) + 2\,\varepsilon_\perp^2$$

$$2\frac{\varepsilon_0^2}{\Gamma_f} \leq F_L \qquad 2\frac{\varepsilon_\parallel^2}{\Gamma_f} \leq F_\parallel \qquad 2\frac{\varepsilon_\perp^2}{\Gamma_f} \leq F_\perp$$

$$\sqrt{2}\pi A_4\Gamma_f = 4\mathcal{F}_0\mathcal{F}_\parallel(r_0 r_\parallel + C_{10}^2) + 4\varepsilon_0\varepsilon_\parallel$$

$$\sqrt{2}A_5\Gamma_f = 3\mathcal{F}_0\mathcal{F}_\perp C_{10}(r_0 + r_\perp)$$

$$A_{FB}\Gamma_f = 3\mathcal{F}_\parallel\mathcal{F}_\perp C_{10}(r_\parallel + r_\perp)$$

$$\sqrt{2}A_7\Gamma_f = 3C_{10}(\mathcal{F}_0\varepsilon_\parallel - \mathcal{F}_\parallel\varepsilon_0)$$

$$\pi A_8\Gamma_f = 2\sqrt{2}(\mathcal{F}_0 r_0\varepsilon_\perp - \mathcal{F}_\perp r_\perp\varepsilon_0)$$

$$\pi A_9\Gamma_f = 3(\mathcal{F}_\perp r_\perp\varepsilon_\parallel - \mathcal{F}_\parallel r_\parallel\varepsilon_\perp)$$

C_{10} and $\mathcal{F}_\lambda$ are real in SM

Define new form factors

Useful definitions

$$\mathrm{P}_1 = \frac{\mathcal{F}_\perp}{\mathcal{F}_\parallel}$$

$$\mathrm{P}_2 = \frac{\mathcal{F}_\perp}{\mathcal{F}_0}$$

$$\mathrm{P}_3 = \frac{\mathcal{F}_\perp}{\mathcal{F}_0 + \mathcal{F}_\parallel} = \frac{\mathrm{P}_1\mathrm{P}_2}{\mathrm{P}_1 + \mathrm{P}_2}$$

Observables recast (last one not independent)

- $F'_{\parallel}\Gamma_f = 2\mathcal{F}_{\parallel}^2(r_{\parallel}^2 + C_{10}^2)$
- $F'_{\perp}\Gamma_f = 2\mathcal{F}_{\perp}^2(r_{\perp}^2 + C_{10}^2)$

$$F'_{\lambda} \equiv F_{\lambda} - \frac{2\varepsilon_{\lambda}^2}{\Gamma_f}$$

- $F'_{L}\Gamma_f = 2\mathcal{F}_{0}^2(r_{0}^2 + C_{10}^2)$
- $(F'_L + F_{\parallel}^2 + \sqrt{2}\pi\, A_4)\Gamma_f = 2(\mathcal{F}_0^2 + \mathcal{F}_{\parallel}^2)(r_{\wedge}^2 + C_{10}^2)$
- $\sqrt{2}A_5\Gamma_f = 3\mathcal{F}_{\perp}\mathcal{F}_0 C_{10}(r_0 + r_{\perp})$
- $A_{FB}\Gamma_f = 3\mathcal{F}_{\perp}\mathcal{F}_{\parallel}C_{10}(r_{\parallel} + r_{\perp})$ $\longrightarrow$ $A_{FB} = 0 \Rightarrow (r_{\parallel} + r_{\perp})$
- $(A_{FB} + \sqrt{2}A_5)\Gamma_f = 3\mathcal{F}_{\perp}(\mathcal{F}_{\parallel} + \mathcal{F}_0)C_{10}\,(r_{\wedge} + r_{\perp})$

where $$r_{\wedge} = \frac{r_{\parallel}P_2 + r_0 P_1}{P_2 + P_1}$$

$$\rightarrow \begin{cases} F'_{\parallel} \to F'_L\,,\ A_{FB} \to \sqrt{2}A_5 \\ \mathcal{F}_{\parallel} \to \mathcal{F}_0 \end{cases}$$

Each set solve for $r_{\perp}, C_{10}$ *and* $(r_{||}, r_0, r_{\wedge})$

We have derived

$$r_\parallel^2 + C_{10}^2 = \frac{F_\parallel \Gamma_f}{2\mathcal{F}_\parallel^2}$$

$$r_\perp^2 + C_{10}^2 = \frac{F_\perp \Gamma_f}{2\mathcal{F}_\perp^2}$$

$$2C_{10}(r_\parallel + r_\perp) = \frac{2}{3}\frac{A_{\rm FB}\Gamma_f}{\mathcal{F}_\perp \mathcal{F}_\parallel}$$

...hence we can write

$$\frac{F_\parallel F_\perp \Gamma_f^2}{4\mathcal{F}_\parallel^2 \mathcal{F}_\perp^2} = (r_\parallel r_\perp - C_{10})^2 + C_{10}^2(r_\parallel + r_\perp)^2$$

$$= (r_\parallel r_\perp - C_{10})^2 + \frac{A_{\rm FB}^2\Gamma_f^2}{9\mathcal{F}_\parallel^2\mathcal{F}_\perp^2}$$

$$r_\parallel r_\perp - C_{10}^2 = \pm\frac{\Gamma_f}{2\mathcal{F}_\parallel\mathcal{F}_\perp}\sqrt{F_\parallel F_\perp - \frac{4A_{\rm FB}^2}{9}}$$

$$2r_\parallel r_\perp - 2C_{10}^2 = \left[(r_\parallel + r_\perp)^2 - \frac{F_\parallel\Gamma_f}{2\mathcal{F}_\parallel^2} - \frac{F_\perp\Gamma_f}{2\mathcal{F}_\perp^2}\right]$$

Or

$$\boxed{r_\parallel + r_\perp = \frac{\pm\sqrt{\Gamma_f}}{\sqrt{2}\mathcal{F}_\perp}\left[\mathsf{P}_1^2 F_\parallel + F_\perp \pm \mathsf{P}_1 Z_1\right]^{1/2}}$$

$$Z_1 = \sqrt{4F_\parallel F_\perp - \tfrac{16}{9}A_{\rm FB}^2}$$

One also obtains

$$r_\parallel^2 - r_\perp^2 = \frac{F_\parallel\Gamma_f}{2\mathcal{F}_\parallel^2} - \frac{F_\perp\Gamma_f}{2\mathcal{F}_\perp^2} \rightarrow r_\parallel - r_\perp = \frac{\pm\sqrt{\Gamma_f}}{\sqrt{2}\mathcal{F}_\perp}\frac{\mathsf{P}_1^2F_\parallel - F_\perp}{\left[\mathsf{P}_1^2F_\parallel + F_\perp \pm \mathsf{P}_1 Z_1\right]^{1/2}}$$

Solve both $r_\parallel$ and $r_\perp$

$$r_\perp = \pm\frac{\sqrt{\Gamma_f}}{\sqrt{2}\mathcal{F}_\perp}\frac{(F'_\perp + \frac{1}{2}\mathsf{P}_1 Z'_1)}{\sqrt{\mathsf{P}_1^2 F'_\parallel + F'_\perp + \mathsf{P}_1 Z'_1}} \qquad Z'_1 = \sqrt{4F'_\parallel F'_\perp - \frac{16}{9}A_{\rm FB}^2}$$

$$r_\perp = \pm\frac{\sqrt{\Gamma_f}}{\sqrt{2}\mathcal{F}_\perp}\frac{(F'_\perp + \frac{1}{2}\mathsf{P}_2 Z'_2)}{\sqrt{\mathsf{P}_2^2 F'_L + F'_\perp + \mathsf{P}_2 Z'_2}} \qquad Z'_2 = \sqrt{4F'_L F'_\perp - \frac{32}{9}A_5^2}$$

$$r_\perp = \pm\frac{\sqrt{\Gamma_f}}{\sqrt{2}\mathcal{F}_\perp}\frac{(F'_\perp + \frac{1}{2}\mathsf{P}_3 Z'_3)}{\sqrt{\mathsf{P}_3^2(F'_\parallel + F'_L + \sqrt{2}\pi A'_4) + F'_\perp + \mathsf{P}_3 Z'_3}}$$

$$Z'_3 = \sqrt{4(F'_L + F'_\parallel + \sqrt{2}\pi A'_4)F'_\perp - \frac{16}{9}(A_{\rm FB} + \sqrt{2}A_5)^2}$$

$$\boldsymbol{F'_\parallel F'_\perp \geq \frac{4}{9}A_{FB}^2}$$

or

$$\boldsymbol{F_\parallel F_\perp \geq \frac{4}{9}A_{FB}^2}$$

$$\boldsymbol{F_L F_\perp \geq \frac{8}{9}A_5^2}$$

$$\mathsf{P}_2 = \frac{2\mathsf{P}_1 A_{\rm FB} F'_\perp}{s\sqrt{2}A_5(2F'_\perp + Z'_1\mathsf{P}_1) - Z'_2\mathsf{P}_1 A_{\rm FB}}$$

$$\mathsf{P}_3 = \frac{2\mathsf{P}_1 A_{\rm FB} F'_\perp}{(A_{\rm FB} + \sqrt{2}A_5)(2F'_\perp + Z'_1\mathsf{P}_1) - Z'_3\mathsf{P}_1 A_{\rm FB}}$$

$$\Rightarrow \mathsf{P}_3 = \frac{\mathsf{P}_1\mathsf{P}_2}{\mathsf{P}_1 + \mathsf{P}_2} \Rightarrow Z'_3 = Z'_1 + Z'_2$$

$Z'_1 + Z'_2 = Z'_3$ *results in the relation*

$$A_4 = \frac{2\sqrt{2}\varepsilon_\parallel \varepsilon_0}{\pi\Gamma_f} + \frac{8A_5A_{\rm FB}}{9\pi(F_\perp - \frac{2\varepsilon_\perp^2}{\Gamma_f})} + \sqrt{2}\frac{\sqrt{(F_L - \frac{2\varepsilon_0^2}{\Gamma_f})(F_\perp - \frac{2\varepsilon_\perp^2}{\Gamma_f}) - \frac{8}{9}A_5^2}\sqrt{(F_\parallel - \frac{2\varepsilon_\parallel^2}{\Gamma_f})(F_\perp - \frac{2\varepsilon_\perp^2}{\Gamma_f}) - \frac{4}{9}A_{\rm FB}^2}}{\pi(F_\perp - \frac{2\varepsilon_\perp^2}{\Gamma_f})}$$

ε_λ can easily be solved in terms of A_7, A_8, A_9

$$\varepsilon_\perp = \frac{\sqrt{2}\pi\Gamma_f}{(r_0 - r_\parallel)\mathcal{F}_\perp}\left[\frac{A_9\mathsf{P}_1}{3\sqrt{2}} + \frac{A_8\mathsf{P}_2}{4} - \frac{A_7\mathsf{P}_1\mathsf{P}_2 r_\perp}{3\pi\hat{C}_{10}}\right]$$

$$\varepsilon_\parallel = \frac{\sqrt{2}\pi\Gamma_f}{(r_0 - r_\parallel)\mathcal{F}_\perp}\left[\frac{A_9 r_0}{3\sqrt{2}r_\perp} + \frac{A_8\mathsf{P}_2 r_\parallel}{4\mathsf{P}_1 r_\perp} - \frac{A_7\mathsf{P}_2 r_\parallel}{3\pi\hat{C}_{10}}\right]$$

$$\varepsilon_0 = \frac{\sqrt{2}\pi\Gamma_f}{(r_0 - r_\parallel)\mathcal{F}_\perp}\left[\frac{A_9\mathsf{P}_1 r_0}{3\sqrt{2}\mathsf{P}_2 r_\perp} + \frac{A_8 r_\parallel}{4r_\perp} - \frac{A_7\mathsf{P}_1 r_0}{3\pi\hat{C}_{10}}\right]$$

Note $\frac{\varepsilon_\lambda}{\sqrt{\Gamma_f}}$ free from the form factor $\mathcal{F}_\lambda$ and Γ_f. Solutions in terms of observables and form factor ratio P_1. However, solutions are iterative.

Solutions for $\frac{\varepsilon_\lambda}{\sqrt{\Gamma_f}}$ with 1σ errors from 8 bin LHCb data

q^2 range in GeV^2	$\varepsilon_\perp/\sqrt{\Gamma_f}$	$\varepsilon_\parallel/\sqrt{\Gamma_f}$	$\varepsilon_0/\sqrt{\Gamma_f}$
$0.1 \le q^2 \le 0.98$	-0.048 ± 0.116	-0.047 ± 0.103	0.020 ± 0.111
$1.1 \le q^2 \le 2.5$	-0.010 ± 0.078	-0.010 ± 0.078	0.078 ± 0.172
$2.5 \le q^2 \le 4.0$	-0.009 ± 0.079	-0.008 ± 0.080	-0.025 ± 0.212
$4.0 \le q^2 \le 6.0$	-0.026 ± 0.097	0.014 ± 0.093	0.032 ± 0.234
$6.0 \le q^2 \le 8.0$	-0.011 ± 0.088	-0.046 ± 0.078	-0.132 ± 0.129
$11.0 \le q^2 \le 12.5$	-0.011 ± 0.050	0.038 ± 0.074	-0.078 ± 0.114
$15.0 \le q^2 \le 17.0$	-0.0003 ± 0.067	-0.027 ± 0.071	0.020 ± 0.072
$17.0 \le q^2 \le 19.0$	0.006 ± 0.076	-0.090 ± 0.090	-0.040 ± 0.088

Equation $Z_1' + Z_2' = Z_3'$ can be used to solve for the observables A_4, A_5, A_{FB}, F_L and $F_\perp$ involved in the relation

$$A_4 = \frac{8\,A_5 A_{FB}}{9\,\pi\,F_\perp} + \frac{Z_1\,Z_2}{2\sqrt{2}\pi\,F_\perp}$$

$$A_5 = \frac{\pi\,A_4\,A_{FB}}{2\,F_{||}} \pm \frac{3Z_1}{8F_{||}}\sqrt{2F_{||}F_L - \pi^2 A_4^2}$$

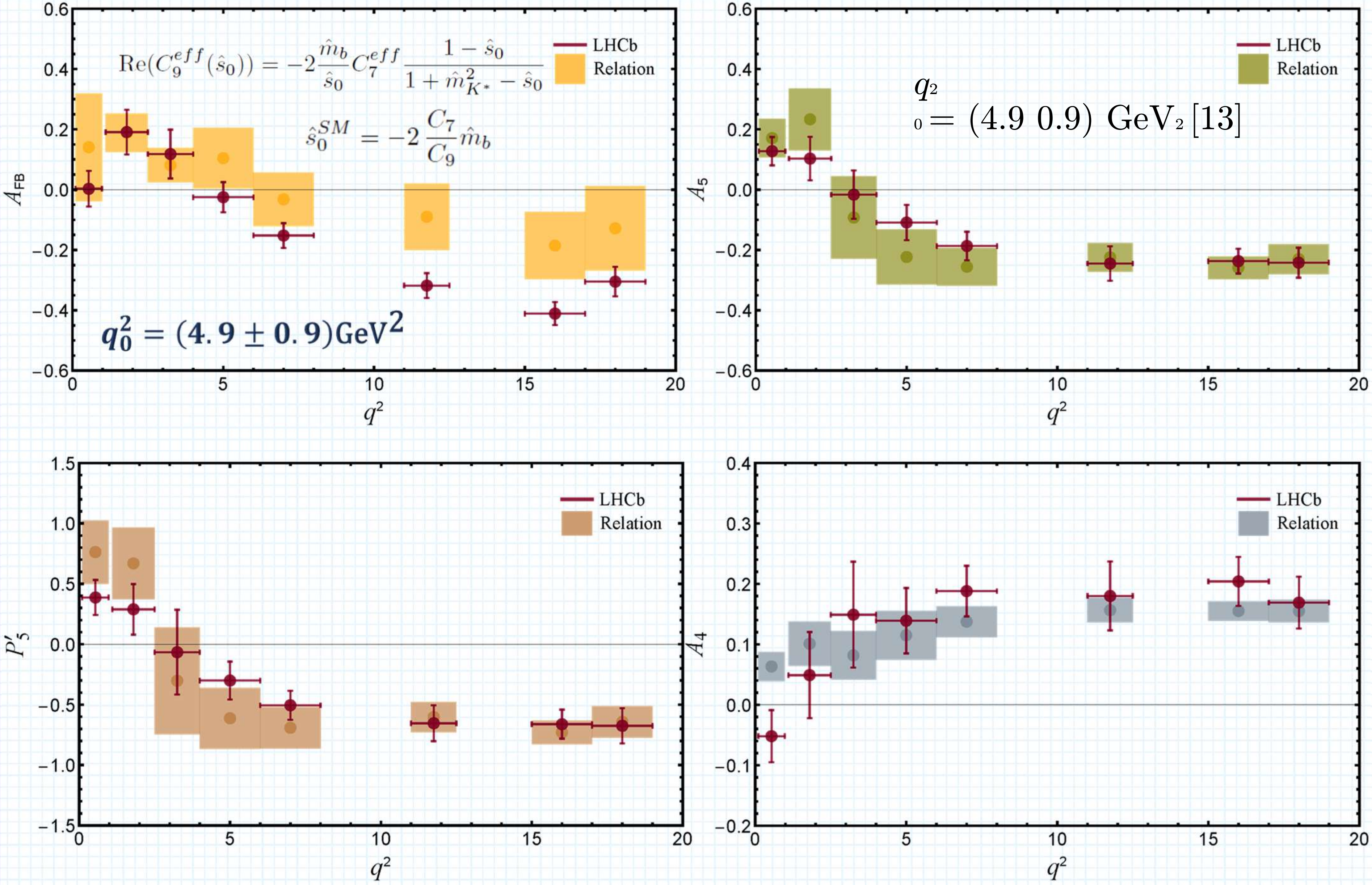
$$A_{FB} = \frac{\pi\,A_4\,A_5}{2\,F_L} \pm \frac{3Z_2}{4\sqrt{2}F_L}\sqrt{2F_{||}F_L - \pi^2 A_4^2}$$

$$\vdots$$

Remember these are one and the same equation. However, sensitivity varies depending on form…

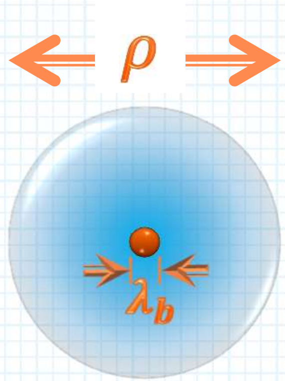
At the zero crossing of A_{FB} one can measure form factor

$$r_{\|} + r_\perp\Big|_{A_{\rm FB}=0} = \pm\frac{\sqrt{\Gamma_f}}{\sqrt{2}\mathcal{F}_\perp}\left(\sqrt{F_\perp} \pm \mathsf{P}_1\sqrt{F_{\|}}\right) = 0 \Rightarrow \boxed{\mathsf{P}_1 = -\frac{\sqrt{F_\perp}}{\sqrt{F_{\|}}}\Big|_{A_{\rm FB}=0}}$$


$\mathrm{Re}(C_9^{eff}(\hat{s}_0)) = -2\frac{\hat{m}_b}{\hat{s}_0}C_7^{eff}\frac{1-\hat{s}_0}{1+\hat{m}_{K^*}^2-\hat{s}_0}$
$\hat{s}_0^{SM} = -2\frac{C_7}{C_9}\hat{m}_b$
$q_0^2 = (4.9 \pm 0.9)\mathrm{GeV}^2$
$q_0^2 = (4.9\ 0.9)$ GeV^2 [13]
LHCb
Relation
A_{FB}
A_5
P'_5
A_4
q^2

Heavy Quark Effective Theory

Flavor



Heavy quark system $M_b \gg \Lambda_{QCD}$

Compton wave length

$$\lambda_b \sim \frac{1}{M_b} \ll \frac{1}{\Lambda_{\mathrm{QCD}}} \sim \rho_{hadron}$$

Spin

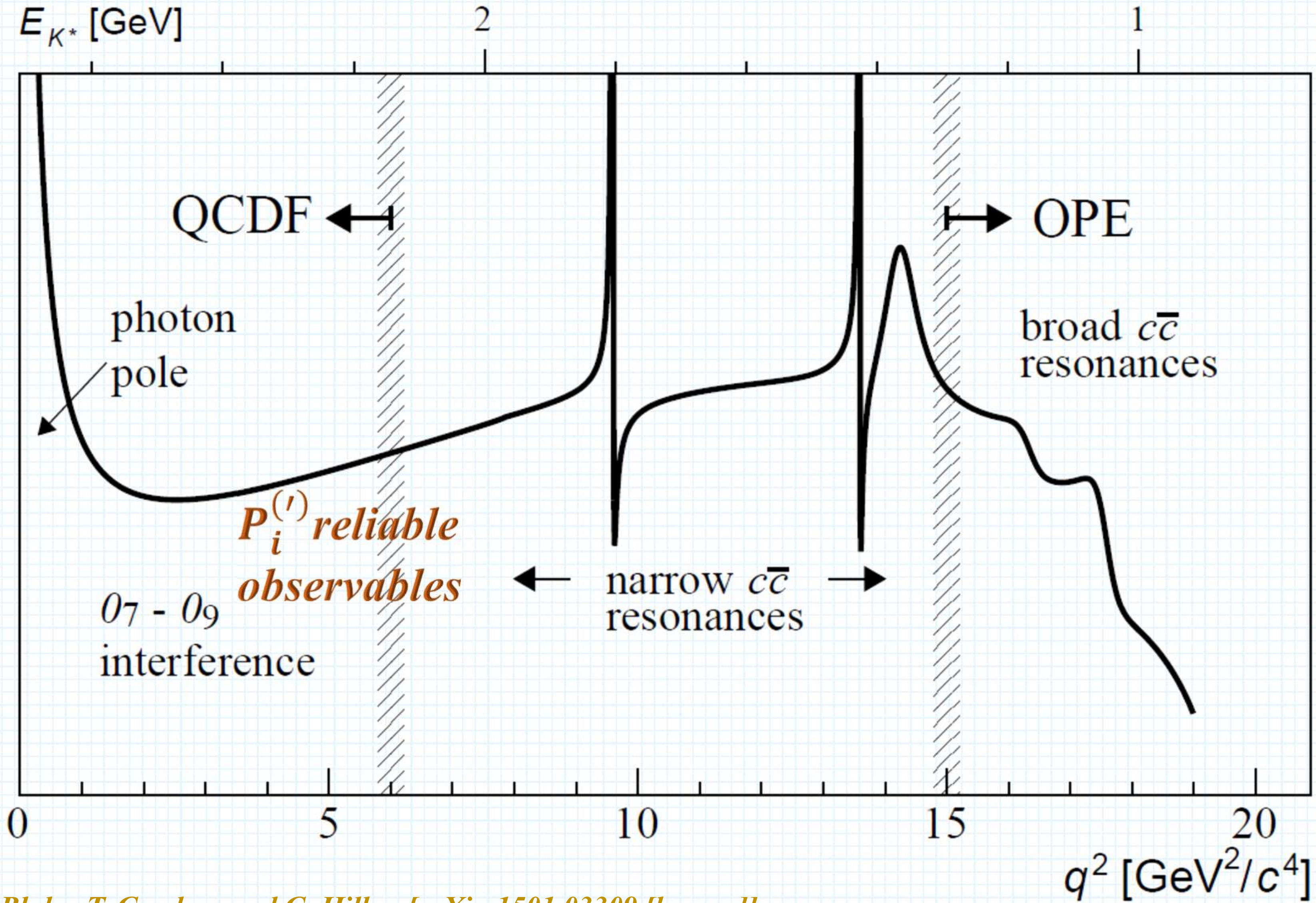
Magnetic moment $\mu_b \sim \frac{1}{M_b} \Rightarrow$ *spin decouples*

In the heavy quark limit, $M_b \to \infty$, configration of light degrees of freedom is independent of the spin and flavor of the heavy quark.

- *Heavy particle carries all the momentum*
- *Momentum exchange between heavy quark and light degrees of freedom is predominantly soft*
- *Heavy quark velocity becomes conserved quantum number*

Rigorous QCD $\Rightarrow$ HQET symmetries in heavy quark limit.

Approaches to estimate hadronic…



T. Blake, T. Gershon and G. Hiller, [arXiv:1501.03309 [hep-ex]].

The seven form factors V, A_0, A_1, A_2, T_1, T_2 and T_3 are calculated via non-perturbative methods like QCD sum rules on the light cone (LCSRs) when K^* energies are large.

P. Ball and R. Zwicky, Phys. Rev. D 71, 014029 (2005) [hep-ph/0412079].

In QCD factorization (QCDF) framework, and within heavy quark and large recoil limit, all seven form factors can be written in terms of only two independent universal factors, namely, $\xi_{\parallel}$ and $\xi_{\parallel}$.

J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D 60, 014001 (1999) [hep-ph/9812358];
M. Beneke and T. Feldmann, Nucl. Phys. B 592, 3 (2001) [hep-ph/0008255];
M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. B 612, 25 (2001) [hep-ph/0106067].
M. Beneke, T. Feldmann and D. Seidel, Eur. Phys. J. C 41, 173 (2005) [hep-ph/0412400].

At leading order in $\frac{1}{m_b}$ and α_s the transversity amplitudes become:

$$A_{\perp}^{L,R} = \sqrt{2} N m_B (1-\hat{s}) \left[(C_9^{eff} + C_9^{'eff}) \mp (C_{10} + C_{10}^{'}) + 2\frac{\hat{m}_b}{\hat{s}} (C_7^{eff} + C_7^{'eff}) \right] \xi_{\perp}(E_{K^*}),$$

$$A_{\parallel}^{L,R} = -\sqrt{2} N m_B (1-\hat{s}) \left[(C_9^{eff} - C_9^{'eff}) \mp (C_{10} - C_{10}^{'}) + 2\frac{\hat{m}_b}{\hat{s}} (C_7^{eff} - C_7^{'eff}) \right] \xi_{\perp}(E_{K^*}),$$

$$A_{0}^{L,R} = -\frac{N m_b}{2\hat{m}_{K^*}\sqrt{\hat{s}}} (1-\hat{s})^2 \left[(C_9^{eff} - C_9^{'eff}) \mp (C_{10} - C_{10}^{'}) + 2\hat{m}_b (C_7^{eff} - C_7^{'eff}) \right] \xi_{\parallel}(E_{K^*}),$$

$$A_t = \frac{N m_b}{\hat{m}_{K^*}\sqrt{\hat{s}}} (1-\hat{s})^2 \left[C_{10} - C_{10}^{'} \right] \xi_{\parallel}(E_{K^*})$$

Large recoil limit

LEET

$$\mathrm{P}_1 = \frac{\mathcal{F}_\perp}{\mathcal{F}_\parallel} = \frac{\sqrt{\lambda(m_B^2, m_{K^*}^2, q^2)}}{(m_B + m_{K^*})^2} \frac{V(q^2)}{A_1(q^2)} = \frac{-\sqrt{\lambda(m_B^2, m_{K^*}^2, q^2)}}{2\, E_{K^*} m_B}$$

$$\frac{V(q^2)}{A_1(q^2)} = \frac{(m_B + m_{K^*})^2}{2E_{K^*} m_B}$$

The transverse helicity amplitudes $H_\pm \propto \left(V \mp \frac{(m_B + m_{K^*})^2}{2 m_B k_{K^*}} A_1 \right)$,

where k_{K^*} *is the momentum of the* K^**. In Large Energy Limit the "+" helicity vanishes up to* $\frac{m_{K^*}^2}{2\, E_{K^*}^2}$. $E_{K^*} = (m_B^2 + m_{K^*}^2 - q^2)/(2m_B)$.

In the HQL the decay of heavy to light quark occurs with helicity of latter inherited by the final vector meson. On the other hand the amplitude to flip the helicity of the fast outgoing light quark is suppressed by $1/E_{K^*}$ *.Thus* α_s *corrections from hard gluon exchange between spectator quark and the fast light quark do not affect the ratio* $\frac{V(q^2)}{A_1(q^2)}$

M. Beneke and T. Feldmann, Nucl. Phys. B 592, 3 (2001) [hep-ph/0008255]; G. Burdman and G. Hiller Phys. Rev. D 2000

P_1 *is a good observable at large recoil limit unaffected by* α_s *order corrections - up to leading order in power corrections*

Right handed currents

In the presence of right-handed currents $\mathcal{A}_\lambda^{L,R} = (\widetilde{\boldsymbol{C}}_9^\lambda \mp \boldsymbol{C}_{10})\boldsymbol{\mathcal{F}}_\lambda - \widetilde{\boldsymbol{\mathcal{G}}}_\lambda$ ***becomes:***

$$\mathcal{A}_\perp^{L,R} = \left((\widetilde{\boldsymbol{C}}_9^\perp + \boldsymbol{C}_9') \mp (\boldsymbol{C}_{10} + \boldsymbol{C}_{10}')\right)\boldsymbol{\mathcal{F}}_\perp - \widetilde{\boldsymbol{\mathcal{G}}}_\perp \qquad \xi = \frac{C_{10}'}{C_{10}}$$

$$\mathcal{A}_{\parallel,0}^{L,R} = \left((\widetilde{\boldsymbol{C}}_9^\parallel - \boldsymbol{C}_9') \mp (\boldsymbol{C}_{10} - \boldsymbol{C}_{10}')\right)\boldsymbol{\mathcal{F}}_{\parallel,0} - \widetilde{\boldsymbol{\mathcal{G}}}_{\parallel,0} \qquad \xi' = \frac{C_9'}{C_{10}}$$

$$F_\perp = 2\,\zeta\,(1+\xi)^2(1+R_\perp^2)$$

$$F_\parallel \mathrm{P}_1^2 = 2\,\zeta(1-\xi)^2(1+R_\parallel^2)$$

$$F_L \mathrm{P}_2^2 = 2\,\zeta(1-\xi)^2(1+R_0^2)$$

$$A_{FB}\mathrm{P}_1 = 3\zeta(1-\xi^2)(R_\parallel + R_\perp)$$

$$\sqrt{2}A_5\mathrm{P}_2 = 3\zeta(1-\xi^2)(R_0 + R_\perp)$$

$$\zeta = \frac{\mathcal{F}_\perp^2 C_{10}^2}{\Gamma_f}$$

$$R_\perp = \frac{r_\perp/C_{10} - \xi'}{1+\xi}$$

$$R_\parallel = \frac{r_\parallel/C_{10} + \xi'}{1-\xi}$$

$$R_0 = \frac{r_0/C_{10} + \xi'}{1-\xi}$$

$$R_\perp = \pm\frac{3}{2}\frac{\left(\frac{1-\xi}{1+\xi}\right)F_\perp + \frac{1}{2}\mathrm{P}_1 Z_1}{\mathrm{P}_1 A_{FB}}$$

$$R_\parallel = \pm\frac{3}{2}\frac{\left(\frac{1+\xi}{1-\xi}\right)\mathrm{P}_1 F_\parallel + \frac{1}{2}Z_1}{A_{FB}}$$

$$R_0 = \pm\frac{3}{2\sqrt{2}}\frac{\left(\frac{1+\xi}{1-\xi}\right)\mathrm{P}_2 F_L + \frac{1}{2}Z_2}{A_5}$$

4 independent observables to solve for 4 parameters

For the moment we assume that the amplitudes are real. Simplicity of expressions. Non-zero imaginary part have also be included.

$$\mathrm{P}_2 = \frac{\left(\frac{1-\xi}{1+\xi}\right)2\mathrm{P}_1 A_{FB}\, F_\perp}{\sqrt{2}A_5\left(\left(\frac{1-\xi}{1+\xi}\right)F_\perp + Z_1\,\mathrm{P}_1\right) - Z_2\mathrm{P}_1 A_{FB}}$$

$$\left(\frac{1+\xi}{1-\xi}\right)\mathrm{P}_1 \to \mathrm{P}_1$$

$$Z_1 = \sqrt{4F_\parallel\, F_\perp - \frac{16}{9}A_{FB}^2} \qquad Z_2 = \sqrt{4F_L\, F_\perp - \frac{32}{9}A_5^2}$$

One extra parameter hence expressions depend on P_1

At $q^2 = q^2_{\max} = (m_B - m_{K^*})^2$ *the* K^* *meson is at rest and the two leptons travel back to back in the* B *meson rest frame. There is no preferred direction in the decay kinematics. Hence, the differential decay distribution in this kinematic limit must be independent of the angles* θ_ℓ *and* ϕ.

- *The entire decay, including the decay* $K^* \to K\pi$ *takes place in a plane resulting in a vanishing contribution to the "⊥" helicity or* $F_\perp = 0$.
- *Since the* K^* *decays at rest, the distribution of* K^* *is isotropic and cannot depend on* θ_K. *It can easily be seen that this is only possible if* $F_\parallel = 2F_L$.

At $q^2 = q^2_{\max}$, $\Gamma_f \to 0$ *as all the transversity amplitudes vanish in this limit. The constraints on the amplitudes result in unique values of the helicity fractions and the asymmetries at this kinematical end-point.*

$$F_L(q^2_{\max}) = \frac{1}{3} \quad F_\parallel(q^2_{\max}) = \frac{2}{3} \quad F_\perp(q^2_{\max}) = 0$$

Hiller, Ziwcky '14

$$A_{FB}(q^2_{\max}) = 0 = \mathrm{A}_{5,7,8,9}(q^2_{\max}) \quad \mathrm{A}_4(q^2_{\max}) = \frac{2}{3\pi}$$

The large q^2 region where the K^* has low-recoil energy has been studied in a modified heavy quark effective theory framework. In the limit $q^2 \to q^2_{\max}$ the hadronic form factors satisfy the conditions

Grinstein, Prijol '04
C. Bobeth, G. Hiller and D. van Dyk, '13

$$\frac{\tilde{\mathcal{G}}_\perp}{\mathcal{F}_\perp} = \frac{\tilde{\mathcal{G}}_\parallel}{\mathcal{F}_\parallel} = \frac{\tilde{\mathcal{G}}_0}{\mathcal{F}_0} = -\kappa\,\frac{2\,m_b\,m_B\,C_7}{q^2} \Rightarrow r_\perp = r_\parallel = r_0 \equiv r$$

Thus only in the presence of right handed currents can one expect

$$R_0 = R_\parallel \neq R_\perp$$

We study the values of R_λ, ζ *and* $\mathrm{P}_{1,2}$ *in the large* q^2 *region and consider the kinematic limit* $q^2 \to \boldsymbol{q^2_{\max}}$.

$$F_\perp(q^2_{\max}) = 0 \;\Rightarrow \zeta = 0 \;\textit{at}\; \boldsymbol{q^2} \to \boldsymbol{q^2_{\max}}$$

$$R_\parallel(\boldsymbol{q^2_{\max}}) = R_0(\boldsymbol{q^2_{\max}}) \;\Rightarrow \mathrm{P}_2 = \sqrt{2}\,\mathrm{P}_1 \;\textit{at}\; \boldsymbol{q^2} \to \boldsymbol{q^2_{\max}}$$

Both P_1 *and* P_2 *go to zero at* $q^2_{\max}$. *Hence take into account limiting values very carefully.*

Taylor expand all observables around the endpoint $\mathbf{q^2_{max}}$ in terms of the variable $\delta \equiv \mathbf{q^2_{max}} - q^2$. Leading power of δ in the Taylor expansion must take into account relative momentum dependence of amplitudes $\mathcal{A}_\lambda^{L,R}$

$$F_L = \frac{1}{3} + F_L^{(1)}\delta + F_L^{(2)}\delta^2 + F_L^{(3)}\delta^3$$

$$F_\perp = F_\perp^{(1)}\delta + F_\perp^{(2)}\delta^2 + F_\perp^{(3)}\delta^3$$

$$A_{FB} = A_{FB}^{(1)}\delta^{1/2} + A_{FB}^{(2)}\delta^{3/2} + A_{FB}^{(3)}\delta^{5/2}$$

$$A_5 = A_5^{(1)}\delta^{1/2} + A_5^{(2)}\delta^{3/2} + A_5^{(3)}\delta^{5/2}$$

Unfortunately, very bad approximation in the strict sense. However, works reasonably well. Resonances cannot be accommodated in a Taylor expansion and there exist resonances. Experimental binned measurements include resonance contributions. We calculate these errors as systematics.

Thank Marcin, Nicola, Danny, Gino… for discussions on this

Compare form-factor generated binned data without resonances with similar data generated using resonances observed in B $\rightarrow K\ell\ell$. Discrepancy will be a rough guide to errors because of resonances. Full study under way.

Taylor expansion of form factors:

$$q^2\,\frac{\tilde{\mathcal{G}}_\lambda}{\mathcal{F}_\lambda} = q^2_{\rm max}\,\frac{\tilde{\mathcal{G}}^{(1)}_\lambda + \delta\left(\tilde{\mathcal{G}}^{(2)}_\lambda - \dfrac{\tilde{\mathcal{G}}^{(1)}_\lambda}{q^2_{\rm max}}\right) + \mathcal{O}(\delta^2)}{\mathcal{F}^{(1)}_\lambda + \delta\mathcal{F}^{(2)}_\lambda + \mathcal{O}(\delta^2)}$$

Assume that relation is valid up to order δ

$$\Rightarrow\ \mathcal{F}^{(1)}_\lambda = c\,\mathcal{F}^{(2)}_\lambda \ \textit{and}$$

$$\left(q^2_{\rm max}\ \mathcal{G}^{(2)}_\lambda - \mathcal{G}^{(1)}_\lambda\right) = c\, q^2_{\rm max}\ \mathcal{G}^{(1)}_\lambda$$

$$\Rightarrow \mathrm{P}^{(1)}_2 = \sqrt{2}\,\mathrm{P}^{(1)}_1 \text{ and } \mathrm{P}^{(2)}_2 = \sqrt{2}\,\mathrm{P}^{(2)}_1$$

The expressions for R_λ in the limit $q^2 \to q^2_{\rm max}$ are

$$
\begin{aligned}
R_\perp(q^2_{\rm max}) &= \frac{8A_{\rm FB}^{(1)}(-2A_5^{(2)} + A_{\rm FB}^{(2)}) + 9(3F_L^{(1)} + F_\perp^{(1)})F_\perp^{(1)}}{8\,(2A_5^{(2)} - A_{\rm FB}^{(2)})\,\sqrt{\frac{3}{2}F_\perp^{(1)} - A_{\rm FB}^{(1)\,2}}} \\
&= \frac{\omega_2 - \omega_1}{\omega_2\sqrt{\omega_1 - 1}}, \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
R_\|(q^2_{\rm max}) &= \frac{3(3F_L^{(1)} + F_\perp^{(1)})\,\sqrt{\frac{3}{2}F_\perp^{(1)} - A_{\rm FB}^{(1)\,2}}}{-8A_5^{(2)} + 4A_{\rm FB}^{(1)} + 3A_{\rm FB}^{(1)}(3F_L^{(1)} + F_\perp^{(1)})} \\
&= \frac{\sqrt{\omega_1 - 1}}{\omega_2 - 1} = R_0(q^2_{\rm max}) \qquad (31)
\end{aligned}
$$

$$
\omega_1 = \frac{3}{2}\frac{F_\perp^{(1)}}{A_{\rm FB}^{(1)\,2}} \quad \text{and} \quad \omega_2 = \frac{4\,(2A_5^{(2)} - A_{\rm FB}^{(2)})}{3\,A_{\rm FB}^{(1)}(3F_L^{(1)} + F_\perp^{(1)})}.
$$

Including the imaginary part

$$\varepsilon_\perp = \frac{\sqrt{2}\pi\Gamma_f}{(r_0 - r_\parallel)\mathcal{F}_\perp}\left[\frac{A_9\mathsf{P}_1}{3\sqrt{2}} + \frac{A_8\mathsf{P}_2}{4} - \frac{A_7\mathsf{P}_1\mathsf{P}_2 r_\perp}{3\pi\hat{C}_{10}}\right]$$

$$\varepsilon_\parallel = \frac{\sqrt{2}\pi\Gamma_f}{(r_0 - r_\parallel)\mathcal{F}_\perp}\left[\frac{A_9 r_0}{3\sqrt{2}r_\perp} + \frac{A_8\mathsf{P}_2 r_\parallel}{4\mathsf{P}_1 r_\perp} - \frac{A_7\mathsf{P}_2 r_\parallel}{3\pi\hat{C}_{10}}\right]$$

$$\varepsilon_0 = \frac{\sqrt{2}\pi\Gamma_f}{(r_0 - r_\parallel)\mathcal{F}_\perp}\left[\frac{A_9\mathsf{P}_1 r_0}{3\sqrt{2}\mathsf{P}_2 r_\perp} + \frac{A_8 r_\parallel}{4 r_\perp} - \frac{A_7\mathsf{P}_1 r_0}{3\pi\hat{C}_{10}}\right]$$

ε_λ can easily be solved in terms of A_7, A_8, A_9. Note $\frac{\varepsilon_\lambda}{\sqrt{\Gamma_f}}$ free from the form factor $\mathcal{F}_\lambda$ and Γ_f.

It leads to a modification of the expressions for ω_1 and ω_2

$$\widehat{\varepsilon}_\perp = \widehat{\varepsilon}_\perp^{(1)}\delta + \widehat{\varepsilon}_\perp^{(2)}\delta^2 + \widehat{\varepsilon}_\perp^{(3)}\delta^3$$

$$\widehat{\varepsilon}_0 = \widehat{\varepsilon}_0^{(0)} + \widehat{\varepsilon}_0^{(1)}\delta + \widehat{\varepsilon}_0^{(2)}\delta^2$$

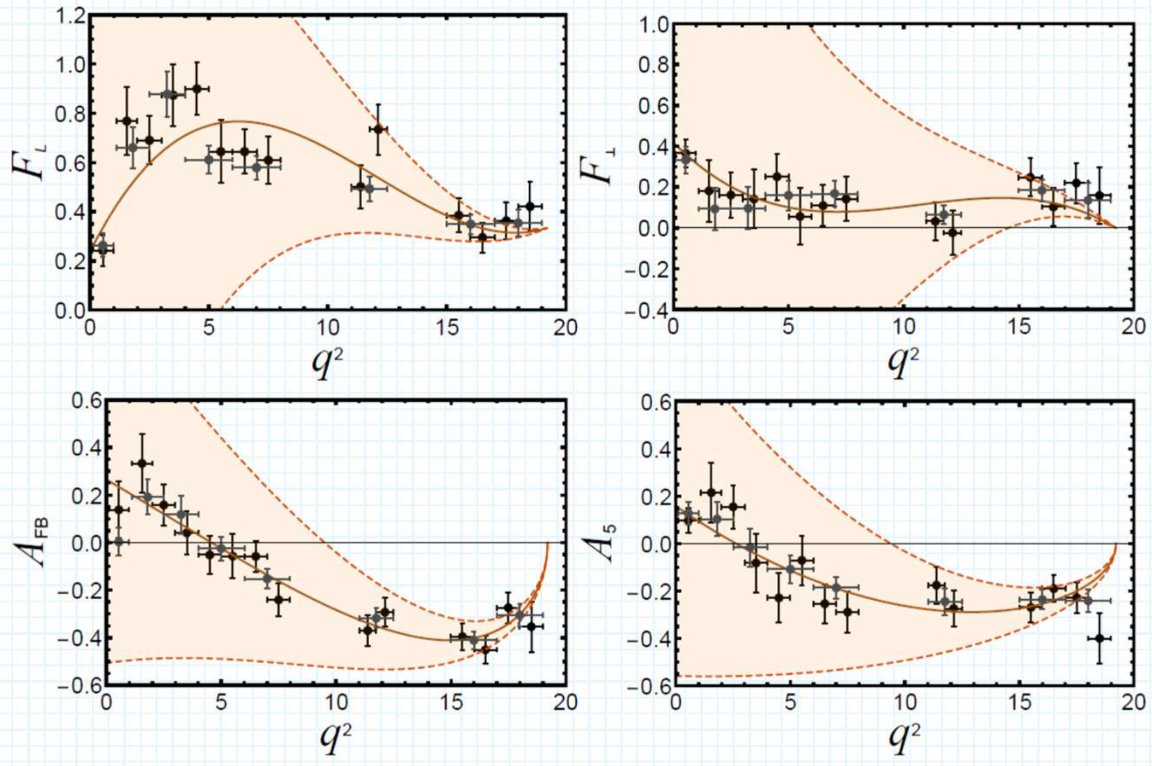
$$\widehat{\varepsilon}_\parallel = \widehat{\varepsilon}_\parallel^{(0)} + \widehat{\varepsilon}_\parallel^{(1)}\delta + \widehat{\varepsilon}_\parallel^{(2)}\delta^2$$

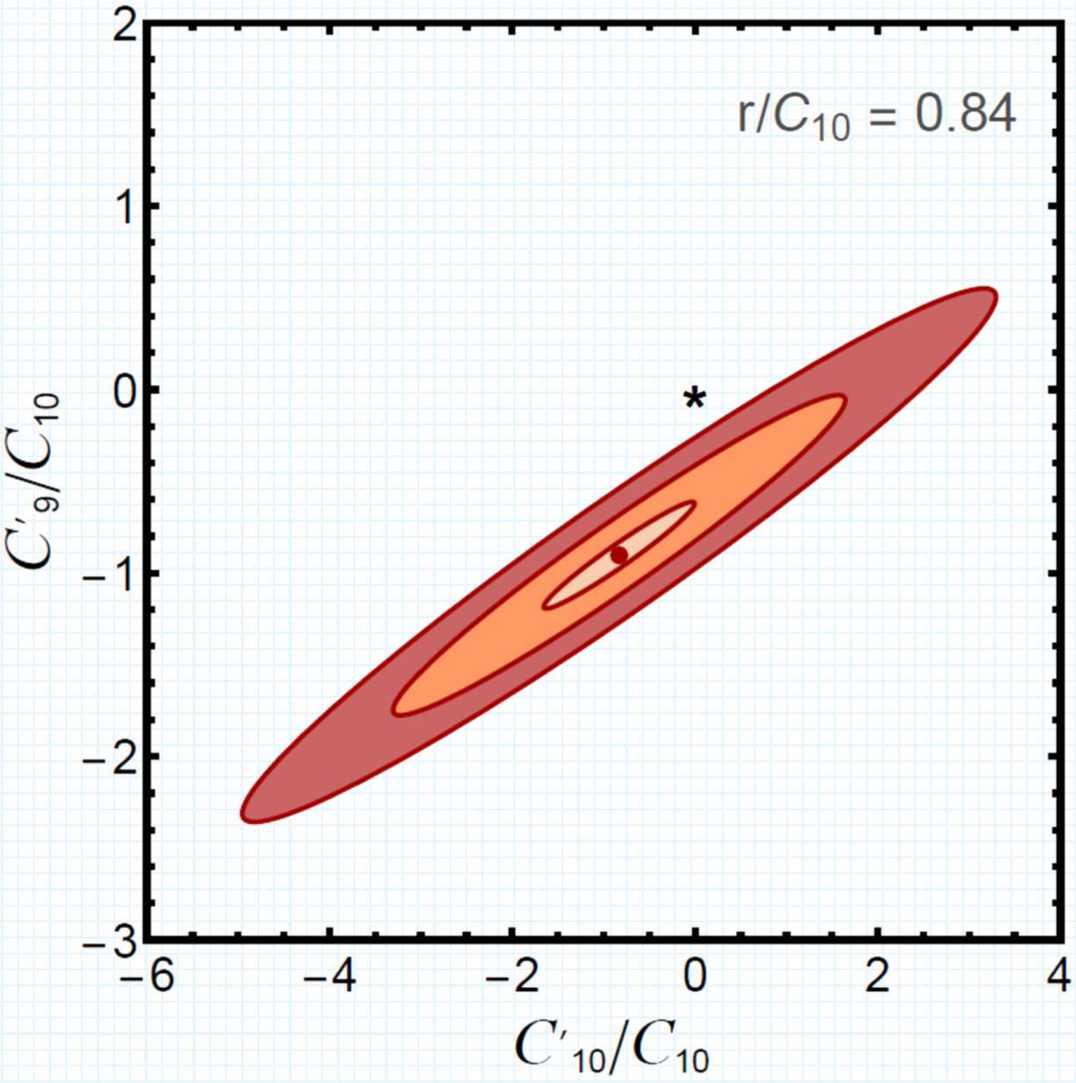
$$\hat{\epsilon}_\lambda \equiv 2\frac{\epsilon_\lambda^2}{\Gamma_f} \qquad \hat{\epsilon}_\parallel^{(0)} = 2\hat{\epsilon}_0^{(0)}$$

$$\omega_1 = \frac{9}{4}\frac{\left(\frac{2}{3} - 2\,\widehat{\varepsilon}_0^{(0)}\right)\left(F_\perp^{(1)} - \widehat{\varepsilon}_\perp^{(1)}\right)}{A_{\mathrm{FB}}^{(1)\,2}}$$

$$\omega_2 = \frac{4\left(2A_5^{(2)} - A_{\mathrm{FB}}^{(2)}\right)\left(1 - 3\,\widehat{\varepsilon}_0^{(0)}\right)}{3\,A_{\mathrm{FB}}^{(1)}\left(3F_L^{(1)} + F_\perp^{(1)} + \widehat{\varepsilon}_\parallel^{(1)} - 2\,\widehat{\varepsilon}_0^{(1)}\right)}$$

	$O^{(1)}(10^{-2})$	$O^{(2)}(10^{-3})$	$O^{(3)}(10^{-4})$
F_L	-2.94 ± 1.36	12.27 ± 2.05	-5.73 ± 0.72
$F_\perp$	6.83 ± 1.75	-9.67 ± 2.59	3.77 ± 0.90
$A_{\rm FB}$	-30.59 ± 2.37	26.75 ± 4.42	-4.00 ± 1.83
A_5	-16.57 ± 2.36	6.77 ± 4.18	1.94 ± 1.61





r/C10 = 0.84
C'9/C10
C'10/C10
2
1
0
−1
−2
−3
−6
−4
−2
0
2
4

Conclusions

1. ***The $B \to K^* \ell^+ \ell^-$ is an excellent mode to study. Sensitive to NP and hints seen.***
2. ***Theoretical issues are well understood. Resonances effects are still remain to be understood.***
3. ***Non-local contributions are difficult to estimate but can be handled by eliminating them in terms of observables.***
4. ***More effort should be put to estimate them. One should look for experimental hints to estimate how large they are.***

THANK YOU