Spectroscopy of heavy-light mesons

N. B. Devlani

Applied Physics Department
The Maharaja Sayajirao University of Baroda, Vadodara, INDIA

Outline

Introduction

Theoretical status

There exist many approaches to study the quark-antiquark dynamics e.g.

- Effective field theories
- Heavy quark effective field theory(HQET)
- Lattice QCD

Phenomelogical approaches such as;

- Bag models
- QCD Sum rules
- Potential models

Potential models

- Use of non-relativistic potential models is justified by the fact that the bottom and, to a lesser extent, the charm masses are large in comparison to Λ_{QCD}, the typical hadronic scale.
- A quantum mechanical description of the system based on two heavy quarks interacting through a suitable potential appears reasonable.
- The potential is usually chosen in a way that at short distances it coincides with the weak coupling QCD one-gluon exchange Coulomb potential and in the long range it incorporates confinement.
- Though there exist many potential models with relativistic and nonrelativistic considerations employed to study the hadron properties based on its quark structure, the most commonly used potential is the coulomb plus linear potential, $V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + Ar$, with the string tension A. Such a form is supported by lattice QCD calculations[1, 2].

Various potential models

Table 1.1: Various interquark potentials.

Cornell potential[3]	$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + Ar$
Quigg-Rosner(Logarithmic) [4]	$V(r) = A\ln(r/r_0)$
ERHM[5]	$V(r) = \frac{1}{2}(1+\gamma_0)A^2r^2 + B$
Martin[6]	$V(r) = A + Br^{0.1}$
Buchmuller and Tye[7]	$V(r) = -\frac{4}{3}\frac{1}{(2\pi)^3}\int d^3q\exp(i\mathbf{q}\cdot\mathbf{x})\frac{4\pi\alpha_s(\mathbf{q}^2)}{\mathbf{q}^2}$
Falkensteiner[8]	$V(r) = -\frac{4}{3}\frac{\alpha_s}{r}\mathrm{erf}(\sqrt{\pi}Ar) + Ar$

Motivation

- Heavy mesons which are regarded as bound states of a heavy quark and an anti-quark are essentially non-relativistic. Therefore, a potential model based approach has been quite successful in estimating various properties of these mesons.
- However, mesons containing light quarks do not fall under the scope of non-relativistic quantum mechanics.
- Recently, mesons containing one light and another heavy quark/anti-quark have created a lot of interest, particularly the D, D_S, B and B_S mesons. These systems may be considered to be semi-relativistic[9, 10].

Motivation

- Recent experimental measurements of the P-wave masses of D and D_S mesons, by many experimental facilities worldwide such as FOCUS(E831) at Fermilab, BELLE and CLEO[11, 12, 13], it is found that most of these theoretical predictions do not agree fully with the available experimental data[14, 15, 16, 17, 18, 19, 20, 21]
- BaBar collaboration in 2009 observed several new excited candidate states for the D and D_S meson.
- In the cases of B and B_S mesons; apart from well established states two $\mathrm{L} = 1$ candidate states were observed in the nineties
- the possible spin-parity quantum numbers of these open charm and beauty states could be as listed in the Table-1.2[22].

Motivation

Table 1.2: Possible J^P of the open charm and beauty states.

State	observed channel	possible J^P
$D_{s1}(2710)$	$DK,\ D^\star K$	1^-
$D_{sJ}(2860)$	$DK,\ D^\star K$	$1^-,\ 3^-, \ldots$
$D_{sJ}(3040)$	$D^\star K$	$0^-,\ 1^+,\ 2^-, \ldots$
$D(2550)$	$D^*\pi$	$0^-, 1^-, 2^-, \ldots$
$D(2600)$	$D\pi, D^*\pi$	$1^-, 3^-, \ldots$
$D(2750)$	$D^*\pi$	$0^-, 1^-, 2^-, 3^-, \ldots$
$D(2760)$	$D\pi$	$0^+, 1^-, 2^+, 3^-, \ldots$
$B_1(5721)^0$	$B^{\star +}\pi^-$	1^+
$B_J^\star(5732)$	$B\pi,\ B^\star\pi$	0^+
$B_2^\star(5747)$	$B^+\pi^-$	2^+
$B_{s1}(5830)^0$	$B^{\star +}K^-$	1^+
$B_{s2}^\star(5840)^0$	B^+K^-	2^+

Motivation

- We would like to test the applicability of a potential model, (Coulomb plus power), to heavy-light flavoured mesons.
- We would like to study how far we can extend this formulation by including the kinematic relativistic corrections within the Hamiltonian of these systems.
- Such a study will be useful to know the applicability of the potential model to heavy-light systems as these system may be considered to be semi-relativistic.
- Moreover, in the limit that the heavy quark mass becomes infinite, the heavy-light meson behaves analogously to the hydrogen atom i.e., the heavier quark does not contribute to the orbital degrees of freedom and the properties of the meson are determined by those of the light quark[23, 11].
- Therefore it will be useful to make a comparative study of the system with a hydrogen like as well as a gaussian wave function as it is known that the hydrogen like wave function is more suitable to the coulombic regime while the gaussian wave function is more suitable to the linear confining region.

Objectives

- To extend the potential model approach to heavy-light mesons by including the kinematic relativistic corrections to the Hamiltonia of these systems with the hope of improving the theoretical predictions.
- To compare the properties of heavy-light flavoured mesons obtained using variational approach with hydrogen-like and gaussian wave functions in order to quantify the role played by the confining interaction within the present scheme.

Mass spectra of heavy-light flavoured mesons

Mass spectra

- The hamiltonian employed is

$$H = \sqrt{\mathbf{p}^2 + m_Q^2} + \sqrt{\mathbf{p}^2 + m_{\bar{q}}^2} + V(\mathbf{r}) \tag{1}$$

- We expand the kinetic energy(K.E.) part of the Hamiltonian up to $\mathcal{O}(p^6)$, and $V(\mathbf{r})$ is the quark-antiquark potential [24, 25, 26],

$$V(r) = -\frac{\alpha_c}{r} + Ar^{\nu} + V_0; \;\; \alpha_c = \frac{4}{3}\alpha_S. \tag{2}$$

- The value of the QCD coupling constant $\alpha_s(M^2)$ is determined through the simplest model with freezing[27, 28], namely

$$\alpha_s(M^2) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right) \ln \frac{M^2 + M_B^2}{\Lambda^2}} \tag{3}$$

where the scale is taken as $M = 2m_Q m_{\bar{q}}/\left(m_Q + m_{\bar{q}}\right)$, $M_B = 0.95$ GeV[27, 28], and $\Lambda = 413$ MeV [29].

Mass spectra

- We have used the hydrogenic radial wave function as well gaussian wave function in the present study. The hydrogenic wave function has the form

$$R_{nl}(r) = \left(\frac{\mu^3(n-l-1)!}{2n(n+l)!}\right)^{1/2} (\mu r)^l e^{-\mu r/2} L_{n-l-1}^{2l+1}(\mu r), \quad (4)$$

and the gaussian wave function has the form

$$R_{nl}(r) = \mu^{3/2} \left(\frac{2\,(n-1)!}{\Gamma\,(n+l+1/2)}\right)^{1/2} (\mu r)^l\, e^{-\mu^2 r^2/2} L_{n-1}^{l+1/2}(\mu^2 r^2). \quad (5)$$

- We employ the Ritz variational scheme. We obtain the expectation values of the Hamiltonian as

$$H\psi = E\psi \quad (6)$$

For a chosen value of ν, the variational parameter, μ is determined for each state using the Virial theorem

$$\langle K.E.\rangle = \frac{1}{2}\left\langle \frac{rdV}{dr} \right\rangle. \quad (7)$$

Mass spectra

- As the interaction potential assumed here does not contain the spin dependent part, Eq(6) gives the spin averaged masses of the system in terms of the power index ν. The spin averaged mass for the ground state is computed for the values of ν from 0.5 to 2.0. The spin-averaged mass is matched with the experimental value for the ground state using the equation [26]

$$M_{SA} = M_P + \frac{3}{4}(M_V - M_P) \tag{8}$$

where M_V and M_P are the experimentally measured vector and pseudoscalar meson ground state masses. This fixes the parameter V_0, for the chosen value of ν . Using this value of V_0 we calculate S, P, and D wave spin-averaged masses of D_s mesons which are listed in Tables **??** and **??**. For the comparison for the n state, we compute the spin-average or the center of weight mass from the respective experimental as well as theoretical values as [26]

$$M_{CW,n} = \frac{\Sigma_J 2(2J+1)M_{nJ}}{\Sigma_J 2(2J+1)} \tag{9}$$

where, $M_{CW,n}$ denotes the spin-averaged mass of the n state and M_{nJ} represents the mass of the meson in the nJ state.

Table 2.1: Value of V_0 (in GeV).

Meson	ν	Hydrogenic	Gaussian	Meson	ν	Hydrogenic	Gaussian
	0.5	-0.157	-0.181		0.5	-0.120	-0.148
	1.0	-0.326	-0.337		1.0	0.268	-0.277
D	1.5	-0.574	-0.518	B	1.5	-0.512	-0.447
	2.0	-0.982	-0.768		2.0	-0.922	-0.696
	0.5	-0.113	-0.138		0.5	-0.090	-0.118
	1.0	-0.268	-0.282		1.0	-0.220	-0.234
D_s	1.5	-0.478	-0.440	B_s	1.5	-0.422	-0.374
	2.0	-0.808	-0.639		2.0	-0.752	-0.693
	0.5	-0.119	-0.157				
	1.0	-0.171	-0.204				
B_c	1.5	-0.228	-0.245				
	2.0	-0.294	-0.281				

Spin-dependent forces

- To take into account the spin dependent and the spin-orbit interaction, causing the splitting of the nL levels one introduces additional term in the Hamiltonian[30, 31, 32]

$$
\begin{aligned}
V_{SD}(\mathbf{r}) &= \left(\frac{\mathbf{L}\cdot\mathbf{S_Q}}{2m_Q^2}+\frac{\mathbf{L}\cdot\mathbf{S_{\bar{q}}}}{2m_{\bar{q}}^2}\right)\left(-\frac{dV(r)}{rdr}+\frac{8}{3}\alpha_S\frac{1}{r^3}\right)+ \\
& \frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar{q}}}\frac{\mathbf{L}\cdot\mathbf{S}}{r^3}+\frac{4}{3}\alpha_S\frac{2}{3m_Q m_{\bar{q}}}\mathbf{S_Q}\cdot\mathbf{S_{\bar{q}}}4\pi\delta(\mathbf{r}) \\
& +\frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar{q}}}\left\{3(\mathbf{S_Q}\cdot\mathbf{n})(\mathbf{S_{\bar{q}}}\cdot\mathbf{n})-(\mathbf{S_Q}\cdot\mathbf{S_{\bar{q}}})\right\}\frac{1}{r^3}; \\
& \mathbf{n}=\frac{\mathbf{r}}{r} \qquad (10)
\end{aligned}
$$

Mass spectrum of the D_s meson

- For the calculation of the mass spectrum of the D_s meson the parameters used were $A = 0.14\ \mathrm{GeV}^{\nu+1}$, $m_s = 0.52$ GeV, $m_c = 1.55$ GeV, $\alpha_s = 0.622$.

Table 2.2: S-wave spin-averaged mass of D_S meson.

		Hydrogenic			Gaussian			Expt.	Theory
nL	ν	μ (GeV)	$\lvert R(0)\rvert$ ($\mathrm{GeV}^{3/2}$)	$E(\mu)$ (GeV)	μ (GeV)	$\lvert R(0)\rvert$ ($\mathrm{GeV}^{3/2}$)	$E(\mu)$ (GeV)	(GeV)	(GeV)
	0.5	0.902	0.606	2.076	0.349	0.309	2.076		2.076[14]
	1.0	1.203	0.933	2.076	0.467	0.480	2.076		2.082[16]
1S	1.5	1.480	1.273	2.076	0.576	0.658	2.076	2.076[21]	2.074[19]
	2.0	1.697	1.564	2.076	0.663	0.811	2.076		2.072[22]
									2.075[15]

Mass spectrum of the D_S meson

		Hydrogenic			Gaussian			
nL	ν	μ (GeV)	$\|R(0)\|$ ($GeV^{3/2}$)	$E(\mu)$ (GeV)	μ (GeV)	$\|R(0)\|$ ($GeV^{3/2}$)	$E(\mu)$ (GeV)	Theory (GeV)
2S	0.5	0.745	0.227	2.364	0.199	0.109	2.346	2.779[14]
	1.0	1.248	0.493	2.682	0.336	0.239	2.713	2.700[16]
	1.5	1.688	0.775	3.163	0.480	0.408	3.207	2.706[19]
	2.0	1.991	0.993	4.022	0.601	0.572	3.905	2.695[22]
								2.720[15]
3S	0.5	0.735	0.149	2.505	0.159	0.070	2.489	3.323[14]
	1.0	1.359	0.373	3.099	0.298	0.179	3.175	3.165[16]
	1.5	1.892	0.613	4.140	0.450	0.334	4.180	3.076[19]
	2.0	2.245	0.793	6.219	0.583	0.488	5.765	3.236[15]
4S	0.5	0.746	0.114	2.605	0.139	0.053	2.594	
	1.0	1.461	0.312	3.451	0.278	0.149	3.567	3.356[19]
	1.5	2.058	0.522	5.108	0.437	0.294	5.086	3.665[15]

Mass spectrum of the D_S meson

Table 2.3: P and D-wave spin averaged mass of D_S meson

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $E(\mu)$ (GeV)	Expt. (GeV)	Theory (GeV)
	0.5	0.721	2.332	0.225	2.309		2.568[14]
	1.0	1.174	2.574	0.365	2.540		2.531[16]
$1P$	1.5	1.628	2.838	0.499	2.806	2.514[21]	2.538[19]
	2.0	2.011	3.131	0.608	3.137		2.511[22]
							2.537[15]
	0.5	0.723	2.487	0.169	2.464		3.142[14]
	1.0	1.326	3.030	0.310	3.026		3.008[16]
$2P$	1.5	1.925	3.798	0.460	3.800		2.954[19]
	2.0	2.358	5.129	0.583	4.975		2.991[22]
							3.119[15]

Mass spectrum of the D_S meson

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $E(\mu)$ (GeV)	Theory (GeV)
	0.5	0.703	2.451	0.186	2.425	2.917[14]
	1.0	1.250	2.892	0.331	2.852	2.873[16]
1D	1.5	1.814	3.421	0.474	3.382	2.850[19]
	2.0	2.291	4.091	0.587	4.129	2.814[22]
						2.950[15]
	0.5	0.726	2.570	0.154	2.544	3.288[16]
	1.0	1.401	3.303	0.296	3.277	3.161[19]
2D	1.5	2.105	4.365	0.449	4.322	3.236[22]
	2.0	2.628	6.235	0.571	6.028	3.436[15]

Mass spectrum of the D_S meson

Table 2.4: Mass spectrum of D_S meson(in GeV)

States	This work		Expt.[21]	Others				
	Hydrogenic	Gaussian		[15]	[19]	[16]	[?]	[22]
1^1S_0	1.801	1.970	1.968	1.969	1.975	1.940	1.965	1.969
1^3S_1	2.248	2.117	2.112	2.111	2.108	2.130	2.113	2.107
$D_{s0}(1^3P_0)$	2.335	2.444	2.318	2.509	2.455	2.380	2.487	2.344
$D'_{s1}(1P'_1)$	2.569	2.540	2.535	2.574	2.522	2.520	2.605	2.510
$D_{s1}(1P_1)$	2.529	2.530	2.460	2.536	2.502	2.510	2.535	2.488
$D_{s2}(1^3P_2)$	2.652	2.566	2.573	2.571	2.586	2.580	2.581	2.559
1^1S_0	2.569	2.684		2.688	2.659	2.610	2.700	2.640
1^3S_1	2.725	2.723	2.710^{+12}_{-7}	2.731	2.722	2.730	2.806	2.714
1^3D_1	2.874	2.873		2.913	2.838	2.820	2.900	2.804
$1D'_2$	2.914	2.896		2.931	2.845	2.860	2.913	2.849
$1D_2$	2.877	2.816		2.961	2.856	2.880	2.953	2.788
1^3D_3	2.891	2.834	2.862^{+6}_{-3}	2.971	2.857	2.900	2.925	2.811

Mass spectrum of the D_S meson

States	This work		Expt.[21]	Others				
	Hydrogenic	Gaussian		[15]	[19]	[16]	[?]	[22]
2^3P_0	2.628	2.947		3.054	2.901	2.900	3.067	2.830
$2P_1$	3.046	3.019	3.044^{+30}_{-9}	3.067	2.928	3.000	3.114	2.958
$2P_1'$	2.913	3.023		3.154	2.942	3.010	3.165	2.995
2^3P_2	3.171	3.048		3.142	2.980	3.060	3.157	3.040
3^1S_0	3.030	3.158		3.219	3.044	3.090	3.259	
3^3S_1	3.123	3.180		3.242	3.087	3.190	3.345	
2^3D_1	3.243	3.292		3.383	3.144	3.250		3.217
$2D_2'$	3.303	3.312		3.403	3.172	3.280		3.260
$2D_2$	3.296	3.248		3.456	3.167	3.290		3.217
2^3D_3	3.318	3.263		3.469	3.157	3.310		3.240
4^1S_0	3.402	3.556		3.652	3.331			
4^3S_1	3.467	3.571		3.669	3.364			

Mass spectrum of the D meson

- For the D meson we have used $A = 0.14\ GeV^{\nu+1}$, $m_q = 0.45\,GeV$ and $m_c = 1.55$ GeV, $\alpha_s = 0.645$.

Table 2.5: S-wave spin-averaged masses of D meson

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $\|R(0)\|$ ($GeV^{3/2}$)	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $\|R(0)\|$ ($GeV^{3/2}$)	Gaussian $E(\mu)$ (GeV)	Expt. (GeV)	Theory (GeV)
	0.5	0.853	0.557	1.975	0.330	0.284	1.975		1.971[14]
	1.0	1.150	0.872	1.975	0.448	0.450	1.975		1.977[16]
1S	1.5	1.409	1.183	1.975	0.553	0.618	1.975	1.975	1.973[19]
	2.0	1.606	1.439	1.975	0.632	0.756	1.975		1.974[22]
									1.975[15]

Mass spectrum of the D meson

		Hydrogenic			Gaussian			
nL	ν	μ	$\|R(0)\|$	$E(\mu)$	μ	$\|R(0)\|$	$E(\mu)$	Theory
		(GeV)	(GeV$^{3/2}$)	(GeV)	(GeV)	(GeV$^{3/2}$)	(GeV)	(GeV)
2S	0.5	0.709	0.211	2.263	0.189	0.101	2.247	2.666[14]
	1.0	1.199	0.464	2.596	0.324	0.226	2.629	2.700[16]
	1.5	1.603	0.718	3.140	0.464	0.388	3.152	2.586[19]
	2.0	1.875	0.908	4.162	0.574	0.534	3.972	2.616[22]
								2.619[15]
3S	0.5	0.702	0.139	2.406	0.152	0.065	2.392	3.205[14]
	1.0	1.306	0.352	3.028	0.288	0.169	3.104	3.165[16]
	1.5	1.788	0.564	4.212	0.437	0.317	4.174	2.936[19]
	2.0	2.106	0.721	6.672	0.554	0.453	6.042	3.087[15]
4S	0.5	0.713	0.106	2.508	0.133	0.049	2.498	
	1.0	1.403	0.294	3.395	0.268	0.141	3.510	3.208[19]
	1.5	1.939	0.477	5.281	0.422	0.279	5.129	3.474[15]

Mass spectrum of the D meson

Table 2.6: P and D-wave spin-averaged mass of D meson

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $E(\mu)$ (GeV)	Expt. (GeV)	Theory (GeV)
	0.5	0.686	2.230	0.214	2.208		2.450[14]
	1.0	1.131	2.481	0.351	2.448		2.417[16]
$1P$	1.5	1.573	2.747	0.482	2.729	2.434	2.426[19]
	2.0	1.923	3.082	0.581	3.118		2.420[22]
							2.414[15]
	0.5	0.690	2.387	0.161	2.366		2.820[14]
	1.0	1.279	2.951	0.299	2.949		2.902[16]
$2P$	1.5	1.842	3.792	0.444	3.775		3.020[19]
	2.0	2.227	5.372	0.555	5.160		2.920[22]
							2.986[15]

Mass spectrum of the D meson

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $E(\mu)$ (GeV)	Theory (GeV)
1D	0.5	0.670	2.351	0.178	2.325	2.801[14]
	1.0	1.206	2.807	0.319	2.768	2.754[16]
	1.5	1.754	3.356	0.458	3.337	2.707[19]
	2.0	2.190	4.131	0.559	4.222	3.175[22]
						2.834[15]
2D	0.5	0.693	2.472	0.147	2.447	
	1.0	1.353	3.231	0.285	3.207	
	1.5	2.019	4.386	0.433	4.330	
	2.0	2.484	6.602	0.542	6.350	

Mass spectrum of the D meson

Table 2.7: Mass spectrum of D meson(in GeV)

State	This work		Expt.[21]	Others				
	Hydrogenic	Gaussian		[15]	[19]	[16]	[14]	[22]
1^1S_0	1.702	1.865	1.869	1.871	1.874	1.850	1.868	1.867
1^3S_1	2.157	2.018	2.010	2.010	2.006	2.020	2.005	2.010
1^3P_0	2.225	2.352	$2.308^{+0.036}_{-0.036}$	2.406	2.341	2.270	2.377	2.252
$1P_1'$	2.477	2.454	$2.441^{+0.032}_{-0.032}$	2.469	2.407	2.410	2.490	2.417
$1P_1$	2.431	2.434	2.422	2.426	2.389	2.400	2.417	2.402
1^3P_2	2.564	2.473	2.461	2.460	2.477	2.460	2.460	2.466
2^1S_0	2.476	2.598	2.539	2.581	2.540	2.500	2.589	2.555
2^3S_1	2.642	2.639	2.637	2.632	2.601	2.620	2.692	2.636
1^3D_1	2.810	2.803		2.788	2.750	2.710	2.795	2.740
$1D_2'$	2.854	2.829		2.850	2.727	2.760	2.833	2.789
$1D_2$	2.766	2.722	2.752	2.806	2.689	2.740	2.775	2.693
1^3D[illegible]	2.799	2.741	2.763	2.863	2.688	2.780	2.799	2.719

Mass spectrum of the D meson

State	This work		Expt.[21]	Others				
	Hydrogenic	Gaussian		[15]	[19]	[16]	[14]	[22]
2^3P_0	2.508	2.868		2.919	2.758	2.780	2.949	2.752
$2P_1$	2.811	2.940		2.932	2.792	2.890	2.995	2.886
$2P_1'$	2.976	2.951		3.021	2.802	2.890	3.045	2.926
2^3P_2	3.108	2.971		3.012	2.860	2.94	3.035	2.971
3^1S_0	2.955	3.087		3.062	2.904	2.980	3.141	
3^3S_1	3.054	3.110		3.096	2.947	3.070	3.226	
2^3D_1	3.172	3.233		3.228	3.052	3.130		3.168
$2D_2'$	3.241	3.256		3.307	3.029	3.170		3.215
$2D_2$	3.213	3.169		3.259	2.997	3.160		3.145
2^3D_3	3.263	3.187		3.335	2.999	3.190		3.170
4^1S_0	3.343	3.498		2.452	3.175	3.370		
4^3S_1	3.413	3.514		3.482	3.208			

Mass spectrum of B meson

- The parameters used were $A = 0.14\ \mathrm{GeV}^{\nu+1}$, $m_q = 0.45$ GeV, $m_b = 4.88$ GeV, $\alpha_s = 0.61$

Table 2.8: S-wave spin-averaged mass of B meson

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $\|R(0)\|$ ($\mathrm{GeV}^{3/2}$)	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $\|R(0)\|$ ($\mathrm{GeV}^{3/2}$)	Gaussian $E(\mu)$ (GeV)	Expt.[21] (GeV)	Theory (GeV)
$1S$	0.5	0.953	0.658	5.314	0.367	0.334	5.314	5.314	5.314[15]
	1.0	1.247	0.985	5.314	0.488	0.513	5.314		5.313[19]
	1.5	1.474	1.266	5.314	0.585	0.673	5.314		5.318[16]
	2.0	1.646	1.493	5.314	0.655	0.796	5.314		5.313[14]

Mass spectrum of the B meson

		Hydrogenic			Gaussian			Theory
nL	ν	μ (GeV)	$\lvert R(0)\rvert$ ($\mathrm{GeV}^{3/2}$)	$E(\mu)$ (GeV)	μ (GeV)	$\lvert R(0)\rvert$ ($\mathrm{GeV}^{3/2}$)	$E(\mu)$ (GeV)	(GeV)
2S	0.5	0.771	0.239	5.607	0.205	0.114	5.588	5.902[15]
	1.0	1.278	0.511	5.918	0.348	0.252	5.942	5.842[19]
	1.5	1.650	0.750	6.456	0.489	0.419	6.415	5.860[16]
	2.0	1.900	0.926	7.748	0.590	0.555	7.226	5.912[14]
3S	0.5	0.758	0.156	5.748	0.164	0.073	5.730	6.385[15]
	1.0	1.383	0.383	6.335	0.307	0.187	6.394	6.131[19]
	1.5	1.829	0.583	7.512	0.459	0.340	7.377	6.232[16]
	2.0	2.126	0.731	9.963	0.567	0.466	9.232	6.340[14]
4S	0.5	0.768	0.119	5.848	0.143	0.055	5.835	6.785[15]
	1.0	1.480	0.318	6.690	0.286	0.155	6.778	6.347[19]
	1.5	1.975	0.491	8.570	0.442	0.299	8.277	
	2.0	2.311	0.621	12.785	0.553	0.417	11.333	

Mass spectrum of the B meson

Table 2.9: P-wave spin-averaged mass of the B meson

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $E(\mu)$ (GeV)	Theory (GeV)
1P	0.5	0.745	5.576	0.232	5.551	5.745[15]
	1.0	1.215	5.803	0.378	5.774	5.696[19]
	1.5	1.657	6.026	0.508	6.027	5.695[16]
	2.0	1.979	6.351	0.598	6.410	5.717[14]
2P	0.5	0.745	5.730	0.174	5.706	6.249[15]
	1.0	1.366	6.251	0.320	6.250	6.030[19]
	1.5	1.909	7.056	0.466	7.010	6.105[16]
	2.0	2.262	8.631	0.568	8.387	6.184[14]

Mass spectrum of the B meson

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $E(\mu)$ (GeV)	Theory (GeV)
1D	0.5	0.723	5.696	0.192	5.667	6.106[15]
	1.0	1.289	6.115	0.341	6.079	5.924[19]
	1.5	1.845	6.598	0.479	6.606	5.970[16]
	2.0	2.248	7.359	0.572	7.486	6.007[14]
2D	0.5	0.746	5.813	0.158	5.785	6.540[15]
	1.0	1.443	6.517	0.305	6.495	6.183[19]
	1.5	2.096	7.612	0.452	7.543	6.306[14]
	2.0	2.523	9.822	0.552	9.556	

Mass spectrum of the B meson

Table 2.10: Mass spectrum of the B meson(in GeV)

State	This work		Expt.[21]	Others				
	Hydro.	Gauss.		[15]	[19]	[16]	[14]	[18]
1^1S_0	5.146	5.266	5.279	5.280	5.277	5.280	5.279	5.279
1^3S_1	5.375	5.330	5.325	5.326	5.325	5.330	5.324	5.324
1^3P_0	5.675	5.746	$5.732^{\pm 0.005}_{\pm 0.02}$	5.749	5.678	5.650	5.706	5.689
$1P_1'$	5.820	5.785		5.774	5.699	5.690	5.742	5.744
$1P_1$	5.749	5.764	5.723	5.723	5.686	5.690	5.700	5.731
1^3P_2	5.851	5.779	5.743	5.741	5.704	5.710	5.714	5.759
2^1S_0	5.870	5.930		5.890	5.822	5.830	5.886	5.892
2^3S_1	5.934	5.946		5.906	5.848	5.870	5.920	5.924
1^3D_1	6.152	6.114		6.119	6.005	5.970	6.025	
$1D_2'$	6.170	6.125		6.121	5.955	5.980	6.037	
$1D_2$	6.076	6.056		6.103	5.920	5.960	5.985	
1^3D[illegible]	6.089	6.060		6.091	5.871	5.970	5.993	

Mass spectrum of the B meson

State	This work		Others			
	Hydro.	Gauss.	[15]	[19]	[16]	[14]
2^3P_0	5.997	6.225	6.221	6.010	6.060	6.163
$2P_1$	6.104	6.243	6.209	6.022	6.100	6.175
$2P_1'$	6.310	6.256	6.281	6.028	6.100	6.194
2^3P_2	6.354	6.255	6.260	6.040	6.120	6.188
3^1S_0	6.308	6.387	6.379	6.117	6.210	6.320
3^3S_1	6.344	6.396	6.387	6.136	6.240	6.347
2^3D_1	6.505	6.522	6.534	6.248	6.240	
$2D_2'$	6.531	6.532	6.554	6.207	6.320	
$2D_2$	6.503	6.476	6.528	6.179	6.310	
2^3D_3	6.522	6.479	6.542	6.140	6.320	
4^1S_0	6.671	6.773	6.781	6.335	6.520	
4^3S_1	6.696	6.779	6.786	6.351		

Mass spectrum of the B_S meson

- The parameters used were $A = 0.14\ \mathrm{GeV}^{\nu+1}$, $m_s = 0.52$ GeV, $m_b = 4.88$ GeV, $\alpha_s = 0.58$.

Table 2.11: S-wave spin-averaged masses of the B_S meson.

		Hydrogenic			Gaussian				
nL	ν	μ (GeV)	$\lvert R(0)\rvert$ ($\mathrm{GeV}^{3/2}$)	$E(\mu)$ (GeV)	μ (GeV)	$\lvert R(0)\rvert$ ($\mathrm{GeV}^{3/2}$)	$E(\mu)$ (GeV)	Expt.[21] (GeV)	Theory (GeV)
1S	0.5	1.017	0.725	5.401	0.391	0.367	5.401	5.401	5.404[15]
	1.0	1.320	1.073	5.401	0.515	0.555	5.401		5.404[19]
	1.5	1.567	1.387	5.401	0.618	0.729	5.401		5.415l[16]
	2.0	1.753	1.642	5.401	0.867	0.850	5.401		5.409[14]

Mass spectrum of the B_S meson

nL	ν	Hydrogenic			Gaussian			Theory
		μ (GeV)	$\lvert R(0)\rvert$ ($GeV^{3/2}$)	$E(\mu)$ (GeV)	μ (GeV)	$\lvert R(0)\rvert$ ($GeV^{3/2}$)	$E(\mu)$ (GeV)	(GeV)
2S	0.5	0.817	0.261	5.693	0.218	0.125	5.672	5.988[15]
	1.0	1.344	0.551	5.987	0.364	0.269	6.011	5.959[19]
	1.5	1.753	0.821	6.458	0.511	0.448	6.448	5.960[16]
	2.0	2.027	1.020	7.314	0.623	0.603	7.128	6.011[14]
3S	0.5	0.801	0.169	5.832	0.173	0.079	5.813	6.473[15]
	1.0	1.454	0.413	6.386	0.321	0.199	6.447	6.259[19]
	1.5	1.948	0.641	7.416	0.480	0.364	7.351	6.332[16]
	2.0	2.273	0.808	9.488	0.600	0.510	8.911	6.442[14]
4S	0.5	0.811	0.129	5.930	0.151	0.060	5.915	6.878[15]
	1.0	1.557	0.343	6.723	0.298	0.165	6.816	6.500[19]
	1.5	2.108	0.541	8.370	0.462	0.319	8.192	
	2.0	2.473	0.688	11.943	0.586	0.455	10.766	

Mass spectrum of the B_S meson

Table 2.12: P-wave spin-averaged masses of the B_S meson.

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $E(\mu)$ (GeV)	Theory (GeV)
1P	0.5	0.790	5.663	0.246	5.637	5.844[15]
	1.0	1.270	5.883	0.395	5.851	5.805[19]
	1.5	1.733	6.098	0.531	6.085	5.802[16]
	2.0	2.088	6.372	0.631	6.405	5.820[14]
2P	0.5	0.788	5.814	0.184	5.789	6.343[15]
	1.0	1.427	6.314	0.334	6.310	6.161[19]
	1.5	2.014	7.032	0.487	7.008	6.208[16]
	2.0	2.407	8.357	0.601	8.163	6.287[14]

Mass spectrum of the B_S meson

		Hydrogenic		Gaussian		
nL	ν	μ (GeV)	$E(\mu)$ (GeV)	μ (GeV)	$E(\mu)$ (GeV)	Theory (GeV)
1D	0.5	0.765	5.780	0.203	5.751	6.200[15]
	1.0	1.345	6.186	0.356	6.147	6.047[19]
	1.5	1.926	6.641	0.502	6.626	6.076[16]
	2.0	2.372	7.281	0.605	7.239	6.114[14]
2D	0.5	0.788	5.896	0.167	5.867	6.635[15]
	1.0	1.506	6.571	0.318	6.546	6.323[19]
	1.5	2.207	7.556	0.473	7.501	6.406[16]
	2.0	2.683	9.417	0.586	9.069	

Mass spectrum of the B_S meson

Table 2.13: Mass spectrum of the B_S meson(in GeV).

State	This work		Expt.[21]	Others			
	Hydro.	Gauss.		[15]	[19]	[16]	[14]
1^1S_0	5.236	5.355	5.366	5.372	5.366	5.370	5.373
1^3S_1	5.461	5.417	5.415	5.414	5.417	5.430	5.421
1^3P_0	5.764	5.820		5.833	5.781	5.750	5.804
$1P_1'$	5.898	5.857	5.853	5.865	5.805	5.790	5.842
$1P_1$	5.835	5.845	5.829	5.831	5.795	5.800	5.805
1^3P_2	5.927	5.859	5.840	5.842	5.815	5.820	5.820
2^1S_0	5.942	5.998		5.976	5.939	5.930	5.985
2^3S_1	6.003	6.016		5.992	5.966	5.970	6.019
1^3D_1	6.209	6.188		6.209	6.094	6.070	6.127
$1D_2'$	6.226	6.199		6.218	6.067	6.080	6.140
$1D_2$	6.156	6.110		6.189	6.043	6.070	6.095
1^3D[illegible]	6.168	6.188		6.191	6.016	6.080	6.103

Mass spectrum of the B_S meson

State	This work		Others			
	Hydro.	Gauss.	[15]	[19]	[16]	[14]
2^3P_0	6.090	6.283	6.318	6.143	6.170	6.264
$2P_1$	6.190	6.306	6.345	6.153	6.200	6.278
$2P_1'$	6.362	6.312	6.321	6.160	6.210	6.296
2^3P_2	6.404	6.317	6.359	6.170	6.220	6.292
3^1S_0	6.360	6.441	6.467	6.254	6.310	6.421
3^3S_1	6.395	6.449	6.475	6.274	6.340	6.449
2^3D_1	6.555	6.579	6.629	6.362	6.340	
$2D_2'$	6.579	6.588	6.651	6.339	6.420	
$2D_2$	6.562	6.517	6.625	6.320	6.410	
2^3D_3	6.579	6.524	6.637	6.298	6.420	
4^1S_0	6.705	6.812	6.874	6.487	6.620	
4^3S_1	6.729	6.818	6.879	6.504		

Mass spectrum of the B_C meson

We have employed $A = 0.175\ \mathrm{GeV}^{\nu+1}$, $m_b = 4.88$ GeV, $m_c = 1.55$ GeV and $\alpha_s = 0.45$.

Table 2.14: S wave spin averaged masses of the B_c meson.

		Hydrogenic			Gaussian			
nL	ν	μ (GeV)	$\|R(0)\|$ $GeV^{3/2}$	$E(\mu)$ (GeV)	μ (GeV)	$\|R(0)\|$ $GeV^{3/2}$	$E(\mu)$ (GeV)	Theory (GeV)
	0.5	1.776	1.674	6.318	0.680	0.843	6.318	6.318[33]
	1.0	2.104	2.158	6.318	0.810	1.095	6.318	6.316,[34]
1S	1.5	2.409	2.643	6.318	0.920	1.326	6.318	6.321[35]
	2.0	2.683	3.108	6.318	1.013	1.532	6.318	6.301[32]
								6.319[30]
	0.5	1.352	0.556	6.659	0.362	0.267	6.627	6.890[33]
	1.0	2.013	1.010	6.890	0.542	0.490	6.893	6.869[34]
2S	1.5	2.621	1.500	7.134	0.716	0.743	7.196	6.879[35]
	2.0	3.129	1.957	7.403	0.875	1.003	7.504	6.893[32]

Mass spectrum of the B_C meson

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $\lvert R(0)\rvert$ $GeV^{3/2}$	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $\lvert R(0)\rvert$ $GeV^{3/2}$	Gaussian $E(\mu)$ (GeV)	Theory (GeV)
3S	0.5	1.300	0.349	6.809	0.283	0.165	6.777	7.248[33]
	1.0	2.155	0.746	7.247	0.542	0.438	7.302	7.224[34]
	1.5	2.964	1.203	7.761	0.665	0.595	7.916	7.266[35]
	2.0	3.615	1.620	8.420	0.850	0.859	8.602	
4S	0.5	1.303	0.263	6.912	0.244	0.123	6.883	7.539[33]
	1.0	2.305	0.619	7.539	0.437	0.293	7.610	
	1.5	3.270	1.046	8.340	0.640	0.520	8.575	
	2.0	4.018	1.424	9.498	0.840	0.782	9.677	
5S	0.5	1.321	0.215	6.993	0.221	0.099	6.969	7.796[33]
	1.0	2.445	0.541	7.796	0.413	0.254	7.901	
	1.5	3.538	0.941	8.899	0.624	0.472	9.199	
	2.0	4.357	1.286	10.653	0.835	0.730	10.743	

Mass spectrum of the B_C meson

Table 2.15: P and D-wave spin-averaged masses of the B_C meson.

		Hydrogenic		Gaussian		
nL	ν	μ (GeV)	$E(\mu)$ (GeV)	μ (GeV)	$E(\mu)$ (GeV)	Theory (GeV)
1P	0.5	1.314	6.629	0.408	6.757	6.818[33]
	1.0	1.899	6.808	0.588	6.765	6.746,[34]
	1.5	2.434	6.987	0.746	6.928	6.751[35]
	2.0	2.917	7.152	0.882	7.075	6.728[32]
						6.736[30]
2P	0.5	1.281	6.792	0.299	6.880	7.183[33]
	1.0	2.097	7.197	0.490	7.174	7.140[34]
	1.5	2.883	7.652	0.676	7.659	7.152[35]
	2.0	3.586	8.135	0.849	8.162	7.122[32]
						7.142[30]

Mass spectrum of the B_C meson

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $E(\mu)$ (GeV)	Theory (GeV)
3P	0.5	1.291	6.901	0.303	6.955	7.486[33]
	1.0	2.269	7.505	0.447	7.509	
	1.5	3.244	8.235	0.646	8.324	
	2.0	4.092	9.119	0.838	9.229	
1D	0.5	1.246	6.758	0.330	6.720	7.086[33]
	1.0	1.982	7.087	0.523	7.037	7.078[34]
	1.5	2.670	7.437	0.699	7.365	7.039[35]
	2.0	3.297	7.775	0.854	7.675	7.008[32]
						7.009[30]
2D	0.5	1.270	6.878	0.268	6.840	7.428[33]
	1.0	2.199	7.430	0.464	7.389	
	1.5	3.122	8.081	0.658	8.043	
	2.0	3.968	8.789	0.839	8.741	

Mass spectrum of the B_C meson

Table 2.16: Mass spectrum of the B_C meson(in GeV).

States	Hydro.	Gauss.	Expt.[21]	Ref.[33]	Ref.[34]	Ref[35]	Ref[32]	Ref[30]
1^1S_0	6.144	6.271	6.277	6.272	6.270	6.271	6.253	6.264
1^3S_1	6.376	6.334		6.333	6.332	6.338	6.317	6.337
1^3P_0	6.784	6.725		6.784	6.699	6.699	6.683	6.700
$1P_1$	6.720	6.759		6.720	6.734	6.741	6.717	6.730
$1P'_1$	6.810	6.763		6.810	6,749	6.750	6.729	6.736
1^3P_2	6.888	6.777		6.888	6,762	6.768	6.743	6.747
2^1S_0	6.850	6.883		6.850	6.835	6.855	6.867	6.856
2^3S_1	6.904	6.896		6.904	6.881	6.887	6.902	6.899
1^3D_1	7.082	7.031		7.082	7.072	7.028	7.008	7.012
$1D_2$	7.076	7.036		7.076	7.077	7.036	7.001	7.009
$1D'_2$	7.090	7.044		7.090	7.079	7.041	7.016	7.012
1^3D_3	7.092	7.037		7.092	7.081	7.045	7.007	7.005

Mass spectrum of the B_C meson

States	Hydro.	Gauss.	Ref.[33]	Ref.[34]	Ref[35]	Ref[32]	Ref[30]
2^3P_0	7.155	7.144	7.155	7.091	7.122	7.088	7.108
$2P_1$	7.067	7.169	7.067	7.126	7.145	7.113	7.135
$2P'_1$	7.206	7.173	7.206	7.145	7.150	7.124	7.142
2^3P_2	7.244	7.174	7.244	7.156	7.164	7.134	7.153
3^1S_0	7.225	7.295	7.225	7.193	7.250		
3^3S_1	7.255	7.305	7.255	7.235	7.272		
2^3D_1	7.426	7.384	7.426				
$2D_2$	7.405	7.387	7.405				
$2D'_2$	7.429	7.395	7.429				
2^3D_3	7.444	7.389	7.444				
3^3P_0	7.445	7.482	7.445				
$3P_1$	7.333	7.505	7.333				
$3P'_1$	7.517	7.509	7.517				
3^3P_2	7.567	7.518	7.567				
4^1S_0	7.524	7.606	7.524				

E1 and M1 transitions of heavy-light flavored mesons

E1 radiative transitions

- Electric transitions do not change quark spin. The lowest nonrelativistic order transition is the electric dipole (E1) transition. These transitions have $\triangle l = \pm 1$ and $\triangle s = 0$.
- The E1 matrix elements are determined by using the variational radial wave functions of initial and final state and explicitly performing the angular integration given by[10]

$$\Gamma_{fi} = \frac{4\alpha}{9}\left(\frac{e_Q m_{\bar q} - e_{\bar q} m_Q}{m_{\bar q} + m_Q}\right)^2 k^3 \left|\langle f\left|r\right| i\rangle\right|^2 \frac{E_f}{M_i} \times \begin{cases} 1 & \text{for } {}^3P_J \to {}^3S_1 \\ 1 & \text{for } {}^1P_1 \to {}^1S_0 \\ (2J+1)/3 & \text{for } {}^3S_1 \to {}^3P_J \\ 3 & \text{for } {}^1S_0 \to {}^1P_1 \end{cases} \quad (11)$$

  Here, α is the fine structure constant, k is the photon energy, $e_{\bar q}$ and e_Q are the quark charges in units of the proton charge, E_f is the energy of the final meson state, M_i is the mass of the initial meson state, and $m_{\bar q}$ and m_Q are the quark masses.
- Here

$$k = \frac{M_i^2 - M_f^2}{M^2}; \quad \langle f\left|r\right| i\rangle = \int dr R_{n_i l_i}(r) R_{n_f l_f}(R) \quad (12)$$

D_S Meson

Table 3.1: E1 transitions in D_s Meson.

Transition	k (MeV)		Γ(keV)		[36]	[10]	[37]	[22]
	Hydrog.	Gauss.	Hydrog.	Gauss.				
$D_{s\mathbf{2}} \to D_s^\star \gamma$	0.374	0.410	9.3	8.7	8.8	44.1	19	
$D'_{s\mathbf{1}} \to D_s^\star \gamma$	0.301	0.388	4.5	6.2	4.76	8.90	5.6	
$D'^{+}_{s\mathbf{1}} \to D_s \gamma$	0.654	0.506	43.8	13.5	3.49	54.5	15	
$D_{s\mathbf{1}} \to D_s \gamma$	0.624	0.498	4.0	2.6	4.90	12.8	6.2	
$D_{s\mathbf{1}} \to D_s^\star \gamma$	0.266	0.380	0.3	1.2	0.13	15.5	5.5	
$D_{s\mathbf{0}} \to D_s^\star \gamma$	0.086	0.305	0.1	3.6	1.0	4.92	1.9	
$2^{\mathbf{3}} S_{\mathbf{1}}[D_s(2710)] \to D_{s\mathbf{2}} \gamma$	0.072	0.153	0.3	0.7				0.1
$2^{\mathbf{3}} S_{\mathbf{1}}[D_s(2710)] \to D_{s\mathbf{0}} \gamma$	0.362	0.264	6.5	0.7				6.9

D Meson

Table 3.2: E1 transitions in D Meson.

Transition	k (GeV)		Γ(keV)		[36]	[37]	Ref[38] and [39]			[40]	
	Hydr.	Gauss.	Hydr	Gauss.			WR	R	Rel	NR	rel
$D_2^+ \to D^{\star +}\gamma$	0.375	0.413	17.99	17.00	51	59	50	7	0.14	15.0	6.49
$D_1'^+ \to D^{\star +}\gamma$	0.299	0.397	08.33	13.77	30.87	36	13	2.6	0.22		
$D_1'^+ \to D^{+}\gamma$	0.653	0.518	83.61	30.20	21.71	14	59	0.69	1.6		
$D_1^+ \to D^{+}\gamma$	0.620	0.503	06.83	02.82	39.59	58	14	1.8			
$D_1^+ \to D^{\star +}\gamma$	0.259	0.381	0.52	01.24	10.25	8.6	9	4.1			
$D_0^+ \to D^{\star +}\gamma$	0.067	0.310	0.10	07.23	17	28	3.3	2.4		76	7.55
$2^3S_1 \to D_2\gamma$	0.077	0.161	0.585	1.585							
$2^3S_1 \to D_0\gamma$	0.384	0.272	14.570	1.515							
$2^1S_0 \to D_1'\gamma$	0.000	0.140	0.000	1.853	76						
$2^1S_0 \to D_1\gamma$	0.045	0.158	0.210	2.692	12.3						

B and B_S mesons

Table 3.3: E1 transitions in the B and B_S mesons

Transition	B Meson				B_s Meson			
	k(MeV)		Γ(keV)		k(MeV)		Γ(keV)	
	Hydrog.	Gauss.	Hydrog.	Gauss.	Hydrog.	Gauss.	Hydrog.	Gauss.
$1^3P_2 \to 1^3S_1\gamma$	0.448	0.425	153.07	131.36	0.448	0.425	153.07	131.36
$1P_1' \to 1^3S_1\gamma$	0.421	0.423	125.95	122.87	0.421	0.423	125.95	122.87
$1P_1' \to 1^1S_0\gamma$	0.625	0.480	410.89	179.35	0.625	0.480	410.89	179.35
$1P_1 \to 1^1S_0\gamma$	0.568	0.469	2.99	8.98	0.568	0.469	2.99	8.98
$1P_1 \to 1^3S_1\gamma$	0.362	0.412	0.78	6.09	0.362	0.412	0.78	06.09
$1^3P_0 \to 1^3S_1\gamma$	0.296	0.389	43.89	100.54	0.296	0.389	43.89	100.54
$2^3S_1 \to 1^3P_2\gamma$	0.075	0.154	7.97	10.33	0.075	0.154	2.91	10.33
$2^3S_1 \to 1^3P_0\gamma$	0.234	0.154	1.59	2.07	0.234	0.154	17.38	2.07
$2^1S_0 \to 1P_1'\gamma$	0.044	0.142	1.00	13.79	0.044	0.142	1.00	13.79
$2^1S_0 \to 1P_1\gamma$	0.106	0.154	0.14	0.94	0.106	0.154	0.14	0.94

B_C meson

Table 3.4: E1 transitions in B_C meson

Transition	k (GeV)		Γ(keV)		[35]	[34]	[32]	[30]
	Hydr.	Gauss.	Hydr.	Gauss.				
$1^3P_2 \to 1^3S_1 + \gamma$	0.493	0.429	302.485	81.955	83	107	102.9	112.6
$1P_1' \to 1^3S_1 + \gamma$	0.420	0.415	7.514	10.625	11	13.6	8.1	0.1
$1P_1 \to 1^3S_1 + \gamma$	0.335	0.412	91.411	62.301	60	78.9	77.8	99.5
$1P_1' \to 1^1S_0 + \gamma$	0.633	0.474	617.743	95.146	80	132	131.1	56.4
$1P_1 \to 1^1S_0 + \gamma$	0.551	0.470	16.945	15.399	13	18.4	11.6	0.0
$1^3P_0 \to 1^3S_1 + \gamma$	0.396	0.380	156.623	57.009	55	67.2	65.3	79.2
$2^3S_1 \to 1^3P_2 + \gamma$	0.016	0.118	0.047	3.417	5.7	5.18	14.8	17.7
$2^3S_1 \to 1P_1' + \gamma$	0.093	0.132	0.225	0.406	0.7	0.63	1.0	0.0
$2^3S_1 \to 1P_1 + \gamma$	0.182	0.136	39.640	2.672	4.7	5.05	12.08	14.5
$2^3S_1 \to 1^3P_0 + \gamma$	0.119	0.169	3.873	2.004	2.9	3.78	7.7	7.8
$2^1S_0 \to 1P_1' + \gamma$	0.040	0.119	1.261	5.408	6.1	3.72	15.9	5.2
$2^1S_0 \to 1P_1 + \gamma$	0.129	0.123	1.772	0.988	1.3	1.02	1.9	0.0

M1 transitions

- The M1 rate for transitions where $\triangle l = 0$ between S-wave levels is given by[22, 41, 42]

$$\Gamma_{M1}(i \to f + \gamma) = \frac{16\alpha}{3}\mu^2 k^3 (2J_f + 1) \left|\langle f | j_0(kr/2) | i \rangle\right|^2 ,$$

where the magnetic dipole moment is

$$\mu = \frac{m_{\bar{q}} e_Q - m_Q e_{\bar{q}}}{4 m_{\bar{q}} m_Q}$$

and k as before is the photon energy.

- Here

$$\langle f | r | i \rangle = \int dr R_{n_i l_i}(r) \, j_0(kr/2) \, R_{n_f l_f}(r) \tag{13}$$

D_S meson

Table 3.5: M1 transitions in D_s Meson.

Transition	k (GeV)		Γ(keV)		[10]	[43]
	Hydro.	Gauss.	Hydro.	Gauss.		
$1^3S_1 \rightarrow 1^1S_0\gamma$	0.403	0.063	5.980	0.3017	1.91	0.2
$2^3S_1 \rightarrow 2^1S_0\gamma$	0.152	0.008	0.352	0.0061		
$3^3S_1 \rightarrow 3^1S_0\gamma$	0.091	0.003	0.078	0.0011		
$4^3S_1 \rightarrow 4^1S_0\gamma$	0.065	0.002	0.029	0.0004		

D meson

Table 3.6: M1 transitions in D Meson.

Transition	k (GeV)		Γ(keV)		[36]	[43]	[40]	
	Hydr.	Gaus.	Hydr.	Gaus.			NR	Rel
$1^3S_1 \to 1^1S_0\gamma$	0.406	0.147	13.10	0.339	1.8	0.90 ± 0.02	1.38	0.08
$2^3S_1 \to 2^1S_0\gamma$	0.160	0.041	0.88	0.007				
$3^3S_1 \to 3^1S_0\gamma$	0.097	0.023	0.20	0.001				
$4^3S_1 \to 4^1S_0\gamma$	0.069	0.016	0.07	0.000				

B and B_S mesons

Table 3.7: M1 transitions in the B and B_S mesons.

Transition	k(MeV)		Γ(keV)		[44]	[45]	[46]	[47]
	Hydr.	Gaus.	Hydr.	Gaus.				
$1^3S_1 \to 1^1S_0\gamma$	0.224	0.064	52.12	1.258	0.19	0.272	0.085	0.22 ± 0.09
$2^3S_1 \to 2^1S_0\gamma$	0.063	0.016	1.24	0.018				
$3^3S_1 \to 3^1S_0\gamma$	0.036	0.009	0.22	0.003				
$4^3S_1 \to 4^1S_0\gamma$	0.025	0.006	0.08	0.001				

B_C meson

Table 3.8: M1 transitions in the B_C meson

Transition	k (GeV)		Γ(eV)		[35]	[34]	[32]	[30]
	Hydr.	Gauss.	Hydr.	Gauss.				
$1^3S_1 \rightarrow 1^1S_0\gamma$	0.228	0.063	5956	23.5	80	33	60	134.5
$2^3S_1 \rightarrow 2^1S_0\gamma$	0.054	0.013	80	2		17	10	
$3^3S_1 \rightarrow 3^1S_0\gamma$	0.030	0.010	14	1				
$4^3S_1 \rightarrow 4^1S_0\gamma$	0.020	0.005	4	0				
$5^3S_1 \rightarrow 5^1S_0\gamma$	0.016	0.004	2	0				
$6^3S_1 \rightarrow 6^1S_0\gamma$	0.009	0.003	0	0				

Decay properties of heavy-light flavored mesons

Decay constants

- The decay constants of mesons are important parameters in the study of leptonic or non-leptonic weak decay processes. The decay constants of pseudoscalar (f_P) and vector (f_V) mesons are obtained by parameterizing the matrix elements of weak current between the corresponding mesons and the vacuum as

$$\langle 0| \bar{Q}\gamma^{\mu}\gamma_5 Q \,|P_{\mu}(k)\rangle = if_P k^{\mu} \tag{14}$$

$$\langle 0| \bar{Q}\gamma^{\mu} Q \,|V(k,\epsilon)\rangle = f_V M_V \epsilon^{\mu} \tag{15}$$

  where k is the meson momentum, ϵ^{μ} and M_V are the polarization vector and mass of the vector meson.

- Incorporating a First order QCD correction factor , we compute the decay constants using the relation,

$$f^2_{P/V} = \frac{12\left|\psi_{P/V}(0)\right|^2}{M_{P/V}}\bar{C}^2(\alpha_S) \tag{16}$$

  Where $\bar{C}^2(\alpha_S)$ is the QCD correction factor given by[48]

$$\bar{C}^2(\alpha_S) = 1 - \frac{\alpha_S}{\pi}\left[2 - \frac{m_Q - m_{\bar{q}}}{m_Q + m_{\bar{q}}}\ln\frac{m_Q}{m_{\bar{q}}}\right] \tag{17}$$

D_S meson

Table 4.1: Pseudoscalar decay constants of D_s mesons(in GeV).

1S		2S		3S		4S	
Hydro.	Gauss.	Hydro.	Gauss.	Hydro.	Gauss.	Hydro.	Gauss.
0.533	0.315	0.286	0.141	0.205	0.098	0.163	0.077
(0.379)	(0.224)	(0.204)	(0.100)	(0.146)	(0.070)	(0.116)	(0.055)
0.254 ± 0.006[49]							
0.248 ± 0.002[50]							
0.235 ± 0.024[51]							

D_S meson

Table 4.2: Vector decay constants of D_s mesons(in GeV).

1S		2S		3S		4S	
Hydro.	Gauss.	Hydro.	Gauss.	Hydro.	Gauss.	Hydro.	Gauss.
0.652	0.329	0.296	0.142	0.208	0.098	0.165	0.077
(0.464)	(0.234)	(0.211)	(0.101)	(0.148)	(0.070)	(0.117)	(0.055)
0.335[52, 53]							
$0.326^{+0.021}_{-0.017}$[54]							
0.254[55]							
0.242[56]							
0.298 ± 0.011[57]							

D meson

Table 4.3: Pseudoscalar decay constants of D mesons(in GeV).

1S		2S		3S		4S	
Hydro.	Gauss.	Hydro.	Gauss.	Hydr.	Gauss.	Hydro.	Gauss.
0.498	0.302	0.274	0.136	0.195	0.094	0.155	0.073
(0.363)	(0.220)	(0.199)	(0.099)	(0.142)	(0.068)	(0.113)	(0.053)
$0.206 \pm 0.008 \pm 0.002$[49]							
0.207 ± 0.004[58]							
0.230 ± 0.025[59]							
0.234[60]							
0.248[61]							

D meson

Table 4.4: Vector decay constants of D meson(in GeV).

1S		2S		3S		4S	
Hydr.	Gauss.	Hydro.	Gauss.	Hydr.	Gauss.	Hydr.	Gauss.
0.626	0.316	0.284	0.137	0.198	0.094	0.156	0.073
(0.456)	(0.231)	(0.207)	(0.100)	(0.145)	(0.068)	(0.114)	(0.054)
$0.223^{+0.023}_{-0.019}$[54]							
0.234[55]							
0.262 ± 0.010[62]							
0.237[56]							

B meson

Table 4.5: Pseudoscalar decay constants of B meson(in GeV).

1S		2S		3S		4S	
Hydro.	Gauss.	Hydro.	Gauss.	Hydr.	Gauss.	Hydro.	Gauss.
0.410	0.216	0.204	0.101	0.149	0.072	0.120	0.058
0.191 ± 0.009[63]							
0.197 ± 0.009[64]							
0.193 ± 0.011[49]							
0.198 ± 0.014[65]							
0.196 ± 0.029[59]							
0.189[60]							
0.195 ± 0.011[66]							
0.231 ± 0.009[57]							

B meson

Table 4.6: Vector decay constants of B meson(in GeV).

1S		2S		3S		4S	
Hydro.	Gauss.	Hydro.	Gauss.	Hydr.	Gauss.	Hydro.	Gauss.
0.420	0.218	0.206	0.101	0.149	0.072	0.120	0.058
$0.196^{+0.028}_{-0.027}$[54]							
0.190[55]							
0.164[56]							
0.194 ± 0.008[67]							
0.252 ± 0.010[57]							

B_S meson

Table 4.7: Pseudoscalar decay constants of B_s meson(in GeV).

1S		2S		3S		4S	
Hydro.	Gauss.	Hydro.	Gauss.	Hydr.	Gauss.	Hydro.	Gauss.
0.443	0.232	0.219	0.107	0.159	0.077	0.129	0.062
0.227 ± 0.010[63]							
0.242 ± 0.010[64]							
0.231 ± 0.015[49]							
0.237 ± 0.017[65]							
0.216 ± 0.032[59]							
0.218[60]							
0.243 ± 0.011[66]							
0.266 ± 0.010[57]							

B_S meson

Table 4.8: Vector decay constants of B_s meson(in GeV).

1S		2S		3S		4S	
Hydro.	Gauss.	Hydro.	Gauss.	Hydr.	Gauss.	Hydro.	Gauss.
0.454	0.234	0.220	0.107	0.160	0.077	0.129	0.062
$0.229^{+0.032}_{-0.031}$[54]							
0.217[55]							
0.225 ± 0.009[56]							
0.289 ± 0.011[57]							

B_C meson

Table 4.9: Pseudoscalar decay constants of B_c.

State	Present work		[68]	[34]	[69]
1S	Hydro	0.826(0.660)	0.402	0.433	0.427(6)(2)
	Gauss.	0.424(0.339)			
2S	Hydro	0.375(0.299)			
	Gauss.	0.182(0.145)			
3S	Hydro	0.270(0.216)			
	Gauss.	0.158(0.126)			
4S	Hydro	0.220(0.176)			
	Gauss.	0.104(0.083)			
5S	Hydro	0.189(0.151)			
	Gauss.	0.088(0.071)			
6S	Hydro	0.142(0.113)			
	Gauss.	0.078(0.062)			

B_C meson

Table 4.10: Vector decay constants of B_c.

State	Present work		[68]	[34]
1S	Hydro	0.843(0.674)	0.431	0.503
	Gauss.	0.426(0.340)		
2S	Hydro	0.376(0.301)		
	Gauss.	0.182(0.146)		
3S	Hydro	0.271(0.216)		
	Gauss.	0.158(0.127)		
4S	Hydro	0.220(0.176)		
	Gauss.	0.104(0.083)		
5S	Hydro	0.189(0.151)		
	Gauss.	0.088(0.071)		
6S	Hydro	0.142(0.113)		
	Gauss.	0.078(0.062)		

Mixing parameters

- The mixing parameters are the integrated oscillation rate (χ_q), x_q and y_q given by

$$\chi_q = \frac{x_q^2 + y_q^2}{2\left(x_q^2 + 1\right)}, \quad \text{where,} \tag{18}$$

$$x_q = \frac{\Delta m_q}{\Gamma_q} = \Delta m_q \tau_{B_q} \qquad y_q = \frac{\Delta\Gamma_q}{2\Gamma_q} = \frac{\Delta\Gamma_q \tau_{B_q}}{2}. \tag{19}$$

- where

$$\triangle m_q = 2\left|M_{12}^q\right| \ \text{ and } \ \triangle\Gamma_q = 2\left|\Gamma_{12}^q\right| \tag{20}$$

- The off-diagonal elements of the mass and the decay matrices are [70]

$$M_{12} = -\frac{G_F^2 m_W^2 \eta_B m_{B_q} B_{B_q} f_{B_q}^2}{12\pi^2} S_0\left(m_t^2/m_W^2\right)\left(V_{tq}^\star V_{tb}\right)^2, \tag{21}$$

$$\Gamma_{12} = \frac{G_F^2 m_b^2 \eta'_B m_{B_q} B_{B_q} f_{B_q}^2}{8\pi}\left[\left(V_{tq}^\star V_{tb}\right)^2\right], \tag{22}$$

The known function $S_0(x_t)$ can be approximated by $0.784 x_t^{0.76}$[71] and V_{ij} are the elements of the CKM matrix. The parameters η_B and η'_B correspond

Mixing parameters

Table 4.11: Mixing parameters for B Meson.

ν	Δm (in ps^{-1})		x		y		χ	
	Hydro.	Gauss.	Hydro.	Gauss	Hydro.	Gauss	Hydro.	Gauss
0.5	0.808	0.213	1.232	0.325	0.0043	0.0011	0.3014	0.0478
1.0	1.735	0.498	2.646	0.759	0.0092	0.0026	0.4375	0.1827
1.5	2.744	0.841	4.185	1.283	0.0146	0.0045	0.4730	0.3111
2.0	3.642	1.161	5.555	1.771	0.0194	0.0062	0.4843	0.3791
Expt.[21]	0.507 ± 0.005		0.774 ± 0.008				0.1873 ± 0.0024	
SM[72]	0.543 ± 0.091							
[73]($\nu = 1$)	0.593		0.9014		0.0046		0.2242	

Mixing parameters

Table 4.12: Mixing paramters in B_s meson

ν	Δm (in ps^{-1})		x		y		χ	
	Hydro.	Gauss.	Hydro.	Gauss	Hydro.	Gauss	Hydro.	Gauss
0.5	20.093	5.289	29.578	7.785	0.1031	0.0271	0.4994	0.4919
1.0	42.266	11.896	62.216	17.511	0.2168	0.0610	0.4999	0.4984
1.5	67.566	20.286	99.457	29.862	0.3466	0.1041	0.5000	0.4994
2.0	90.437	31.425	133.123	46.257	0.4639	0.1612	0.5000	0.4998
Expt.	17.77 ± 0.10[74]		26.2 ± 0.5[21]				0.4993[21]	
SM[72]	17.30 ± 2.6							
[73]($\nu = 1$)	23.26		33.95		0.1722		0.4996	

Conculsion

Conclusions

In this work we used potential model to describe various properties of the heavy-light mesons using the gaussian and hydrogen like wave function. The conclusions to be drawn from the present work are the following:

- In the present work various properties of heavy-light mesons are successfully reproduced using the potential model scheme by incorporating the kinematic relativistic correction. It can be inferred that one can successfully extend the non-relativistic potential model approach to heavy-light mesons with suitable corrections even though the light quark is relativistic.
- Looking at the mass spectra of heavy-light mesons it is observed that the spin-averaged masses as well as the complete mass spectra are in better agreement with the gaussian wave function. Since the gaussian wave function is more suitable to the confining interaction; based on the present study it can be concluded that the confining interaction seems to play an important role particularly in the case of heavy-light flavoured mesons.

Conclusions

- The results for E1 and M1 dipole transitions due to the gaussian wave function are in satisfactory agreement with other theoretical and experimental predictions for some heavylight flavoured mesons. Although more precise experimental measurements are required in most cases.
- In the case of decay constants we find that the results due to hydrogen like wave function are overestimated while the results due to gaussian wave function are in better agreement with other theoretical estimates.
- Overall it can be concluded that the gaussian wave function can be successfully used to obtain various properties of heavy-light flavoured mesons.

References

[1] G.S. Bali et al. ((SESAM and TχL Collaborations)), Phys. Rev. D **62**, 054503 (2000)

[2] C. Alexandrou, P. de Forcrand, O. Jahn, Nucl.Phys.Proc.Suppl. **119**, 667 (2003)

[3] E. Eichten et al., Phys. Rev. D **17**(11), 3090 (1978)

[4] C. Quigg, J.L. Rosner, Phys.Rept. **56**, 167 (1979)

[5] P. Vinodkumar et al., Eur.Phys.J. **A4**, 83 (1999)

[6] A. Martin, Phys.Lett. **B93**, 338 (1980)

[7] W. Buchmüller, S.H.H. Tye, Phys. Rev. D **24**, 132 (1981)

[8] P. Falkensteiner, D. Flamm, F. Schoberl, Phys.Lett. **B131**, 450 (1983)

[9] D.P. Stanley, D. Robson, Phys. Rev. D **21**, 3180 (1980)

[10] S.F. Radford, W.W. Repko, M.J. Saelim, Phys. Rev. D **80**(3), 034012 (2009)

[11] J. Link et al. (FOCUS Collaboration), Phys.Lett. **B586**, 11 (2004)

[12] K. Abe et al. ((Belle Collaboration)), Phys. Rev. D **69**, 112002 (2004)

[13] S. Anderson et al. (CLEO), Nucl. Phys. **A663**, 647 (2000)

[14] M. Di Pierro, E. Eichten, Phys. Rev. D **64**(11), 114004 (2001)

[15] D. Ebert, R. Faustov, V. Galkin, Eur.Phys.J. **C66**, 197 (2010)

[16] J. Zeng, J.W. Van Orden, W. Roberts, Phys. Rev. D **52**(9), 5229 (1995)

[17] S. Godfrey, R. Kokoski, Phys. Rev. D **43**(5), 1679 (1991)

[18] S.N. Gupta, J.M. Johnson, Phys. Rev. D **51**(1), 168 (1995)

[19] T. Lahde, C. Nyfalt, D. Riska, Nucl.Phys. **A674**, 141 (2000)

[20] Y.b. Dai, C.S. Huang, H.Y. Jin, Phys.Lett. **B331**, 174 (1994)

[21] J. Beringer et al. (Particle Data Group), Phys.Rev. **D86**, 010001 (2012)

[22] D.M. Li, P.F. Ji, B. Ma, Eur.Phys.J. **C71**, 1 (2011), ISSN 1434-6044

[23] N. Isgur, M.B. Wise, Phys. Rev. Lett. **66**(9), 1130 (1991)

[24] A.K. Rai, R.H. Parmar, P.C. Vinodkumar, J. Phys. G: Nucl. Part. Phys. **28**(8), 2275 (2002)

[25] A.K. Rai, J.N. Pandya, P.C. Vinodkumar, J. Phys. G: Nucl. Part. Phys. **31**(12), 1453 (2005)

[26] A.K. Rai, B. Patel, P.C. Vinodkumar, Phys. Rev. C **78**(5), 055202 (2008)

[27] A.M. Badalian, A.I. Veselov, B.L.G. Bakker, Phys. Rev. D **70**, 016007 (2004)

[28] Y.A. Simonov, Physics of Atomic Nuclei **58**, 107 (1995)

[29] D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Rev. D **79**, 114029 (2009)

[30] E.J. Eichten, C. Quigg, Phys.Rev. **D49**, 5845 (1994)

[31] D. Gromes, Z.Phys. **C26**, 401 (1984)

[32] S. Gershtein et al., Phys.Usp. **38**, 1 (1995)

[33] D. Ebert, R. Faustov, V. Galkin, Eur.Phys.J. **C71**, 1825 (2011)

[34] D. Ebert, R. Faustov, V. Galkin, Phys.Rev. **D67**, 014027 (2003)

[35] S. Godfrey, Phys.Rev. **D70**, 054017 (2004)

[36] F.E. Close, E.S. Swanson, Phys. Rev. D **72**(9), 094004 (2005)

[37] S. Godfrey, Phys.Rev. **D72**, 054029 (2005)

[38] Fayyazuddin, Riazuddin, Phys. Rev. D **48**, 2224 (1993)

[39] Fayyazuddin, Riazuddin, Journal of Physics G: Nuclear and Particle Physics **24**(1), 23 (1998)

[40] O. Lakhina, E.S. Swanson, Phys. Lett. **B650**, 159 (2007)

[41] W.A. Bardeen, E.J. Eichten, C.T. Hill, Phys. Rev. D **68**(5), 054024 (2003)

[42] N. Brambilla et al., Eur.Phys.J. **C71**, 1534 (2011)

[43] H.M. Choi, C.R. Ji, Phys. Rev. D **59**(7), 074015 (1999)

[44] D. Ebert, R. Faustov, V. Galkin, Phys.Lett. **B537**, 241 (2002)

[45] A.H. Orsland, H. Hogaasen, Eur.Phys.J. **C9**, 503 (1999)

[46] J. Goity, W. Roberts, Phys.Rev. **D64**, 094007 (2001)

[47] P. Colangelo, F. De Fazio, G. Nardulli, Phys.Lett. **B316**, 555 (1993)

[48] E. Braaten, S. Fleming, Phys. Rev. D **52**(1), 181 (1995)

[49] D. Asner et al. (Heavy Flavor Averaging Group) (2010), arXiv:hep-ex/1010.1589

[50] C.T.H. Davies et al. (HPQCD Collaboration), Phys. Rev. D **82**, 114504 (2010)

[51] S. Narison, Phys.Lett. **B520**, 115 (2001)

[52] D. Ebert, V.O. Galkin, R.N. Faustov, Phys. Rev. D **57**(9), 5663 (1998)

[53] D. Ebert, V.O. Galkin, R.N. Faustov, Phys. Rev. D **59**(1), 019902 (1998)

[54] C. Albertus et al., Phys. Rev. D **71**(11), 113006 (2005)

[55] K. Bowler et al. (UKQCD Collaboration), Nucl.Phys. **B619**, 507 (2001)

[56] A. Abd El-Hady, A. Datta, J.P. Vary, Phys. Rev. D **58**(1), 014007 (1998)

[57] D.S. Hwang, G.H. Kim, Phys. Rev. D **55**(11), 6944 (1997)

[58] E. Follana et al. (HPQCD and UKQCD Collaborations), Phys. Rev. Lett. **100**, 062002 (2008)

[59] G. Cvetic et al., Phys.Lett. **B596**, 84 (2004)

[60] D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Lett. **B635**, 93 (2006)

[61] B. Patel, P.C. Vinodkumar, CPC(HEP & NP) **34(9)**, 1497 (2010), 0908.2212

[62] D.S. Hwang, G.H. Kim, Phys. Rev. D **55**, 6944 (1997)

[63] H. Na et al., Phys.Rev. **D86**, 034506 (2012)

[64] E. Neil et al. (Fermilab Lattice Collaboration, MILC Collaboration), PoS **LATTICE2011**, 320 (2011)

[65] M.Z. Yang, Eur.Phys.J. **C72**, 1880 (2012), arXiv:hep-ph/1104.3819

[66] C. Bernard et al., PoS **LATTICE2008**, 278 (2008), 0904.1895

[67] D.S. Hwang, C. Kim, W. Namgung, Phys.Lett. **B406**, 117 (1997)

[68] D. Hwang, G. Kim, J.Korean Phys.Soc. **29**, S251 (1996)

[69] C. McNeile et al., Phys.Rev. **D86**, 074503 (2012)

[70] A. Buras, W. Slominski, H. Steger, Nucl.Phys. **B245**, 369 (1984)

[71] T. Inami, C. Lim, Prog.Theor.Phys. **65**, 297 (1981)

[72] A. Lenz, U. Nierste (2011), arXiv:hep-ph/1102.4274

[73] B. Patel, P.C. Vinodkumar, Journal of Physics G: Nuclear and Particle Physics **36**(11), 115003 (2009)

[74] A. Abulencia et al. (CDF Collaboration), Phys.Rev.Lett. **97**, 242003 (2006)