Strong-Field QED Processes in Intense Laser Pulses

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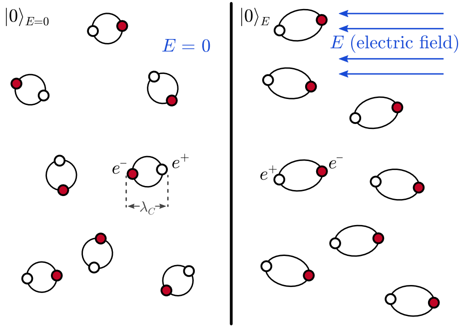
Outline

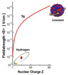
What are Strong EM Fields?



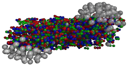
Compare field strength F with QED scales e, m: $eE_{\text{QED}}\lambda_C = m(c^2)$

$$E_{\text{QED}} = 1.8 \times 10^{18}\text{V/m}\,, \qquad B_{\text{QED}} = 4 \times 10^{9}\text{T}\,, \qquad I_{\text{QED}} = 2 \times 10^{29}\text{W/cm}^2$$

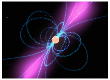
Sources of Strong EM Fields



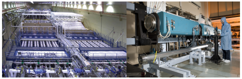
Coulomb Field in highly charged ions (U^{91+}): $E \sim 10^{18}$ V/m


[Uni Frankfurt]

Heavy ion collisions (chiral magnetic effect): $B \sim 10^{14}$ T


[NASA]

Pulsars and Magnetars (rotating neutron stars): $B \sim 10^{8} \dots 10^{11}$ T


[LLNL,RAL]

Optical (multi)-PW high-intensity lasers $I = 10^{22} \dots 10^{23}$ W/cm^2
Ultra-short duration (single-cycle, few fs), pulse shape and CEP control

Strong Field Parameters

- (Plane-wave) laser field is characterized by Gauge potential A^μ and wave vector $k^\mu = (\omega, 0, 0, -\omega)$:

$$A^\mu = A^\mu(\phi) = A(\phi)\epsilon^\mu$$

  with phase $\phi = k \cdot x$ and transversality property $A \cdot k = 0$

- For intense lasers the field invariants

$$\mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}\,, \qquad \mathcal{G} = \frac{1}{4} F_{\mu\nu}\, {}^\star F^{\mu\nu}$$

  are (almost) zero: Laser beams are (almost) null-fields: $\mathcal{F} = \mathcal{G} = 0$.

- Null fields cannot produce pairs, the vacuum is stable.
- We need probe to see nonlinear strong-field effects: matter/photons
  In other words: We need additional momenta to form non-vanishing invariants

Strong Field Parameters II

- Classical nonlinearity parameter/normalized vector potential:

$$a_0 = \frac{eA}{m} = \frac{eE\lambda_L}{m} = \frac{eE\lambda_C}{\omega}$$

  - Electron kinetic energy equals rest mass
  - # of laser photons absorbed in Compton wavelength
  - Multi-Photon effects
  - Inverse Keldysh adiabadicity parameter

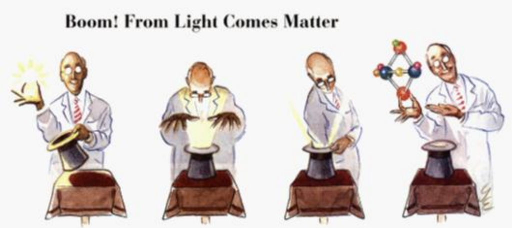
- Quantum nonlinearity/efficiency parameter for a probe particle with momentum p:

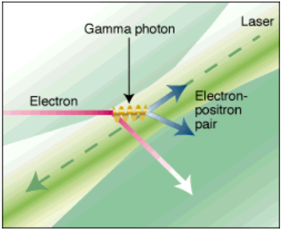
$$\chi = \frac{e}{m^3}\sqrt{(F_{\mu\nu}p^\nu)^2} \sim 2\gamma\frac{E}{E_{\mathrm{QED}}}$$

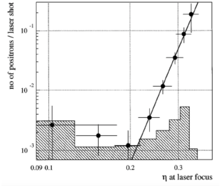
- Classical and Quantum nonlinearity parameter are related by quantum energy parameter:

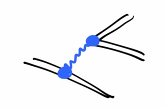
$$b_0 = \frac{k \cdot p}{m^2}\,, \qquad \chi = a_0 b_0$$

The SLAC E-144 Experiment









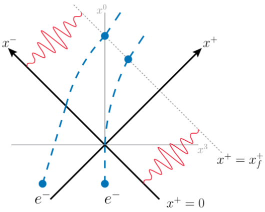
$$e + n\gamma_L \to e' + \gamma'$$
$$\gamma' + n'\gamma_L \to e^+e^-$$

$$a_0 = 0.6 (= \eta)$$
$$\chi \sim 1$$

[https://www.slac.stanford.edu/exp/e144/e144.html]

[D. Burke et al., Phs. Rev. Lett. **79**, 1626 (1997).]

Light-Front Physics



- Laser field depends only on the phase variable $\phi = \omega(t+z) = k \cdot x = \omega x^+$
- Light-front coordinates (l.f.c.) are the coordinates of choice $x^+ = t \pm z$: $x^\mu = (x^+, x^-, x^1, x^2)$
- Laser wavevector has just one component in l.f.c. $k^- = 2\omega$
- Three cyclic coordinates $\Rightarrow$ three conserved canonical momenta
- Non-diagonal scalar product:

$$a \cdot b = \frac{1}{2} a^+ b^- + \frac{1}{2} a^- b^+ - \boldsymbol{a}^\perp \cdot \boldsymbol{b}^\perp$$

- Light-front mass shell $p^2 = m^2$: $E = \sqrt{\boldsymbol{p}^2 + m^2} \Leftrightarrow p^- = \frac{(\boldsymbol{p}^\perp)^2 + m^2}{p^+}$

[P. A. M. Dirac, "Forms of Relativistic Dynamics" Rev. Mod. Phys. (1949).]

Classical EOM of an Electron

Electrons in Intense Laser Fields are Always Relativistic!

- Consider the classical equation of motion of an electron in a plane wave laser field

$$\frac{d\pi^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu}(\phi)\pi_\nu, \qquad m\frac{dx^\mu}{d\tau} = \pi^\mu$$

- Three conserved momenta: p^+, $\boldsymbol{p}^\perp \to$ integrability
  Replace proper time derivative by laser phase derivative $\frac{d\phi}{d\tau} = \omega p^+/m$
- Solution for classical kinetic momentum

$$\pi^\mu(\phi) = p^\mu - eA^\mu + k^\mu \left(\frac{eAp}{kp} - \frac{e^2A^2}{2kp} \right)$$

Note: Mass shell condition: $\pi^2 = m^2$

Hamilton Jacobi

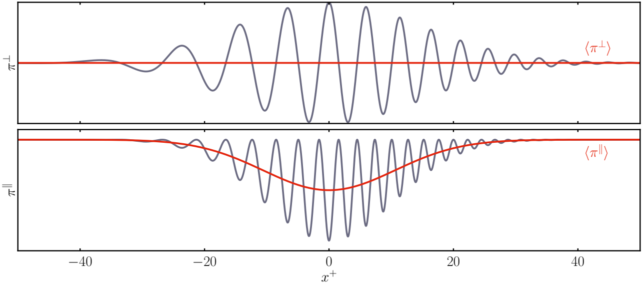
- Classical dynamics can be described using Hamilton-Jacobi equation

$$(\partial_\mu S_{HJ} + eA_\mu)(\partial^\mu S_{HJ} + eA^\mu) = m^2$$

  with the kinetic momentum $\pi^\mu = -\partial^\mu S_{HJ} - eA^\mu$.
- The solution of the Hamilton Jacobi action is given by

$$S_{HJ} = -p \cdot x - (2k \cdot p)^{-1} \int d\phi (2eA \cdot p - e^2 A^2)$$

Classical Motion of an Electron



- Cycle averages:

  $\langle \pi^\mu(\phi) \rangle = p^\mu + U^\mu(\phi) = q^\mu(\phi)\,, \qquad U^\mu(\phi) = \frac{e^2 \langle -A \cdot A \rangle}{2k \cdot p} k^\mu$
- Electron motion in average rest frame
  - Linear Laser Polarization: Figure-8
  - Circular Laser Polarization: Micro-Synchrotron

[N. B. Narozhny and M. S. Fofanov, JETP **83**, 14 (1996).]

Limits of the Classical Description

✓ Radiation emitted by accelerated electron:

$$\mathbb{P}^{\mu} \sim \int d^4k\, \delta(k^2)\, \mathrm{sign}(k^0)\, j(k) \cdot j(-k)$$

$$j(k) \sim \int d\phi\, \pi(\phi)\, \exp\left\{ i \int d\phi \frac{k \cdot \pi}{k_L \cdot p} \right\}$$

✓ (Non-linear) Thomson Sources, ...

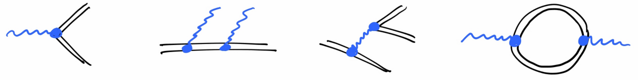
✓ Radiative losses → "friction" term in force equation (LAD, LL, ...)

× Electron Recoil: $b_0 = \gamma\omega_L/m \sim 1$

× Stochasticity in Photon Emission

× Pair Production: $\gamma \to e^+e^-$

× Coherent Multi-Photon Emission, Higer-order Processes, Loops, ...



Radiation Back-reaction

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- But: The Lorentz-force equation does not take this into the back-reaction!

Self-consistent solution of EOM and radiation

$$m_0 \frac{du^\mu}{d\tau} = e(\partial^\mu A^\nu_{\text{total}} - \partial^\mu A^\nu_{\text{total}}) u_\nu$$

$$\oplus$$

$$\Box A_{\text{total}} = e \int d\tau u^\nu(\tau) \delta(x - x(\tau))$$

$$A_{\text{total}} = A_{\text{ext}} + A_{\text{radiation}}$$

Green's function method

$$A^\mu_{\text{total}} = A^\mu_{\text{ext}} + \int d^4x' D(x - x') j^\mu(x')$$

[Paul A. M. Dirac, "Classical theory of radiating electrons", Proc. Roy. Soc. of London, (1938).]

Lorentz-Abraham-Dirac (LAD) equation

$$\underbrace{(m_0 + \delta m)}_{m_{\text{physical}}} \frac{du^\mu}{d\tau} = eF^{\mu\nu}_{\text{ext}} u_\nu + \frac{2}{3}\alpha \left(\ddot{u}^\mu + \dot{u} \cdot \dot{u}\, u^\mu \right)$$

Radiation field completely eliminated $\rightarrow$ effective self-force terms

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- Runaway solutions, pre-acceleration, ...

Landau and Lifshitz argument (LL)

- Whenever $|F_{\mathrm{ext}}| \ll E_{\mathrm{CED}}, B_{\mathrm{CED}}$ in the instantaneous rest frame: Series expansion applicable (in charge e): Replace the four-acceleration in the RR-force by the zero-th order result: Lorentz-force equation

$$m\dot{u}^\mu = eF^{\mu\nu}u_\nu + \frac{2}{3}\alpha\left(\frac{e}{m}(u^\alpha\partial_\alpha F^{\mu\nu})u_\nu - \frac{e^2}{m^2}F^{\mu\nu}F_{\alpha\nu}u^\alpha + \frac{e^2}{m^2}(F^{\alpha\nu}u_\nu)^2u^\mu\right)$$

$F = F_{\mathrm{ext}}$, $m = m_{\mathrm{physical}}$

- Field strengths $F_{\mathrm{CED}} = F_{\mathrm{QED}}/\alpha = \frac{m^2}{e\alpha}$
- Therefore: Low order $\hbar$ corrections are much more important than higher order e corrections!
- H. Spohn, Europhys. Lett. **50**, 287 (2000): all physical solutions of LAD are solutions of LL

Quantum Description: Why Naive Pert Theory Fails

- Lagrangian for QED

$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\Psi}\gamma_\mu\Psi A^\mu$$

- Split the vector potential: $A^\mu \rightarrow A^\mu_{\mathrm{laser}} + A^\mu_{\mathrm{non-laser}}$
- Perturbation Theory: Interaction picture $\Rightarrow$ Feynman diagrams $\Rightarrow$ S-matrix elements

$$S \sim \langle n'_e, n'_{\mathrm{laser}}, n'_{\mathrm{non-laser}} | H_{\mathrm{int}} | n_e, n_{\mathrm{laser}}, n_{\mathrm{non-laser}} \rangle$$

- Estimate for number of laser photons: $n_{\mathrm{laser}} \sim 10^{14} a_0^2/\lambda^3$

Nonlinear Scattering in the presence of quantized radiation fields:

[J. Bergou and S. Varr, J. Phys. A **14** 1469 (1981).]

[J. Bergou and S. Varr, J. Phys. A **14** 2281 (1981).]

This is super-complicated!!!

Coherent Photon States

- Laser light is *coherent light*. Number of photons scattered out of the coherent mode much smaller than the number in the laser mode.
- Coherent states $|c\rangle$ are Eigenstates of annihilation operators:

$$\hat{a}|c\rangle = c|c\rangle$$

- They are displaced vacuum states, with displacement operator $\hat{D} = e^{c\hat{a}^\dagger - c^*\hat{a}}$:

$$|c\rangle = \hat{D}|0\rangle$$

- Generalization to multi-mode fields $\hat{a} \to \hat{\mathcal{A}}^{(+)}$, $c \to C(\boldsymbol{k})$, then:

$$\hat{D}^\dagger\, \hat{\mathcal{A}}\, \hat{D} = \hat{\mathcal{A}} + \underbrace{FT[C]}_{A_{class}}, \qquad \langle C|\hat{\mathcal{A}}|C\rangle = A_{class}$$

- Matrix Elements:

$$\langle f; C|S[\Psi, \bar{\Psi}, \mathcal{A}]|i; C\rangle = \langle f|S[\Psi, \bar{\Psi}, \mathcal{A} + A_{\text{class}}]|i\rangle$$

[T. W. B. Kibble, Phys. Rev. **138**, B740 (1965).]

[C. Harvey et. al. Phys. Rev. A **79**, 063407 (2009).]

Transition to the Furry Picture

- Lagrangian of QED with background field A:

$$
\begin{aligned}
L_e &= \bar{\Psi}(i\not{\partial} - m)\Psi & & & L_e^{(F)} &= \bar{\Psi}(i\not{\partial} - e\not{A} - m)\Psi \\
L_r &= -\tfrac{1}{4}F^2 & &\rightarrow & L_r^{(F)} &= -\tfrac{1}{4}F^2 \\
L_i &= -e\bar{\Psi}\gamma_\mu\Psi(\mathcal{A}^\mu + A^\mu) & & & L_i^{(F)} &= -e\bar{\Psi}\gamma_\mu\Psi\mathcal{A}^\mu
\end{aligned}
$$

- Volkov States are solutions of the Dirac equation in the laser background field A^μ

$$(i\not{\partial} - e\not{A} - m)\Psi = 0$$

- Laser field A^μ is taken into account nonperturbatively
- Use Volkov states as basis set for expansion of the electron field operator

$$\hat{\Psi}_{(F)}(x) = \sum\!\!\!\!\!\!\int_\alpha \; \hat{c}_\alpha \Psi_\alpha^{(+)}(x) + \hat{d}_\alpha^\dagger \Psi_\alpha^{(-)}(x)$$

[D. M. Volkov, Z. Phys **94**, 250 (1935).]

Volkov States

- Volkov States can be written in the following form:

$$\Psi_p(x, A) = \Omega_p[A(\phi)]\psi_p(x)$$

- Ω is *unitary matrix* that acts on the *free* electron wavefunction $\psi_p(x)$

$$\Omega(\phi) = \left(1 + \frac{e\not{k}\not{A}(\phi)}{2kp}\right) \exp\left\{-i\frac{1}{2kp}\int d\phi(2epA - e^2A^2)\right\}$$

- Another representation uses the classical Hamilon-Jacobi action:
$\Psi = \Gamma(\phi)\, e^{-iS_{HJ}}\, u_p$

Volkov States

Explicit solution of the Dirac Equation

- Three momentum components are conserved $\boldsymbol{p}^{\perp}$ and p^{+}, the corresponding operators commute with the Hamiltonian. The quantum number p is the asymptotic electron momentum outside the laser pulse.
- Transform into 2nd order equation
- Ansatz for wave function $\Psi_p(x, A) = \Omega_p(\phi)\, e^{-ipx} u_p$, find solution for Ω_p
- ODE for unknown function $\Omega_p(\phi)$

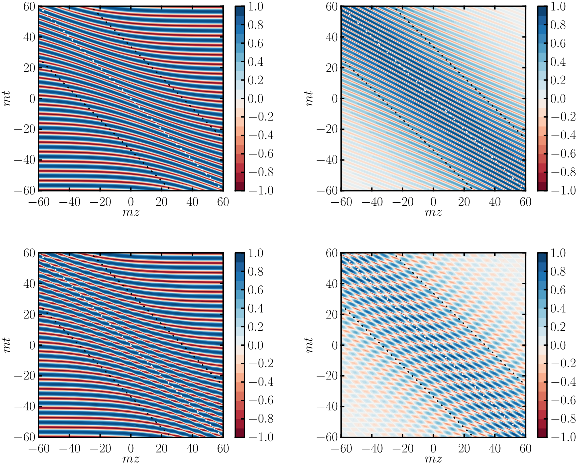
$$2i(kp)\frac{d\Omega_p}{d\phi} = \left(2e(pA) - e^2 A^2 - ie\not{k}\not{A}'\right)\Omega_p$$

- BLACKBOARD

[Landau Lifshitz, Volume 4]

Properties of Volkov States

$$\mathcal{S} = \mathrm{tr}\left[e^{-ipx}\Omega_p(\phi)\right] \qquad \mathcal{T}_{\mu\nu} = \mathrm{tr}\left[e^{-ipx}\sigma_{\mu\nu}\Omega_p(\phi)\right]$$



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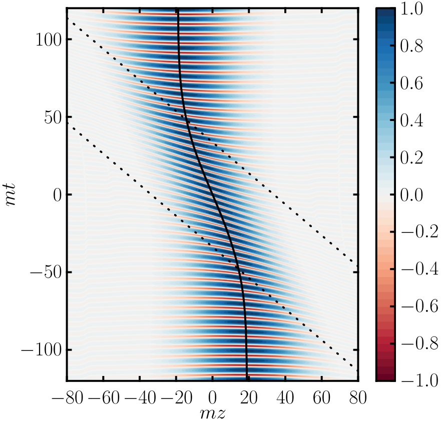
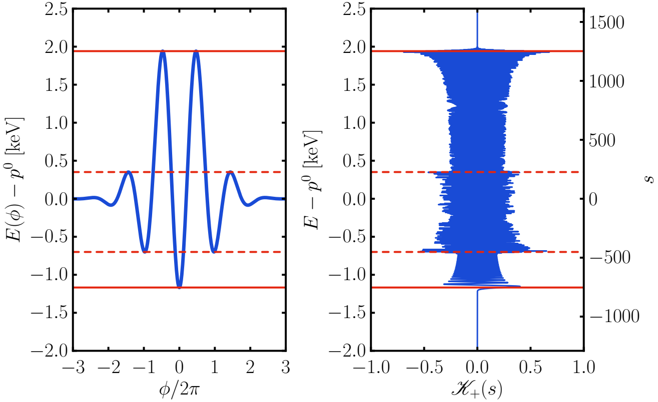


$$\Phi_S(x) = \int \frac{dp^+}{p^+}\, h(p^+)\, e^{-ipx}\, \mathrm{tr}\, \Omega_p(\phi)$$

Momentum Spectrum of Volkov States

$$\Omega_p(\phi) = \int \frac{ds}{2\pi}\, e^{-is\phi} \sum_{j=0,+,-} d_j \mathscr{K}_j(s)$$

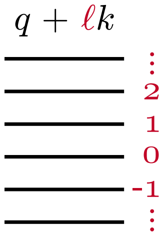


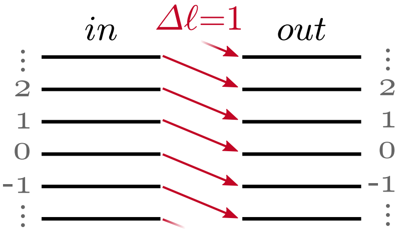
IPW Limit of the Volkov States

IPW limit: Discrete spectrum

$$\mathscr{K}_j(s) \to \sum_n \delta(s - n - U_p^- / k^-) K_j^{(n)}(s)$$

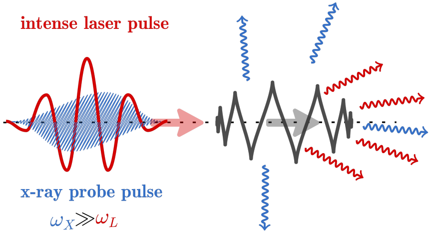
$$\Psi = \sum_n e^{-i(q+nk)x} \Psi_n$$

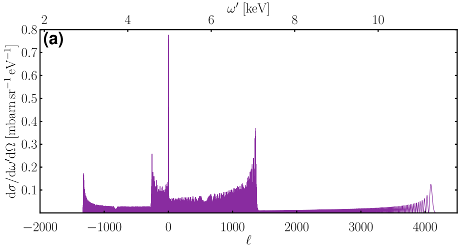




Probing the Momentum Structure of Volkov States

Using x-ray Compton Scattering: Pump-probe set-up





[DS et al, New J. Phys. **18**, 023044 (2016).]

Lippmann-Schwinger Equation

Transform the Dirac equation into an integral equation

$$\Psi_p(x, A) = \psi_p(x) + \int d^4 z G_0(x - z)\, e\not{A}(z)\, \Psi_p(z, A)$$

with iterative solution

$$\Psi_p^{(N)}(x, A) = \sum_n \left[G_0\, e\not{A}(z)\right]^n \psi_p$$