Hadrons and Mutiquark States in Holographic QCD

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Plan of the Talk

- Introduction
- Hadron Stucture in Holographic QCD
  - – Mesons
  - – Baryons
  - – Multiquarks (Deuteron, Tetraquarks)
- Conclusions

Introduction

- 1993 ’t Hooft   Holographic Principle

  Information about string theory contained in some region of space can be represented as “Hologram” (theory which lives on the boundary of that region)

- 1997-1998 Maldacena, Polyakov, Witten et al   AdS/CFT correspondence
  Duality of 4D conformal supersymmetric Yang-Mills and supersting theories

  - Matching partition functions gives relation between parameters

  Strings g_s – coupling, l_s – length, R – AdS radius

  SU(N) YM g_{YM} – coupling, ’t Hooft coupling $\lambda = g_{YM}^2 N$

  $2\pi g_s = g_{YM}^2\,, \qquad \frac{R^4}{l_s^4} = 2\, g_{YM}^2\, N$

  - Symmetry arguments: Conformal group acting in boundary theory isomorphic to $SO(4,2)$ – the isometry group of AdS_5 space

Introduction

- 't Hooft limit (large N at λ fixed) $g_{YM} = \frac{\lambda}{N} \ll 1$

  corresponds $g_s \ll 1$ (tree-level perturbative string threory)

  “Conformal Field side” of duality works

- Strong coupling limit $\lambda \gg 1$ means $l_s \ll R$

  Supergravity limit (closed strings shrink to point-like particles)

  “String Theory side” of duality works

Introduction

- AdS/CFT $\to$ AdS/QCD upon breaking conformal invariance
- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space
- HQCD models reproduce main features of QCD at low and high energies: chiral symmetry, confinement, power scaling of hadron form factors
- Physical interpretation of extra 5th dimension as Scale

Introduction

- Top-down approaches Low-energy approximation of string theory trying to find a gravitational background with features similar to QCD (e.g. Sakai-Sugimoto model)

- Bottom-up approaches More phenomenological use the features of QCD to construct 5d dual theory including gravity on AdS space

- Towards to QCD:
  – Break conformal invariance and generate mass gap
  – Tower of normalized bulk fields (Kaluza-Klein modes) $\leftrightarrow$ Hadron wave functions
  – Spectrum of Kaluza-Klein modes $\leftrightarrow$ Hadrons spectrum

- Hard-wall:
  AdS geometry is cutted by two branes UV $(z = \epsilon \to 0)$ and IR $(z = z_{\rm IR})$
  Analogue of quark bag model, linear dependence on $J(L)$ of hadron masses
  Covariant constituent quark model with infrared confinement (Mikhail Ivanov et al)

- Soft-wall:
  Soft cuttoff of AdS space by dilaton field $e^{-\varphi(z)}$
  Analytical solution of EOM, Regge behavior $M^2 \sim J(L)$

Introduction

- AdS metric   Poincaré form

  $$ds^2 = g_{MN}(z)\, dx^M dx^N = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)$$ R – AdS radius

- Metric Tensor $g_{MN}(z) = \epsilon^a_M(z)\epsilon^b_N(z)\eta_{ab}$

- Vielbein $\epsilon^a_M(z) = \frac{R}{z}\,\delta^a_M$ (relates AdS and Lorentz metric)

- Manifestly scale-invariant $x \to \lambda x$, $z \to \lambda z$.

- z – extra dimensional (holographic) coordinate;
  $z = 0$ is UV boundary , $z = \infty$ is IR boundary

- Five Dimensions: L = Length, W = Width, H = Height, T = Time, S = Scale

Introduction

- Action for scalar field

$$S_\Phi = \frac{1}{2}\int d^dx dz\,\sqrt{g}\,e^{-\varphi(z)}\left(\partial_M\Phi(x,z)\partial^M\Phi(x,z) - m^2\,\Phi^2(x,z)\right)$$

- Dilaton field $\varphi(z) = \kappa^2 z^2$
- $g = |\mathrm{det} g_{MN}|$
- m – 5d mass, $m^2 R^2 = \Delta(\Delta - 4)$, $\Delta = 3$ conformal dimension
- Kaluza-Klein (KK) expansion $\Phi(x,z) = \sum\limits_n \phi_n(x)\,\Phi_n(z)$
- Tower of KK modes $\phi_n(x)$ dual to 4-dimensional fields describing hadrons
- Bulk profiles $\Phi_n(z)$ dual to one-dimensional hadronic wave functions

Introduction

- Use $-\partial_\mu \partial^\mu \phi_n(x) = M_n^2 \phi_n(x)$
- Substitute $\Phi_n(z) = \left(\frac{R}{z}\right)^{1-d} \phi_n(z)$
- Identify $\Delta = \tau = N + L$ (here $N = 2$ – number of partons in meson)

  Here τ is twist = Canonical dimension - Sum of spins

  Examples: mesons $\tau = 2 \times 3/2 - 2 \times 1/2 = 2$
  baryons $\tau = 3 \times 3/2 - 3 \times 1/2 = 3$

$$\left[-\frac{d^2}{dz^2} + \frac{4L^2 - 1}{4z^2} + \kappa^4 z^2 - 2\kappa^2 \right] \phi_n(z) = M_n^2 \phi_n(z)$$

- Solutions:
  $\phi_{nL}(z) = \phi_{n,\tau-2}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+1)}}\, \kappa^{L+1}\, z^{L+1/2}\, e^{-\kappa^2 z^2/2}\, L_n^L(\kappa^2 z^2)$
- $M_{nL}^2 = 4\kappa^2 \left(n + \frac{L}{2}\right) = 4\kappa^2 \left(n + \frac{\tau}{2} - 1\right)$
- Massless pion $M_\pi^2 = 0$ for $n = L = 0$ Brodsky, Téramond

Introduction

- Why dilaton is quadratic ?
- It generates quadratic confinement potential $U(z) = \kappa^4 z^2$ in the squared equation of motion which equivalent to squared Lattice QCD linear potential
  Trawinski et al, PRD90 (2014) 074017
  Gutsche et al, PRD90 (2014) 096007
- $U(z) = \kappa^4 z^2 \quad \longleftrightarrow \quad U^2_{\rm Lattice}(z) = \sigma^2 z^2$
- Therefore $\kappa^2 = \sigma$ (string tension)
- In both approaches $\kappa \sim \sqrt{\sigma} \sim 0.5$ GeV

Introduction

- “Positive dilaton”: Brodsky, Téramond

$$S_\Phi^+ = \frac{1}{2}\int d^dx dz\,\sqrt{g}\,e^{\varphi(z)}\left[\partial_M\Phi_+\partial^M\Phi_+ - m^2\,\Phi_+^2\right]$$

- “Negative dilaton”: Gutsche, Lyubovitskij, Schmidt, Vega PRD 85 (2012) 076003

$$S_\Phi^- = \frac{1}{2}\int d^dx dz\,\sqrt{g}\,e^{-\varphi(z)}\left[\partial_M\Phi_-\partial^M\Phi_- - (m^2 + U(z))\,\Phi_-^2\right]$$

Potential

$$U(z) = \frac{z^2}{R^2}\left(\varphi''(z) + \frac{1-d}{z}\varphi'(z)\right)$$

- “No-wall”

$$S_\Phi = \frac{1}{2}\int d^dx dz\sqrt{g}\left[\partial_M\Phi\partial^M\Phi - (m^2 + V(z))\Phi^2\right]$$

Potential

$$V(z) = \frac{z^2}{R^2}\left(\frac{1}{2}\varphi''(z) + \frac{1}{4}(\varphi'(z))^2 + \frac{1-d}{2z}\varphi'(z)\right)$$

- All 3 actions are equivalent upon field rescaling $\Phi_\pm = e^{\mp\varphi(z)}\Phi_\mp = e^{\mp\varphi(z)/2}\Phi$

Introduction

- Extension to AdS fermions (baryons)

$$S_\psi = \int d^d x dz\, \sqrt{g}\, \bar{\Psi}(x,z) \Big(\not{D} - \mu - \varphi(z)/R \Big) \Psi(x,z)$$

- Field decomposition (left/right) and KK expansion

$$\Psi(x,z) = \Psi_L(x,z) + \Psi_R(x,z) \quad \Psi_{L/R} = \frac{1 \mp \gamma^5}{2} \Psi$$

$$\Psi_{L/R}(x,z) = \sum_n \Psi^n_{L/R}(x)\, F^n_{L/R}(z)$$

- EOM

$$\left[-\partial_z^2 + \kappa^4 z^2 + 2\kappa^2 \Big(\mu R \mp \tfrac{1}{2} \Big) + \frac{\mu R(\mu R \pm 1)}{z^2} \right] F^n_{L/R}(z) = M_n^2\, F^n_{L/R}(z)$$

Solutions (for $d = 4$ and $\mu R = L + 3/2$)

- Bulk profiles

$$F^n_L(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+3)}}\, \kappa^{L+3}\, z^{L+9/2}\, e^{-\kappa^2 z^2/2}\, L_n^{L+2}(\kappa^2 z^2)$$

$$F^n_R(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+2)}}\, \kappa^{L+2}\, z^{L+7/2}\, e^{-\kappa^2 z^2/2}\, L_n^{L+1}(\kappa^2 z^2)$$

- Mass spectrum: $M^2_{nL} = 4\kappa^2 \Big(n + L + 2 \Big)$

Introduction

- Extension to higher-spin AdS fields

  Fradkin, Vasiliev, Buchbinder, Lavrov, Metzaev, Pashnev, ...

  E.g. boson case $\Phi \ \rightarrow \ \Phi_{M_1 M_2 \cdots M_J}$

  5d mass $m^2 R^2 \ \rightarrow \ m_J^2 R^2 = (\Delta - J)(\Delta + J - 4)$

  Dilaton potential

$$U_J(z) = \frac{z^2}{R^2} \left(\varphi''(z) + \frac{1 + 2J - d}{z} \varphi'(z) \right)$$

  Solutions

- $\phi_{nL}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L)}} \, \kappa^{L+1} \, z^{L+1/2} \, e^{-\kappa^2 z^2/2} \, L_n^L(\kappa^2 z^2)$

- $M_{nLJ}^2 = 4\kappa^2 \Big(n + \frac{L+J}{2} \Big) \rightarrow 4\kappa^2 \Big(n + J \Big)$ at large J

Introduction

- Scattering problem for AdS field gives information about propagation of external field from z to the boundary $z = 0$ — bulk-to-boundary propagator $\Phi_{\rm ext}(q, z)$ [Fourier-trasform of AdS field $\Phi_{\rm ext}(x, z)$]:
  $\Phi_{\rm ext}(q, z) = \int d^dx e^{-iqx}\, \Phi_{\rm ext}(x, z)$

- Vector field as example
  $$\partial_z \left(\frac{e^{-\varphi(z)}}{z} \partial_z V(q, z) \right) + q^2\, \frac{e^{-\varphi(z)}}{z} V(q, z) = 0\,.$$
  $$V(Q, z) \;=\; \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right)$$
  Consistent with GI, fulfills UV and IR boundary conditions :
  $V(Q, 0) = 1\,,\;\; V(Q, \infty) = 0$

- Hadron form factors

$$F_\tau(Q^2) = \langle \phi_\tau | \hat{V}(Q) | \phi_\tau \rangle = \int\limits_0^\infty dz\, \phi_\tau^2(z)\, V(Q, z) = \frac{\Gamma(\tau)\,\Gamma(a + 1)}{\Gamma(a + \tau)}$$

is implemented by a nontrivial dependence of AdS fields on 5-th coordinate

Introduction

- Power scaling at large Q^2

$$F_\tau(Q^2) \sim \frac{1}{(Q^2)^{\tau-1}}$$

Quark counting rules: Matveev-Muradyan-Tavhelidze-Brodsky-Farrar 1973

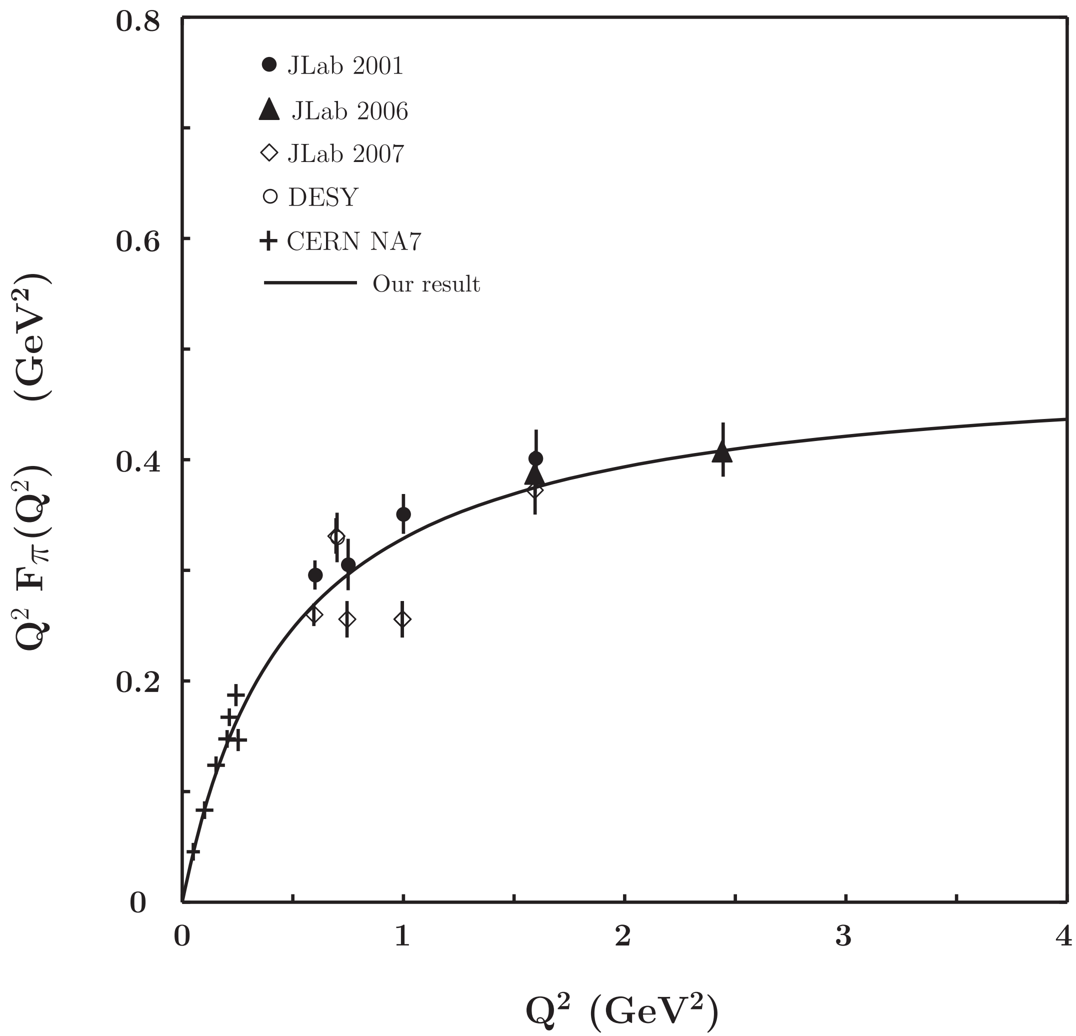
$$\mathrm{Pion}: \quad \frac{1}{Q^2}$$

$$\mathrm{Nucleon(Dirac)}: \quad \frac{1}{Q^4}$$

$$\mathrm{Nucleon(Pauli)}: \quad \frac{1}{Q^6}$$

$$\mathrm{Deuteron(Charge)}: \quad \frac{1}{Q^{10}}$$

Mesons: pion form factor



Before done in quark models: A. Dorokhov et al., M. Ivanov et al., C. Roberts et al.

LFWFs motivated by holographic QCD

- Matching matrix elements (e.g. form factors) in HQCD and LF QCD

- Drell-Yan-West formula

$$F_\tau(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \, \psi_\tau^\dagger(x, \mathbf{k}'_\perp) \, \psi_\tau(x, \mathbf{k}_\perp) \, ,$$

where $\psi(x, \mathbf{k}_\perp) \equiv \psi(x, \mathbf{k}_\perp; \mu_0)$, $\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$, and $Q^2 = \mathbf{q}_\perp^2$

- HQCD

$$F_\tau(Q^2) = \int_0^\infty dz \, V(Q, z) \, \varphi_\tau^2(z) = \frac{\Gamma(\frac{Q^2}{4\kappa^2} + 1) \, \Gamma(\tau)}{\Gamma(\frac{Q^2}{4\kappa^2} + \tau)} \, .$$

- Result for effective LFWF at the initial scale μ_0

$$\psi_\tau(x, \mathbf{k}_\perp) = \sqrt{\tau - 1} \, \frac{4\pi}{\kappa} \, \sqrt{\log(1/x)} \, (1-x)^{\frac{\tau-4}{2}} \, \exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2} \, \frac{\log(1/x)}{(1-x)^2} \right]$$

Mesons: Light-Front Wave Function

- Mesonic WF longitudinal part and quark masses

$$\phi_{nJ}(z,x,m_1,m_2) = \phi_{nL}(z)\, f(x,m_1,m_2)$$

- Modified meson mass formula

$$M_{nJ}^2 = 4\kappa^2\left(n + \frac{L+J}{2}\right) + \int\limits_0^1 dx\left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right) f^2(x,m_1,m_2)$$

- Leptonic decay constants $P^- \to \ell^- \bar{\nu}_\ell$

$$f_M = \kappa \frac{\sqrt{6}}{\pi} \int\limits_0^1 dx \sqrt{x(1-x)}\, f(x,m_1,m_2)$$

- Find $f(x,m_1,m_2)$ to fulfill the following constraints

- In sector of light quarks (consistency with ChPT):

Gell-Mann-Oakes-Renner (GMOR) $M_\pi^2 = 2\hat{m}\, B$
Gell-Mann-Okubo (GMO) $4M_K^2 = M_\pi^2 + 3M_\eta^2$

Mesons: Light-Front Wave Function

- In sector of heavy quarks (consistency with HQET)
- Leptonic decay constants

$f_{Q\bar{q}} \sim 1/\sqrt{m_Q}$ heavy–light mesons

$f_{Q\bar{Q}} \sim \sqrt{m_Q}$ heavy quarkonia

$f_{c\bar{b}} \sim m_c/\sqrt{m_b}$ at $m_c \ll m_b$

- Mass spectrum

  Expansion

$$\begin{aligned} M_{Q\bar{q}} &= m_Q + \bar{\Lambda} + \mathcal{O}(1/m_Q) \\ M_{Q\bar{Q}} &= 2m_Q + E + \mathcal{O}(1/m_Q) \end{aligned}$$

Splitting

$$M^V_{Q\bar{q}} - M^P_{Q\bar{q}} \sim \frac{1}{m_Q}$$

Light Mesons

- Following 't Hooft NPB 75 (1974) 461

$$f(x, m_1, m_2) = N\, x^{\alpha_1}\, (1-x)^{\alpha_2}$$

where N is the normalization constant

$$1 = \int_0^1 dx\, f^2(x, m_1, m_2)$$

α_1, α_2 are parameters fixed in order to get consistency with QCD.

- Light quark sector $\alpha_i = m_i/(2B)$

$$B = |\langle 0|\bar{u}u|0\rangle|/F_\pi^2$$

is the quark condensate parameter

- Leptonic decay constants in chiral limit

$$f_\pi = f_K = f_\rho = 3f_\omega = \frac{3f_\phi}{\sqrt{2}} = \kappa\frac{\sqrt{6}}{8}\,.$$

Heavy Mesons

- Heavy–light mesons $\alpha_Q = \alpha = \mathcal{O}(1)$

$$\alpha_q = \frac{2\alpha_Q}{m_Q}\left(1 + \frac{\bar{\Lambda}}{2m_Q}\right) - \frac{1}{2}\ .$$

Leds to

$$M_{Qq} = m_Q + \bar{\Lambda} + \mathcal{O}(1/m_Q)$$

$$M^2_{Q\bar{q}} = 4\kappa^2\left(n + \frac{L+J}{2}\right) + (m_Q + \bar{\Lambda})^2$$

$$f_{Q\bar{q}} = \frac{\kappa\sqrt{6}}{\pi}\,\frac{2\sqrt{\alpha}}{\alpha + \frac{3}{2}}\sqrt{\frac{\bar{\Lambda}}{m_Q}} \sim \sqrt{\frac{1}{m_Q}}$$

Heavy Mesons

- Heavy Quarkonia

$$\alpha_{Q_i} = \frac{m_{Q_i}}{4E}\left(1 - \frac{E}{2(m_{Q_1} + m_{Q_2})}\right) + \mathcal{O}\Big(\frac{1}{m_{Q_i}}\Big)$$

$$\kappa = \beta\left(\frac{\mu_{Q_1 Q_2}}{E}\right)^{1/4}\left(\frac{m_{Q_1} + m_{Q_2}}{E}\right)^{1/2},$$

where $\beta = \mathcal{O}(1)$ and $\mu_{Q_1 Q_2} = m_{Q_1} m_{Q_2}/(m_{Q_1} + m_{Q_2})$.

$$M^2_{Q_1 \bar{Q}_2} = 4\kappa^2\left(n + \frac{L+J}{2}\right) + (m_{Q_1} + m_{Q_2} + E)^2$$

and

$$f_{Q\bar{Q}} \sim \sqrt{\frac{m_Q}{E}}\,.$$

Mesons Masses: choice of parameters

- Dilaton parameter $\kappa = 500$ MeV
- Current quark massses

$$m_{u/d} = 7 \text{ MeV}\,, \quad m_s = 24 m_{u/d} = 168 \text{ MeV}$$
$$m_c = 1.275 \text{ GeV}\,, \quad m_b = 4.18 \text{ GeV}$$

Mesons Masses: Results

Masses of light mesons

Meson	n	L	S	Mass [MeV]			
π	0,1,2,3	0	0	140	1010	1421	1738
K	0	0,1,2,3	0	495	1116	1498	1801
η	0,1,2,3	0	0	566	11494	1523	1822
$f_0[\bar{n}n]$	0,1,2,3	1	1	721	1233	1587	1876
$f_0[\bar{s}s]$	0,1,2,3	1	1	985	1404	1723	1993
$\rho(770)$	0,1,2,3	0	1	721	1233	1587	1876
$\omega(782)$	0,1,2,3	0	1	721	1233	1587	1876
$\phi(1020)$	0,1,2,3	0	1	985	1404	1723	1993
$a_1(1260)$	0,1,2,3	1	1	1010	1421	1738	2005

Mesons Masses: Results

Masses of heavy-light mesons and heavy quarkonia

Meson	J^{P}	n	L	S	Mass [MeV]			
$D(1870)$	0^-	0	0,1,2,3	0	1870	2000	2121	2235
$D^*(2010)$	1^-	0	0,1,2,3	1	2000	2121	2235	2345
$D_s(1969)$	0^-	0	0,1,2,3	0	1970	2093	2209	2320
$D_s^*(2107)$	1^-	0	0,1,2,3	1	2093	2209	2320	2425
$B(5279)$	0^-	0	0,1,2,3	0	5280	5327	5374	5420
$B^*(5325)$	1^-	0	0,1,2,3	1	5336	5374	5420	5466
$B_s(5366)$	0^-	0	0,1,2,3	0	5370	5416	5462	5508
$B_s^*(5413)$	1^-	0	0,1,2,3	1	5416	5462	5508	5553

Mesons Masses: Results

Masses of heavy quarkonia

Meson	J^{P}	n	L	S	Mass [MeV]			
$\eta_c(2980)$	0^-	0,1,2,3	0	0	2975	3477	3729	3938
$\psi(3097)$	1^-	0,1,2,3	0	1	3097	3583	3828	4032
$\chi_{c0}(3415)$	0^+	0,1,2,3	1	1	3369	3628	3843	4038
$\chi_{c1}(3510)$	1^+	0,1,2,3	1	1	3477	3729	3938	4129
$\chi_{c2}(3555)$	2^+	0,1,2,3	1	1	3583	3828	4032	4219
$\eta_b(9390)$	0^-	0,1,2,3	0	0	9337	9931	10224	10471
$\Upsilon(9460)$	1^-	0,1,2,3	0	1	9460	10048	10338	10581
$\chi_{b0}(9860)$	0^+	0,1,2,3	1	1	9813	10110	10359	10591
$\chi_{b1}(9893)$	1^+	0,1,2,3	1	1	9931	10224	10471	10700
$\chi_{b2}(9912)$	2^+	0,1,2,3	1	1	10048	10338	10581	10808
$B_c(6277)$	0^-	0,1,2,3	0	0	6277	6719	6892	7025

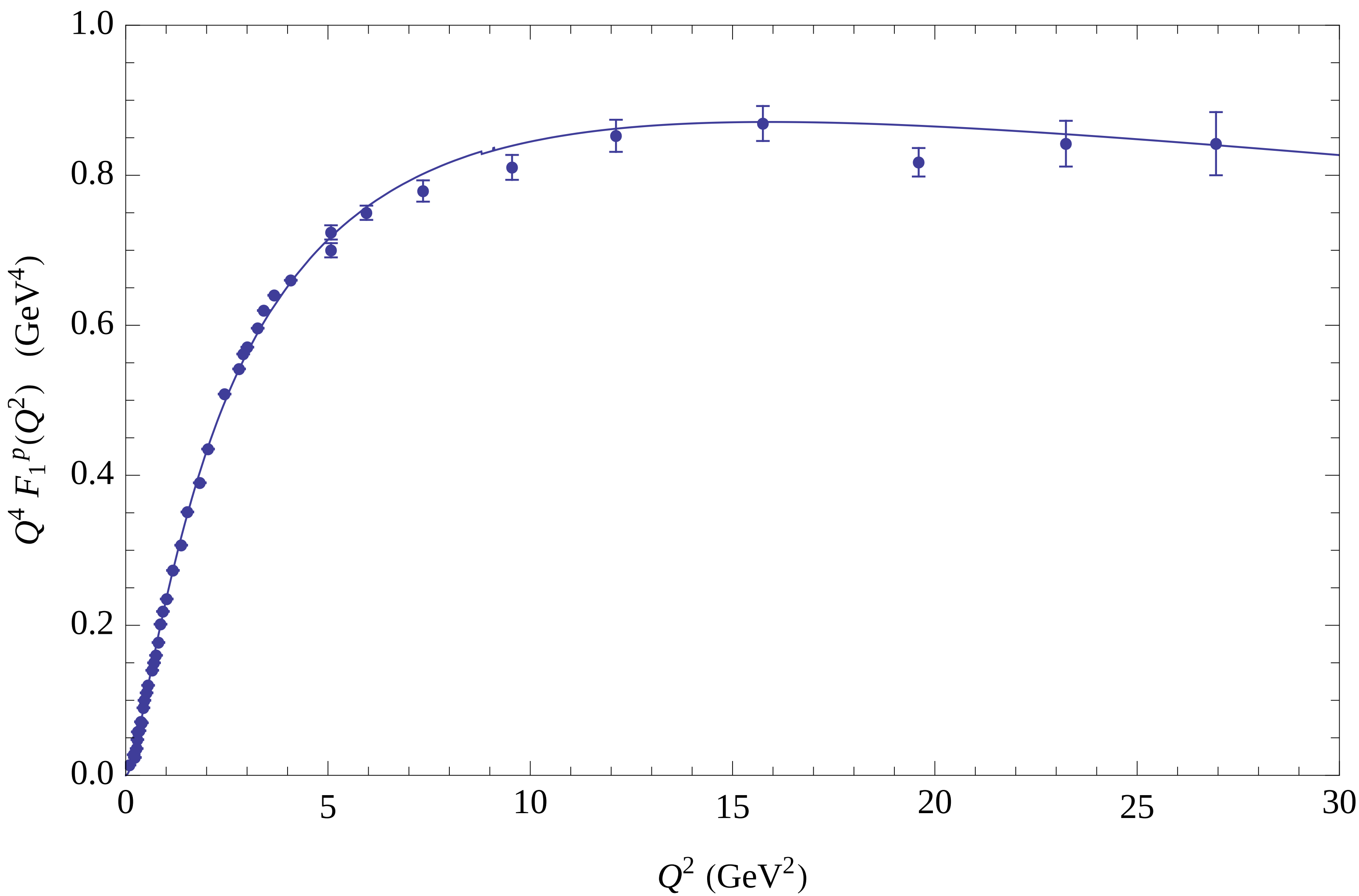
Electromagnetic stucture of nucleons

Scale parameter $\kappa = 383$ MeV

Mass and electromagnetic properties of nucleons

Quantity	Our results	Data
m_p (GeV)	0.93827	0.93827
μ_p (in n.m.)	2.793	2.793
μ_n (in n.m.)	-1.913	-1.913
r_E^p (fm)	0.840	0.8768 $\pm$ 0.0069
$\langle r_E^2 \rangle^n$ (fm^2)	-0.117	-0.1161 $\pm$ 0.0022
r_M^p (fm)	0.785	0.777 $\pm$ 0.013 $\pm$ 0.010
r_M^n (fm)	0.792	0.862$^{+0.009}_{-0.008}$
r_A (fm)	0.667	0.67$\pm$0.01

Nucleon as quark-scalar diquark in LFQM



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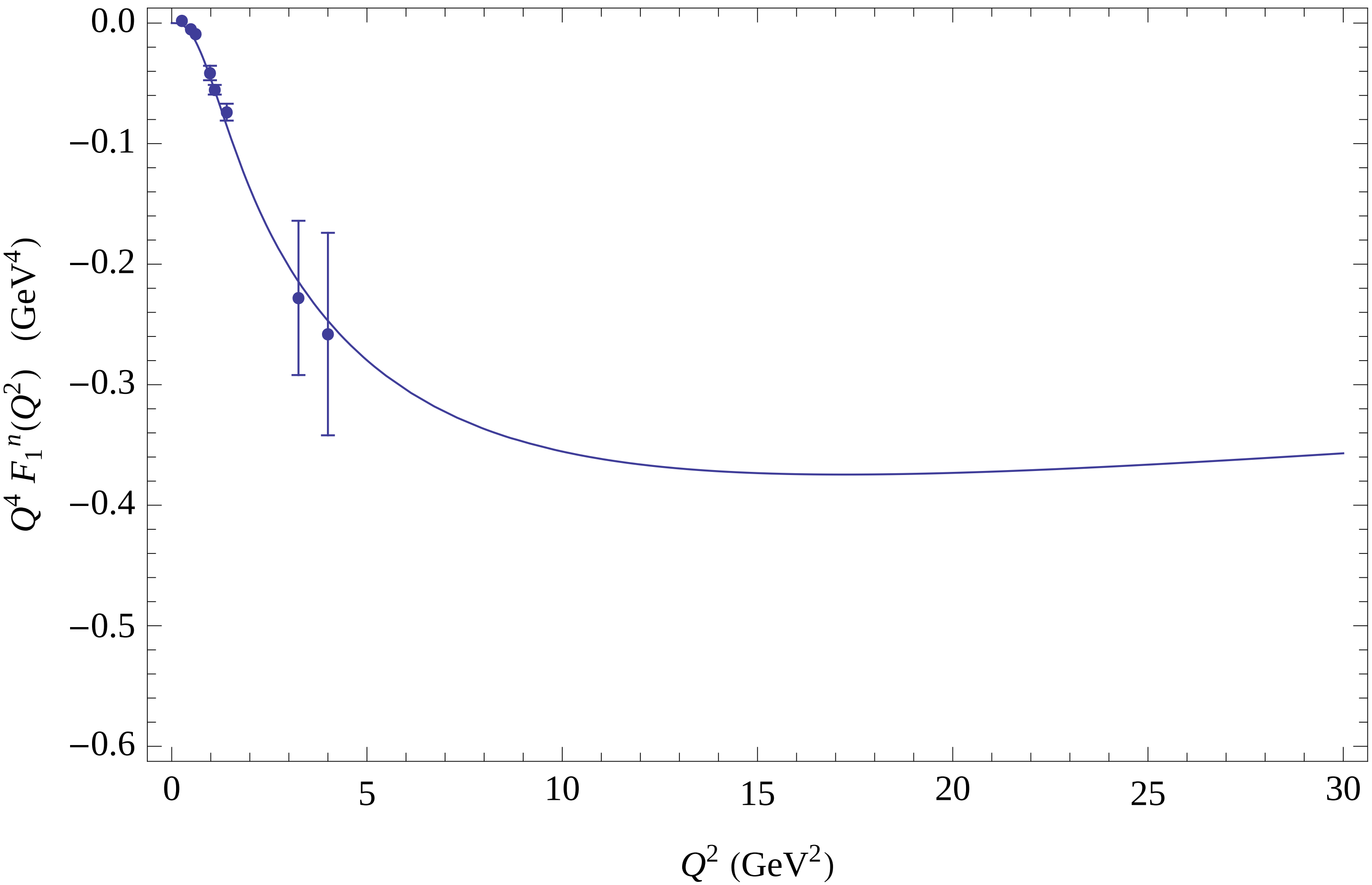

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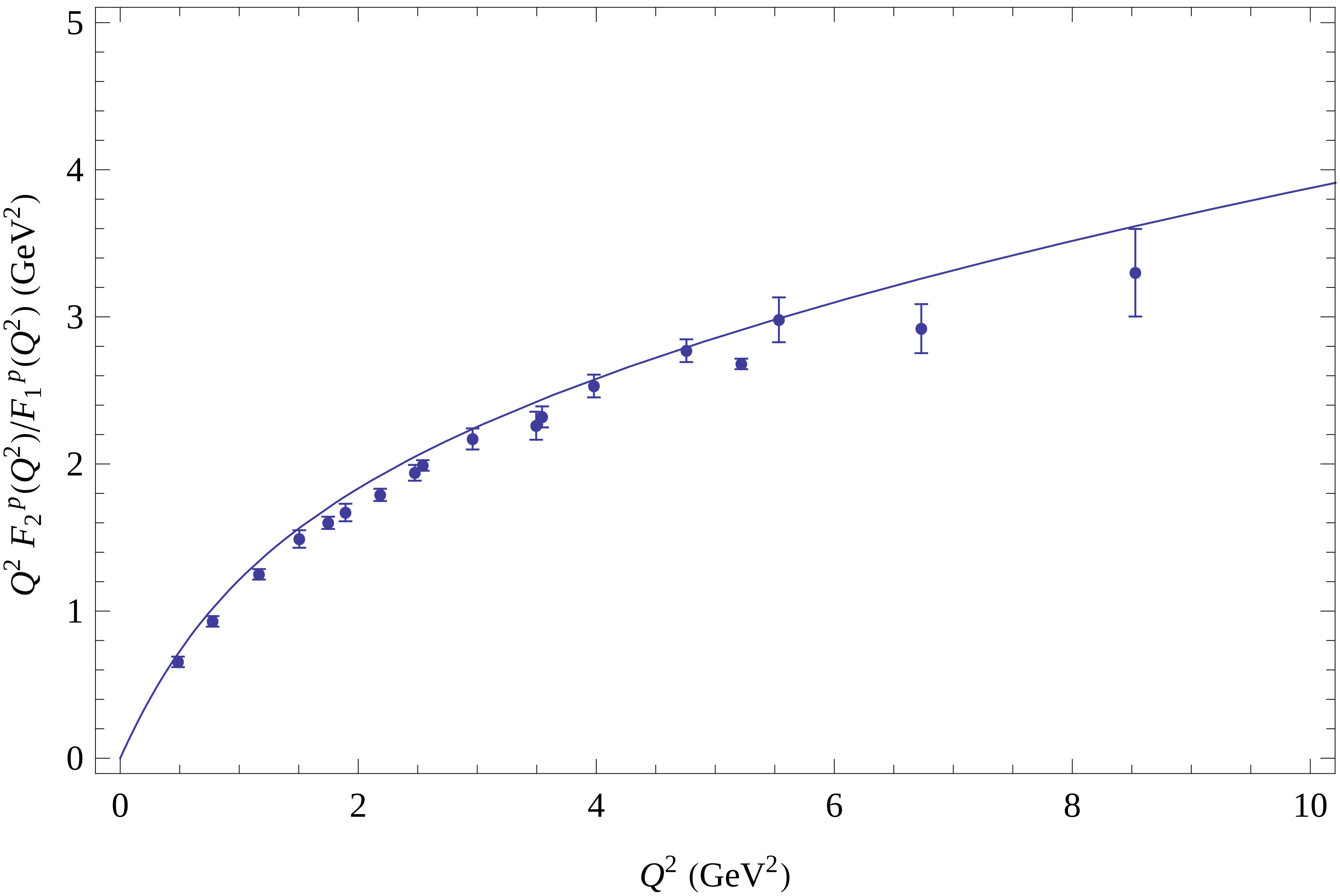

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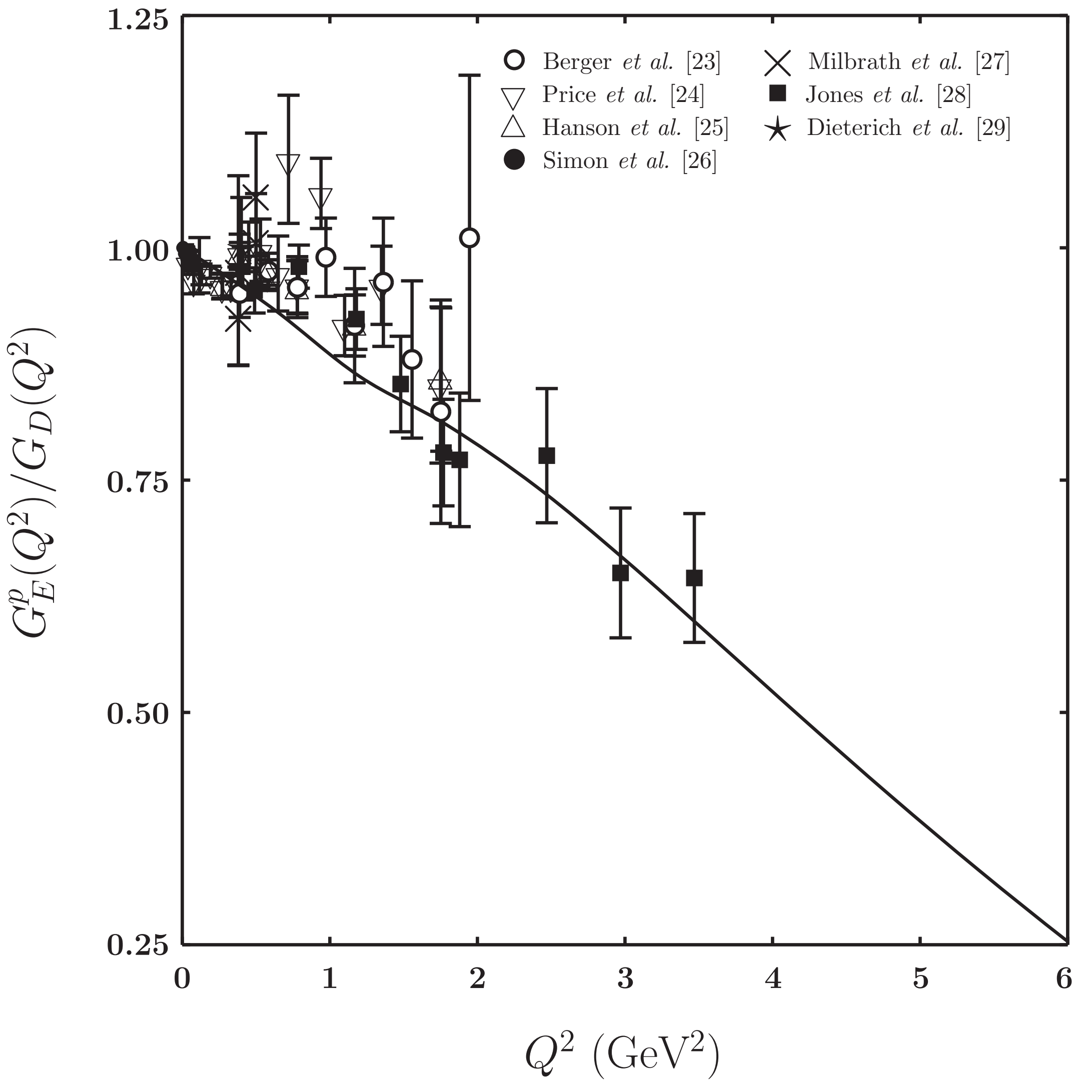

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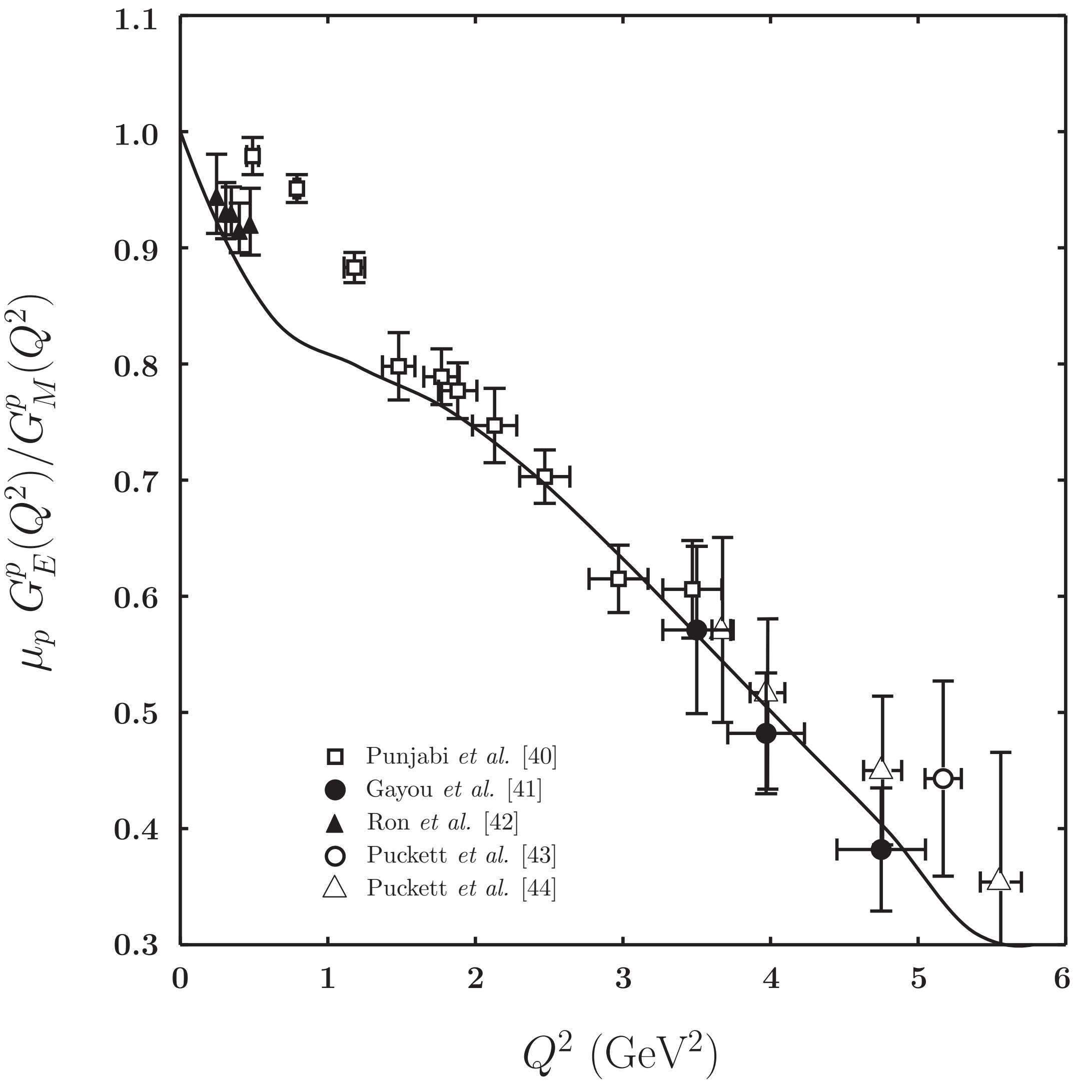

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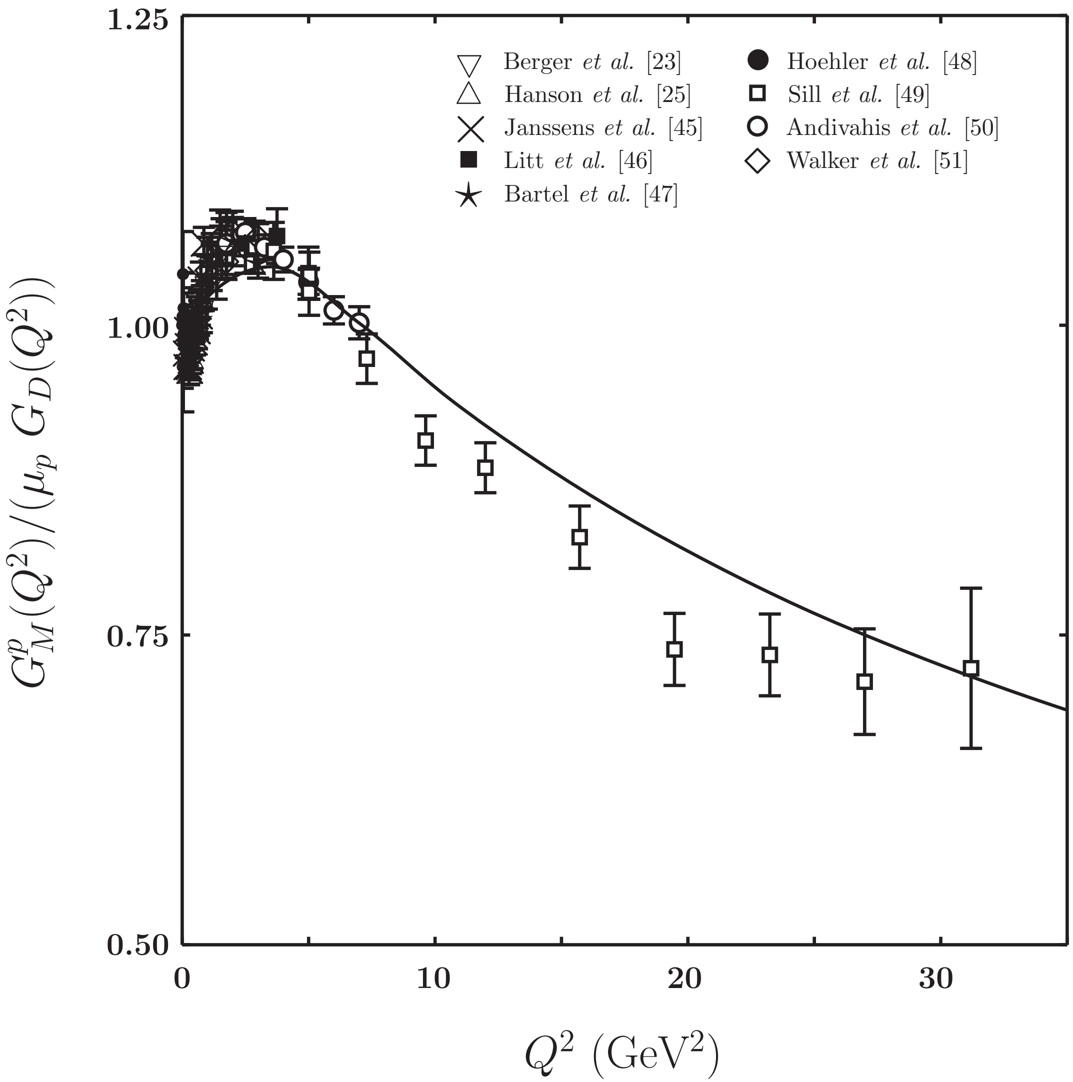
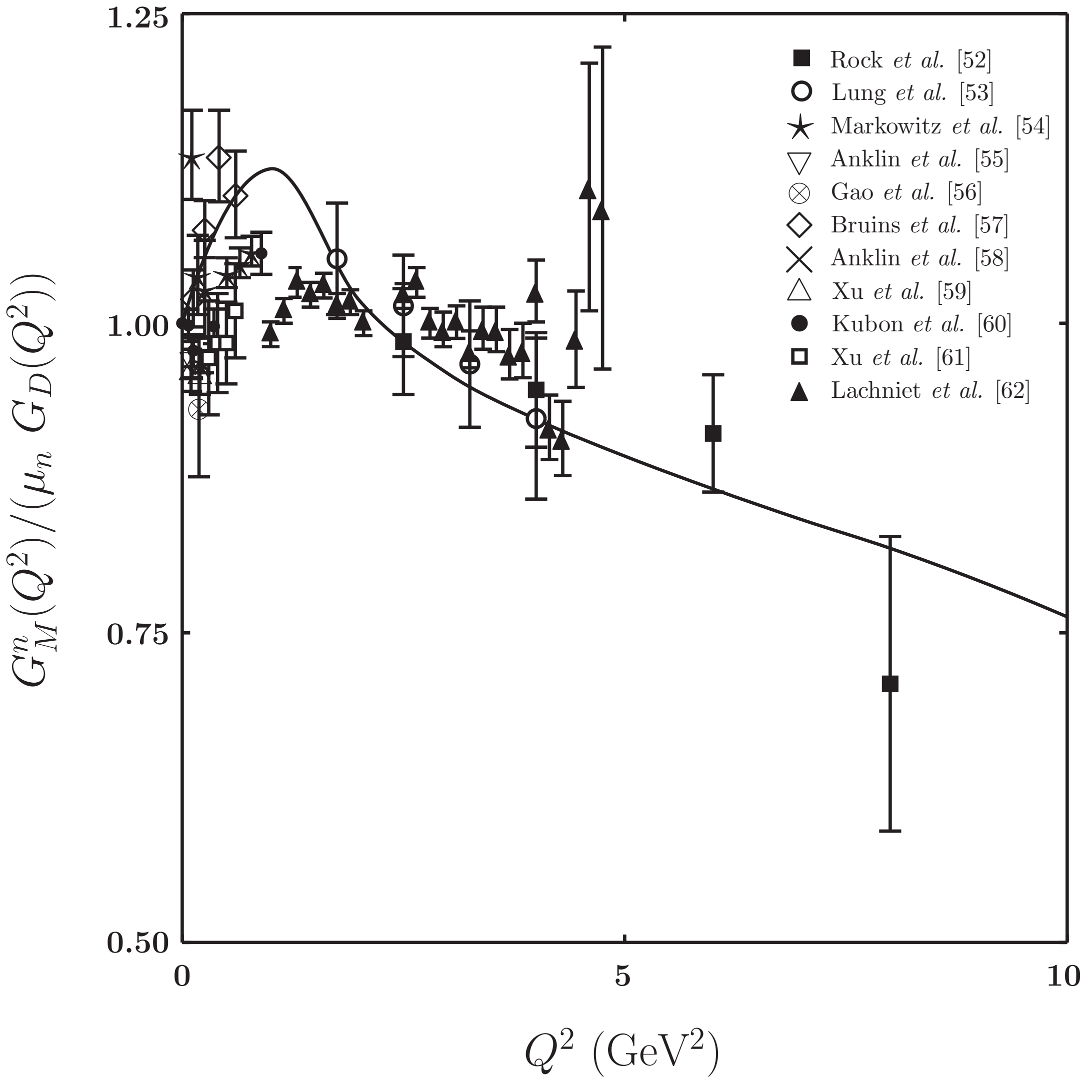
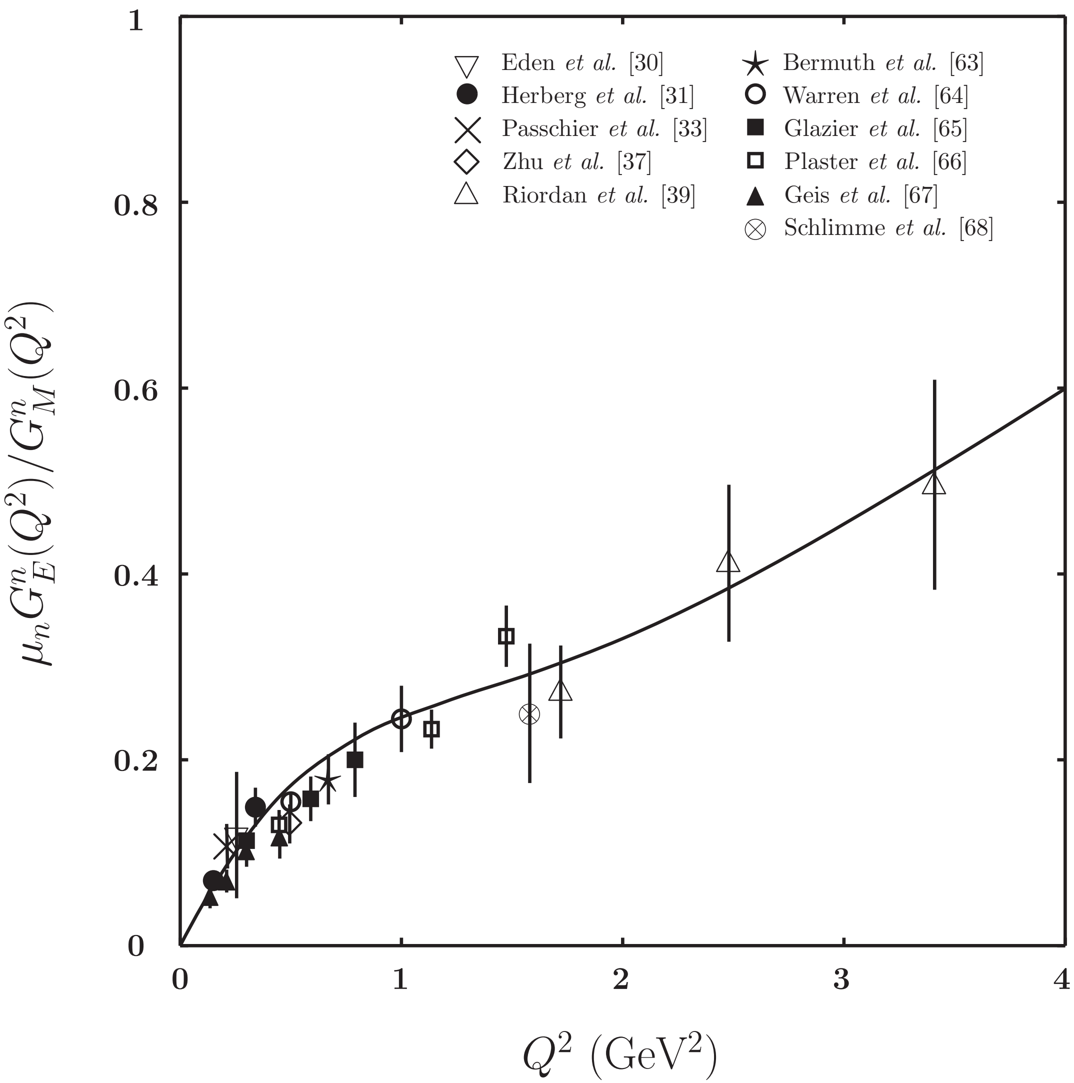


Nucleon as quark-scalar diquark in LFQM



Nucleon as quark-scalar diquark in LFQM



Roper resonance $N(1440)$

- Put $n = 1$ and get solutions dual to Roper:

$$M_{\mathcal{R}} \simeq 1440 \text{ MeV}$$

- $N \to R + \gamma$ transition

$$M^{\mu} = \bar{u}_{\mathcal{R}} \left[\gamma_{\perp}^{\mu} \, F_1(q^2) \, + \, i\sigma^{\mu\nu} \, \frac{q_{\nu}}{M_{\mathcal{R}}} \, F_2(q^2) \right] u_N \, , \quad \gamma_{\perp}^{\mu} = \gamma^{\mu} - q^{\mu} \, \frac{\not{q}}{q^2}$$

- Helicity amplitudes

$$\begin{aligned} H_{\pm\frac{1}{2}0} &= \sqrt{\frac{Q_-}{Q^2}} \left(F_1 M_+ - F_2 \frac{Q^2}{M_{\mathcal{R}}} \right) \\ H_{\pm\frac{1}{2}\pm1} &= -\sqrt{2Q_-} \left(F_1 + F_2 \frac{M_+}{M_{\mathcal{R}}} \right) \end{aligned}$$

- Alternative set [Weber, Capstick, Copley et al]

$$\begin{aligned} A_{1/2} &= -b \, H_{\frac{1}{2}1} \, , \quad S_{1/2} \, = \, b \, \frac{|\mathbf{p}|}{\sqrt{Q^2}} \, H_{\frac{1}{2}0} \, , \\ Q_{\pm} &= M_{\pm}^2 + Q^2 \, , \quad M_{\pm} = M_{\mathcal{R}} \pm M_N \, , \quad b \, = \, \sqrt{\frac{\pi\alpha}{2EM_{\mathcal{R}}M_N}} \end{aligned}$$

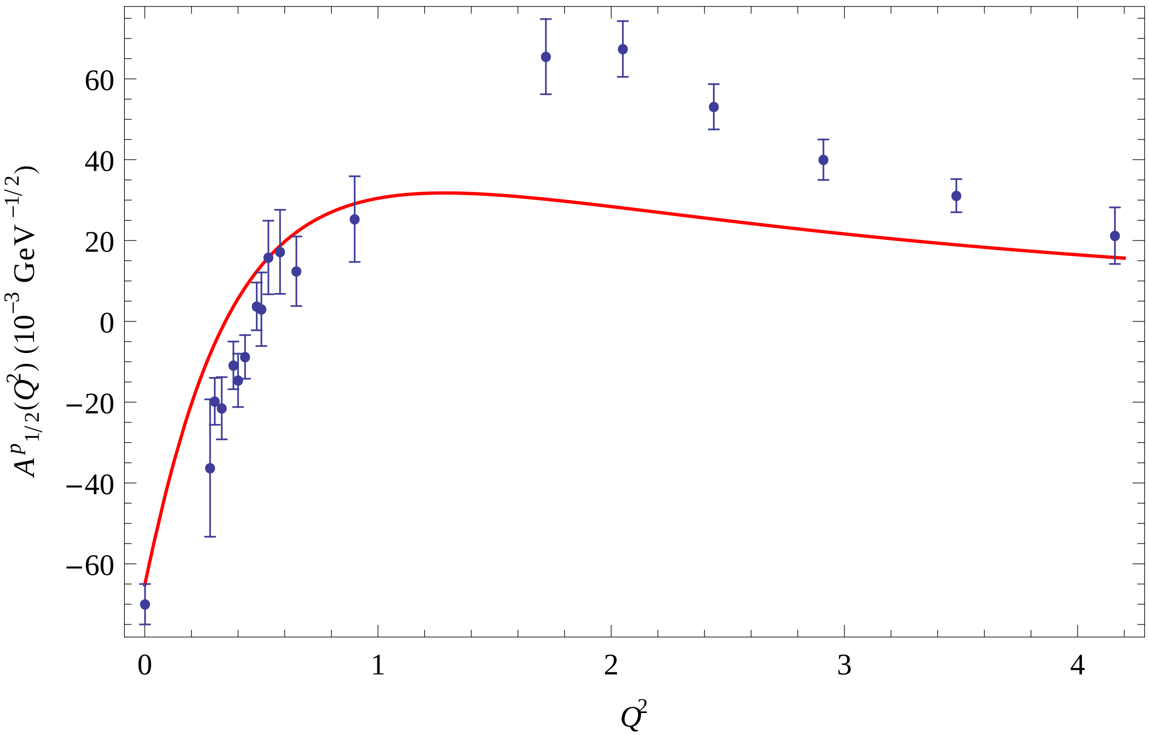
Roper resonance $N(1440)$

Helicity amplitudes $A^N_{1/2}(0)\,,\ S^N_{1/2}(0)$

Quantity	Our results	Data
$A^p_{1/2}(0)$ (GeV$^{-1/2}$)	-0.065	-0.065 $\pm$ 0.004
$A^n_{1/2}(0)$ (GeV$^{-1/2}$)	0.040	0.040 $\pm$ 0.010
$S^p_{1/2}(0)$ (GeV$^{-1/2}$)	0.040	
$S^p_{1/2}(0)$ (GeV$^{-1/2}$)	-0.040	

Roper resonance $N(1440)$

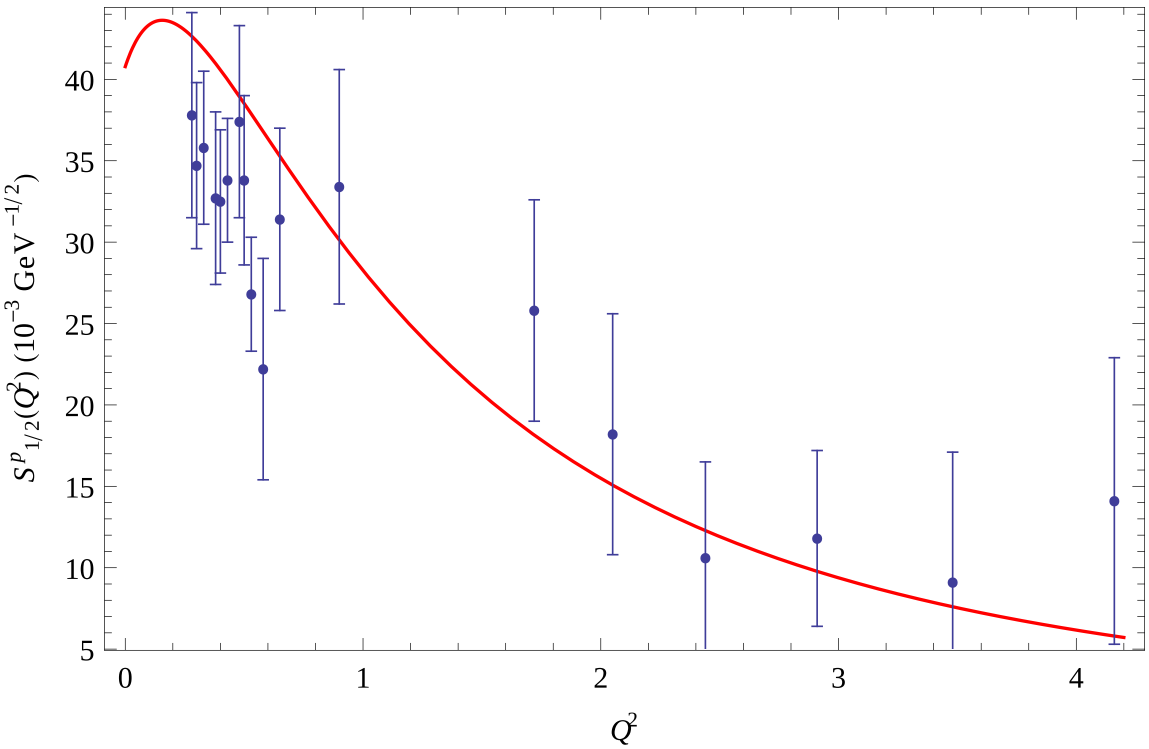
Helicity amplitude $A^p_{1/2}(Q^2)$



Data: CLAS Coll at JLab, Mokeev et al, PRC86 (2012) 035203

Roper resonance $N(1440)$

Helicity amplitude $S^p_{1/2}(Q^2)$



Data: CLAS Coll at JLab, Mokeev et al, PRC86 (2012) 035203

Deuteron

- Effective action in terms of AdS fields $d^M(x,z)$ and $V^M(x,z)$
- $d^M(x,z)$ – dual to Fock component contributing to deuteron with twist $\tau = 6$
- $V^M(x,z)$ – dual to the electromagnetic field

$$\begin{aligned}
S &= \int d^4x dz\, e^{-\varphi(z)} \Big[-\frac{1}{4} F_{MN}(x,z) F^{MN}(x,z) - D^M d^\dagger_N(x,z) D_M d^N(x,z) \\
&- i c_2(z) F^{MN}(x,z) d^\dagger_M(x,z) d_N(x,z) \\
&+ \frac{c_3(z)}{4M_d^2} \partial^M F^{NK}(x,z) \Big(-d^\dagger_M(x,z) \overleftrightarrow{D}_K\, d_N(x,z) + \mathrm{H.c.} \Big) \\
&+ d^\dagger_M(x,z) \Big(\mu^2 + U(z) \Big)\, d^M(x,z) \Big]
\end{aligned}$$

Deuteron

- $F^{MN} = \partial^M V^N - \partial^N V^M$ - stress tensor of vector field

  $D^M = \partial^M - ieV^M(x,z)$ - covariant derivative

  $\mu^2 R^2 = (\Delta - 1)(\Delta - 3)$ - five-dimensional mass

  $\Delta = 6 + L$ is the dimension of $d^M(x,z)$

  L is the maximum value orbital angular momentum

  $U(z) = U_0\,\varphi(z)/R^2$ is the confinement potential

  U_0 is constant fixed the deuteron mass.

  Use axial gauge for both vector fields $d^z(x,z) = 0$ and $V^z(x,z) = 0$

Deuteron

- First perform Kaluza-Klein (KK) decomposition for vector AdS field dual to deuteron

$$d^\mu(x,z) = \exp\left[\frac{\varphi(z) - A(z)}{2}\right] \sum_n d_n^\mu(x)\Phi_n(z)\,,$$

$d_n^\mu(x)$ is the tower of the KK fields dual to the deuteron fields with radial quantum number n and twist-dimension $\tau = 6$, and $\Phi_n(z)$ are their bulk profiles.
Then we derive the Schrödinger-type equation of motion for the bulk profile

$$\left[-\frac{d^2}{dz^2} + \frac{4(L+4)^2 - 1}{4z^2} + \kappa^4 z^2 + \kappa^2 U_0\right]\Phi_n(z) = M_{d,n}^2 \Phi_n(z)\ .$$

Deuteron

- The analytical solutions of this EOM read

$$\Phi_n(z) = \sqrt{\frac{2n!}{(n+L+4)!}}\,\kappa^{L+5}\,z^{L+9/2}\,e^{-\kappa^2 z^2/2}\,L_n^{L+4}(\kappa^2 z^2)\,,$$

$$M_{d,n}^2 = 4\kappa^2\left[n + \frac{L+5}{2} + \frac{U_0}{4}\right],$$

  where $L_n^m(x)$ are the generalized Laguerre polynomials.

- Restricting to the ground state $(n = 0,\ L = 0)$ we get $M_d = 2\kappa\sqrt{\frac{5}{2} + \frac{U_0}{4}}$
- Using central value for deuteron mass $M_d = 1.875613$ GeV and $\kappa = 190$ MeV (fitted from data on electromagnetic deuteron form factors),
  we fix $U_0 = 87.4494$.

Deuteron

- We can compare this value for the deuteron scale parameter to the analogous one of κ_N defining the nucleon properties - mass and electromagnetic form factors. In description of nucleon case we fixed the value to $\kappa_N \simeq 380$ MeV, which is 2 times bigger than the deuteron scale parameter κ.

- Difference between the nucleon and deuteron scale parameters can be related to the change of size of the hadronic systems - the deuteron as a two-nucleon bound state is 2 times larger than the nucleon.

Deuteron

- The gauge-invariant matrix element describing the interaction of the deuteron with the external vector field (dual to the electromagnetic field) reads

$$
\begin{aligned}
M^{\mu}_{\rm inv}(p,p') &= -\left(G_1(Q^2)\epsilon^*(p')\cdot\epsilon(p) - \frac{G_3(Q^2)}{2M_d^2}\,\epsilon^*(p')\cdot q\,\epsilon(p)\cdot q\right)(p+p')^{\mu} \\
&- G_2(Q^2)\left(\epsilon^{\mu}(p)\,\epsilon^*(p')\cdot q - \epsilon^{*\mu}(p')\,\epsilon(p)\cdot q\right)
\end{aligned}
$$

where $\epsilon(\epsilon^*)$ and $p(p')$ are the polarization and four-momentum of the initial (final) deuteron, and $q = p' - p$ is the momentum transfer.

Deuteron

- Three EM form factors $G_{1,2,3}$ of the deuteron are related to the charge G_C, quadrupole G_Q and magnetic G_M form factors by
- Expressions for the form factors

$$G_C = G_1 + \frac{2}{3}\tau_d G_Q\,, \quad G_M \;=\; G_2\,, \quad G_Q = G_1 - G_2 + (1+\tau_d)G_3, \quad \tau_d = \frac{Q^2}{4M_d^2}$$

These form factors are normalized at zero recoil as

$$G_C(0) = 1\,, \quad G_Q(0) = M_d^2 \mathcal{Q}_d = 25.83\,, \quad G_M(0) = \frac{M_d}{M_N}\mu_d = 1.714$$

- $\mathcal{Q}_d = 7.3424$ GeV^{-2} and $\mu_d = 0.8574$ – quadrupole and magnetic moments of the deuteron.

Deuteron

- Differential cross section for the elastic $e - D$ scattering (Rosenbluth formula)
  $d\sigma/d\Omega \sim \left[A(Q^2) + B(Q^2)\tan^2\frac{\theta}{2}\right]$
- Structure functions

$$
\begin{aligned}
A(Q^2) &= G_C^2(Q^2) + \frac{2}{3}\tau_d G_M^2(Q^2) + \frac{8}{9}\tau_d^2 G_Q^2(Q^2)\,, \\
B(Q^2) &= \frac{4}{3}\tau_d(1+\tau_d)G_M^2(Q^2)\,.
\end{aligned}
$$

- Scaling at large Q^2 (Brodsky et al., Carlson et al.)

$$
\begin{aligned}
\text{Leading}: &\qquad \sqrt{A(Q^2)} \sim \sqrt{B(Q^2)} \sim G_C(Q^2) \sim 1/Q^{10} \\
\text{Subleading}: &\qquad G_M(Q^2) \sim G_Q(Q^2) \sim 1/Q^{12}
\end{aligned}
$$

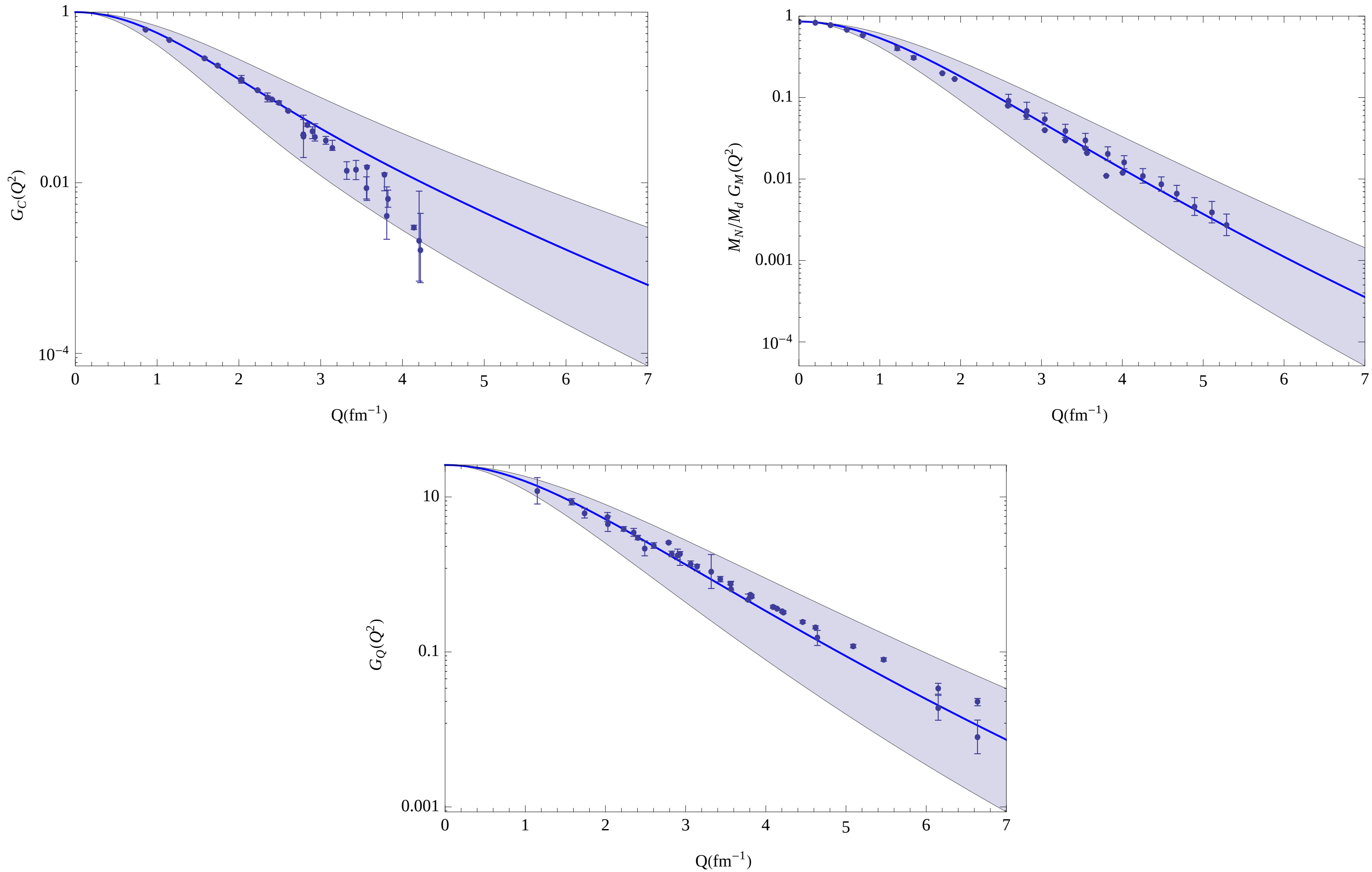
It fixes the z dependence of $c_2(z)$ and $c_3(z)$

$$
c_2(z) = \frac{M_d}{30M_N}\mu_d\,\kappa^2 z^2\,, \quad c_3(z) = \left(M_d^2\mathcal{Q}_d - 1 + \frac{M_d}{30M_N}\mu_d\right)\kappa^2 z^2
$$

Deuteron

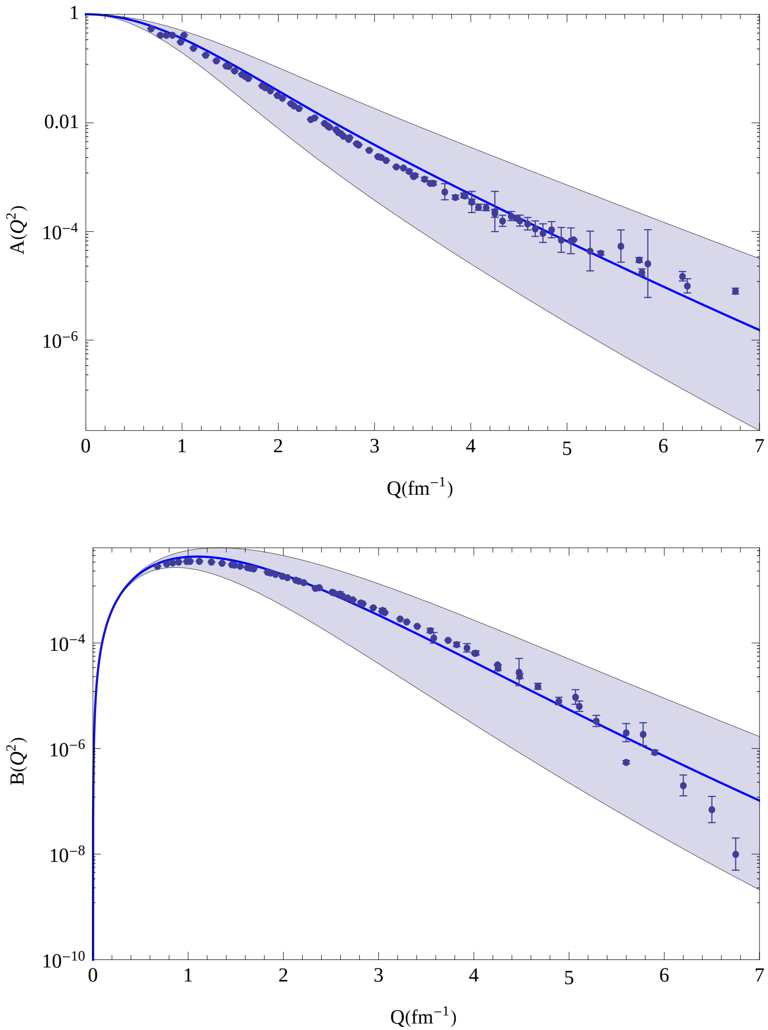
- Numerical results for $G_C(Q^2)$ (charge), $G_Q(Q^2)$ (quadrupole) and $G_M(Q^2)$ (magnetic)
- Shaded band corresponds to values of κ in range of 150 MeV $< \kappa <$ 250 MeV.
- Increase of the parameter κ leads to an enhancement of the form factors.
- The best description of the data on the deuteron form factors is obtained for $\kappa = 190$ MeV and is shown by the solid line.

Deuteron



Deuteron form factors

Deuteron



Structure Functions $A(Q^2)$ and $B(Q^2)$

Deuteron

Charge radius
$r_C = (-6G'_C(0))^{1/2} = 1.92$ fm
Data: $r_C = 2.13 \pm 0.01$ fm

Magnetic radius $r_M = (-6G'_M(0)/G_M(0))^{1/2} = 2.24$ fm
Data $r_M = 1.90 \pm 0.14$ fm.

Tetraquarks

- Under study at CERN, KEK, Fermilab, etc.
- Theoretical study in Covariant Confined Tetraquark Model (Mikhail Ivanov et al.)
- N_c QCD:
  Mesons $q^a \bar{q}^a$ and under $SU(N_c)$ the $\bar{q}^a$ transforms similar to
  $$\epsilon^{aa_1 \ldots a_{N_c-1}} \underbrace{q_{a_1} \ldots q_{N_c-1}}_{N_c-1}$$
- Baryons $\epsilon^{a_1 \ldots a_{N_c}} \underbrace{q_{a_1} \ldots q_{N_c}}_{N_c}$
- q^a transforms similar to $\epsilon^{aa_1 \ldots a_{N_c-1}} \underbrace{\bar{q}_{a_1} \ldots \bar{q}_{N_c-1}}_{N_c-1}$
- Multiquarks $\epsilon_{aa_1 \ldots a_{N_c-1}} \epsilon_{ab_1 \ldots b_{N_c-1}} \underbrace{q^{a_1} \ldots q^{a_{N_c-1}}}_{N_c-1} \underbrace{\bar{q}^{b_1} \ldots \bar{q}^{b_{N_c-1}}}_{N_c-1}$
- Limit to real QCD: $N_c = 3$

  Tetraquark $T = D^a \, \bar{D}^a = \left(\epsilon^{aa_1 a_2} \; q_{a_1} q_{a_2}\right) \left(\epsilon^{ab_1 b_2} \; \bar{q}_{b_1} \bar{q}_{b_2}\right)$
  is color diquark-antidiquark bound state

Tetraquarks

- Equation of motion from mesons case by rescaling $\tau \to \tau + 2$
- Solutions: $\phi_{nL}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+3)}}\, \kappa^{L+3}\, z^{L+7/2}\, e^{-\kappa^2 z^2/2}\, L_n^{L+2}(\kappa^2 z^2)$
- $M^2_{nJL} = 4\kappa^2\Big(n + \frac{L+J}{2} + 1\Big)$
- Agreement with COMPASS Coll. at SPS (CERN)
  for $a_1(1414)$ with spin-parity $J^{PC} = 1^{++}$ discovered in 2015
- Put $n = 0, L = 1, J = 1$ and get $M_{a_1} = 2\kappa\sqrt{2}$
- Using $\kappa = 0.5$ GeV get $M_{a_1} = \sqrt{2}$ GeV $\simeq 1.414$ GeV
- Brodsky-Teramond (superconformal case) $M^2_{nLS} = 4\kappa^2\Big(n + L + \frac{S}{2} + 1\Big)$
- Our $M^2_{nJL} = 4\kappa^2\Big(n + \frac{L+J}{2} + 1\Big)$
- Degenerate at $J = L + S$ when all three decouple
- Specifically for $a_1(1414)$ with $J^{PC} = 1^{++}$ we have $J = L = S = 1$

Tetraquarks

- Another important result - lower limit for tetraquark masses
- Put $n = L = J = 0$ and get $M_T \geq 2\kappa \simeq 1$ GeV using Lattice QCD constraint for κ

Summary

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD:
  attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space

- HQCD models reproduce main features of QCD at low and high energies

- Soft–wall holographic approach – covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances

- Mesons, baryons, exotics from unified point view and including high Fock states