Quantum regime of laser-matter interactions at extreme intensities

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Dedicated to memory of Nikolay Narozhny[1] *(1940-2016)*

Professor Nikolay Borisovich Narozhny, the eminent Russian theoretical physicist, Head of the Department for Theoretical Nuclear Physics and Vice Head of Academic Council at the National Research Nuclear University MEPhI, **died on February 15, 2016** in Moscow. With his passing away, physics of intense electromagnetic fields lost one of the most outstanding representatives...

[1] SV Popruzhenko. *Tribute to Nikolay Narozhny, In: The International Committee on Ultra-high Intensity Lasers (ICUIL) newsletter*. 2016. URL: http://www.icuil.org/newsletter.html?download=255:icuil-news-volume-7-june-2016; EN Avrorin et al. *In memory of Nikolay Borisovich Narozhny [official obituary in Russian]*. 2016. URL: http://ufn.ru/dates/inmemoria/narozhny.pdf.

Research activity

- 1963–1978 member of VI Ritus research unit at FIAN
  - calculation of *probabilities for photon emission and pair photoproduction in circularly polarized electromagnetic wave*[2)]
  - first calculation of *polarization operator in a constant crossed field*[3)]
  - first *direct* calculation of *spontaneous pair production in electric field*[4)]
  - ...
- 1979–1980 visit to J Eberly group at University of Rochester, USA
  - *effect of collapses and revivals in cavity QED*[5)]
- 1982 defense of Dr.Sc. dissertation; 1983–2016 Head of Department of Theoretical Nuclear Physics at MEPhI
- ∼1995–2016 leader of own research group at Department of Theoretical Nuclear Physics at MEPhI

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2) NB Narozhnyi, AI Nikishov, and VI Ritus. "Quantum processes in the field of a circularly polarized electromagnetic wave". In: *Sov. Phys. JETP* 20 (1965), p. 622.

3) NB Narozhny. "Propagation of plane electromagnetic waves in a constant field". In: *Sov. Phys. JETP* 28 (1969), p. 371.

4) NB Narozhny and AI Nikishov. "The simplest Processes in a Pair-Producing Field". In: *Soviet. J. Nucl. Phys* 11 (1970), p. 596.

5) JH Eberly, NB Narozhny, and JJ Sanchez-Mondragon. "Periodic spontaneous collapse and revival in a simple quantum model". In: *Physical Review Letters* 44 (1980), p. 1323; NB Narozhny, JJ Sanchez-Mondragon, and JH Eberly. "Coherence versus incoherence: Collapse and revival in a simple quantum model". In: *Physical Review A* 23 (1981), p. 236.

Overview

Introduction: characteristic levels of laser intensity

Parameters of state-of-the-art high-power laser facilities

- Carrier wavelength and frequency: $\lambda \simeq 1\mu\text{m}, \quad \nu = \frac{c}{\lambda} \simeq 10^{15}\text{Hz}, \quad \hbar\omega \simeq 1\text{eV}$
- Average pulse energy and duration: $W_\text{L} \simeq 0.1\text{kJ}, \quad \tau \simeq$ 100fs [6)] – tiny (!) due to CPA[7)]
- Peak power:

$$P_\text{L} \simeq \frac{W_L}{\tau} \simeq \frac{100\text{J}}{100 \times (10^{-15}\text{s})} = 10^{15}\text{W} \equiv 1\text{PW} - \text{HUGE(!)}$$

- Peak intensity:[8)]

$$I_\text{L} \simeq \frac{P_\text{L}}{R^2} \simeq \frac{P_\text{L}}{\lambda^2} \simeq \frac{10^{15}\text{W}}{(10^{-4}\text{cm})^2} \simeq 10^{23}\text{W/cm}^2 \quad - \text{HUGE(!)}$$

- Repetition rate: $\nu_\text{R} \simeq 10^{-4} \div 10$ Hz (i.e. **average** power and intensity are both rather low)

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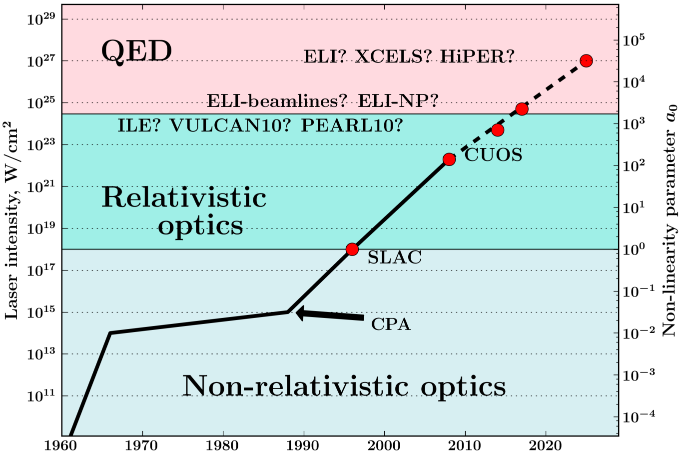
6) $1\text{fs} \equiv 10^{-15}\text{s}$

7) D Strickland and G Mourou. "Compression of amplified chirped optical pulses". In: *Optics communications* 56 (1985), p. 219.

8) V Yanovsky et al. "Ultra-high intensity-300-TW laser at 0.1 Hz repetition rate." In: *Optics Express* 16 (2008), p. 2109.

Leap of laser-focused intensity vs time for tabletop systems[9)]



9) T Tajima and G Mourou. "Zettawatt-exawatt lasers and their applications in ultrastrong-field physics". In: *Physical Review Special Topics-Accelerators and Beams* 5 (2002), p. 031301; NB Narozhny and AM Fedotov. "Extreme light physics". In: *Contemporary Physics* 56 (2015), p. 249.

Characteristic levels of laser intensity I

- Validity of external (classical) field concept $\hat{A}_\mu \to \mathcal{A}_\mu$:
  - Laser field is a coherent state of photons ($|c\rangle = e^{-|c|^2/2}\sum_{n=0}^{\infty}\frac{c^n}{\sqrt{n!}}|n\rangle$) with $\bar{N}_\gamma = \langle c|\hat{c}^\dagger\hat{c}|c\rangle = |c|^2$
  - $W_{\mathsf{L}} \simeq \frac{E^2+H^2}{8\pi}V \simeq \frac{E^2}{4\pi}V, \quad \bar{N}_\gamma \simeq \frac{W_{\mathsf{L}}}{\hbar\omega} \gg 1 \quad \Longrightarrow \quad E \gg \sqrt{\frac{\hbar\omega}{V}}$ (always valid for $\omega = 0$ or $V = \infty$!)
  - For $V \simeq \lambda^3$: $E \gg \omega^2\sqrt{\frac{\hbar}{c^2}}$ or $\boxed{I_L = \dfrac{c}{4\pi}E^2 \gtrsim 10^5 \mathsf{W/cm^2}}$
- Strong field concept in atomic physics:
  - Atomic length: $l_{\mathsf{at}} = \frac{\hbar^2}{Zme^2} = 5.3\times10^{-9}$cm (for $Z=1$)
  - Atomic energy: $\mathscr{E}_{\mathsf{at}} \simeq \frac{Ze^2}{l_{\mathsf{at}}} = \frac{mZ^2e^4}{\hbar^4} \simeq 10$eV (for $Z=1$)
  - $eEl_{\mathsf{at}} \gtrsim \mathscr{E}_{\mathsf{at}} \quad\Longrightarrow\quad E \gtrsim E_{\mathsf{at}} \equiv \frac{Ze}{l_{\mathsf{at}}^2} = \frac{m^2Z^3e^5}{\hbar^4} = 5\times10^9$V/cm (for $Z=1$) or $\boxed{I_{\mathsf{L}} \gtrsim \dfrac{c}{4\pi}E_{\mathsf{at}}^2 = 3\times10^{16}\mathsf{W/cm^2}}$
  - **For such laser intensities material targets become ionized (i.e. plasma).** Laser-plasma interactions are usually simulated with Particle-In-Cell (PIC) codes.

Characteristic levels of laser intensity II

- Relativistic intensity – classical interpretation:
  - Equation of motion:

$$\frac{d\vec{p}}{dt} = e\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{H}\right) \ (*)$$

$\Longrightarrow \ p_\perp \simeq \frac{eE}{\omega}$ – momentum of quiver oscillation

  - $a_0 \equiv \frac{p_\perp}{mc} \simeq \frac{eE}{m\omega c} \gtrsim 1$ – this motion becomes relativistic ($\gamma \simeq a_0 \gtrsim 1$)
  - This corresponds to $E \gtrsim E_{\mathsf{rel}} \equiv \frac{m\omega c}{e}$ or $\boxed{I_{\mathsf{L}} \gtrsim \frac{c}{4\pi} E_{\mathsf{rel}}^2 \simeq 3 \times 10^{18} \mathsf{W/cm}^2}$
  - Lorentz- and gauge-invariant[10)] definition for plane wave:

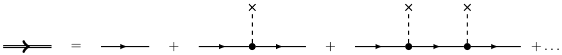
$$\boxed{a_0 = \frac{e}{mc}\sqrt{-\mathscr{A}_\mu \mathscr{A}^\mu}}$$

  - For $a_0 \gtrsim 1$ **equation of motion (∗) is nonlinear** – harmonics generation! (hence a_0 is often called **classical parameter of nonlinearity**)
  - In laser physics community, people often use to point field strength and intensity by dimensionless a_0 ($a_0 \approx 6 \times 10^{-10} \lambda\,[\mu\mathsf{m}]\sqrt{I_{\mathsf{L}}[\mathsf{W/cm}^2]}$, **currently attained level** $\leftrightarrow$ $\boldsymbol{a_0 \simeq 10^2}$)

[10)] This is indeed invariant under gauge transformations $\delta A^\mu \propto k^\mu$ of a plane wave.

Characteristic levels of laser intensity III

- Relativistic intensity – quantum (QED) interpretation:
  - In QED, motion of electron in external field $\mathscr{A}_\mu$ is described by sum of diagrams:



(**)

  - Each vertex corresponds to $-ie\gamma^\mu \mathscr{A}_\mu \simeq e\mathscr{A}$, each electron line (free propagator) – to $iS_0 = \frac{1}{\gamma p - m} \simeq \frac{1}{m}$. Hence, the expansion parameter $\simeq \frac{e\mathscr{A}}{m} \equiv a_0 \implies a_0 \gtrsim 1$ corresponds to **non-perturbative with respect to $\mathscr{A}_\mu$ (or multiphoton) interaction**
  - Pictorial interpretation: due to high density of photons in external field

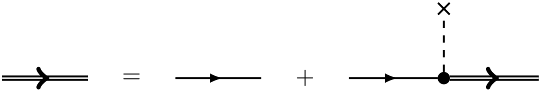
$$\text{Vertex weight} \sim \sqrt{\alpha} \to \sqrt{\alpha} \times \sqrt{\bar{N}_\gamma} \simeq \sqrt{\alpha} \times \sqrt{l_C^2 \times \lambda \times \bar{n}_\gamma} \simeq$$

$$\simeq \frac{e}{\sqrt{\hbar c}} \times \sqrt{\left(\frac{\hbar}{mc}\right)^2 \times \frac{2\pi c}{\omega} \times \frac{E^2}{4\pi\hbar\omega}} \simeq \frac{eE}{m\omega c} \simeq a_0$$

Note a_0 is **indeed purely classical** (as $\hbar$ totally cancels)

Characteristic levels of laser intensity IV

- In my talk **I will always assume $a_0 \gtrsim 1$ (in fact even $a_0 \gg 1$)**
- **IFQED approach: all-order summation (with respect to interaction with external field)**:
  - This results in closed equation



or in usual notation $\boxed{\{i\gamma^\mu(\partial_\mu - ie\mathcal{A}_\mu) - m\}S(x,x') = \delta^{(4)}(x-x')}$ for exact (with respect to interaction with external field), or 'dressed', electron propagator
  - This equation can be solved analytically for a few particular cases (constant field, plane wave, Coulomb field, etc.)
  - Amplitudes of the processes are then formulated as in ordinary QED, **but with free fermion lines and propagators replaced with the 'dressed' ones**
  - In IFQED this approach was actually tested in late 90's in famous E144 SLAC experiment[11)]

[11)] DL Burke et al. "Positron production in multiphoton light-by-light scattering". In: *Physical Review Letters* 79 (1997), p. 1626; C Bamber et al. "Studies of nonlinear QED in collisions of 46.6 GeV electrons with intense laser pulses". In: *Physical Review D* 60 (1999), p. 092004.

Characteristic levels of laser intensity V

- Account for **classical radiation reaction**:
  - Radiation reaction force acting on electron:

$$\vec{F}_{\mathsf{rad}} \simeq -\frac{2e^4\left(\vec{E}+\frac{\vec{v}}{c}\times\vec{H}\right)_\perp^2\gamma^2}{3m^2c^5}\vec{v}$$

    (assuming $\gamma \simeq a_0$, $E_\perp$, $H \simeq E$) becomes $\gtrsim eE$ for $E \gtrsim \left(\frac{m^4\omega^2c^6}{e^5}\right)^{1/3}$, or $a_0 \gtrsim \left(\frac{mc^3}{e^2\omega}\right)^{1/3} \simeq 400$, $\boxed{I_{\mathsf{L}} \gtrsim 5\times 10^{23}\mathsf{W/cm}^2}$.
  - In this regime one should take account for **(classical!) RR** in simulations of laser-matter interaction.
- **Relativistic ions**: $a_{0i} = \frac{(Ze)E}{M\omega c} \gtrsim 1$, or $a_0 = \frac{eE}{m\omega c} \gtrsim \frac{M}{Zm} \simeq \frac{2M_p}{m} \sim 4\times 10^3$, corresponding to $\boxed{I_{\mathsf{L}} \gtrsim 5\times 10^{25}\mathsf{W/cm}^2}$

- Self-energy correction

$$\mathscr{E}_{\text{em}} = \frac{1}{2}\int d^3r \int d^3r' \, \frac{\rho(\vec{r})\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} \simeq \frac{e^2}{r_0} \gtrsim mc^2$$

at $\boxed{r_0 \lesssim r_e \equiv \frac{e^2}{mc^2}}$ (r_e – *'classical electron radius'*);

- Radiation reaction force (in proper reference frame 'p'):

$$F_{\text{rad}} = \frac{2}{3}\frac{e^4}{m^2c^4}E_{\text{p}}^2$$

produces across distance r_e the work

$$A = F_{\text{rad}} r_e \simeq \frac{e^6 E_{\text{p}}^2}{m^3 c^6} \gtrsim mc^2 \quad \text{at} \quad \boxed{E_{\text{p}} \gtrsim E_{\text{cr}} \equiv \frac{m^2c^4}{e^3}}$$

- **The distance r_e and the field strength E_{cr} are considered as limits of applicability of Classical Electrodynamics.**

- sQED as example: quantized scalar charged field:

$$\hat{\Psi}(x) = \sum_{\vec{p}} \frac{1}{\sqrt{2V\varepsilon_{\vec{p}}}} \left(e^{-ipx}\hat{a}_{\vec{p}} + e^{ipx}\hat{b}^{\dagger}_{\vec{p}} \right)$$

- 4-current operator:

$$\hat{j}_{\mu}(x) = ie \; : \hat{\Psi}^{\dagger}(x) \overleftrightarrow{\partial}_{\mu} \hat{\Psi}(x) : =$$

$$= \sum_{\vec{p},\vec{p}'} \frac{e}{2V\sqrt{\varepsilon_{\vec{p}}\varepsilon_{\vec{p}'}}} \left\{ (p_{\mu} + p'_{\mu}) \left[\underbrace{e^{i(p'-p)x}\hat{a}^{\dagger}_{\vec{p}'}\hat{a}_{\vec{p}}}_{\text{particle current}} - \underbrace{e^{-i(p'-p)x}\hat{b}^{\dagger}_{\vec{p}}\hat{b}_{\vec{p}'}}_{\text{antiparticle current}} \right] + \right.$$

$$\left. + (p_{\mu} - p'_{\mu}) \left[\underbrace{e^{-i(p'+p)x}\hat{b}_{\vec{p}'}\hat{a}_{\vec{p}} - e^{i(p'+p)x}\hat{a}^{\dagger}_{\vec{p}'}\hat{b}^{\dagger}_{\vec{p}}}_{\text{non-diagonal terms}} \right] \right\}$$

Non-diagonal part: (annihilation/creation of virtual pair).

Radiation corrections in QED II

- Self-energy correction $\left(\vec{r}=\vec{R}+\frac{\vec{\xi}}{2},\ \vec{r}'=\vec{R}-\frac{\vec{\xi}}{2}\right)$:

$$\mathscr{E}_{\text{em}}=\frac{1}{2}\int d^3\xi\,\frac{C(\vec{\xi})}{|\vec{\xi}|},$$

$$C(\vec{\xi})=\int d^3R\,\langle 1_{\text{rest}}|\hat{j}^0\left(\vec{R}+\frac{\vec{\xi}}{2}\right)\hat{j}^0\left(\vec{R}-\frac{\vec{\xi}}{2}\right)-:\quad:|1_{\text{rest}}\rangle=$$

$$=\frac{e^2}{2}\int\frac{d^3p}{(2\pi)^3}\underbrace{\left(1+\frac{m}{\varepsilon_{\vec{p}}}\right)e^{i\vec{p}\vec{\xi}}}_{\text{from }\hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a}}-\frac{e^2}{2}\int\frac{d^3p}{(2\pi)^3}\underbrace{\left(1-\frac{m}{\varepsilon_{\vec{p}}}\right)e^{i\vec{p}\vec{\xi}}}_{\text{from }\hat{b}\hat{a}^\dagger\hat{a}\hat{b}^\dagger}$$

- In classical limit $(m\to\infty)$ the first *(particle)* contribution reduces to $e^2\delta^{(3)}(\vec{\xi})$, as expected, resulting in linear divergency:

$$\mathscr{E}_{\text{em}}\propto\frac{e^2}{2}\int d^3\xi\,\frac{\delta^{(3)}(\vec{\xi})}{|\vec{\xi}|}=\frac{e^2}{2r_0},\quad r_0\to 0$$

Radiation corrections in QED III

- However, in general setting the leading divergency *is canceled by the virtual pairs contribution*:[12)]

$$C(\vec{\xi}) = \frac{e^2}{2}\left(\cancel{\delta^{(3)}(\vec{\xi})} - \frac{m^2}{2\pi^2|\vec{\xi}|}K_1(m|\vec{\xi}|)\right) - \frac{e^2}{2}\left(\cancel{\delta^{(3)}(\vec{\xi})} + \frac{m^2}{2\pi^2|\vec{\xi}|}K_1(m|\vec{\xi}|)\right) = -\frac{e^2m^2}{2\pi^2|\vec{\xi}|}K_1(m|\vec{\xi}|) \simeq \frac{e^2 m}{\pi^2|\vec{\xi}|^2}, \quad \vec{\xi}\to 0$$

so that

$$\boxed{\mathscr{E}_{\text{em}} \simeq \frac{e^2 m}{\pi^2}\int\frac{d^3\xi}{|\vec{\xi}|^3} \propto e^2 m\log\left(\frac{1}{mr_0}\right), \quad r_0\to 0}$$

- **Thus, a pointlike charge is effectively replaced by a cloud of virtual pairs of size $\simeq l_{\text{C}} = \frac{1}{m} \simeq 137 r_e$ (or $\frac{\hbar}{mc} \simeq 4\times 10^{-11}$cm in conventional units)**

12) VF Weisskopf. "On the self-energy and the electromagnetic field of the electron". In: *Physical Review* 56.1 (1939), p. 72.

QED strong field

- In QED the analogue of E_{cr} is **Sauter-Schwinger critical field**

$$E_{\text{S}} \equiv \frac{m^2c^3}{e\hbar} = 1.3\times 10^{16}\text{V/cm}, \quad I_{\text{L}} = \frac{c}{4\pi}E_{\text{S}}^2 \simeq 5\times 10^{29}\text{W/cm}^2$$

defined as $eE_{\text{S}}l_{\text{C}} \simeq mc^2$. Note that $E_{\text{S}} \simeq \frac{E_{\text{cr}}}{137}$

  - $E \sim E_{\text{S}}$ also arises in heavy ion collisions with $Z_{\text{tot}} \simeq 137$; $H \sim \frac{m^2c^3}{e\hbar} \simeq 4\times 10^{13}$G are anticipated around compact astrophysical objects

- **But in which reference frame???** (as field strength is frame-dependent) For vacuum problems the criterion should be formulated in Lorentz-invariant manner:
  - Two field invariants: $E^2 - H^2 = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu}$, $\vec{E}\cdot\vec{H} = \frac{1}{8}\epsilon_{\mu\nu\lambda\varkappa}F^{\mu\nu}F^{\lambda\varkappa}$, or equivalent dimensionless $\varepsilon \equiv E_{\parallel}/E_{\text{S}}$, $\eta \equiv H_{\parallel}/E_{\text{S}}$, where

$$E_{\parallel}, H_{\parallel} = \sqrt{\frac{E^2-H^2}{2} \pm \sqrt{\left(\frac{E^2-H^2}{2}\right)^2 + \left(\vec{E}\cdot\vec{H}\right)^2}}$$

  – field strengths in a reference frame where $\vec{E} \parallel \vec{H}$

  - **Then Sauter-Schwinger critical field is defined by demanding ε, $\eta \simeq 1$**

Quantum dynamical parameter χ and quantum regime

- However, in presence of particle (p^μ) one extra invariant can be defined:

$$\chi = \frac{e\hbar}{m^3c^4}\sqrt{-(F_{\mu\nu}p^\nu)^2} = \frac{\gamma\sqrt{\left(\vec{E}+\frac{\vec{v}\times\vec{H}}{c}\right)^2-\frac{(\vec{v}\cdot\vec{E})^2}{c^2}}}{E_S} = \frac{E_P}{E_S}$$

–proper acceleration (in Compton units)

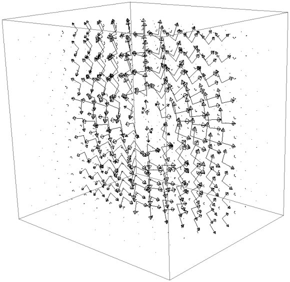
  - As I will show, for $\chi \lesssim 1$ it is also emitted photon energy-to-particle energy ratio, hence $\boldsymbol{\chi} \gtrsim \mathbf{1}$ **indicates significance of quantum recoil**
  - Hence **quantum regime of laser-matter interaction is naturally defined by $\boldsymbol{\chi} \gtrsim \mathbf{1}$**
  - $E_{L\parallel} \sim E_{L\perp} \Rightarrow E_{P\parallel} \sim E_{L\parallel},\ E_{P\perp} \sim \gamma E_{L\perp} \Rightarrow \boxed{E_P \sim \gamma E_{L\perp}}$
  - Generally speaking, $a_0 = \frac{eE}{m\omega c}$ and $\chi \simeq \frac{E_\perp \gamma}{E_S}$ are independent:

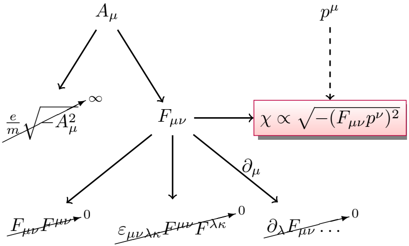
Regime	$a_0 \ll 1$	$a_0 \gg 1$
$\chi \ll 1$	classical non-relativistic	classical relativistic
$\chi \gtrsim 1$	perturbative QED	IFQED

  For instance, in SLAC experiment they were $a_0 \sim 1$ and $\chi \sim 1$
  - However, if we assume $E_\perp \sim E$, $\gamma \sim a_0 \gg 1$, then $\chi \simeq \frac{\hbar\omega}{mc^2}a_0^2 \gtrsim 1$ for $a_0 \gtrsim \sqrt{\frac{mc^2}{\hbar\omega}} \simeq 700$ or $\boxed{I_{\mathrm{L}} \gtrsim 10^{24}\mathrm{W/cm}^2}$

Locally constant crossed field (CCF) approximation

- **General fact**:[13] for $a_0 \gg 1$ **any** EM field in the proper reference frame of ultra-relativistic particle looks as **constant** ($\omega = 0$) **crossed** ($E \approx H$, $\vec{E} \cdot \vec{H} \approx 0$)

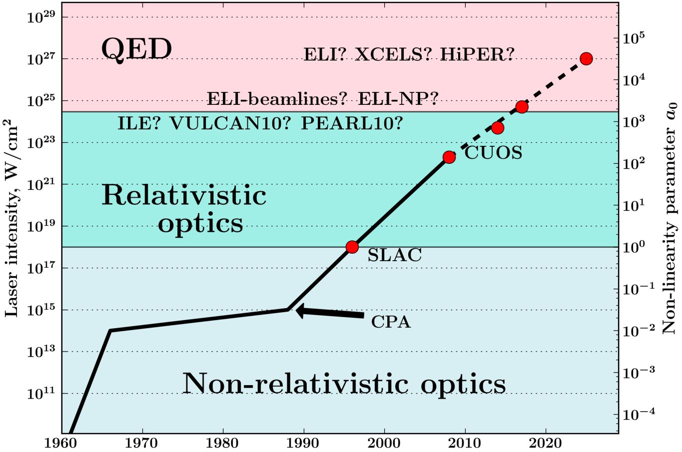




$$\chi = \frac{e\hbar}{m^3c^4}\sqrt{-(F_{\mu\nu}p^\nu)^2} \simeq \frac{E_\perp \gamma}{E_S} \gg \varepsilon, \eta \quad \text{for} \quad \gamma \gg 1$$

[13] AI Nikishov and VI Ritus. "Quantum processes in the field of a plane electromagnetic wave and in a constant field. I". In: *Sov. Phys. JETP* 19.2 (1964), p. 529.

Leap of laser-focused intensity vs time for tabletop systems[14)]



14) Tajima and Mourou, "Zettawatt-exawatt lasers and their applications in ultrastrong-field physics"; Narozhny and Fedotov, "Extreme light physics".

IFQED elementary processes

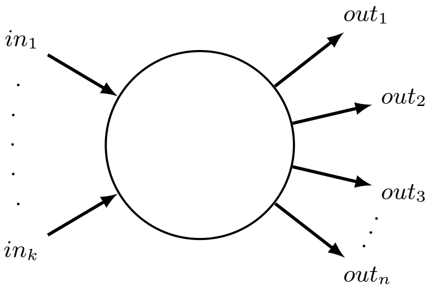
Motivation

- Intense Field QED is a well-developed (at least, theoretically) research area.
- However, most results were being obtained by **extremely bulky calculations**.
- Merely everybody would agree that **qualitative considerations** always **allow to gain deeper insight** into a problem.
- Surprisingly, **qualitative considerations in IFQED have been almost never discussed in literature** in general setting.

  Notable exceptions (discussions of important selected aspects):
  - A. B. Migdal, "Vacuum Polarization in Strong Inhomogeneous Fields", Sov. Phys. JETP **35**, 845 – 853 (1972) [see also A.B. Migdal, "Fermions and bosons in strong fields" [in Russian] (Nauka, Moscow, 1978)]
  - E. Kh. Akhmedov, "Beta Decay and Other Processes in Strong Electromagnetic Fields", Physics of Atomic Nuclei **74**, 12991315 (2011) [arXiv:1011.3776].
- I am going to demonstrate[15)] how at least some of known simple asymptotic expressions for probability rates of basic processes in strong external field could receive a **simple-man explanation (analysis of kinematics + uncertainty principle + dimensional arguments)**.

15) AM Fedotov. "Qualitative considerations in Intense Field QED". In: *arXiv:1507.08512* (2015).

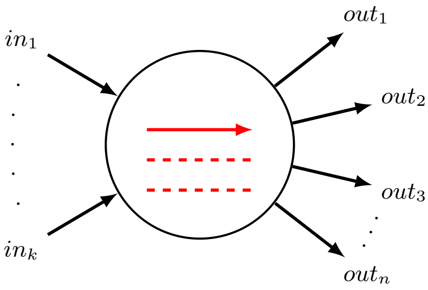
General framework



- Energy lack:

$$\Delta\varepsilon = \sum \varepsilon_f - \sum \varepsilon_i > 0$$

General framework

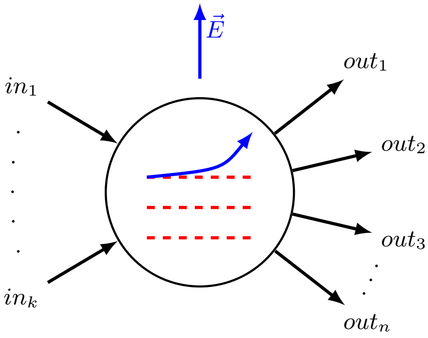


- Energy lack:

$$\Delta\varepsilon = \sum \varepsilon_f - \sum \varepsilon_i > 0$$

- Virtual particles:

$$t \lesssim t_q \simeq \frac{1}{\Delta\varepsilon}$$



- Energy lack:

$$\Delta\varepsilon = \sum \varepsilon_f - \sum \varepsilon_i > 0$$

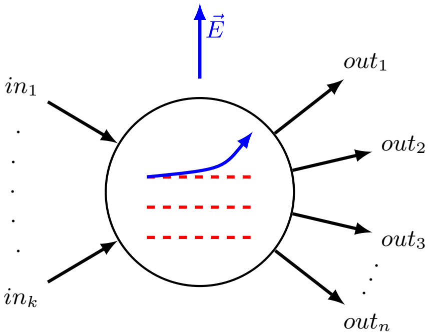
- Virtual particles:

$$t \lesssim t_q \simeq \frac{1}{\Delta\varepsilon}$$

- Energy balance: $t \gtrsim t_e$

$$e \int_0^{t_e} \vec{E} \cdot d\vec{s} \simeq \Delta\varepsilon$$

General framework



- Energy lack:

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$$t \lesssim t_q \simeq \frac{1}{\Delta\varepsilon}$$

- Energy balance: $t \gtrsim t_e$

$$e \int_0^{t_e} \vec{E} \cdot d\vec{s} \simeq \Delta\varepsilon$$

- $t_e \lesssim t_q$ $\rightarrow$ process is 'allowed' (quantum regime!)
- $t_e \gtrsim t_q$ $\rightarrow$ process is 'suppressed' $\propto e^{-t_e/t_q}$ (quasiclassical regime!)

Quasiclassical evaluation of exponential suppression

- Purely electric constant field, time gauge: $\vec{A}(t) = -\vec{E}t$
- Quasiclassical solutions ($E \ll E_S = m^2/e$):

$$\Psi(\vec{r},t) \propto \exp\left\{ i\vec{p}\vec{r} - i\int_0^t \varepsilon(t')\,dt' \right\},\ \varepsilon(t) = \sqrt{\left(\vec{p} - e\vec{A}(t)\right)^2 + m^2}$$

- Quantum amplitude of the process:

$$c_{i\to f} = -i\int_{-\infty}^{+\infty} dt\, V_{fi} \exp\left\{ i\int_0^t \Delta\varepsilon(t')\,dt' \right\}, \quad V_{fi} \propto \delta^{(3)}\left(\Delta\vec{p}\right)$$

- Landau (1932), but $t \leftrightarrow x$:

$$c_{i\to f} \propto \exp\left\{ -\int_0^{t_*} \Delta\varepsilon(it')\,dt' \right\}, \quad \Delta\varepsilon(it_*) = 0, \quad \Delta\vec{p} = 0$$

- It turns out that $t_* \simeq t_e$, so that $c_{i\to f} \simeq e^{-t_e/t_q}$!

Spontaneous pair creation in constant electric field

- Characteristic time scales:

$$\Delta\varepsilon = 2m, \quad eEt_e \simeq 2m \quad \Longrightarrow \quad \boxed{t_e \simeq \frac{m}{eE}} \quad \boxed{t_q \simeq \frac{1}{\Delta\varepsilon} \simeq \frac{1}{m}}$$

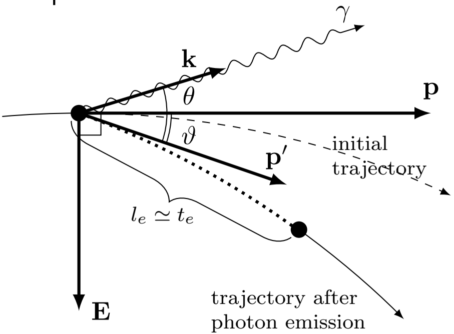
- For $E \ll E_{\mathsf{S}} = m^2/e$ we have $t_e \gg t_q, \quad \Longrightarrow$ process is 'suppressed' (quasiclassical regime).
- Note also that $t_e = \frac{1}{\omega a_0} \ll \frac{1}{\omega}$ for $a_0 \gg 1 \Longrightarrow$ validity of **locally constant field approximation**
- Expected suppression factor $e^{-t_e/t_q} \simeq e^{-E_{\mathsf{S}}/E}$.
- More precisely, for $\vec{p}_\perp = 0$ (for sake of simplicity only):

$$\Delta\varepsilon(t) = 2\sqrt{m^2 + e^2E^2t^2}, \quad \Delta\varepsilon(it_*) = 0 \implies t_* = \frac{m}{eE} \simeq t_e,$$

$$W_{e^-e^+} = \left|\exp\left\{-2\int_0^{m/eE} \sqrt{m^2 - e^2E^2t'^2}\,dt'\right\}\right|^2 = e^{-\pi m^2/eE}$$

- Correct pre-exponential factor $N_{\mathsf{loops}} \simeq \frac{VT}{t_q^2 t_e^2} \simeq e^2E^2VT \simeq 10^{28}$ for optical lasers ($V \sim \lambda^3$ and $T \sim \omega^{-1}$)!
- **Hence actual threshold is $E \simeq 0.1E_{\mathsf{S}}$ ($I_{\mathsf{L}} \simeq 10^{27\div 28}$W/cm^2)!**

Photon emission by relativistic electron -I



- Assume: $p, k, p-k \gg m, eEt$
- Initial energy ($\vec{p} \perp \vec{E}$ for simplicity):

$$\varepsilon_{\vec{p}}(t) = \sqrt{(\vec{p} - e\vec{A})^2 + m^2} = \\ = \sqrt{p^2 + e^2E^2t^2 + m^2} \approx \\ \approx p + \frac{e^2E^2t^2 + m^2}{2p}$$

- Final momentum: $\vec{p}' = \vec{p} - \vec{k}$
- ${p'}^2 = p^2 + k^2 - 2pk\cos\theta = (p-k)^2 + 4pk\sin^2(\theta/2)$
- Final energy (let $\vec{k} \in \mathrm{Span}\,(\vec{p}, \vec{E})$ for simplicity):

$$\varepsilon_{\vec{p}'}(t) = \sqrt{(\vec{p}' - e\vec{A})^2 + m^2} = \sqrt{{p'}^2 - 2e\vec{E}\cdot\vec{k}t + e^2E^2t^2 + m^2} \approx \\ \approx p - k + \frac{e^2E^2t^2 + 2eEkt\sin\theta + m^2 + 4pk\sin^2(\theta/2)}{2(p-k)}$$

Photon emission by relativistic electron -II

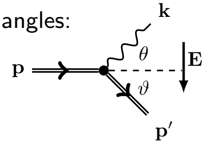
- Energy lack:

$$\Delta\varepsilon(t) = \varepsilon_{\vec{p}'}(t) + k - \varepsilon_{\vec{p}}(t) \approx$$
$$\approx \not{p} - \not{k} + \frac{e^2E^2t^2 + 2eEkt\sin\theta + m^2 + 4pk\sin^2(\theta/2)}{2(p-k)} +$$
$$+ \not{k} - \left(\not{p} + \frac{e^2E^2t^2 + m^2}{2p}\right) =$$
$$= \boxed{\frac{k\left[e^2E^2t^2 + 2eEpt\sin\theta + m^2 + 4p^2\sin^2(\theta/2)\right]}{2p(p-k)}}$$

Photon emission – case (i): $t \lesssim m/eE$

$$\Delta\varepsilon = \frac{k\left[\cancel{e^2E^2t^2} + \cancel{2eEpt\sin\theta} + m^2 + \cancel{4p^2\sin^2(\theta/2)}\right]}{2p(p-k)} \simeq \frac{km^2}{p(p-k)}$$

- Estimate of angles:



$$\theta \lesssim \frac{m}{p} = \frac{1}{\gamma} \ll 1$$

$$k_\perp = p'_\perp, \quad p' \approx p - k$$

$$\vartheta \simeq \frac{k}{p-k}\theta \lesssim \frac{km}{(p-k)p} \ll 1$$

- Characteristic times scales:

$$t_q \simeq \frac{1}{\Delta\varepsilon} \simeq \frac{p(p-k)}{m^2k} \gtrsim t_e \simeq \frac{\Delta\varepsilon}{eE\vartheta} \simeq \frac{m}{eE}$$

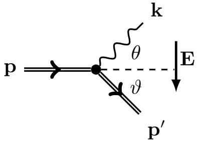
- Radiation frequency range: $k \lesssim \frac{eEp}{m}p = \chi p \lesssim p \Longrightarrow \boxed{\chi \lesssim 1}$
- Emission probability and radiation reaction:

$$\boxed{W_\gamma \overset{(?)}{\simeq} e^2/t_e \sim (e^2m^2/p)\chi}, \quad \boxed{F_{RR} \simeq kW_\gamma \sim e^2m^2\chi^2}$$

Photon emission – case (ii): $t \gg m/eE$

$$\boxed{\Delta\varepsilon = \frac{k\left[e^2E^2t^2 + \cancel{2eEpt\sin\theta} + \cancel{m^2} + \cancel{4p^2\sin^2(\theta/2)}\right]}{2p(p-k)} \simeq \frac{e^2E^2t^2}{p}}$$

- Estimate of angles:



$$\boxed{k \sim p}$$

$$\vartheta, \theta \lesssim \frac{eEt}{p} \ll 1$$

$\boxed{eE\vartheta t_e \simeq \Delta\varepsilon}$ identically!!!

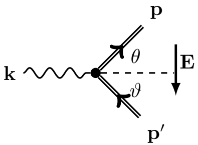
- Time scales analysis:

$$t_q \simeq \frac{1}{\Delta\varepsilon} \simeq \frac{p}{e^2E^2t_q^2} \implies \boxed{t_q \simeq \left(\frac{p}{e^2E^2}\right)^{1/3} = \frac{m}{eE}\chi^{1/3}} \quad (\chi \gg 1)$$

- Emission probability and radiation reaction:

$$\boxed{W_\gamma \overset{(?)}{\simeq} e^2/t_q \sim (e^2m^2/p)\chi^{2/3}}, \quad \boxed{F_{RR} \simeq kW_\gamma \sim e^2m^2\chi^{2/3}}$$

Pair photoproduction by hard photon -I



- Since kinematic is basically the same as above, let us for the sake of simplicity neglect transverse motion from the beginning
- Energy lack:

$$\Delta\varepsilon(t) = \varepsilon_{\vec{k}-\vec{p}}(t) + \varepsilon_{\vec{p}}(t) - k \overset{\text{1D}}{\approx}$$

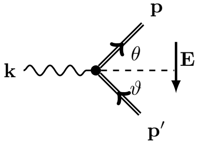
$$\overset{\text{1D}}{\approx} \sqrt{(k-p)^2 + e^2E^2t^2 + m^2} + \sqrt{p^2 + e^2E^2t^2 + m^2} - k \approx$$

$$\approx \not{k} - \not{p} + \frac{e^2E^2t^2 + m^2}{2(k-p)} + \not{p} + \frac{e^2E^2t^2 + m^2}{2p} - \not{k} =$$

$$= \frac{k\left(e^2E^2t^2 + m^2\right)}{2p(k-p)} \gtrsim \boxed{\frac{2\left(e^2E^2t^2 + m^2\right)}{k}}$$

$$\left(\text{minimum is attained at } p = p' = \frac{k}{2}\right)$$

Pair photoproduction: case (i) $t \lesssim m/eE$



$$\boxed{\Delta\varepsilon(t) = \frac{2\left(\cancel{e^2E^2t^2} + m^2\right)}{k}}$$

- Estimation for angles: $\boxed{\theta, \vartheta \simeq m/k \ll 1}$
- Characteristic time scales:

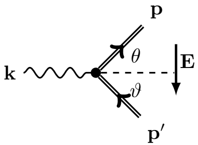
$$\boxed{t_q \simeq \frac{1}{\Delta\varepsilon} \simeq \frac{k}{m^2}} \ll \boxed{t_e \simeq \frac{\Delta\varepsilon}{eE\vartheta} \simeq \frac{m}{eE}}$$

- Thus process is suppressed ($\propto e^{-t_e/t_q}$) for $\varkappa = \frac{eEk}{m^3} \lesssim 1$
- Stationary point: $\Delta\varepsilon(it_*) = \frac{2\left(-e^2E^2t_*^2 + m^2\right)}{k} = 0 \implies t_* = \frac{m}{eE} \simeq t_e$ (!!!)
- Suppression factor:

$$W_{e^-e^+} \propto \left|\exp\left(-\int_0^{t_*} \Delta\varepsilon(it)\,dt\right)\right|^2 = \left|\exp\left(-\int_0^{m/eE} \frac{2\left(-e^2E^2t^2 + m^2\right)}{k}\,dt\right)\right|^2$$

$$= \left|e^{-4m^3/3eEk}\right|^2 = \boxed{e^{-8/3\varkappa}} \quad !!!$$

Pair photoproduction: case (ii) $t \gg m/eE$



$$\Delta\varepsilon(t) = \frac{2\left(e^2E^2t^2 + \cancel{m^2}\right)}{k}$$

- Estimation for angles: $\boxed{\theta, \vartheta \simeq eEt/k \ll 1}$
- t_e is arbitrary ($eE\vartheta t \simeq \Delta\varepsilon(t)$ identically)
- Time scale analysis:

$$t_q \simeq \frac{1}{\Delta\varepsilon(t_q)} \simeq \frac{k}{e^2E^2t_q^2} \implies \boxed{t_q \simeq \left(\frac{k}{e^2E^2}\right)^{1/3} \simeq \frac{m}{eE}\varkappa^{1/3}}$$

- Hence, for $\varkappa \gg 1$

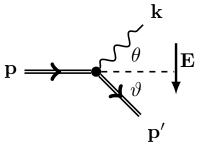
$$\boxed{W_{e^-e^+} \overset{(?)}{\simeq} \frac{e^2}{t_q} \sim \frac{e^2m^2}{k}\varkappa^{2/3}}$$

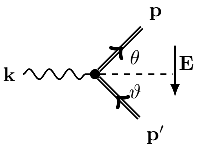
Discussion

- Qualitative arguments (**kinematical + uncertainty principle + dimensional analysis**) suffice for deeper intuitive understanding of various formulas of IFQED previously obtained by formal manipulations
- The key parameters are the **formation time and length** of a process
- If they are smaller than the scale of variation of the field, the **locally constant field approximation** is valid

Self-sustained QED cascades

- The non-perturbative theory of the simplest quantum processes in a constant crossed field is rather well developed since 60s [V.I. Ritus, Trudy FIAN, Vol. 111, pp. 5-151, 1979]. In particular, the energy distributions and the total probabilities are well known.[16)] **In the limit $\chi \gg 1$ they scale universally**:





$$W_{rad}(\chi \gg 1) \approx 1.46 \frac{\alpha m^2 c^4}{\hbar \varepsilon} \chi^{2/3}$$

$$W_{rad}(\chi \ll 1) \leftrightarrow \text{class. electrodynamics}$$

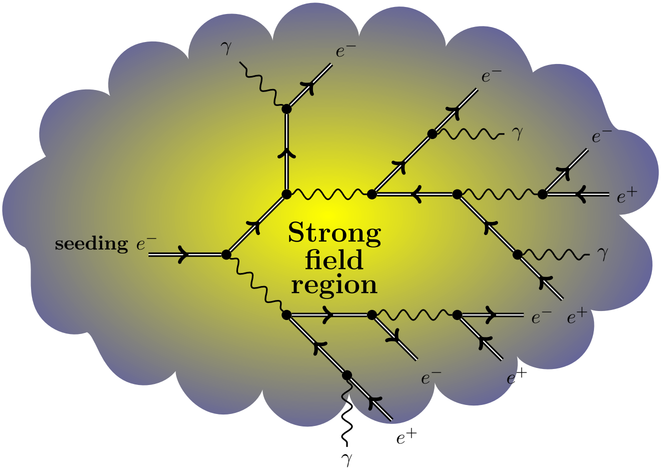
$$W_{cr}(\chi_\gamma \gg 1) \approx 0.23 \frac{\alpha m^2 c^4}{\hbar^2 \omega} \varkappa^{2/3}$$

$$W_{cr}(\chi_\gamma \lesssim 1) \propto e^{-8/3\varkappa}$$

-locked for $\varkappa \lesssim 1$

16) Nikishov and Ritus, "Quantum processes in the field of a plane electromagnetic wave and in a constant field. I".

QED cascade in strong field



Two types of cascades[17]

QED cascades

S(hower)-type:

- **produced by energy ε_0 of seed particle**
- proceeds until particles energy loss ($N_{e^+e^-} \propto \varepsilon_0$)
- similar to Extensive Air Showers
- *almost* observed in E-144 SLAC experiment

A(valanche)-type, or **self-sustained:**

- **field donates energy by reaccelerating secondary particles**
- $\boxed{N_{e^+e^-}(t) \simeq e^{\Gamma t}}$ until
  - . . . field depletion
  - . . . particles escape
  - . . . thermolization
- similar to gas or dielectric discharge

[17] AM Fedotov et al. "Limitations on the attainable intensity of high power lasers". In: *Physical Review Letters* 105 (2010), p. 080402; AA Mironov, NB Narozhny, and AM Fedotov. "Collapse and revival of electromagnetic cascades in focused intense laser pulses". In: *Physics Letters A* 378 (2014), p. 3254.

Toy model: uniformly rotating electric field[18)]

Initially slow $p(0) \ll mc$ particle in a "relativistic field" $a_0 = \frac{eE_0}{mc\omega} \gg 1$

$$\chi(0) \sim E_0/E_S \ll 1$$

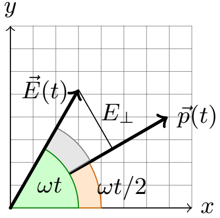
$$\frac{mc}{eE_0} \ll t \ll \frac{1}{\omega}$$

$$\vec{E}(t) = \{E_0 \cos\omega t, E_0 \sin\omega t\} \approx E_0\{1, \omega t\}$$

$$\frac{d\vec{p}(t)}{dt} = e\vec{E}(t), \quad \vec{p}(0) = 0$$

angle $\frac{\omega t}{2}$

$$\vec{p}(t) = \cancel{\vec{p}(0)}^{\,0} + \int_0^t e\vec{E}(t)\,dt = eE_0\left\{t, \frac{\omega t^2}{2}\right\}$$



$$E_\perp \sim E_0\frac{\omega t}{2}, \quad \chi(t) \sim \frac{E_\perp \gamma}{E_S} \sim \underbrace{\frac{E_0}{E_S}}_{\text{small}} \times \underbrace{\frac{\omega t}{2}}_{\text{small}} \times \underbrace{\frac{eE_0 t}{mc}}_{\text{very large!}}$$

χ can attain unity rather quickly: $t \ll \omega^{-1}$!!! But how general is that?

[18)] Fedotov et al., "Limitations on the attainable intensity of high power lasers"; AR Bell and JG Kirk. "Possibility of prolific pair production with high-power lasers". In: *Physical Review Letters* 101 (2008), p. 200403.

General case (numerical simulations[19])

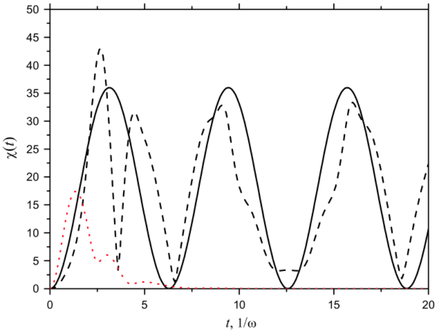


FIG. 1 (color online). Evolution of quantum dynamical parameter χ along the particle trajectory for $a_0 = 3 \times 10^3$, $\hbar\omega = 1$ eV in three cases: head-on collision of two elliptically polarized plane waves (solid line); collision at 90° of two linearly polarized plane waves with orthogonal linear polarizations (dashed line); single tightly focused e-polarized laser beam (dotted line).

[19] Fedotov et al., "Limitations on the attainable intensity of high power lasers".

Estimation of a typical "acceleration time"

At the initial part of trajectory:

$$\chi(t) \simeq \frac{E_\perp}{E_S}\gamma \sim \frac{E\omega t}{E_S} \times \frac{eEt}{mc} = \left(\frac{E}{E_S}\right)^2 \frac{mc^2\omega}{\hbar} t^2 \sim 1$$

Typical "acceleration time":

$$t \sim t_{acc} = \frac{\hbar}{\alpha mc^2 \mu}\sqrt{\frac{mc^2}{\hbar\omega}}$$

Hereinafter it is suitable to use the dimensionless parameter $\boxed{\mu = E/E_*}$

$$\boxed{E_* = \alpha E_S \approx \frac{E_S}{137} \;\Leftrightarrow\; I_* \sim 2.5 \times 10^{25} \mathrm{W/cm^2}}$$

Estimation of the free path time for $e^{\pm} \to e^{\pm}\gamma$, $\gamma \to e^{+}e^{-}$

$$W(t_{free}) \sim \frac{1}{t_{free}} \sim \frac{\alpha m^2 c^4}{\hbar\, \varepsilon}\, \chi^{2/3}$$

$$\chi(t_{free}) \sim \left(\frac{E}{E_S}\right)^2 \frac{mc^2\omega}{\hbar}\, t_{free}^2$$

"Acceleration" stage

$$\varepsilon(t_{free}) \sim eEc\, t_{free}$$

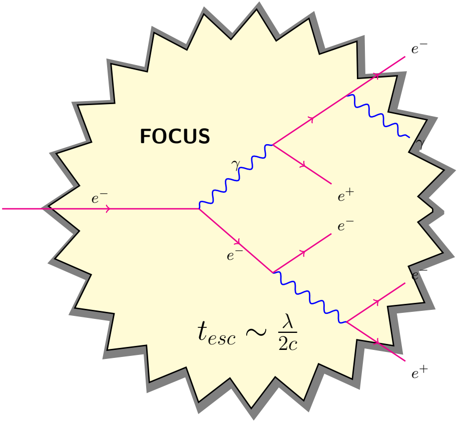
$$t_{free} \sim \frac{\hbar}{\alpha m c^2 \mu^{1/4}} \sqrt{\frac{mc^2}{\hbar\omega}}$$

$$\chi \sim \mu^{3/2}, \quad \varepsilon \sim mc^2 \mu^{3/4} \sqrt{\frac{mc^2}{\hbar\omega}}, \quad \vartheta \sim \omega t_{free} \sim \frac{1}{\alpha\mu^{1/4}} \sqrt{\frac{\hbar\omega}{mc^2}}$$

The "escape" time. Hierarchy of scales for $\mu \gtrsim 1$ ($I \gtrsim 10^{25}$ W/cm^2)

$$\frac{t_{free}}{t_{acc}} \sim \mu^{3/4}, \quad \frac{\pi/\omega}{t_{free}} \sim \pi\mu^{1/4}\sqrt{\frac{\alpha^2 mc^2}{\hbar\omega}} \quad (0.5\alpha^2 mc^2 = 13.6\text{eV} \Longrightarrow \sqrt{\ } \sim 5)$$

$$t_{acc} \lesssim t_{free} \ll t_{esc}$$



$$N_{e^-e^+} \sim e^{t_{esc}/t_{free}}$$

Crucial points

- Optical field
$$\hbar\omega[\sim \text{eV}] \ll mc^2[0.5\text{MeV}];$$
- High intensity $a_0 = eF/m\omega c \gg 1$, so that
$$\chi \sim \frac{F}{E_S} \times \gamma \sim \frac{F}{E_S} \times a_0 \sim \left(\frac{F}{E_S}\right)^2 \times \frac{mc^2}{\hbar\omega}$$
can become non-small well below E_S;
- Generality of the field configuration (non-constant, non-plane wave field – is always satisfied in a tightly focused field

However, the threshold value $I_* \sim 10^{25}\text{W/cm}^2$ should not be understood literally. According to the previous estimations, even at $I \sim I_*$ we have $N_{e-e^+} \sim e^{15} \sim 10^6$, but universal scaling of probabilities is only setting in and the whole estimation may be not reliable. In fact the threshold may be even less especially for particular field configurations (e.g for weakly focused colliding pulses, see below).

Kinetic description of generated $e^-e^+\gamma$- plasma (EPPP)

Phase space distributions of EPPP:

$$f_-(\vec{r},\vec{p},t), \quad f_+(\vec{r},\vec{p},t), \quad f_\gamma(\vec{r},\vec{p},t)$$

$$\frac{df_a}{dt} = \text{GAIN} - \text{LOSS}$$

Currently neglected:

- (Possible) degeneracy of EPPP:
  $\varepsilon_e \gg \varepsilon_F = (3\pi^2)^{1/3}\hbar c n_e^{1/3} \Longrightarrow n_e \ll (\varepsilon_e/\hbar c)^3$;
- Recombination processes $\mathcal{O}(n_a^2)$ ($e^\pm\gamma \to e^\pm$, $e^+e^- \to \gamma$):
  $n_e \ll (mc/\hbar)^2 \times (\varepsilon_e/\hbar c)$;
- "Trident" processes ($e^\pm \to e^\pm e^- e^+$, $e^\pm \to e^\pm\gamma\gamma$);
- Other $\mathcal{O}(\alpha^2)$ processes ($e^\pm\gamma \to e^\pm\gamma$, $e^+e^- \to \gamma\gamma$, $\gamma\gamma \to e^+e^-$,...);
- ...

Kinetic (cascade) equations

$$\left\{\frac{\partial}{\partial t}+\frac{\vec{p}}{\varepsilon}\cdot\nabla\pm e\left(\vec{E}+\frac{\vec{p}}{\varepsilon}\times\vec{H}\right)\cdot\frac{\partial}{\partial\vec{p}}\right\}f_{\pm}(\vec{p},t)=$$

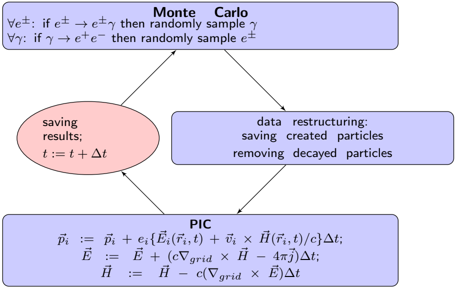
$$=\underbrace{\underbrace{\int f_{\pm}(\vec{p}+\vec{k},t)w_{rad}(\vec{p}+\vec{k}\to\vec{k})d^3k}_{\text{gain } e^{\pm}\to e^{\pm}\gamma}-\underbrace{f_{\pm}(\vec{p},t)\int w_{rad}(\vec{p}\to\vec{k})d^3k}_{\text{loss } e^{\pm}\to e^{\pm}\gamma}}_{\text{quantum radiation reaction (friction)}}+$$

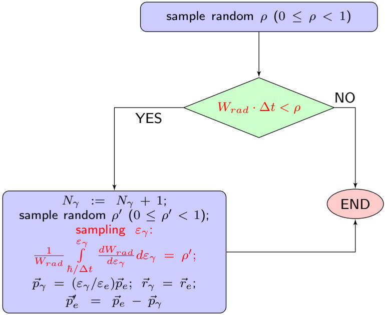
$$+\underbrace{\int f_{\gamma}(\vec{k},t)w_{cr}(\vec{k}\to\vec{p})d^3k}_{\text{gain } \gamma\to e^-e^+}$$

$$\left\{\frac{\partial}{\partial t}+\frac{\vec{k}}{\omega}\cdot\nabla\right\}f_{\gamma}(\vec{k},t)=\underbrace{\int[f_{+}(\vec{p},t)+f_{-}(\vec{p},t)]w_{rad}(\vec{p}\to\vec{k})d^3p}_{\text{gain } e^{\pm}\to e^{\pm}\gamma}-$$

$$-\underbrace{f_{\gamma}(\vec{k},t)\int w_{cr}(\vec{k}\to\vec{p})d^3p}_{\text{loss } \gamma\to e^-e^+}$$

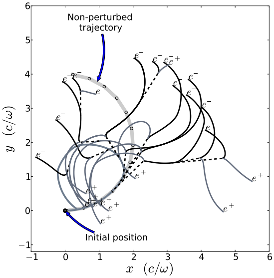
General structure of PIC-MC code[20]

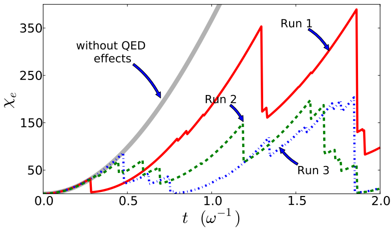


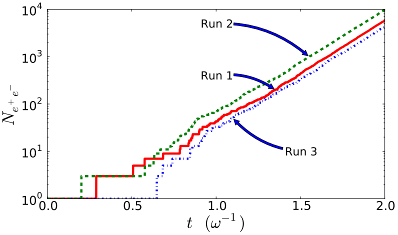


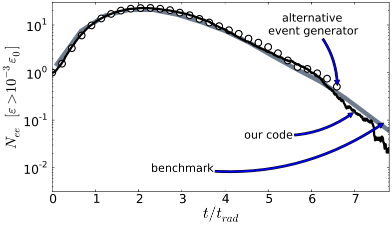
[20] NV Elkina et al. "QED cascades induced by circularly polarized laser fields". In: *Physical Review Special Topics-Accelerators and Beams* 14 (2011), p. 054401.

Simulations of cascade dynamics[21)]









21) Elkina et al., "QED cascades induced by circularly polarized laser fields".

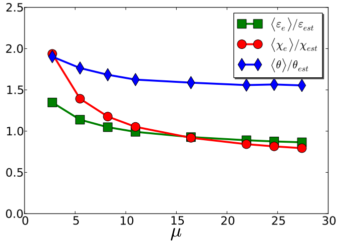
Proof of scaling

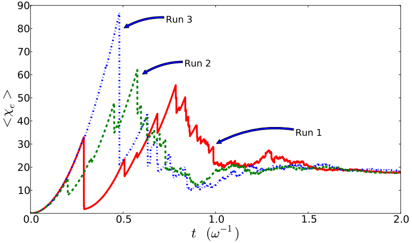
$$\chi_{est} \sim \mu^{3/2}$$

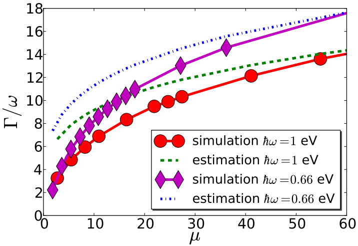
$$\varepsilon_{est} \sim mc^2\mu^{3/4}\sqrt{\frac{mc^2}{\hbar\omega}}$$

$$\vartheta_{est} \sim \frac{1}{\alpha\mu^{1/4}}\sqrt{\frac{\hbar\omega}{mc^2}}$$

$$\Gamma \sim \alpha\mu^{1/4}\sqrt{\frac{mc^2\omega}{\hbar}}$$







Laser energy absorption by EPPP

$$\dot{\vec{p}}_\pm = \pm e\vec{E} \overset{\vec{E}\propto e^{-i\omega t}}{\Longrightarrow} \vec{v}_\pm = \frac{\vec{p}_\pm c^2}{\varepsilon_\pm} = \pm\frac{e\vec{E}c^2}{-i\omega\varepsilon_\pm}$$

$$\vec{j} = \begin{cases} e\sum\limits_\pm n_\pm \vec{v}_\pm \\ -i\omega\vec{P} = -i\omega\frac{\epsilon-1}{4\pi}\vec{E} \end{cases}$$

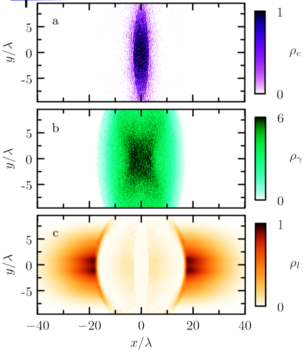
$$\epsilon = 1 - \frac{4\pi e^2c^2}{\omega^2}\sum_\pm \frac{n_\pm}{\varepsilon_\pm} = 1 - \frac{8\pi e^2c^2n_e}{\omega^2\varepsilon_e}$$

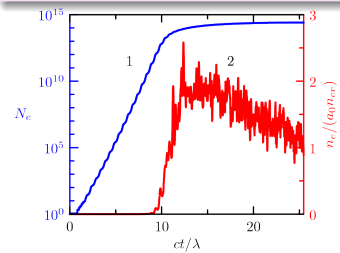
$$\vec{E},\ \vec{H} \propto e^{i\vec{k}\vec{r}-i\omega t}, \quad k = \sqrt{\epsilon}\frac{\omega}{c} \Longrightarrow \epsilon < 0$$

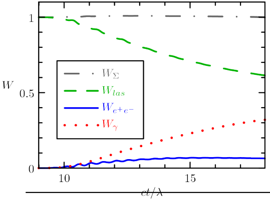
Absorption threshold: $n_e > \frac{\omega^2}{8\pi e^2c^2}\varepsilon_e = n_{cr}\gamma \simeq n_{cr}a_0$

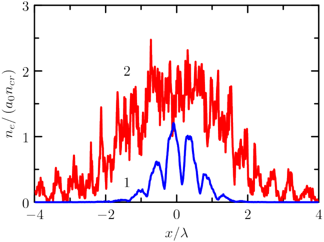
2D self-consistent simulation with backreaction[22)]

$I_L = 3 \times 10^{24}$W/cm^2, $\lambda_L = 0.8\mu$m
$R_f = 5\mu$m, $\tau = 100$fs









[22)] EN Nerush et al. "Laser field absorption in self-generated electron-positron pair plasma". In: *Physical Review Letters* 106 (2011), p. 035001.

Some parallel/successive simulations

- J.G. Kirk et al., "Pair production in counter-propagating laser beams", Plasma Phys. Control. Fusion **51**, 085008 (2009).
- E.N. Nerush, et al., "Laser field absorption in self-generated electron-positron pair plasma", PRL **106**, 035001 (2011).
- R. Duclous et al., "Monte Carlo calculations of pair production in high-intensity laserplasma interactions", Plasma Phys. Control. Fusion **53**, 015009 (2011).
- C. P. Ridgers, et al., "Dense Electron-Positron Plasmas and Ultraintense γ-rays from Laser-Irradiated Solids", PRL **108**, 165006 (2012).
- J.G. Kirk et al., "Pair plasma cushions in the hole-boring scenario", Plasma Phys. Control. Fusion **55** 095016 (2013).
- V.F. Bashmakov, et al., "Effect of laser polarization on quantum electrodynamical cascading", Physics of Plasmas, **21** 013105 (2014).
- C. S. Brady et al., "Synchrotron radiation, pair production, and longitudinal electron motion during 10-100 PW laser solid interactions", Phys. Plasmas **21**, 033108 (2014).
- A. Gonoskov, et al., "Extended particle-in-cell schemes for physics in ultrastrong laser fields: Review and developments", PRE **92**, 023305 (2015).
- M. Lobet, et al., "Modeling of radiative and quantum electrodynamics effects in PIC simulations of ultra-relativistic laser-plasma interaction", J. Phys. Conf. series **688**, 012058 (2016).
- T. Grismayer, et al., "Laser absorption via quantum electrodynamics cascades in counter propagating laser pulses", Physics of Plasmas, **23**, 056706 (2016).
- M. Jirka, et al, "Electron dynamics and γ and e^-e^+ production by colliding laser pulses", PRE **93**, 023207 (2016).
- ...

Observation:

Typically, in more realistic simulations, self-sustained regime of QED cascades is already observed at intensities $10^{23\div24}$W/cm^2, $1\div2$ orders lower than 5×10^{25}W/cm^2 $\leftrightarrow$ $\boxed{E=\alpha E_S}$.

Of ultimate importance for ELI, XCELS, etc.!

- If $R\gg1/\omega$, then $t_{esc}\simeq R\gg1/\omega$ (if **radiative trapping** also takes place [Gonoskov et al., PRL 2014; Ji et al., PRL 2014; AF et al., PRA 2014], then even $t_{esc}\gg R$!);
- Any estimate of Γ always underestimates cascade multiplicity: $\langle e^{\Gamma t}\rangle > e^{\langle\Gamma\rangle t}$, and even $\langle e^{\Gamma t}\rangle \gg e^{\langle\Gamma\rangle t}$ for $t\gg\Gamma^{-1}$;
- Originally, we assumed $\varkappa\gtrsim1$ as rough condition for pair production (this also approved usage of universal asymptotic for W). However, $W_{e^+e^-}(\varkappa\ll1)=\mathcal{O}(e^{-8/3\varkappa})$ remains non-negligible for even smaller values $\varkappa\gtrsim0.1$

Radiative impenetrability *of strong field region*

Pomeranchuk theorem:[23]

$$m\frac{d\gamma}{dt} \approx -\frac{2}{3}\frac{e^4}{m^2}\underbrace{\left[(\vec{E}+\vec{v}\times\vec{H})^2-(\vec{v}\cdot\vec{E})^2\right]\Big|_{\vec{r}=\vec{\rho}+\vec{v}t}}_{F_\perp^2(t)}\gamma^2,$$

$$-\int\limits_{\gamma_i}^{\gamma_f}\frac{d\gamma}{\gamma^2}=\frac{1}{\gamma_f}-\frac{1}{\gamma_i}=\frac{2}{3}\frac{e^4}{m^3}\int\limits_{t_i}^{t_f}F_\perp^2(t)\,dt,$$

$$\gamma_f^{(max)}=\frac{3}{2}\frac{m^3}{\alpha^2\int\limits_{-\infty}^{0}F_\perp^2(t)\,dt}\sim\frac{3}{2}\frac{m^3}{e^4\frac{1}{2}\left(\frac{m\omega a_0}{e}\right)^2R}=\frac{3m}{e^2a_0^2\omega^2R}$$

Condition for **focus impenetrability**:[24]

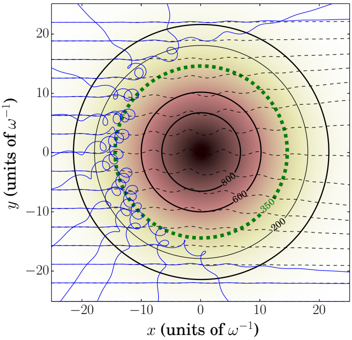
$$\gamma_f^{(max)}\lesssim a_0, \quad \text{or} \quad \boxed{a_0\gtrsim\left(\frac{3m}{e^2\omega^2R}\right)^{1/3}}$$

[23] IY Pomeranchuk. "On the maximum energy which the primary electrons of cosmic rays can have on the earths surface due to radiation in the earths magnetic field". In: *J. Phys. (USSR)* 2 (1940), p. 65.

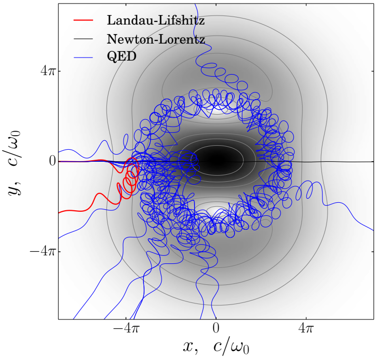
[24] AM Fedotov et al. "Radiation friction versus ponderomotive effect". In: *Physical Review A* 90 (2014), p. 053847.

Perfect agreement with simulations[25)]

Classical simulation
$(\gamma_i = 10^3,\ a_0 = 10^3,\ R = 10/\omega)$



Quantum MC simulation
$(\gamma_i = 10^4,\ a_0 = 10^3,\ R = 10/\omega)$



25) Fedotov et al., "Radiation friction versus ponderomotive effect".

- Still, **hard γ-quanta** emitted by high-energy particles approaching focus can penetrate inside and initiate cascades!
- Matching conditions:

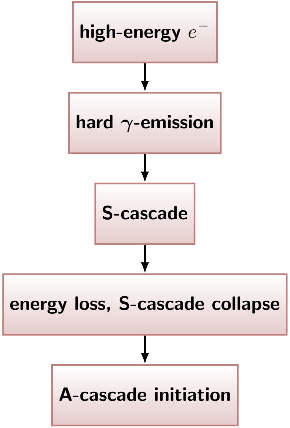
$$\boxed{a_0 \gtrsim a_{th,A}}$$

$$t_S \simeq t_{free} \cdot n \simeq$$

$$\simeq \underbrace{\frac{\hbar \varepsilon_0}{\alpha m^2 c^4} \chi_i^{-2/3}}_{t_{free} \simeq W^{-1}} \cdot \underbrace{\log_2 \left(\frac{\chi_i}{\cancel{\chi_f}_{\simeq 0.1}} \right)}_{n}$$

$$\boxed{t_{free} \ll t_S \ll \tau_L}$$

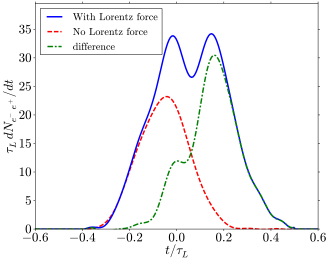
Collapse&revival scenario:

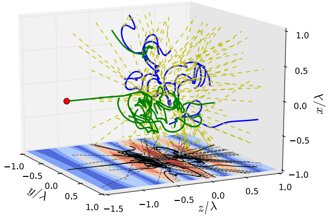


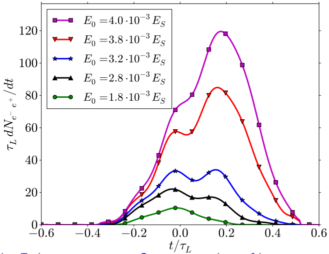
26) Mironov, Narozhny, and Fedotov, "Collapse and revival of electromagnetic cascades in focused intense laser pulses".

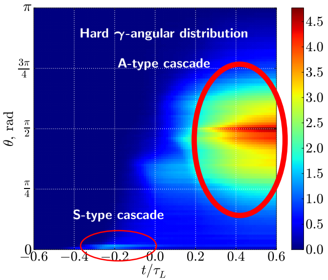
Example simulation

2×counterpropagating CP $10fs$ laser pulses + 3GeV seeding e^--beam;
$E_0 = 3.2\times10^{-3}E_S$ (i.e., $a_0 = 1600, I \simeq 5\times10^{24}$W/cm^2)









Discussion

- At $I \gtrsim 10^{24\div25}$W/cm^2 a **new physical regime** of laser - matter interaction should be revealed, characterized by **massive production of QED ($e^- e^+ \gamma$) cascades** [with macroscopic multiplicity!]
  - There may be though some problems with injection of seed particles (e.g. due to radiative impenetrability of strong field region)
  - One possible solution – conversion of S-cascades to A-cascades (as hard photons may easily access focus)
- At $I \gtrsim 10^{26\div27}$W/cm^2 even focusing of laser pulses in vacuum would become unstable due to spontaneous pair creation and subsequent cascades development
- This process of fast depletion of a focused laser field in vacuum due to production of $e^- e^+ \gamma$-plasma may very likely **prevent attainability of the Sauter-Schwinger critical electric field**

$$E_S = \frac{m^2 c^3}{e\hbar} = 1.3 \times 10^{16}\text{V/cm}$$

  with laser fields capable for pair creation
- However, for more definite predictions **further simulations of this regime are required.**

Radiation corrections

Motivation

- Most of NB Narozhny's pioneer works are well recognized:
  - calculation of *probabilities for photon emission and pair photoproduction in circularly polarized electromagnetic wave*
  - first calculation of *polarization operator in a constant crossed field*
  - first *direct* calculation of *spontaneous pair production in electric field*
  - or the *effect of collapses and revivals in cavity QED*
- However, his probably *the most deep and significant contribution* (at least those claimed as such in his Dr.Sc. dissertation back in 1982), the α^3-order calculations[27)] proving the original Ritus conjecture[28)] of possible **break-down of perturbative QED at $\alpha\chi^{2/3} \gtrsim 1$**, still remains rather unknown.
- Here I am going to give the review of that old idea, in particular:
  - to explain some known arguments in favor of the conjecture;
  - give several insights into its meaning;
  - stress its significance for the near future progress of laser-matter interaction studies at extreme intensities.

[27)] NB Narozhny. "Radiation corrections to quantum processes in an intense electromagnetic field". In: *Physical Review D* 20 (1979), p. 1313; NB Narozhny. "Expansion parameter of perturbation theory in intense-field quantum electrodynamics". In: *Physical Review D* 21 (1980), p. 1176; DA Morozov, NB Narozhnyj, and VI Ritus. "Vertex function of an electron in a constant electromagnetic field". In: *Sov. Phys. JETP* 53 (1981), p. 1103.

[28)] VI Ritus. "Radiative effects and their enhancement in an intense electromagnetic field". In: *Sov. Phys. JETP* 30 (1970), p. 1181.

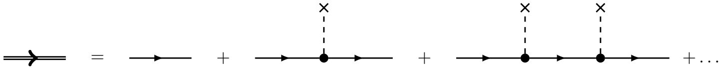
Reminder: Radiation corrections in QED

- As mentioned in Introduction, in Classical Electrodynamics self-field energy diverges as $\mathscr{E}_{\text{em}} \simeq \frac{e^2}{r_0}$ and in QED it is still present but is *much weaker* (logarithmic, $\mathscr{E}_{\text{em}} \simeq e^2 m \log\left(\frac{1}{mr_0}\right)$, vs linear) than in Classical Electrodynamics.
- After renormalization (which is all the same required for physical reasons, albeit $\mathscr{E}_{\text{em}} \simeq \alpha m \log\left(\frac{1}{mr_0}\right) \ll m$ for any reasonable value of r_0!), the coupling constant becomes effectively *'running'*, and its energy dependence essentially mimics the nature of divergency: $\alpha(\varepsilon) \simeq \alpha \log(\varepsilon/m)$, $\varepsilon \gg m$ (high energy 'stripping'). Note that $\alpha(\varepsilon)$ *remains small for all reasonable values of energy!*
- Review and classification of the variety of high-energy QED processes[29)] demonstrates that all the cross sections remain small $\sigma(\varepsilon) \lesssim \alpha^n r_e^2 \log^k(\varepsilon/m)$ within all the **reasonable** energy range.
- Thus, **perturbation theory in ordinary QED works pretty well for all the reasonable values of parameters.**

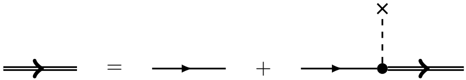
[29)] VG Gorshkov. "Electrodynamic processes in colliding beams of high-energy particles". In: *Physics-Uspekhi* 16 (1973), p. 322; VN Baier et al. "Inelastic processes in high energy quantum electrodynamics". In: *Physics Reports* 78 (1981), p. 293.

Intense Field QED

- However, in **external field** with $a_0 \gg 1$ perturbation theory **with respect to interaction with that field** breaks down and all-order summation is needed, which reduces to replacement of free propagators by the exact ones in external field:



- For several cases (including the most important paradigmatic case of **constant crossed field**, which corresponds to $a_0 \gg 1$ and relativistic motion across the field) the equation



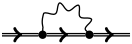
can be solved in closed form.

- Note that in CCF electrons /photons are characterized by a single Lorentz- and gauge-invariant parameter $\chi = \frac{e}{m^3}\sqrt{-(F_{\mu\nu}p^\nu)^2}$ / $\varkappa = \frac{e}{m^3}\sqrt{-(F_{\mu\nu}k^\nu)^2}$ - for electrons this is just proper acceleration in Compton units.

Radiation corrections in Intense Field QED I

- It was noticed already at its birth[30)] that in IFQED radiation corrections **are growing surprisingly fast** with χ or $\varkappa$ (i.e. with **both energy and field strength**):

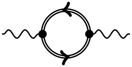
$$M^{(2)}(\chi) =$$



$$\simeq \alpha m \chi^{2/3}, \quad \chi \gg 1;$$

$$W_{e^\pm \to e^\pm \gamma}(\chi) = \frac{2m}{p_0} \operatorname{Im} M^{(2)} \simeq \frac{\alpha m^2}{p_0} \chi^{2/3}, \quad \chi \gg 1;$$

$$\mathscr{P}^{(2)}(\varkappa) =$$



$$\simeq \alpha m^2 \varkappa^{2/3}, \quad \varkappa \gg 1;$$

$$W_{\gamma \to e^+ e^-}(\varkappa) = \frac{2}{k_0} \operatorname{Im} \mathscr{P}^{(2)} \simeq \frac{\alpha m^2}{k_0} \varkappa^{2/3}, \quad \varkappa \gg 1;$$

[30)] Nikishov and Ritus, "Quantum processes in the field of a plane electromagnetic wave and in a constant field. I"; Narozhny, "Propagation of plane electromagnetic waves in a constant field"; Ritus, "Radiative effects and their enhancement in an intense electromagnetic field".

Radiation corrections in Intense Field QED II

- This implies that for $\chi, \varkappa \gtrsim \alpha^{-3/2} \simeq 1.6 \times 10^3$ ($E_{\mathsf{p}} \gtrsim 12 E_{\mathsf{cr}} \simeq 1600 E_S$ – recall $E_{\mathsf{cr}} \simeq 137 E_S$ is the **classical critical field**!):

$$M^{(2)} \simeq m, \quad \mathscr{P}^{(2)} \simeq m^2$$

  and that in proper reference frame

$$t_e \sim W^{-1}_{e^\pm \to e^\pm \gamma} \simeq t_C, \quad t_\gamma \sim W^{-1}_{\gamma \to e^+ e^-} \simeq t_C$$

- These means that **radiation corrections become not small** and **radiation-free motion** could show up only at **Compton scale** (where localization is all the same impossible).

Reference values for $\alpha\chi^{2/3} \sim 1$

- For high-energy e^- counterpropagating laser pulse $\chi \sim \frac{E\gamma_{\text{in}}}{E_S}$

$\varepsilon_{\text{in}} = m\gamma_{\text{in}}$, GeV	800	80	8	0.8
E/E_S	10^{-3}	10^{-2}	0.1	1
I_L,W/cm^2	5×10^{23}	5×10^{25}	5×10^{27}	5×10^{29}

Observe that this threshold could be **almost overcome experimentally** by combining state-of-the-art laser systems with the future ILC–class TeV lepton colliders.

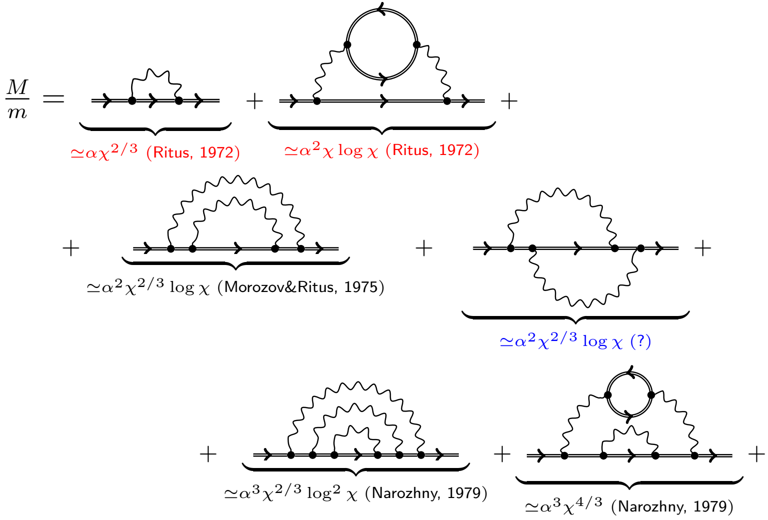
- Note: the table assumes **transverse** propagation across the field. For self-sustained (A-type) cascades[31] $E \gtrsim \alpha E_S$ and

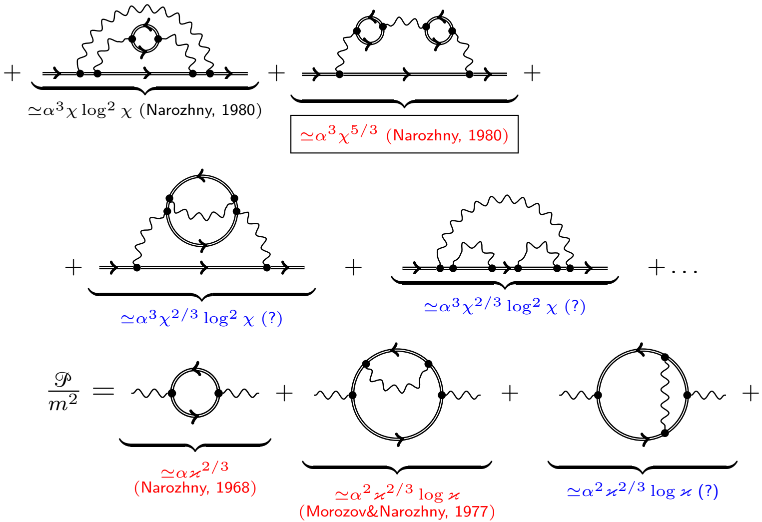
$$\measuredangle(\vec{p},\vec{E}) \sim \left(\frac{\alpha E_S}{E}\right)^{1/4} \lesssim 1, \quad \chi \sim \left(\frac{E}{\alpha E_S}\right)^{3/2} \gtrsim 1,$$

but $\boxed{\alpha\chi^{2/3} \sim \frac{E}{E_S} \ll 1}$

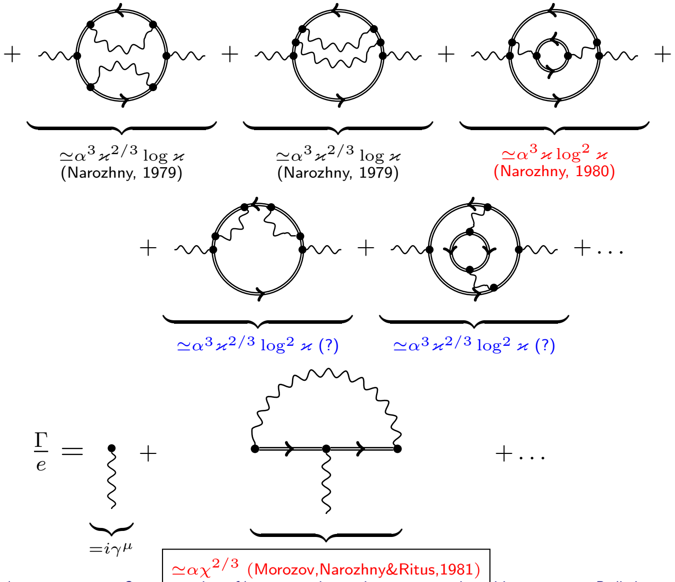
[31] Fedotov et al., "Limitations on the attainable intensity of high power lasers"; Elkina et al., "QED cascades induced by circularly polarized laser fields".

Higher order radiation corrections in IFQED I




+
+
+
$\simeq\alpha^3\chi\log^2\chi$ (Narozhny, 1980)
$\simeq\alpha^3\chi^{5/3}$ (Narozhny, 1980)
+
+
+ . . .
$\simeq\alpha^3\chi^{2/3}\log^2\chi$ (?)
$\simeq\alpha^3\chi^{2/3}\log^2\chi$ (?)
$\frac{\mathscr{P}}{m^2} =$
+
+
+
$\simeq\alpha\varkappa^{2/3}$
(Narozhny, 1968)
$\simeq\alpha^2\varkappa^{2/3}\log\varkappa$
(Morozov&Narozhny, 1977)
$\simeq\alpha^2\varkappa^{2/3}\log\varkappa$ (?)

Higher order radiation corrections in IFQED III



Physical interpretation I

- On defense, Nikolay Borisovich was asked for a 'simple words' **physical reasoning (interpretation)** for appearance of the parameter $\alpha\varkappa^{2/3}$. And his answer was about that in ultrarelativistic case $\mathscr{P} = \alpha m^2 F(\varkappa)$ should not depend on m. Then, since $\varkappa \propto m^{-3}$, it should be $F(\varkappa) \propto \varkappa^{2/3}$ unambiguously for $\varkappa \gg 1$. But this argument doesn't work for M, Γ and higher orders.
- However, recently a more visual and direct explanation was seemingly found:[32)]
- Consider for definiteness[33)] formation times and lengths for the polarization operator $\mathscr{P}^{(2)}(\varkappa \gg 1)$, given by the **virtual process** $\gamma \to e^- e^+ \to \gamma$.
- Assume that initially $\vec{k} \perp \vec{E}$, then the energy uncertainty of the virtual process

$$\Delta\varepsilon(t) \;=\; \sqrt{p^2 + e^2E^2t^2 + m^2} + \sqrt{(k-p)^2 + e^2E^2t^2 + m^2} - k$$

32) Fedotov, "Qualitative considerations in Intense Field QED".

33) Ultrarelativistic kinematics is in fact similar for all the processes.

Physical interpretation II

- Assuming k and p large, we have:

$$\Delta\varepsilon(t) \simeq \not{p} + \frac{e^2E^2t^2+m^2}{2p} + \not{k} - \not{p} + \frac{e^2E^2t^2+m^2}{2(k-p)} - \not{k} =$$

$$= \frac{k\left(e^2E^2t^2+m^2\right)}{2p(k-p)} \geq \frac{2\left(e^2E^2t^2+m^2\right)}{k}$$

- Assume (to be confirmed by result) that $eEt \gg m$. Then $\Delta\varepsilon(t) \simeq \frac{e^2E^2t^2}{k}$ and from the uncertainty principle

$$\Delta\varepsilon \cdot t \sim 1 \quad \Longrightarrow \quad t, l_{\parallel} \simeq \left(\frac{k}{e^2E^2}\right)^{1/3} \equiv \frac{k}{m^2\varkappa^{2/3}} \equiv \frac{m}{eE}\varkappa^{1/3},$$

where $\varkappa = eEk/m^3$. This key simple estimate exactly coincides with direct derivation of the effective formation region from quantum amplitude![34] And indeed, $eEt \simeq m\varkappa^{1/3} \gg m$, as expected.

[34] DA Morozov and VI Ritus. “Elastic electron scattering in an intense field and two-photon emission”. In: *Nuclear Physics B* 86 (1975), p. 309.

Physical interpretation III

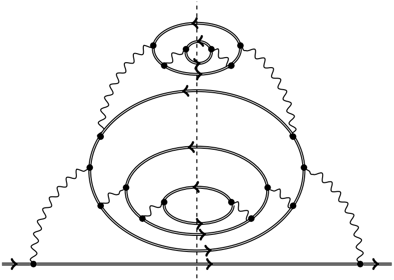
- Transverse separation $l_\perp \simeq \frac{eEt^2}{k} \simeq \frac{1}{m\varkappa^{1/3}} \equiv \frac{1}{(eEk)^{1/3}}$ (note it is m-independent). Maybe a bit counterintuitively, charge separation **reduces** (rather than increases) with the field – this is a quantum effect due to that t reduces too fast.
- Moreover, strong ($\varkappa \gg 1$) field is capable for confining virtual pairs to distances smaller than $l_C = \frac{1}{m}$! (this is very reminiscent to the Ritus's observation[35] of **strong field–small distance correspondence**).
- In **'proper'** reference frame[36] $l'_\parallel \sim \frac{m}{k} l_\parallel \sim \frac{1}{m\varkappa^{2/3}} \ll l_\perp$, thus $\boldsymbol{l'_\parallel}$ **is the smallest scale**. Surprisingly, for $\alpha\varkappa^{2/3} \sim 1$ it coincides to the classical electron radius r_e!
- Now polarization operator should be defined by these scales: $\mathscr{P}(\varkappa) \simeq e^2/{l_\perp}^2(\varkappa)$. Similarly, $M \simeq e^2/l'_\parallel(\varkappa)$. The parameter $\alpha\chi^{2/3} \equiv \frac{e^2/l'_\parallel}{m}$ – Coulomb to rest energy ratio.

---

[35] VI Ritus. "Lagrangian of an intense electromagnetic field and quantum electrodynamics at small distances". In: *Sov. Phys. JETP* 42 (1975), p. 774.

[36] I.e. where the photon is 'soft' ($k' \sim m$) and $E_P \sim \varkappa E_S$.

Nonperturbative regime and QED cascades



- At $\chi, \varkappa \gg 1$

$$A \simeq eEl_\perp \simeq \Delta\varepsilon$$

– intermediate virtual states are close to mass shell, $\mathrm{Im}\,M \sim \mathrm{Re}\,M$, $\mathrm{Im}\,\mathscr{P} \sim \mathrm{Re}\,\mathscr{P}$.

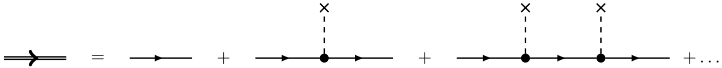
- Optical theorem:

$$W_{\text{e-seeded cascade}} \simeq \frac{m}{p_0}\mathrm{Im}\,M,$$
$$W_{\gamma\text{-seeded cascade}} \simeq \frac{1}{p_0}\mathrm{Im}\,\mathscr{P}$$

- In self-sustained regime $\alpha\chi^{2/3} \simeq \frac{E}{E_S}$
- Non-attainability of E_S (due to self-sustained cascades)?
- Time variation of the field required?

Discussion

- The **conjecture** that radiation corrections in IFQED are growing as a power of energy and field strength is really puzzling and **challenging for theoreticians:**
  - In such a regime **QED** may become a truly **non-perturbative theory**: all the numerous results published by now may become invalid!
  - In particular, the whole IFQED approach we got used to, should also break down, as the **external field** lines used in 'exact' propagators from the beginning



    should be radiatively corrected as well!
  - Possible hints: for $\alpha\chi^{2/3} \sim 1$ (i) domination of polarization loops? (ii) $l'_{\|} \simeq r_e$; (iii) for self-sustained cascades $E \simeq E_S$?
- The regime $\alpha\chi^{2/3} \gtrsim 1$ may 'soon' appear observable for experimentalists.
- Unfortunately, potential **significance** of consequences of the **conjecture has still been underestimated** by the community.

Conclusion

Conclusion

- There is still plenty of unsolved theoretical problems in IFQED. Among them:
  - Understanding of physical meaning of complicated calculations made previously, which is mostly absent (also strongly needed for generalizations and further development)
  - Problem of principle attainability of Sauter-Schwinger fields with lasers
  - Completely unexplored non-perturbative Ritus-Narozhny regime at $\alpha\chi^{2/3} \gtrsim 1$
- These and other theoretical challenges should be urgently addressed due to some near-future prospects of further radical increase of experimental capabilities (with ELI, XCELS, etc.)
- These problems may be also sound in other fields (e.g. in astrophysics)

Thank you for attention!