NonLeptonic Two-Body Decays of Charmed Mesons and CP Violation

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Physics of Heavy Quarks and Hadrons
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Experimental Results on Direct CPV in D^0 Decays

LHCb collaboration, at the end of 2011, measured

$$\Delta A_{\rm CP} = a_{CP}(K^+K^-) - a_{\rm CP}(\pi^+\pi^-)$$

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&= (-0.87 \pm 0.41 \pm 0.06)\% && (\text{Belle}\,(2012)),\\
&= (+0.24 \pm 0.62 \pm 0.26)\% && (\text{BaBaR}\,(2008)),\\
&= (-0.34 \pm 0.15 \pm 0.10)\% && (\text{LHCb}\,(2013)),\\
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By taking into account the last data from LHCb (2016)

$$\begin{aligned}
\Delta a_{\rm CP} &= (-0.32 \pm 0.22)\% && \textit{PDG}\,2014 \\
&= (-0.137 \pm 0.070)\% && \textit{HFAG Feb}\,2016
\end{aligned}$$

Plan of the talk

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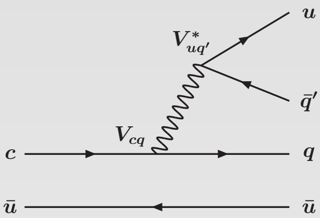
3. CP violations
4. Conclusions

Hadronic two-body Decays of D Meson (1)

In the Standard Model flavour changing transitions are induced by exchange of W bosons:



- CKM hierarchy leads to two-generation dominance

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - \imath\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-\imath\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

	UT$_{fit}$
λ	0.22534 ± 0.00065
A	0.821 ± 0.012
ρ	0.136 ± 0.024
η	0.361 ± 0.014

UTFit Collaboration

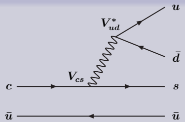
Hadronic two-body Decays of D Meson (2)

Due to the CKM hierarchy we can classify decay processes into three classes

Cabibbo Favoured (CF)

$|V_{cs} V_{ud}^*| \approx 1$ as,

for example, $D^0 \to K^- \pi^+$



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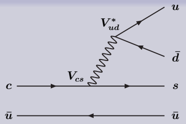
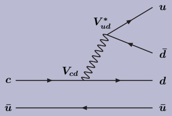


Singly Cabibbo Suppressed (SCS)

$|V_{cd}V_{ud}^*| \approx \lambda$
($D^0 \to \pi^+\pi^-$)



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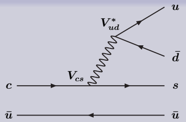

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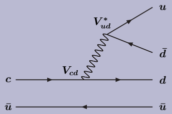

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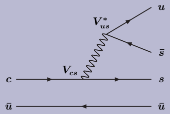

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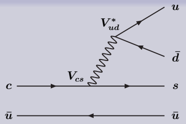

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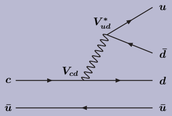

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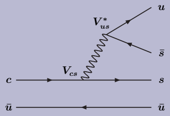
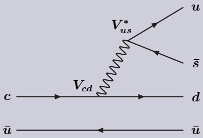


Double Cabibbo Suppressed (DCS)

$|V_{cd}V_{us}^{*}| \approx \lambda^2$
($D^0 \to K^+\pi^-$)



Weak Effective Hamiltonian(1)

The Charged Current weak interaction Lagrangean

$$\mathscr{L}^{CC} = -\frac{g}{2\sqrt{2}} V_{ij} \left(\bar{\mathscr{U}}_i \gamma^\mu (1-\gamma_5) \mathscr{D}_j \right) W_\mu^\dagger + h.c.$$

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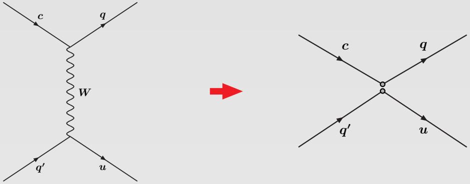
The Effective Field Theory approach allows to build an effective hamiltonian in which short and long distance contributions are separate:



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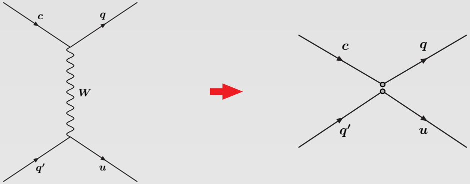

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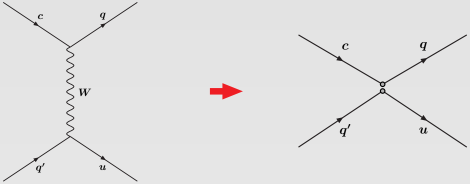


$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) O_i(\mu)$$

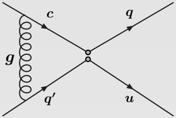
1. H_{eff} should be μ-independent;
2. The Wilson coefficients $C_i(\mu)$ summarize the physics contributions from scales higher than μ and due to asymptotic freedom of QCD they can be calculated in perturbation theory as long as μ is not too small.
3. It is customary to choose μ to be of the order of the mass of the decaying hadron $\Rightarrow$ Large logs $\ln m_W/\mu \Rightarrow$ renormalization group improved expansion

Weak Effective Hamiltonian (2)

We start with the operator ($q, q' \in \{d, s\}$)

$$O_2 = \left[\bar{q}^{\alpha}\gamma^{\mu}(1-\gamma_5)c_{\alpha}\right]\left[\bar{u}^{\beta}\gamma_{\mu}(1-\gamma_5)q'_{\beta}\right]$$

and



We have the operator

$$O_1 = \left[\bar{q}^{\alpha}\gamma^{\mu}(1-\gamma_5)c_{\beta}\right]\left[\bar{u}^{\beta}\gamma_{\mu}(1-\gamma_5)q'_{\alpha}\right]$$

and so

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{CKM}\left[C_1(\mu)\,O_1(\mu) + C_2(\mu)\,O_2(\mu)\right]$$

Weak Effective Hamiltonian (3)

We have another kind of Feynman diagrams

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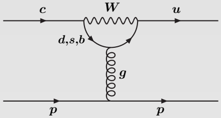
The QCD penguins operators



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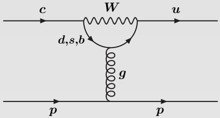

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O_3 &= \left[\bar{u}^\alpha \gamma^\mu (1-\gamma_5) c_\alpha\right] \sum_{p=u,d,s} \left[\bar{p}^\beta \gamma_\mu (1-\gamma_5) p_\beta\right] \\
O_4 &= \left[\bar{u}^\alpha \gamma^\mu (1-\gamma_5) c_\beta\right] \sum_{p=u,d,s} \left[\bar{p}^\beta \gamma_\mu (1-\gamma_5) p_\alpha\right] \\
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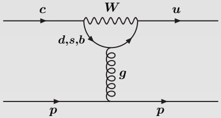

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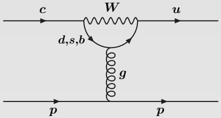

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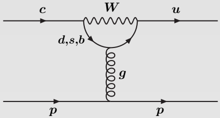

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Weak Effective Hamiltonian: CF and DCS

$$H_{\rm W}^{\rm CF} = \frac{G_F}{\sqrt{2}} V_{ud}\, V_{cs}^{*} \Big[C_1(\mu) \big[\bar{u}^{\beta} \gamma^{\mu} (1-\gamma_5) c_{\alpha}\big] \big[\bar{s}^{\alpha} \gamma_{\mu} (1-\gamma_5)\, d_{\beta}\big](\mu) + C_2(\mu) \big[\bar{s}^{\alpha} \gamma^{\mu} (1-\gamma_5) c_{\alpha}\big] \big[\bar{u}^{\beta} \gamma_{\mu} (1-\gamma_5)\, d_{\beta}\big](\mu) \Big] + h.c.$$

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Hadronic Matrix Elements

We have to evaluate

$$A(D \to f) = \langle f | H_{\mathrm{W}} | D \rangle = \frac{G_F}{\sqrt{2}} V V^* C_j(\mu) \boxed{\langle f | O_j(\mu) | D \rangle}$$

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- The hadronic matrix elements are dominated by non-perturbative QCD: they summarize the physics contributions to the amplitude from scales lower than μ
  - HQET is expected to do not work well, due to the large Λ_{QCD}/m_c corrections
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- Models of calculations can be useful to estimate order of magnitudes
  - Factorization & Final state Interactions
  - Flavour symmetries ($SU(3)_F$, isospin, U-spin, etc.)

Factorization: A Simple Model to Evaluate Matrix Elements

The idea (due to Feynman) is

$$\langle M_1\, M_2 |\, J^\mu J'_\mu\, | D\rangle \approx \langle M_1 |\, J^\mu\, | D\rangle \langle M_2 |\, J'_\mu\, | 0\rangle$$

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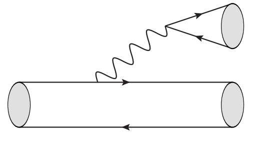
Color allowed external W emission tree amplitude: T $\rightarrow$



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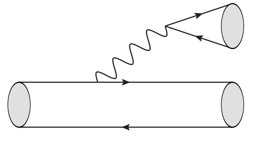
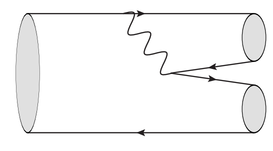


Color suppressed internal W emission tree amplitude: C $\rightarrow$

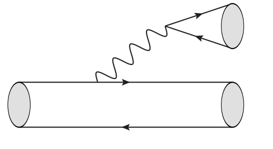


Factorization: A Simple Model to Evaluate Matrix Elements

The idea (due to Feynman) is

$$\langle M_1\, M_2 |\, J^\mu J'_\mu \,| D\rangle \approx \langle M_1 |\, J^\mu \,| D\rangle \langle M_2 |\, J'_\mu \,| 0\rangle$$

Color allowed external W emission tree amplitude: T $\rightarrow$



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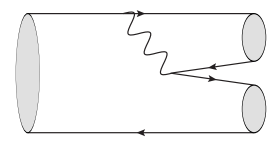
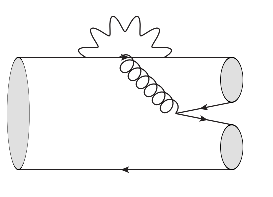


QCD Penguin amplitude: P $\rightarrow$



Factorization: Simple Model to Evaluate Matrix Elements (1)

$$\langle M_1\, M_2 |\, J^\mu J'_\mu \,| D\rangle \approx ... + \langle 0 |\, J^\mu \,| D\rangle \langle M_1\, M_2 |\, J'_\mu \,| 0\rangle$$

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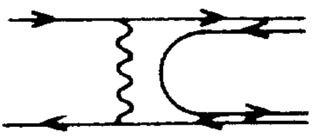

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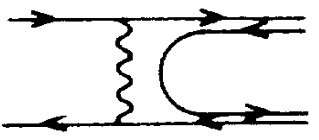
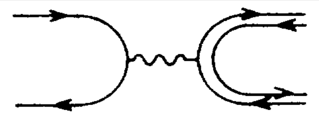


Annihilation amplitude



Factorization: Decay Constants and Form Factors

$$\langle P_i(p)|\, A^\mu \,|0\rangle \quad = \quad -i\, f_{P_i}\, p^\mu$$

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\end{aligned}
$$

$$
\begin{aligned}
q &= p_j - p_i \\
f_+(0) &= f_0(0)
\end{aligned}
$$

Factorization: The $D^0 \to \pi^-\pi^+$ Amplitude

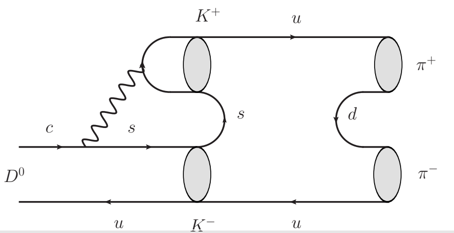
$$
\begin{aligned}
A(D^0 \to \pi^-\pi^+) \quad &= \\
-\frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* \quad &\times \quad \Big[(C_2 + \xi\, C_1)\, \langle \pi^- | \bar{d}\gamma_\mu c | D^0 \rangle\, \langle \pi^+ | \bar{u}\gamma^\mu \gamma_5 d | 0 \rangle \Big] \\
+\frac{G_F}{\sqrt{2}} V_{ub} V_{cb}^* \quad &\times \quad \Big[(C_4 + \xi\, C_3)\, \langle \pi^- | \bar{d}\gamma_\mu c | D^0 \rangle\, \langle \pi^+ | \bar{u}\gamma^\mu \gamma_5 d | 0 \rangle \\
&\quad -2\,(C_6 + \xi\, C_5)\, \langle \pi^- \pi^+ | \bar{u} u | 0 \rangle\, \langle 0 | \bar{u}\gamma_5 c | D^0 \rangle \\
&\quad +2\,(C_6 + \xi\, C_5)\, \langle \pi^- | \bar{d} c | D^0 \rangle\, \langle \pi^+ | \bar{u}\gamma_5 d | 0 \rangle \Big]
\end{aligned}
$$

$$
\begin{aligned}
\langle 0 | \bar{u}\gamma_5 c | D^0 \rangle \quad &= \quad -\imath \frac{f_D\, m_D^2}{m_u + m_c} \\
\langle \pi^+ | \bar{u}\gamma_5 d | 0 \rangle \quad &= \quad \frac{f_\pi\, m_\pi^2}{m_u + m_d}
\end{aligned}
$$

At the end we should consider effects of Final State Interaction to compare predictions and experimental data.

Final State Interaction Effects

These long-distance effects are dominated by resonances with the correct quantum numbers and masses very near the one of charmed mesons.



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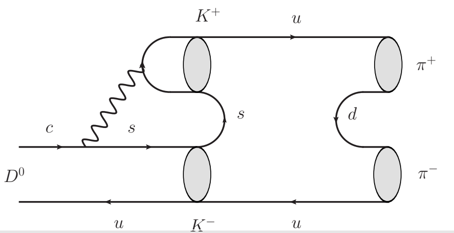

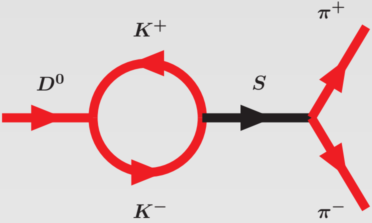

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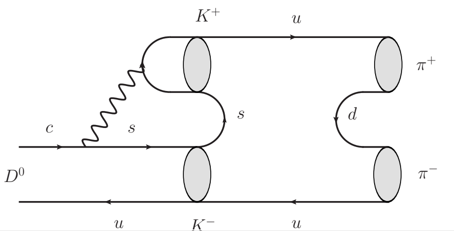

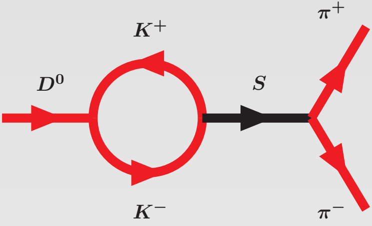

for the PP final state a scalar octet, S_c with $J^P = 0^+$

$$g_{888}\, d_{abc}\, P_a\, P_b\, S_c$$

$$\tan\delta = \frac{\Gamma(\tilde{S})}{2\,(m_{\tilde{S}} - m_D)}$$

Results

This kind of approach gives:

- a quite good agreement with the experimental data (at that time) on the branching ratios;
- direct CP violation effects of the order of 10^{-3};

In particular

$$\Delta a_{\mathrm{CP}} = a_{\mathrm{CP}}^{\mathrm{dir}}(K^+K^-) - a_{\mathrm{CP}}^{\mathrm{dir}}(\pi^+\pi^-) \simeq 0.11 \times 10^{-3}$$

Buccella, Lusignoli, Miele, Pugliese, P.S., Phys. Rev. D51 (1995) 3478

Singly Cabibbo Suppressed D^0 Decays

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PDG 2014

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(Franco Mishima Silvestrini JHEP 1205 (2012) 140)

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Large $SU(3)_F$ Violation

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Buccella, Lusignoli, Pugliese, P.S., PRD88(2013) 074011

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$$\begin{aligned} H_{\Delta U=1} &= \frac{G_F}{2\sqrt{2}}(V_{us} V_{cs}^* - V_{ud} V_{cd}^*)[C_1(O_1^s - O_1^d) + C_2(O_2^s - O_2^d)] \\ &\simeq \frac{G_F}{\sqrt{2}} \sin\theta_C \cos\theta_C [C_1(O_1^s - O_1^d) + C_2(O_2^s - O_2^d)]. \end{aligned}$$

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More interestingly the two independent combinations of S-wave states having U=1 can be written in terms of two representations of $SU(3)_F$

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\begin{aligned}
|8, U=1\rangle = \frac{\sqrt{3}}{2\sqrt{5}} \quad \Big\{ \quad & |K^+K^- > + |K^-K^+ > - |\pi^+\pi^- > - |\pi^-\pi^+ > \\
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|27, U=1\rangle = \frac{1}{\sqrt{10}} \quad \Big\{ \quad & |K^+K^- > + |K^-K^+ > - |\pi^+\pi^- > - |\pi^-\pi^+ > \\
+ \quad & \frac{3}{2} \left[|\pi^0\pi^0 > - |\eta_8\eta_8 > - \frac{1}{\sqrt{3}}(|\pi^0\eta_8 > + |\eta_8\pi^0 >) \right] \Big\}.
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$\Rightarrow$

$$A(D^0 \to K^+K^-) = \alpha \left(T' - \frac{2}{3}C' \right) + \beta \left(T' + C' \right)$$
$$A(D^0 \to \pi^+\pi^-) = \gamma \left(T' - \frac{2}{3}C' \right) + \delta \left(T' + C' \right)$$

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New Parameters

- To take into account these discrepancies we explicitly break SU(3) symmetry by allowing a different parameters for the color connected (T) and color suppressed (C) amplitudes in the CF and DCS channels: i.e. $T' = T(1+\varepsilon)$ and $C' = C(1-\varepsilon)$
- For D^+ we have tree independent amplitudes $\langle 8|\, H^{\bar{6}} \,|D^+\rangle$, $\langle 8|\, H^{15} \,|D^+\rangle$ and $\langle 27|\, H^{15} \,|D^+\rangle$ we have a new parameter we called D.
- Another parameter (K) takes into account the non conservation of the current $\bar{s}\gamma_\mu(1-\gamma_5)q$ with an opposite sign in the CF and in the DCS.

Final State Interactions in D Decays

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In other words, strong phases are generated by the resonances responsible for rescattering of final states.

Assuming no exotic resonances belonging to the 27 representation, the possible resonances have $SU(3)_F$ and isospin quantum numbers $(8, I=1)$, $(8, I=1/2)$, $(8, I=0)$ and $(1, I=0)$. Moreover, the two states with $I=0$ can be mixed, yielding two resonances:

$$\begin{aligned} |f_0> &= +\sin\phi \; |8, I=0> + \cos\phi \; |1, I=0> \\ |f_0'> &= -\cos\phi \; |8, I=0> + \sin\phi \; |1, I=0> \end{aligned}$$

Final State Interactions in D Decays

We describe the FSI as the effect of resonances in the scattering of the final particles.

In other words, strong phases are generated by the resonances responsible for rescattering of final states.

Assuming no exotic resonances belonging to the 27 representation, the possible resonances have $SU(3)_F$ and isospin quantum numbers $(8, I=1)$, $(8, I=1/2)$, $(8, I=0)$ and $(1, I=0)$. Moreover, the two states with $I=0$ can be mixed, yielding two resonances:

$$
\begin{aligned}
|f_0> &= +\sin\phi\ |8, I=0> + \cos\phi\ |1, I=0> \\
|f_0'> &= -\cos\phi\ |8, I=0> + \sin\phi\ |1, I=0>
\end{aligned}
$$

The mixing angle ϕ and the strong phases δ_0, δ_0', δ_1 and $\delta_{1/2}$ are our model parameters together with the amplitudes T, T', C, C', D, and K. Moreover, we assume that

$$T' = T(1+\varepsilon) \text{ and } C' = C(1-\varepsilon)$$

The phases for the decay modes of D_s^+ is expected to be different from those coming in the D^0 and D^+ decay modes as an effect of $SU(3)$ breaking. So we have parameterized this with ε_δ such that

$$\delta_1' = \delta_1(1-\varepsilon_\delta) \text{ and } \delta_{\frac{1}{2}}' = \delta_{\frac{1}{2}}(1-\varepsilon_\delta)$$

Buccella, Franco, Lusignoli, Paul, Pugliese, P.S. , Silvestrini, in preparation

Some Amplitudes Expressions

$$A(D^0 \to \pi^+ K^-) = \frac{1}{5}\Big(3T - 2C - K\Big)e^{i\delta_{\frac{1}{2}}} + \frac{2}{5}\Big(T + C\Big)$$

$$A(D^+ \to \pi^+ \bar{K}^0) = \Big(T + C\Big)$$

$$A(D_s^+ \to K^+ \bar{K}^0) = -\frac{1}{5}\Big(2T - 3C + D\Big)e^{i\delta_1} + \frac{2}{5}\Big(T + C\Big)$$

$$\begin{aligned} A(D^0 \to \pi^+\pi^-) &= \left(T' - \frac{2}{3}C'\right)\left\{-\frac{3}{10}\left(e^{\imath\delta_0} + e^{\imath\delta_0'}\right) + \left(-\frac{3}{10}\cos(2\phi) + \frac{3}{4\sqrt{10}}\sin(2\phi)\right)\left(e^{\imath\delta_0'} - e^{\imath\delta_0}\right)\right\} \\ &- \left(T' + C'\right)\ \frac{2}{5}\,, \end{aligned}$$

$$\begin{aligned} A(D^0 \to K^+K^-) &= \left(T' - \frac{2}{3}C'\right)\left\{\frac{3}{20}\left(e^{\imath\delta_0} + e^{\imath\delta_0'}\right) + \left(\frac{3}{20}\cos(2\phi) + \frac{3}{4\sqrt{10}}\sin(2\phi)\right)\left(e^{\imath\delta_0'} - e^{\imath\delta_0}\right)\right. \\ &\left. + \frac{3}{10}e^{\imath\delta_1}\right\} + \left(T' + C'\right)\ \frac{2}{5}\,. \end{aligned}$$

Fit to the Experimental Data ($\varepsilon_\delta = 0$)

Channel	Fit ($\times 10^{-3}$)	Exp. ($\times 10^{-3}$)
CF		
$\mathrm{BR}(D^+ \to \pi^+ K_S)$	15.72 ± 0.41	15.3 ± 0.6
$\mathrm{BR}(D^+ \to \pi^+ K_L)$	14.27 ± 0.38	14.6 ± 0.5
$\mathrm{BR}(D^0 \to \pi^+ K^-)$	39.33 ± 0.40	39.3 ± 0.4
$\mathrm{BR}(D^0 \to \pi^0 K_S)$	12.02 ± 0.35	12.0 ± 0.4
$\mathrm{BR}(D^0 \to \pi^0 K_L)$	9.48 ± 0.28	10.0 ± 0.7
SCS		
$\mathrm{BR}(D^0 \to \pi^+ \pi^-)$	1.42 ± 0.03	1.421 ± 0.025
$\mathrm{BR}(D^0 \to \pi^0 \pi^0)$	0.83 ± 0.04	0.826 ± 0.035
$\mathrm{BR}(D^+ \to \pi^+ \pi^0)$	1.24 ± 0.06	1.24 ± 0.06
$\mathrm{BR}(D^0 \to K^+ K^-)$	4.00 ± 0.07	4.01 ± 0.07
$\mathrm{BR}(D^0 \to K_S K_S)$	0.17 ± 0.04	0.18 ± 0.04
$\mathrm{BR}(D^+ \to K^+ K_S)$	2.99 ± 0.14	2.95 ± 0.15
DCS		
$\mathrm{BR}(D^+ \to \pi^0 K^+)$	0.166 ± 0.011	0.189 ± 0.025
$\mathrm{BR}(D^0 \to \pi^- K^+)$	0.140 ± 0.003	0.1399 ± 0.0027

Numerical Results for the Free Parameters ($\varepsilon_\delta = 0$)

Parameter	mean $\pm$ rms	Parameter	mean $\pm$ rms
T	0.408 $\pm$ 0.003	δ_0	-2.721 $\pm$ 0.203
C	-0.231 $\pm$ 0.003	δ_0'	-1.038 $\pm$ 0.107
ε	0.057 $\pm$ 0.009	$\delta_{\frac{1}{2}}$	-1.599 $\pm$ 0.031
D	-0.003 $\pm$ 0.057	δ_1	-1.301 $\pm$ 0.090
K	0.097 $\pm$ 0.012	ϕ	0.346 $\pm$ 0.053

Note that

$$T' = T(1+\varepsilon) \text{ and } C' = C(1-\varepsilon)$$

$$\chi^2/NdF = 5/3 \approx 1.7$$

Our Predictions ($\varepsilon_\delta = 0$)

Channel	Fit ($\times 10^{-3}$)	Exp. ($\times 10^{-3}$)	pull
CF			
$\mathrm{BR}(D^0 \to K_S \eta)$	3.59 ± 0.10	4.85 ± 0.3	-3.98
$\mathrm{BR}(D_s^+ \to K^+ K_S)$	17.2 ± 1.9	15.0 ± 0.5	1.12
$\mathrm{BR}(D_s^+ \to \pi^+ \eta)$	34.1 ± 1.8	17.0 ± 0.9	8.49
SCS			
$\mathrm{BR}(D^0 \to \eta\eta)$	1.01 ± 0.11	1.70 ± 0.20	-3.02
$\mathrm{BR}(D^+ \to \pi^+ \eta)$	3.27 ± 0.51	3.66 ± 0.22	-0.70
$\mathrm{BR}(D^0 \to \pi^0 \eta)$	0.74 ± 0.06	0.69 ± 0.07	0.54
$\mathrm{BR}(D_s^+ \to \pi^0 K^+)$	1.18 ± 0.10	0.63 ± 0.21	2.36
$\mathrm{BR}(D_s^+ \to \pi^+ K_S)$	1.24 ± 0.08	1.22 ± 0.06	0.2
$\mathrm{BR}(D_s^+ \to K^+ \eta)$	1.29 ± 0.07	1.77 ± 0.35	-1.34
DCS			
$\mathrm{BR}(D^+ \to K^+ \eta)$	0.050 ± 0.003	0.112 ± 0.018	-3.40

Here we identify $\eta = \eta_8$

Fit to the Experimental Data ($\varepsilon_\delta \neq 0$)

Channel	Fit ($\times 10^{-3}$)	Exp. ($\times 10^{-3}$)
CF		
$\mathrm{BR}(D^+ \to \pi^+ K_S)$	15.72 ± 0.41	15.3 ± 0.6
$\mathrm{BR}(D^+ \to \pi^+ K_L)$	14.34 ± 0.37	14.6 ± 0.5
$\mathrm{BR}(D^0 \to \pi^+ K^-)$	39.31 ± 0.40	39.3 ± 0.4
$\mathrm{BR}(D^0 \to \pi^0 K_S)$	11.9 ± 0.33	12.0 ± 0.4
$\mathrm{BR}(D^0 \to \pi^0 K_L)$	9.39 ± 0.27	10.0 ± 0.7
$\mathrm{BR}(D_s^+ \to K^+ K_S)$	15.0 ± 0.5	15.0 ± 0.5
SCS		
$\mathrm{BR}(D^0 \to \pi^+ \pi^-)$	1.42 ± 0.03	1.421 ± 0.025
$\mathrm{BR}(D^0 \to \pi^0 \pi^0)$	0.83 ± 0.04	0.826 ± 0.035
$\mathrm{BR}(D^+ \to \pi^+ \pi^0)$	1.22 ± 0.06	1.24 ± 0.06
$\mathrm{BR}(D^0 \to K^+ K^-)$	4.02 ± 0.06	4.01 ± 0.07
$\mathrm{BR}(D^0 \to K_S K_S)$	0.17 ± 0.04	0.18 ± 0.04
$\mathrm{BR}(D^+ \to K^+ K_S)$	2.89 ± 0.12	2.95 ± 0.15
$\mathrm{BR}(D_s^+ \to \pi^0 K^+)$	1.03 ± 0.04	0.63 ± 0.21
$\mathrm{BR}(D_s^+ \to \pi^+ K_S)$	1.24 ± 0.06	1.22 ± 0.06
DCS		
$\mathrm{BR}(D^+ \to \pi^0 K^+)$	0.155 ± 0.005	0.189 ± 0.025
$\mathrm{BR}(D^0 \to \pi^- K^+)$	0.140 ± 0.003	0.1399 ± 0.0027

Numerical Results for the Free Parameters ($\varepsilon_\delta \neq 0$)

Parameter	mean $\pm$ rms	Parameter	mean $\pm$ rms
T	0.407 $\pm$ 0.003	δ_0	-2.628 $\pm$ 0.176
C	-0.230 $\pm$ 0.003	δ_0'	-0.995 $\pm$ 0.101
ε	0.055 $\pm$ 0.008	$\delta_{\frac{1}{2}}$	-1.590 $\pm$ 0.030
ε_δ	0.057 $\pm$ 0.04	δ_1	-1.245 $\pm$ 0.062
D	-0.058 $\pm$ 0.024	ϕ	0.368 $\pm$ 0.050
K	0.0952 $\pm$ 0.024		

Note that

$$T' = T(1+\varepsilon) \qquad C' = C(1-\varepsilon)$$

$$\delta_1' = \delta_1(1-\varepsilon_\delta) \qquad \delta_{\frac{1}{2}}' = \delta_{\frac{1}{2}}(1-\varepsilon_\delta)$$

$$\chi^2/NdF = 9/5 \approx 1.8$$

Our Predictions ($\varepsilon_\delta \neq 0$)

Channel	Fit ($\times 10^{-3}$)	Exp. ($\times 10^{-3}$)	pull
CF			
$\mathrm{BR}(D^0 \to K_S \eta)$	3.56 ± 0.1	4.85 ± 0.3	-4.08
$\mathrm{BR}(D_s^+ \to \pi^+ \eta)$	34.1 ± 1.5	17.0 ± 0.9	9.75
SCS			
$\mathrm{BR}(D^0 \to \eta\eta)$	0.96 ± 0.1	1.70 ± 0.20	-3.31
$\mathrm{BR}(D^+ \to \pi^+ \eta)$	2.84 ± 0.22	3.66 ± 0.22	-2.64
$\mathrm{BR}(D^0 \to \pi^0 \eta)$	0.70 ± 0.04	0.69 ± 0.07	0.124
$\mathrm{BR}(D_s^+ \to K^+ \eta)$	1.14 ± 0.07	1.77 ± 0.35	-1.77
DCS			
$\mathrm{BR}(D^+ \to K^+ \eta)$	0.047 ± 0.002	0.112 ± 0.018	-19.8

Here we identify $\eta = \eta_8$

Standard Model and CP Violation

In the SM CP Violation can emerge in the interaction involving charged currents.

$$\overline{\psi}_1 \gamma_\mu \psi_2 \xrightarrow{\text{CP}} -\overline{\psi}_2 \gamma^\mu \psi_1$$

$$\overline{\psi}_1 \gamma_\mu \gamma_5 \psi_2 \xrightarrow{\text{CP}} -\overline{\psi}_2 \gamma^\mu \gamma_5 \psi_1$$

$$W_\mu \xrightarrow{\text{CP}} -W^{\dagger\mu}$$

$$\mathscr{L} = gV_{12}\, \overline{\psi}_1 \gamma_\mu (1-\gamma_5) \psi_2 W^\mu + gV_{12}^*\, \overline{\psi}_2 \gamma_\mu (1-\gamma_5) \psi_1 W^{\dagger\mu}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\mathscr{L}^{\text{CP}} = gV_{12}\, \overline{\psi}_2 \gamma^\mu (1-\gamma_5) \psi_1 W_\mu^\dagger + gV_{12}^*\, \overline{\psi}_1 \gamma^\mu (1-\gamma_5) \psi_2 W_\mu$$

Neutral Flavoured Mesons

A generic flavoured neutral meson M^0 (K^0, D^0, B_d^0 and B_s^0) with non-zero eigenvalue of flavor F and its antiparticle $\bar{M}^0$ are defined by

$$F\left|M^0\right\rangle = +\left|M^0\right\rangle \qquad F\left|\bar{M}^0\right\rangle = -\left|\bar{M}^0\right\rangle$$

Moreover,

$$CP\left|M^0\right\rangle = \left|\bar{M}^0\right\rangle \qquad CP\left|\bar{M}^0\right\rangle = \left|M^0\right\rangle$$

Weak interactions don't conserve flavour quantum numbers and so M^0 and $\bar{M}^0$ cannot be physical states.
But, if CP is conserved, the physical states are

$$M_\pm = \frac{1}{\sqrt{2}}\left[\left|M^0\right\rangle \pm \left|\bar{M}^0\right\rangle\right] \qquad CP|M_\pm\rangle = \pm|M_\pm\rangle$$

Neutral Flavoured Mesons:Time Evolution

The exact time evolution of $\bar{M}^0$ and M^0 is prohibitively complicated: M^0 and $\bar{M}^0$ couple together and can decay into other states.
Starting from initial states which are linear combinations of $\bar{M}^0$ and M^0, we can study the time evolution of the coefficients by considering the weak interactions as perturbation to the strong ones. At the second order in the weak interactions and in the subspace $M^0 - \bar{M}^0$, the effective hamiltonian can be written as

$$\imath\hbar \frac{d}{dt}|\psi\rangle = \boldsymbol{H}|\psi\rangle \qquad \boldsymbol{H} = \boldsymbol{M} - \frac{\imath}{2}\boldsymbol{\Gamma} \qquad \boldsymbol{M} = \boldsymbol{M}^\dagger \quad \boldsymbol{\Gamma} = \boldsymbol{\Gamma}^\dagger$$

Note that

$$\frac{d}{dt}\langle\psi|\,\psi\rangle = -\frac{1}{\hbar}\langle\psi|\,\boldsymbol{\Gamma}\,|\psi\rangle$$

Moreover,

$$\boldsymbol{H} = H_{strong} + H_{e.m.} + H_{weak} = H_{\Delta F=0} + H_{\Delta F=1}$$

Neutral Flavoured Mesons:Time Evolution (1)

A generic state $|\psi\rangle = a(t)\left|M^0\right\rangle + b(t)\left|\bar{M}^0\right\rangle$ satisfy the equation

$$\imath\hbar\frac{d}{dt}|\psi\rangle = \boldsymbol{H}|\psi\rangle \quad \longrightarrow \quad \imath\hbar\frac{d}{dt}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \left(\boldsymbol{M} - \frac{\imath}{2}\boldsymbol{\Gamma}\right)\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

where

$$\left(\boldsymbol{M} - \frac{\imath}{2}\boldsymbol{\Gamma}\right) = \begin{pmatrix} M_{11} - (\imath/2)\Gamma_{11} & M_{12} - (\imath/2)\Gamma_{12} \\ M_{21} - (\imath/2)\Gamma_{21} & M_{22} - (\imath/2)\Gamma_{22} \end{pmatrix}$$

with

Mass matrix elements

$$M_{11} = M_{11}^* = m_0 + \left\langle M^0\right| H_w \left|M^0\right\rangle + \sum_n \mathscr{P}\frac{|\langle n| H_w \left|M^0\right\rangle|^2}{m_0 - E_n}$$

$$M_{22} = M_{22}^* = m_0 + \left\langle \bar{M}^0\right| H_w \left|\bar{M}^0\right\rangle + \sum_n \mathscr{P}\frac{|\langle n| H_w \left|\bar{M}^0\right\rangle|^2}{m_0 - E_n}$$

$$M_{12} = M_{21}^* = \underbrace{\left\langle M^0\right| H_w \left|\bar{M}^0\right\rangle}_{=0} + \sum_n \mathscr{P}\frac{\left\langle M^0\right| H_w |n\rangle \langle n| H_w \left|\bar{M}^0\right\rangle}{m_0 - E_n}$$

Decay matrix elements

$$\Gamma_{11} = \Gamma_{11}^* = 2\pi\sum_n \delta(m_0 - E_n)|\langle n| H_w \left|M^0\right\rangle|^2$$

$$\Gamma_{22} = \Gamma_{22}^* = 2\pi\sum_n \delta(m_0 - E_n)|\langle n| H_w \left|\bar{M}^0\right\rangle|^2$$

$$\Gamma_{12} = \Gamma_{21}^* = 2\pi\sum_n \delta(m_0 - E_n)\times \left\langle M^0\right| H_w |n\rangle \langle n| H_w \left|\bar{M}^0\right\rangle$$

Neutral Flavoured Mesons:Time Evolution(2)

CPT symmetry

$$M_{11} = M_{22}$$
$$\Gamma_{11} = \Gamma_{22}$$

$$\begin{aligned} |M_a\rangle &= p|M^0\rangle + q|\bar{M}^0\rangle \\ |M_b\rangle &= p|M^0\rangle - q|\bar{M}^0\rangle \end{aligned}$$

$$\frac{q}{p} = \pm\sqrt{\frac{H_{21}}{H_{12}}} = \pm\sqrt{\frac{M_{12}^* - (\imath/2)\Gamma_{12}^*}{M_{12} - (\imath/2)\Gamma_{12}}}$$

$$|M_{a,b}(t)\rangle = e^{-\imath\lambda_{a,b}t}|M_{a,b}(0)\rangle = e^{-\imath m_{a,b}t}e^{-\Gamma_{a,b}t/2}|M_{a,b}(0)\rangle$$

$$\begin{aligned} \lambda_a &\equiv m_a - (\imath/2)\Gamma_a = H_{11} + \frac{q}{p}H_{12} = H_{11} + \sqrt{H_{12}H_{21}} \\ \lambda_b &\equiv m_b - (\imath/2)\Gamma_b = H_{11} - \frac{q}{p}H_{12} = H_{11} - \sqrt{H_{12}H_{21}} \\ m_{a,b} &= M_{11} \pm \Re\sqrt{H_{12}H_{21}} \\ \Gamma_{a,b} &= \Gamma_{11} \mp 2\Im\sqrt{H_{12}H_{21}} \end{aligned}$$

Neutral Flavoured Mesons:Time Evolution(3)

It is very simple to evaluate the time evolution of the flavour eigenstates:

$$\begin{aligned} \left|M^0(t)\right\rangle &= f_+(t)\left|M^0\right\rangle + \frac{q}{p} f_-(t)\left|\bar{M}^0\right\rangle \\ \left|\bar{M}^0(t)\right\rangle &= f_+(t)\left|\bar{M}^0\right\rangle + \frac{p}{q} f_-(t)\left|M^0\right\rangle \end{aligned}$$

where

$$f_\pm(t) = \frac{1}{2} e^{-\imath m_a t} e^{-\Gamma_a t/2} \left[1 \pm e^{-\imath \Delta m t} e^{-\Delta\Gamma t/2}\right]$$

Probability to find at time t the same flavour eigenstate which it had at time $t = 0$

$$P[M^0(t) \to M^0] = P[\bar{M}^0(t) \to \bar{M}^0] = |f_+(t)|^2$$

Probability that an initial M^0 becomes $\bar{M}^0$ and *viceversa*

$$\begin{aligned} P[M^0(t) \to \bar{M}^0] &= \left|\frac{q}{p}\right|^2 |f_-(t)|^2 \\ P[\bar{M}^0(t) \to M^0] &= \left|\frac{p}{q}\right|^2 |f_-(t)|^2 \end{aligned}$$

Mixing Parameters

$$P[M^0(t) \to M^0] = \frac{1}{2} e^{-\Gamma t} (\cosh(y\Gamma t) + \cos(x\Gamma t))$$

$$P[M^0(t) \to \bar{M}^0] = \frac{1}{2} \left|\frac{q}{p}\right|^2 e^{-\Gamma t} (\cosh(y\Gamma t) - \cos(x\Gamma t))$$

$$x \equiv \frac{m_b - m_a}{\Gamma} = \frac{\Delta m}{\Gamma}$$

$$y \equiv \frac{\Gamma_b - \Gamma_a}{2\Gamma} = \frac{\Delta\Gamma}{2\Gamma}$$

$$\Gamma \equiv \frac{\Gamma_b + \Gamma_a}{2}$$

In the case of charm

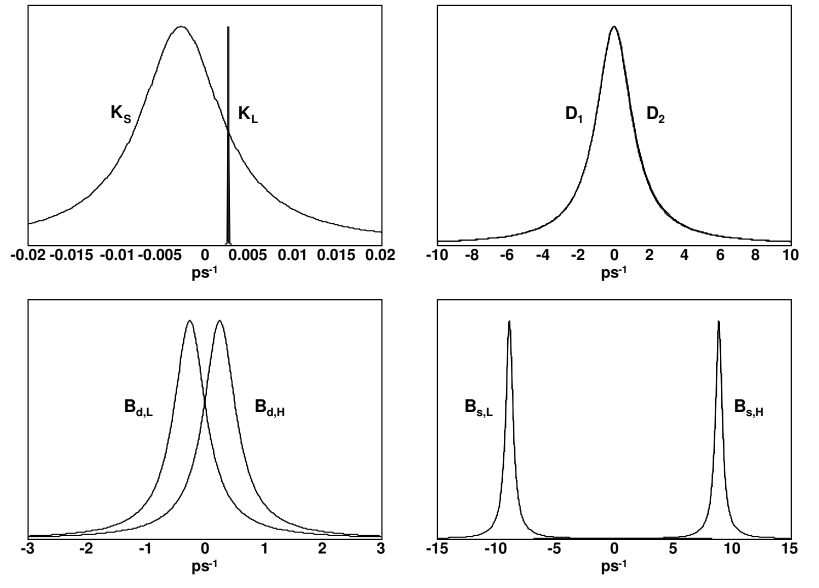
$$x = \frac{1}{\Gamma}\left[\left\langle \bar{D}^0\right| \mathscr{H} \left|D^0\right\rangle + \mathscr{P}\sum_n \frac{\left\langle D^0\right| \mathscr{H} \left|n\right\rangle \left\langle n\right| \mathscr{H} \left|\bar{D}^0\right\rangle + \left\langle \bar{D}^0\right| \mathscr{H} \left|n\right\rangle \left\langle n\right| \mathscr{H} \left|D^0\right\rangle}{m_D^2 - E_n^2}\right]$$

$$y = \frac{1}{2\Gamma}\sum_n \rho_n \left[\left\langle D^0\right| \mathscr{H} \left|n\right\rangle \left\langle n\right| \mathscr{H} \left|\bar{D}^0\right\rangle + \left\langle \bar{D}^0\right| \mathscr{H} \left|n\right\rangle \left\langle n\right| \mathscr{H} \left|D^0\right\rangle\right]$$

$$x, y \approx \lambda^2 [SU(3) breaking]^2$$

Falk, Grossman, Ligeti, Petrov (2001)

Widths and Mass Differences



M.Gersabeck, arXiv:1207.2195 [hep-ex]

Mixing Parameters: the $D^0 \div \bar{D}^0$ System

$$R_M = \frac{1}{2}(x^2 + y^2)$$

$$2\,y_{CP} = \big(|q/p| + |p/q|\big)y\cos\phi - \big(|q/p| - |p/q|\big)x\sin\phi$$
$$2\,A_\Gamma = \big(|q/p| - |p/q|\big)y\cos\phi - \big(|q/p| + |p/q|\big)x\sin\phi$$

$$x_{K^0\pi\pi} = x$$
$$y_{K^0\pi\pi} = y$$
$$|q/p|_{K^0\pi\pi} = |q/p|$$
$$\mathrm{Arg}\,(q/p)_{K^0\pi\pi} = \phi$$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix}_{K^+\pi^-\pi^0} = \begin{pmatrix} \cos\delta_{K\pi\pi} & \sin\delta_{K\pi\pi} \\ -\sin\delta_{K\pi\pi} & \cos\delta_{K\pi\pi} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$A_M = \frac{|q/p|^2 - |p/q|^2}{|q/p|^2 + |p/q|^2}$$

$$x'^{\pm} = \left(\frac{1 \pm A_M}{1 \mp A_M}\right)^{1/4} (x'\cos\phi \pm y'\sin\phi)$$
$$y'^{\pm} = \left(\frac{1 \pm A_M}{1 \mp A_M}\right)^{1/4} (y'\cos\phi \mp x'\sin\phi)$$

$$\frac{\Gamma(D^0\to K^+\pi^-) + \Gamma(\overline{D}^0\to K^-\pi^+)}{\Gamma(D^0\to K^-\pi^+) + \Gamma(\overline{D}^0\to K^+\pi^-)} = R_D$$
$$\frac{\Gamma(D^0\to K^+\pi^-) - \Gamma(\overline{D}^0\to K^-\pi^+)}{\Gamma(D^0\to K^+\pi^-) + \Gamma(\overline{D}^0\to K^-\pi^+)} = A_D$$
$$\frac{\Gamma(D^0\to K^+K^-) - \Gamma(\overline{D}^0\to K^+K^-)}{\Gamma(D^0\to K^+K^-) + \Gamma(\overline{D}^0\to K^+K^-)} = A_K + \frac{\langle t\rangle}{\tau_D}\mathcal{A}_{CP}^{\mathrm{indirect}}$$
$$\frac{\Gamma(D^0\to \pi^+\pi^-) - \Gamma(\overline{D}^0\to \pi^+\pi^-)}{\Gamma(D^0\to \pi^+\pi^-) + \Gamma(\overline{D}^0\to \pi^+\pi^-)} = A_\pi + \frac{\langle t\rangle}{\tau_D}\mathcal{A}_{CP}^{\mathrm{indirect}}$$

Parameter	No CPV	No direct CPV in DCS decays	CPV-allowed	95% CL Interval
x (%)	$0.49\,^{+0.14}_{-0.15}$	$0.44\,^{+0.14}_{-0.15}$	0.37 ± 0.16	$[0.06, 0.67]$
y (%)	0.61 ± 0.08	0.60 ± 0.07	$0.66\,^{+0.07}_{-0.10}$	$[0.46, 0.79]$
$\delta_{K\pi}$ (°)	$6.9\,^{+9.7}_{-11.2}$	$3.6\,^{+10.4}_{-12.1}$	$11.8\,^{+9.5}_{-14.7}$	$[-21.1, 29.3]$
R_D (%)	0.349 ± 0.004	0.348 ± 0.004	0.349 ± 0.004	$[0.342, 0.357]$
A_D (%)	–	–	$-0.39\,^{+1.01}_{-1.05}$	$[-2.4, 1.5]$
$\lvert q/p\rvert$	–	1.002 ± 0.014	$0.91\,^{+0.12}_{-0.08}$	$[0.77, 1.14]$
ϕ (°)	–	-0.07 ± 0.6	$-9.4\,^{+11.9}_{-9.8}$	$[-28.3, 12.9]$
$\delta_{K\pi\pi}$ (°)	$18.1\,^{+23.3}_{-23.8}$	$20.3\,^{+24.0}_{-24.3}$	$27.3\,^{+24.4}_{-25.4}$	$[-23.3, 74.8]$
A_π	–	0.10 ± 0.14	0.10 ± 0.15	$[-0.19, 0.38]$
A_K	–	-0.14 ± 0.13	-0.15 ± 0.14	$[-0.42, 0.12]$
x_{12} (%)	–	$0.44\,^{+0.14}_{-0.15}$		$[0.13, 0.69]$
y_{12} (%)	–	0.60 ± 0.07		$[0.45, 0.74]$
ϕ_{12}(°)	–	0.2 ± 1.7		$[-4.1, 4.6]$

NO CPV $\equiv \{A_D = A_K = A_\pi = 0$ and $|q/p| = 1\ \phi = 0\}$
NO Direct CPV $\equiv \{A_D = 0$ and a relation between $|q/p|$, x, $y\}$

HFAG, July 2015

Neutral Flavoured Mesons: Types of CP Violation

CP Violation in the Mixing

This occurs when the physical states do not coincide with CP eigenstates,

$$|q| \neq |p|$$

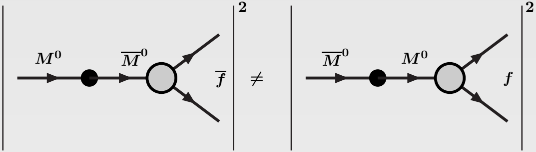
For example if

$$M^0 \to f \not\leftarrow \bar{M}^0 \quad \text{or} \quad M^0 \not\to f \leftarrow \bar{M}^0$$

As in the case of semileptonic decay modes

$$M^0 \to \ell^+ X \not\leftarrow \bar{M}^0 \quad \text{and} \quad M^0 \not\to \ell^- X \leftarrow \bar{M}^0$$

$$\frac{\Gamma(M^0(t) \to \ell^- X) - \Gamma(\bar{M}^0(t) \to \ell^+ X)}{\Gamma(M^0(t) \to \ell^- X) + \Gamma(\bar{M}^0(t) \to \ell^+ X)} = \frac{1 - |p/q|^4}{1 + |p/q|^4}$$



This kind of CPV is of the indirect type

Neutral Flavoured Mesons: Types of CP Violation(1)

CP Violation in the Decays (Direct)

This occurs when the decay amplitudes for CP conjugate processes into final states f and $\bar{f}$ are different in modulus

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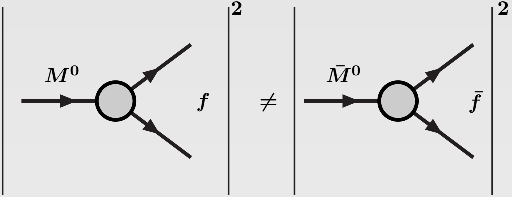

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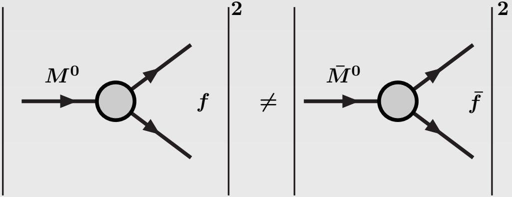


This kind of CPV is the only one is also possible for charged particles, which are forbidden to mix by charge conservation.

Neutral Flavoured Mesons: Types of CP Violation(2)

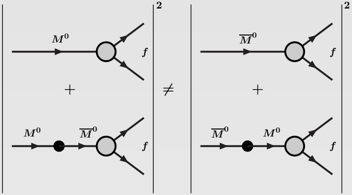
CPV in the interference of mixing and decays

This occurs when both, M^0 and $\bar{M}^0$, decay into the same final state

$$M^0 \to f \leftarrow \bar{M}^0$$

- This is the case of $CPf = \pm f$: $D^0 \to KK, \pi\pi \leftarrow \bar{D}^0$
- but not only: for example $D^0(\overline{D}^0) \to K^- \pi^+$

$$A(M^0 \to f) + A(M^0 \to \bar{M}^0) A(\bar{M}^0 \to f)$$



Useful definition

$$\lambda_f = \frac{\langle \bar{M}^0 | \, M_a \rangle}{\langle M^0 | \, M_a \rangle} \frac{A(\bar{M}^0 \to f)}{A(M^0 \to f)} = \frac{q}{p} \frac{\bar{A}_f}{A_f}$$

CP Symmetry implies

$$\lambda_f = \frac{1}{\lambda_f} \Rightarrow \lambda_f = 1$$

- $|\lambda_f| \neq 1$ CPV in mixing or decay
- $\Im(\lambda_f) \neq 0$ CPV in interf. mixing and decay

Direct CPV in SCS D^0 Decays

A nonzero direct CP asymmetry is present only when the decay amplitude is

$$\mathscr{A} = A\, e^{\imath\delta_A} + B\, e^{\imath\delta_B}$$

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What about the amplitude B?

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The amplitude B is provided by

$$\langle f | H_{\Delta U=0} | D^0 \rangle$$

where

$$H_{\Delta U=0} = -\frac{G_F}{\sqrt{2}} V_{ub} V_{cb}^* \left\{ \underbrace{\sum_{i=3}^{6} C_i O_i}_{\text{Penguins}} + \underbrace{\frac{1}{2} \left[C_1 (O_1^s + O_1^d) + C_2 (O_2^s + O_2^d) \right]}_{\text{Tree } (T'', C'')} \right\}$$

But

$$|T''/T'| = |C''/C'| = \left| \frac{V_{ub} V_{cb}^*}{\sin\theta_C \cos\theta_C} \right| \simeq 10^{-4}$$

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Large CPV can be due only to the Penguin terms

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Neglecting the contribution of the terms containing T'' and C''

$$\mathscr{A}(K^+K^-) \simeq T' f_T(\delta_i, \phi, C'/T') + P\, f_P(\delta_i, \phi)$$

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- The ratio $|P/T'|$ could be large as in the case of $\Delta I = 1/2$ in the K decays
- The FSI could be large

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$$a_{CP} \sim 10^{-2}$$

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In any case

$$\frac{a_{CP}^{\rm dir}(\pi^+\pi^-)}{a_{CP}^{\rm dir}(K^+K^-)} \approx -2$$

Conclusions

We analyzed the decays into two pseudoscalar mesons of D^0, D^+, and D_s in the framework of a model that ascribes the most of the observed $SU(3)_F$ violations to final state interactions.

We were able to give an accurate description of decay branching ratios

The values of the strong phases are in principle suitable to predict consistent CP violations in the decay.

The experimental situation regarding the CP violating asymmetries seems to be rather clear: there is no significant CP violation in the SCS decays at the level of 10^{-3}.

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CP violations asymmetries can be written in terms of the ratio $\Im(P)/T'$, and

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Backup Slides

$$\mathbf{3} = \left(\begin{array}{c} u \\ d \\ s \end{array} \right), \qquad \bar{\mathbf{3}} = \left(\begin{array}{c} \bar{d} \\ -\bar{u} \\ \bar{s} \end{array} \right). \tag{1}$$

$$K^+ = +u\bar{s}, \qquad K^0 = +d\bar{s}, \qquad \bar{K}^0 = +s\bar{d}, \qquad K^- = -s\bar{u}, \tag{2}$$

$$\pi^+ = +u\bar{d}, \qquad \pi^0 = -\frac{1}{\sqrt{2}}(+u\bar{u} - d\bar{d}), \qquad \pi^- = -d\bar{u}, \tag{3}$$

$$\eta_8 = -\frac{1}{\sqrt{6}}(+u\bar{u} + d\bar{d} - 2s\bar{s}), \qquad \eta_0 = -\frac{1}{\sqrt{3}}(+u\bar{u} + d\bar{d} + s\bar{s}), \tag{4}$$

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}, \tag{5}$$

$$(\mathbf{8} \otimes \mathbf{8})_S = \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{27}. \tag{6}$$

$$\bar{\mathbf{3}} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{3} \oplus \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}. \tag{7}$$

The Grinstein-Lebed Approach

Grinstein, Lebed PRD 53 (1996) 6344

$$
\begin{aligned}
H \quad = \quad & \frac{V_{cd}^* V_{us}}{\sqrt{2}} H_0^6 + \frac{(V_{cd}^* V_{ud} - V_{cs}^* V_{us})}{2} H_{1/2}^6 - \frac{V_{cs}^* V_{ud}}{\sqrt{2}} H_1^6 - \frac{(V_{cd}^* V_{ud} - 3 V_{cs}^* V_{us})}{2\sqrt{6}} H_{1/2}^{15} \\
& + \frac{(V_{cd}^* V_{us} + V_{cs}^* V_{ud})}{\sqrt{2}} H_1^{15} + \frac{V_{cd}^* V_{ud}}{\sqrt{3}} H_{3/2}^{15}
\end{aligned}
$$

From which we have, for example

$$
\begin{aligned}
A(D^0 \to \pi^+ \pi^-) \quad &= \quad -\frac{1}{2}\sqrt{\frac{3}{5}} R_{8,1/2}^6 + \frac{3}{2\sqrt{10}} R_{8,1/2}^{15} + \frac{1}{3}\sqrt{\frac{5}{6}} R_{27,3/2}^{15} - \frac{1}{6\sqrt{15}} R_{27,1/2}^{15} \\
A(D^0 \to K^+ K^-) \quad &= \quad \frac{1}{2}\sqrt{\frac{3}{5}} R_{8,1/2}^6 + \frac{1}{\sqrt{5}} R_{8,3/2}^{15} + \frac{7}{6\sqrt{15}} R_{27,1/2}^{15} + \frac{1}{3\sqrt{30}} R_{27,3/2}^{15} - \frac{1}{2\sqrt{10}} R_{8,1/2}^{15}
\end{aligned}
$$

where $R_{b,c}^a$ is the reduced matrix elements of H^a which transform the initial D meson into the representation of dimension b and with $\Delta I = c$