Strong-Field QED Processes in Intense Laser Pulses
Part 2: Nonlinear Compton Scattering

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Reminder 1: Classical Motion

Electrons in Intense Laser Fields are Always Relativistic!

- Consider the classical equation of motion of an electron in a plane wave laser field
$$\frac{d\pi^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu}(\phi)\pi_\nu, \qquad m\frac{dx^\mu}{d\tau} = \pi^\mu$$
- Three conserved momenta: p^+, $\boldsymbol{p}^\perp$ $\rightarrow$ integrability
  Replace proper time derivative by laser phase derivative $\frac{d\phi}{d\tau} = \omega p^+/m$
- Solution for classical kinetic momentum
$$\pi^\mu(\phi) = p^\mu - eA^\mu + k^\mu \left(\frac{eAp}{kp} - \frac{e^2A^2}{2kp} \right)$$

  Note: Mass shell condition: $\pi^2 = m^2$

Reminder 2: Volkov States

- Dirac equation in a plane wave background field

$$(i\not{\partial} - e\not{A} - m)\Psi_p = 0$$

- Volkov States can be written in the following form:

$$\Psi_p(x, A) = \Omega_p[A(\phi)]\psi_p(x)$$

- Ω is *unitary matrix* that acts on the *free* electron wavefunction $\psi_p(x) = e{-}ip \cdot x u_p$

$$\Omega(\phi) = \left(1 + \frac{e\not{k}\not{A}(\phi)}{2kp}\right) \exp\left\{-i\frac{1}{2kp}\int d\phi(2epA - e^2A^2)\right\}$$

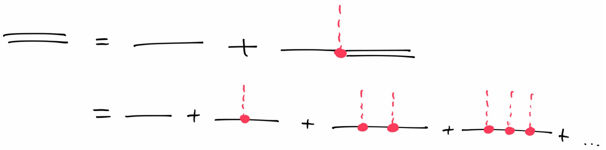
Reminder 3: Lippmann-Schwinger Equation

Transform the Dirac equation into an integral equation

$$\Psi_p(x, A) = \psi_p(x) + \int d^4 z G_0(x - z)\, e \not{A}(z)\, \Psi_p(z, A)$$

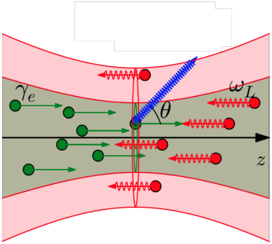
with iterative solution

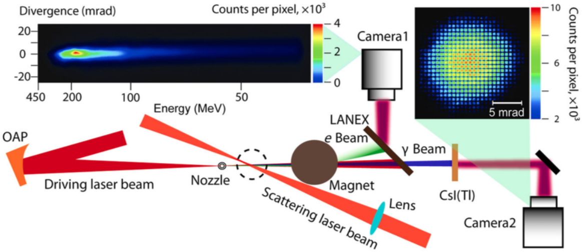
$$\Psi_p^{(N)}(x, A) = \sum_n \left[G_0\, e \not{A}(z) \right]^n \psi_p$$



Describes both the emission and absorption of background field photons.

Non-linear Compton Scattering (NLC)





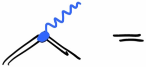
- "Inverse" Compton Scattering: $\omega' > \omega_L$ if incident electron high energetic
- $\omega' \sim 4\gamma^2\omega$
- Beam aperture $\sim 1/\gamma$
- Ultra-short, narrowband, high-brightness source of high-energy photons (x and gamma rays)

[S. G. Rykovanov et al, PRSTAB (2014).]

[S. Chen et al, PRL (2013).]

Weak-Field Epansion of NLC

For small values of $a_0 \ll 1$ we can expand the strong-field diagrams:



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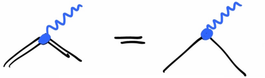

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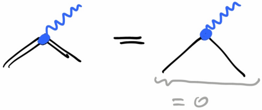

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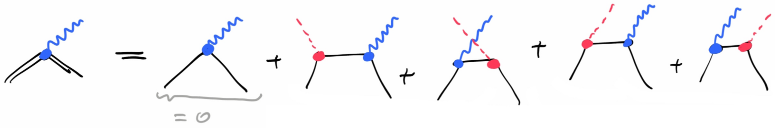

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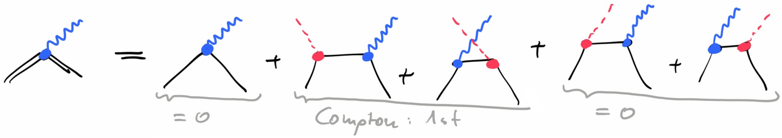

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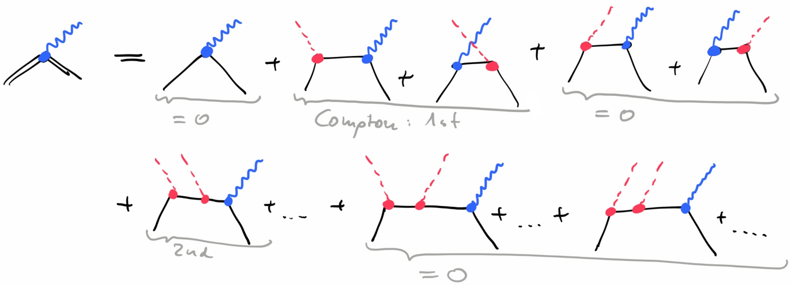

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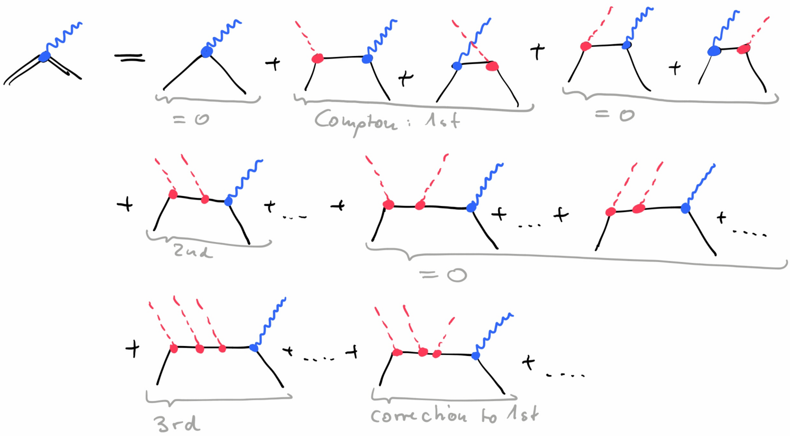


probabilities scale as $\mathbb{P}_n \sim a_0^{2n}$

Matrix Element for NLC

Draw strong-field Feynman diagram, translate into S-matrix using Feynman rules:

$$S = \int d^4x \bar{\Psi}_{p'}(x)(-ie\not{A}_{k'})\Psi_p(x)$$
$$= -ie \int d\ell d\phi \, \delta(p + \ell k - p' - k') \, R(\phi) \, \exp\left\{ i \int d\phi \frac{k' \cdot \pi(\phi)}{k \cdot p'} \right\}$$

ℓ is the longitudinal light-front momentum fraction

$$\left.\begin{array}{rl} p^+ & = k'^+ + p'^+ \\ \boldsymbol{p}^\perp & = (\boldsymbol{k}')^\perp + (\boldsymbol{p}')^\perp \\ p^- + \ell k^- & = k'^- + p'^- \end{array}\right\} \Rightarrow \omega' = \frac{\ell\omega}{1 + \frac{\ell\omega}{m}(1 + \cos\vartheta)}$$

Third equation is only formally a conservation law as ℓ is not determined, we need to integrate over all ℓ!

Matrix Element for NLC

$$S = const. \times \delta_{l.f.c}\, \mathcal{M}(\ell)\,,$$

$$\mathcal{M}(\ell) = \sum T_n \mathscr{C}_n(\ell)\,,$$

$$\begin{Bmatrix} \mathscr{C}_0 \\ \mathscr{C}_{\pm 1} \\ \mathscr{C}_2 \end{Bmatrix} = \int d\phi \, \begin{Bmatrix} 1 \\ g(\phi)e^{\mp i\phi} \\ g^2(\phi) \end{Bmatrix} \exp\left\{ i\ell \int d\phi \frac{n' \cdot \pi(\phi)}{n' \cdot p} \right\}$$

- Integrals over the laser phase $\Rightarrow$ amplitudes at different phases interfere
- For $a_0 \gg 1$ these interferences vanish, the formation region goes like $1/a_0 \Rightarrow$ constant crossed field

Photon Emission Probability:

$$\frac{d\mathbb{P}}{d\omega' d\Omega} = \frac{\alpha\omega'}{16\pi^2}|\mathcal{M}(\ell)|^2\,, \qquad \ell \equiv \frac{k' \cdot p}{k \cdot p'}$$

Convergence of $\mathscr{C}_0$

- Problem: The Phase integral $\mathscr{C}_0$ is an integral over a pure phase and does not converge (numerically)!
- Solution: Use the Gauge Invariance or the matrix element:

$$\mathcal{M} = \mathcal{M}^\mu \varepsilon'_\mu$$

$$\mathcal{M}^\mu k'_\mu = 0$$

- This gives a relation between the phase integrals in the form:

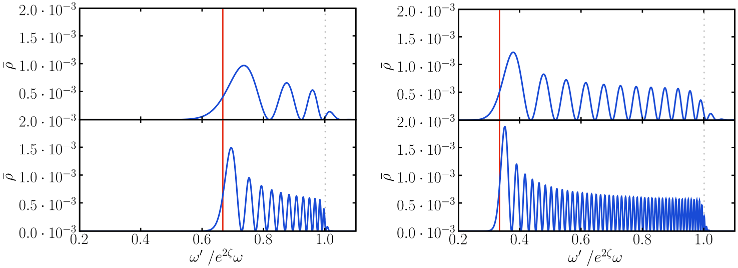
$$\mathcal{M}^\mu k'_\mu = \bar{u}_{p'} \not{k} u_p \left[\underbrace{\ell \mathscr{C}_0(\ell) - w_+ \mathscr{C}_+ - w_- \mathscr{C}_- - w_2 \mathscr{C}_2}_{=0} \right] = 0$$

- Alternative: Boca-Florescu transformation (integration by parts with convergence factor) give the same relation

[Anton Ilderton, PRL (2011).]

[M. Boca and V. Florescu, PRA (2009).]

Ponderomotively Broadened Spectrum



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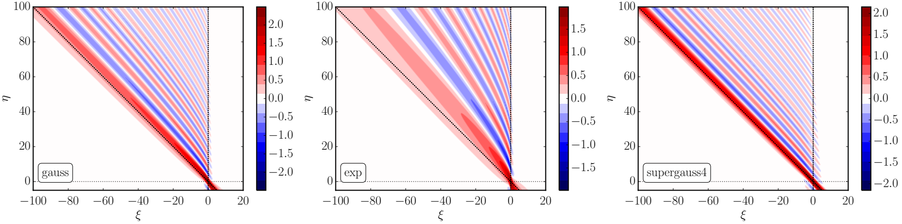
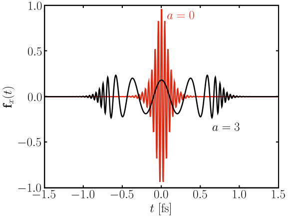
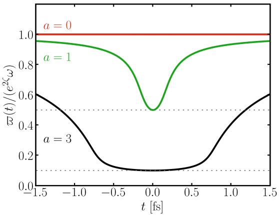
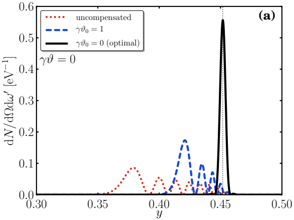


Compensation of Ponderomotive Broadening







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