Distribution Amplitudes of Heavy Hadrons: Theory and Applications

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Outline

Introduction

- Physics of heavy hadrons still remains a hot topic thanks to the existing experimental facility — LHC at CERN and the one under construction — Belle II at KEK
- Many new results have come from the LHC collaborations during last several years and are waiting from Run II now
- There are also interesting results from the theory side in spectroscopy, production processes and decays of heavy hadrons
- Some intersting results on the heavy-hadrons wave-functions are obtained and effectively used in calculations of processes

Naive Factorization

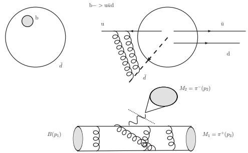
- Color-Transparancy Argument for *B*-Meson Decays
  [Bjorken; Brodsky and Lepage (1980)]

  Since *b*-quark decays into energetic light quarks ($E > 1$ GeV), the produced quark-antiquark pair does not have enough time to evolve to the real size hadronic entity, but remains a small size bound state with a correspondingly small chromomagnetic moment which suppress the QCD interaction between final state mesons

  - Naive Factorization Approach
    [Bauer, Stech, Wirbel (1985)]

    Factorized part only was defined

    Example: $\bar{B}^0 \to \pi^+\pi^-$ decay



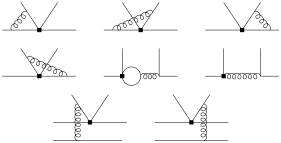
$$\langle\pi^-(p_2)\pi^+(p_3)|(\bar{d}u)_{V-A}(\bar{u}b)_{V-A}|\bar{B}^0(p_1)\rangle =$$

$$\langle\pi^-(p_2)|(\bar{d}u)_{V-A}|0\rangle\langle\pi^+(p_3)|(\bar{u}b)_{V-A}|\bar{B}^0(p_1)\rangle$$

$$f_\pi \qquad \otimes \qquad f^{B\to\pi}(q^2 = m_\pi^2)$$

QCD Factorization

- Basic idea of QCDF [Beneke, Buchalla, Neubert, Sachrajada]
- Example: $\bar{B} \to \pi\pi$ decay
- Energetic π-mesons $E_\pi \sim M_B/2$; soft gluons with momenta $\sim \Lambda_{\rm QCD}$ are decouple in $\Lambda_{\rm QCD}/M_B$



- Effective Hamiltonian for $b \to d$ transitions

$$\mathcal{H}_{\rm eff}^{b\to d} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pd}^* \left[C_1 Q_1^p + C_1 Q_1^p + \sum_{i=3}^{10} C_i Q_i + C_{8g} Q_{8g} \right]$$

- QCDF for $\bar{B} \to M_1 M_2$: with recoiled M_1 and emitted M_2
- In $m_b \to \infty$ limit; hard inter. between $(\bar{B} M_1)$ and M_2 only soft effects are confined to $(\bar{B} M_1)$ system
- Hadronic MEs of four-quark operators are simplified

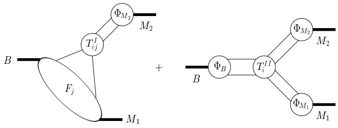
$$\langle M_1 M_2 | j_1^{(i)} \times j_2^{(i)} | \bar{B} \rangle = \langle M_1 | j_1^{(i)} | \bar{B} \rangle \langle M_2 | j_2^{(i)} | 0 \rangle \left[1 + \sum_n r_n \alpha_s^n + \mathcal{O}(\Lambda_{\rm QCD}/m_b) \right]$$

- Nonfactorizable effects are calculable

QCD Factorization

Structure of $\mathcal{O}(\alpha_s)$ corrections:

1. Vertex corrections:
   $\langle M_1 | j_1^{(i)} | \bar{B} \rangle \Longrightarrow f^{B \to M_1}(q^2)$
   $\langle M_2 | j_2^{(i)} | 0 \rangle \Longrightarrow f_{M_2} \phi_{M_2}(x)$
2. Hard-spectator corrections:
   $\langle M_1 | j_1^{(i)} | \bar{B} \rangle \Longrightarrow f_B \phi_B(\xi)\, f_{M_1} \phi_{M_1}(u)$

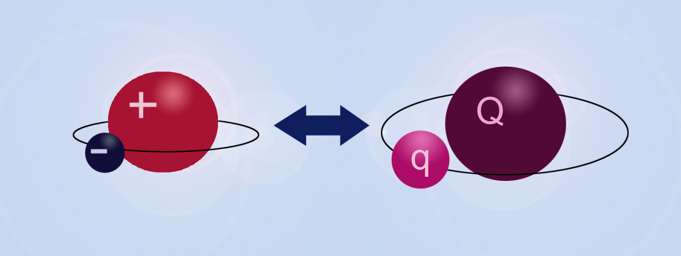


Factorization formula for $\bar{B} \to M_1 M_2$ hadronic matrix element is proven to $\mathcal{O}(\alpha_s)$ and leading twist

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = f^{B \to M_1}(m_{M_2}^2) \int_0^1 dx\, T_i^{\mathrm{I}}(x)\, f_{M_2} \phi_{M_2}(x)$$
$$+ \int_0^1 dx \int_0^1 du \int d\xi\, T_i^{\mathrm{II}}(x, u, \xi)\, f_{M_1} \phi_{M_1}(x)\, f_{M_2} \phi_{M_2}(u)\, f_B \phi_B(\xi)$$

- $T_i^{\mathrm{I}}(x)$ and $T_i^{\mathrm{II}}(x, u, \xi)$ are perturbatively calculable HSK
- Annihilation and Charm-penguin contrib. are subleading
- Infrared logarithms appear at subleading powers; Breakdown of factorization

Hadrons within Heavy Quark Effective Theory



- Heavy Quark Effective Theory (HQET) is useful tool in theoretical analysis of heavy hadrons
- Effective mass of heavy meson $\bar{\Lambda} = m_M - m_Q$
- For the lowest-mass *D*- and *B*-mesons: $\bar{\Lambda} \simeq 0.5$ GeV
- Heavy-quark spin decouples in the Heavy-Quark-Symmetry (HQS) limit
- Heavy quark is assumed to be a scalar particle

Heavy-Meson Interpolating Currents

- Heavy meson is a spinor-like particle in this approach
- Realistic quantum numbers and wave-function can be obtained after construction with the heavy-quark spin
- Heavy-meson bilocal operator [Grozin & Neubert (1996)]

$$\widetilde{O}(z) = Q^*(0)\, E(0,z)\, q(z)$$

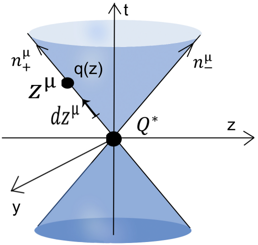
- Link between quarks — Wilson line (path-ordered exponential)

$$E(0,z) = \mathcal{P} \exp \left\{ -i g_{\rm st} \int_0^z dz'^\mu A^a_\mu(z') \frac{\lambda^a}{2} \right\}$$

- Gluonic field $A^a_\mu(z)$, strong coupling $g_{\rm st}$
- Used notation: $A_\mu = A^a_\mu \lambda^a/2$; the Gell-Mann matrices λ^a
- The rule for path-ordering along a line

$$\mathcal{P}\left\{A_\mu(z_1)\, A_\nu(z_2)\right\} = \begin{cases} A_\nu(z_2)\, A_\mu(z_1) & \text{if } z_2 > z_1\,, \\ A_\mu(z_1)\, A_\nu(z_2) & \text{if } z_2 < z_1\,. \end{cases}$$

Light-Cone Decomposition



$$n_\pm^\mu = \frac{1}{\sqrt{2}}(1, 0, 0, \mp 1)$$

$$n_\pm^2 = 0, \qquad (n_+ n_-) = 1$$

$$dz'^\mu = dz'_- n_+^\mu$$

$$A_\pm = n_\pm^\mu A_\mu$$

$$A^\mu = A_+ n_-^\mu + A_- n_+^\mu + A_\perp^\mu$$

- Path is a straight line on the light cone

$$E(0, z) = \mathcal{P} \exp \left\{ -i g_{\rm st} \int_0^{z_-} A_+^a(z'_-) \frac{\lambda^a}{2} dz'_- \right\}$$

- Fock-Schwinger gauge is used

$$A_+(z) = 0 \quad \Rightarrow \quad E(0, z) = 1$$

Heavy-Meson Distribution Amplitudes

- Meson-to-vacuum transition at $z^2 = 0$ [Grozin & Neubert (1996)]

$$\langle 0|Q^*(0)\, E(0,z)\, q(z)|M(v)\rangle = f_M \left\{ \tilde{\varphi}_+(t) + [\tilde{\varphi}_-(t) - \tilde{\varphi}_+(t)]\, \frac{\hat{z}}{2t} \right\} U(v)$$

- f_M is a constant with dimension of a mass
- $t = (vz)$ is the time in heavy-meson rest frame
- Four-velocity of heavy meson at rest $v^\mu = (1,0,0,0)$
- $U(v)$ is non-relativistic wave-function of spinor-like meson
- $\tilde{\varphi}_\pm(t)$ are distribution amplitudes (DAs) of heavy meson
- Fourier transformations of DAs are required in constructing matrix elements

$$\tilde{\varphi}_\pm(t) = \int_0^\infty d\omega\, \mathrm{e}^{-i\omega t} \phi_\pm(\omega)$$

Projection Operator onto $B^{(*)}$-Meson State

- Heavy B- and B^*-mesons are degenerate in this approach
- B-meson-to-vacuum transition at $z^2 = 0$ in real world [Beneke & Feldmann (2001)]

$$\langle 0|\bar{q}_\alpha(z)\, E(0,z)\, h_{v,\beta}(0)|\bar{B}(v)\rangle = -\frac{i f_B m_B}{4} \times \left[(1+\hat{v})\left\{\tilde{\varphi}_+^B(t) - \left[\tilde{\varphi}_+^B(t) - \tilde{\varphi}_-^B(t)\right]\frac{\hat{z}}{2t}\right\}\gamma_5\right]_{\beta\alpha}$$

- B^*-meson-to-vacuum transition at $z^2 = 0$ in real world

$$\langle 0|\bar{q}_\alpha(z)\, E(0,z)\, h_{v,\beta}(0)|\bar{B}^*(v,\varepsilon)\rangle = \frac{i f_B m_{B^*}}{4} \times \left[(1+\hat{v})\left\{\tilde{\varphi}_+^B(t) - \left[\tilde{\varphi}_+^B(t) - \tilde{\varphi}_-^B(t)\right]\frac{\hat{z}}{2t}\right\}\hat{\varepsilon}\right]_{\beta\alpha}$$

- HQS results $f_{B^*} = f_B$ and $\tilde{\varphi}_\pm^{B^*}(t) = \tilde{\varphi}_\pm^B(t)$

Projection Operator onto Three-Particle State

- B-meson-to-vacuum transition at $z^2 = 0$ [Khodjamirian, Mannel & Offen (2007)]

$$\langle 0|\bar{q}_\alpha(z)\, G_{\lambda\rho}(uz)\, h_{v,\beta}(0)|\bar{B}(v)\rangle = \frac{f_B m_B}{4}$$

$$\times \Big[(1+\hat{v})\Big\{(v_\lambda\gamma_\rho - v_\rho\gamma_\lambda)\left[\tilde{\Psi}_A^B(t,u) - \tilde{\Psi}_V^B(t,u)\right] - i\sigma_{\lambda\rho}\,\tilde{\Psi}_V^B(t,u)$$

$$+ \frac{v_\lambda z_\rho - v_\rho z_\lambda}{t}\,\tilde{X}_A^B(t,u) + \frac{z_\lambda\gamma_\rho - z_\rho\gamma_\lambda}{t}\,\tilde{Y}_A^B(t,u)\Big\}\gamma_5\Big]_{\beta\alpha}$$

- Similar projection operator exists for B^*-meson-to-vacuum transition matrix element at $z^2 = 0$
- Fourier-transforms of distribution amplitudes are required

$$\tilde{\Psi}_{V,A}^B(t,u) = \int_0^\infty d\omega \int_0^\infty d\xi\, e^{-i(\omega+u\xi)t}\,\Psi_{V,A}^B(\omega,\xi)$$

EoM Relations Among *B*-Meson LCDAs

- Heavy- and light-quark equation of motion relates DAs [Kawamura, Kodaira, Qiao & Tanaka (2001)]

$$\phi_+^B(\omega) + \omega\,\frac{d\phi_-^B(\omega)}{d\omega} = I(\omega), \qquad (\omega - 2\bar\Lambda)\,\phi_+^B(\omega) + \omega\,\phi_-^B(\omega) = J(\omega)$$

- $I(\omega)$ and $J(\omega)$ are determined by three-particle DAs

$$I(\omega) = 2\frac{d}{d\omega}\int_0^\infty d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi}\,\frac{\partial}{\partial\xi}\left[\Psi_A^B(\rho,\xi) - \Psi_V^B(\rho,\xi)\right]$$

- Wandzura-Wilczek relation follows when $I(\omega) = 0$

$$\phi_-^B(\omega) = \int_\omega^\infty \frac{\phi_+^B(\omega')}{\omega'}\,d\omega'$$

Evolution of B-Meson LCDA

- Evolution equation $\quad (C_F = 4/3)$
  [Lange & Neubert, PRL (2003) 102001]

$$\frac{d\varphi_+^B(\omega;\mu)}{d\ln\mu} = -\frac{\alpha_{\rm st}(\mu)\,C_F}{\pi}\int_0^\infty d\omega'\,\gamma^{\rm LN}(\omega,\omega';\mu)\,\varphi_+^B(\omega';\mu)$$

- The Lange-Neubert anomalous dimension

$$\gamma^{\rm LN}(\omega,\omega';\mu) = \left(\ln\frac{\mu}{\omega} - \frac{5}{4}\right)\delta\,(\omega-\omega') - \Gamma_{\rm LN}(\omega',\omega)$$

$$\Gamma_{\rm LN}(\omega',\omega) = \left[\frac{\omega}{\omega'}\,\frac{\Theta\,(\omega'-\omega)}{\omega'-\omega} + \frac{\Theta\,(\omega-\omega')}{\omega-\omega'}\right]_\oplus$$

- O-plus convention

$$\int_0^\infty d\omega' f(\omega')\,[\gamma(\omega',\omega)]_\oplus = \int_0^\infty d\omega'\,[f(\omega')-f(\omega)]\,\gamma(\omega',\omega)$$

Evolution of *B*-Meson LCDA

- LN kernel factorizes in the space of moments

$$\tilde{\Gamma}_{\rm LN}(N) = \int_0^\infty d\omega \left(\frac{\omega}{\omega'}\right)^{N-1} \Gamma_{\rm LN}(\omega', \omega) = -\Psi(N) - \Psi(-N) - 2\gamma_{\rm E}$$

  $\gamma_{\rm E} = 0.577216$ is the Euler's constant
  $\Psi(x) = \Gamma'(x)/\Gamma(x)$ is the digamma function
- Analitic solution of the evolution equation

$$\varphi_+^B(\omega; \mu) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt\, \varphi_0(t)\, f(\omega, \mu, \mu_0, it)$$

- $f(\omega, \mu, \mu_0, it)$ can be calculated in perturbation theory
- Function $\varphi_0(t)$ is fixed only by $\varphi_0(0) = \lambda_B^{-1}$
- Can be determined after a model is specified
- $SL(2)$ symmetry of the evolution kernel is determined [Braun & Manashov, PLB 731 (2014)]
- Kernel is diagonalized in terms of eigenfunctions of the generator $S_+ = z^2\, \partial/\partial z + 2z$

$B^+ \to \ell^+ \nu_\ell \gamma$ Decay: Leading Order

- Decay amplitude

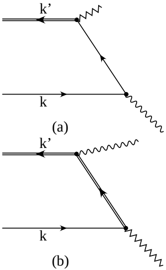
$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} V_{ub}^* \left\langle \gamma \left| \bar{b} \gamma_\mu (1 - \gamma_5) u \right| B^+ \right\rangle L^\mu$$

- Leptonic current

$$L^\mu = \bar{u}(q_\ell) \gamma^\mu (1 - \gamma_5) u(q_\nu)$$

- The photon energy

$$E_\gamma = (vp) = m_B \left(1 - q^2/m_B^2\right)/2$$



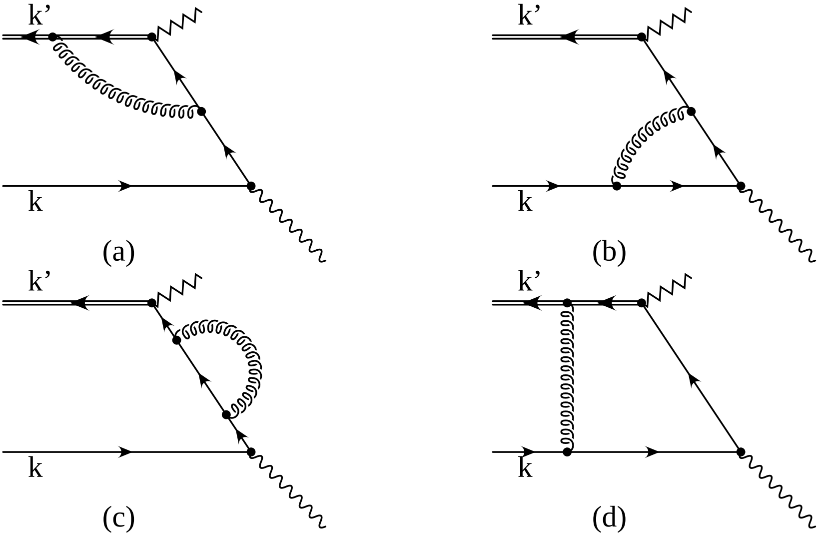
- Transition matrix element [Korchemsky, Pirjol & Yan (2000)]

$$\left\langle \gamma(p,\epsilon) \left| \bar{b}(0) \gamma^\mu (1 - \gamma_5) u(0) \right| B^+(v) \right\rangle =$$

$$= \sqrt{4\pi\alpha} \left\{ \varepsilon^{\mu\rho\sigma\tau} v_\rho p_\sigma \epsilon_\tau^* F_V(E_\gamma) - i \left[(vp) \epsilon^{*\mu} - (v\epsilon^*) p^\mu \right] F_A(E_\gamma) \right\}$$

- The kinematical region $E_\gamma \gg \Lambda_{\rm QCD}$ where perturbative QCD methods for exclusive processes can be applied

$B^+ \to \ell^+ \nu_\ell \gamma$ Decay: Next-to-Leading Order



$B^+ \to \ell^+ \nu_\ell \gamma$ Decay: Theory

- Soft-Collinear Effective Theory (SCET) was suggested for implementation of QCDF
- Differential width [Descotes-Genon & Sachrajada (2003)]

$$\frac{d\Gamma}{dE_\gamma} = \frac{\alpha G_F^2 f_B^2 |V_{ub}|^2 m_B^4}{54\pi^2} \left(C_3^{\rm SCET}\right)^2 \frac{x_\gamma (1 - x_\gamma)}{\Lambda_B^2(E_\gamma)}$$

- Reduced photon energy $x_\gamma = 2E_\gamma / m_B$
- Coefficient $C_3^{\rm SCET}$ results after matching QCD and SCET operators at $\mu_F = m_B$

$$C_3^{\rm SCET} = 1 + \frac{\alpha_s(m_B) C_F}{4\pi} \left[-2\log^2 x_\gamma - 2\,{\rm Li}_2(1 - x_\gamma) + \frac{3x_\gamma - 2}{x_\gamma - 1} \log x_\gamma - 6 - \frac{\pi^2}{12} \right]$$

- The non-perturbative parameter

$$\Lambda_B^{-1}(E_\gamma) = e^{-S(E_\gamma;\mu_F)} \int \frac{dk_+}{k_+} \phi_+^B(k_+;\mu_F) \left\{ 1 + \frac{\alpha_s C_F}{4\pi} \left[\log^2 \frac{2E_\gamma k_+}{\mu_F^2} - \frac{\pi^2}{12} - 1 \right] \right\}$$

- Exponential factor $e^{-S(E_\gamma;\mu_F)}$ appears after resummation of large Sudakov logarithms

$B^+ \to \ell^+ \nu_\ell \gamma$ Decay: Experiment

- Corrections of order of $\mathcal{O}(1/m_B, 1/(2E_\gamma))$ are also calculated [Beneke, Rohrwild (2011)]
- Experimental analysis for partial BF ($E_\gamma^{\rm sig} = [1\ {\rm GeV}, m_B/2]$) with full dataset of $(771.6 \pm 10.6) \times 10^6$ $B\bar{B}$ pairs have been reported by the Belle Collab. [PRD 91 (2015) 112009]
- Recent upper limits (@ 90% C.L.):

$$\mathcal{B}(B^+ \to e^+ \nu_e \gamma) < 6.1 \times 10^{-6}$$
$$\mathcal{B}(B^+ \to \mu^+ \nu_\mu \gamma) < 3.4 \times 10^{-6}$$
$$\mathcal{B}(B^+ \to \ell^+ \nu_\ell \gamma) < 3.5 \times 10^{-6}$$

- The last limit translates into the restriction

$$\lambda_B = \left[\int_0^\infty \frac{dk_+}{k_+} \, \phi_+^B(k_+) \right]^{-1} > 238\ {\rm MeV}$$

- Agrees with the theoretical estimates

$$300\ {\rm MeV} < \lambda_B < 600\ {\rm MeV}$$

$B^+ \to \gamma$ Form Factors

- With account of soft contribution, vector and axial-vector form factors are known [Braun, Khodjamirian (2013)]
- Neglecting radiative and $1/m_B$ corrections, both form factors are equal $F_V^{(0)} = F_A^{(0)} = F_{B\to\gamma^*}^{(0)}$
- Soft contribution was estimated by LCSRs method [Braun, Khodjamirian (2013)]

$$F_{B\to\gamma}^{(0)}(E_\gamma) = \frac{Q_u f_B m_B}{2E_\gamma \lambda_B(\mu)} + \frac{Q_u f_B m_B}{2E_\gamma} \int_0^{\omega_0} d\omega \left[\frac{2E_\gamma}{m_\rho^2}\, e^{-(2E_\gamma\omega - m_\rho^2)/M^2} - \frac{1}{\omega} \right] \phi_+^B(\omega,\mu)$$

- Contains two non-perturbative parameters:
  1. the mass m_ρ of vector ρ-meson
  2. the effective threshold $s_0 = 1.2$ GeV; $\omega_0 = s_0/(2E_\gamma)$
- Difference between ρ- and ω-mesons is neglected

$B_q \to \gamma\gamma$ and $B_q \to \ell^+\ell^-\gamma$ Decays

- Universality of non-perturbative effects in radiative decays
- Width $B_q \to \gamma\gamma$ [Descotes-Genon & Sachrajada (2003)]

$$\Gamma = \frac{\alpha^2 G_F^2 f_B^2 m_B^5}{144\pi^3} \left| V_{tq} V_{tb}^* \right|^2 \left| C_7^{\rm eff} C_9^{\rm SCET} \right|^2 \frac{1}{\Lambda_B^2(m_B/2)}$$

- $B_q \to \ell^+\ell^-\gamma$ differential decay width

$$\frac{d\Gamma}{dE_\gamma} = \frac{\alpha^3 G_F^2 f_B^2 m_B^4}{1728\pi^4} \left| V_{tq} V_{tb}^* \right|^2 \frac{x_\gamma\,(1-x_\gamma)}{\Lambda_B^2(E_\gamma)} \times \left[\left| C_9^{\rm eff} C_3^{\rm SCET} + \frac{2C_7^{\rm eff}}{1-x_\gamma} C_9^{\rm SCET} \right|^2 + \left| C_{10} C_3^{\rm SCET} \right|^2 \right]$$

- Coefficient $C_9^{\rm SCET}$ results after matching QCD and SCET operators at $\mu_F = m_B$

$$C_9^{\rm SCET} = 1 + \frac{\alpha_s(m_B) C_F}{4\pi} \left[\log \frac{m_B^2}{\mu_R} - 2\log^2 x_\gamma + 2\log x_\gamma - 2\,{\rm Li}_2(1-x_\gamma) - 6 - \frac{\pi^2}{12} \right]$$

The $B^+ \to \pi^+ \ell^+ \ell^-$ differential branching fraction

Detailed perturbative analysis in full kinematical region of lepton pair invariant mass squared q^2 was undertaken [Ali, Parkhomenko & Rusov (2014)]

Differential branching fraction

$$\frac{d\mathrm{Br}\left(B^+ \to \pi^+ \ell^+ \ell^-\right)}{dq^2} = \frac{G_F^2 \alpha_{\mathrm{em}}^2 \tau_B}{1024 \pi^5 m_B^3} |V_{tb} V_{td}^*|^2 \sqrt{\lambda(q^2)} \sqrt{1 - \frac{4 m_\ell^2}{q^2}} F(q^2)$$

Dynamical function

$$F(q^2) = \frac{2}{3} \lambda(q^2) \left(1 + \frac{2 m_\ell^2}{q^2}\right) \left| C_9^{\mathrm{eff}} f_+(q^2) + \frac{2 m_b}{m_B + m_\pi} C_7^{\mathrm{eff}} f_T(q^2) \right|^2 + \frac{2}{3} \lambda(q^2) \left(1 - \frac{4 m_\ell^2}{q^2}\right) |C_{10}^{\mathrm{eff}}|^2 f_+^2(q^2) + \frac{4 m_\ell^2}{q^2} \left(m_B^2 - m_\pi^2\right)^2 |C_{10}^{\mathrm{eff}}|^2 f_0^2(q^2)$$

C_i^{eff} are effective Wilson coefficients:
specific combinations of Wilson coefficients

Effective Weak Hamiltonian

Effective Hamiltonian

$$H_{\rm eff}^{(b\to d)} = -\frac{4G_F}{\sqrt{2}}\Big[V_{tb}^* V_{td}\sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$

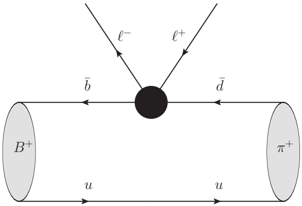
$$+V_{ub}^* V_{ud}\sum_{i=1}^{2} C_i(\mu)\Big(\mathcal{O}_i(\mu)-\mathcal{O}_i^{(u)}(\mu)\Big)\Big] + {\rm h.c.}$$

G_F is the Fermi constant
$C_i(\mu)$ are Wilson coefficients
$\mathcal{O}_i(\mu)$ are the dimension-six operators
V_{ij} are Cabibbo-Kobayashi-Maskawa (CKM) matrix elements

$V_{tb}^* V_{td} \sim V_{ub}^* V_{ud} \sim \lambda^3$ are of the same order in $\lambda = \sin\theta_{12}$

$B \to \pi$ transition matrix elements

Momentum transfered
$q = p_B - p_\pi = p_{\ell^+} + p_{\ell^-}$



$$\langle \pi(p_\pi)|\bar{b}\gamma^\mu d|B(p_B)\rangle = f_+(q^2)\left(p_B^\mu + p_\pi^\mu\right) + \left[f_0(q^2) - f_+(q^2)\right]\frac{m_B^2 - m_\pi^2}{q^2}q^\mu$$

$$\langle \pi(p_\pi)|\bar{b}\sigma^{\mu\nu}q_\nu d|B(p_B)\rangle = \frac{i f_T(q^2)}{m_B + m_\pi}\left[\left(p_B^\mu + p_\pi^\mu\right)q^2 - q^\mu\left(m_B^2 - m_\pi^2\right)\right]$$

Form factors $f_+(q^2)$, $f_0(q^2)$, $f_T(q^2)$ are non-pert. scalar functions

Heavy-Quark Symmetry (HQS) relations

Consider the large-recoil limit (small q^2-values)
Relations to NLO order worked out by Beneke & Feldmann (2000)

$$f_0(q^2) = \left(\frac{m_B^2 + m_\pi^2 - q^2}{m_B^2}\right)\left[\left(1 + \frac{\alpha_s(\mu)C_F}{4\pi}\left(2 - 2L(q^2)\right)\right) f_+(q^2) + \frac{\alpha_s(\mu)C_F}{4\pi}\frac{m_B^2(q^2 - m_\pi^2)}{(m_B^2 + m_\pi^2 - q^2)^2}\Delta F_\pi\right],$$

$$f_T(q^2) = \left(\frac{m_B + m_\pi}{m_B}\right)\left[\left(1 + \frac{\alpha_s(\mu)C_F}{4\pi}\left(\ln\frac{m_b^2}{\mu^2} + 2L(q^2)\right)\right) f_+(q^2) - \frac{\alpha_s(\mu)C_F}{4\pi}\frac{m_B^2}{m_B^2 + m_\pi^2 - q^2}\Delta F_\pi\right],$$

$$L(q^2) = \left(1 + \frac{m_B^2}{m_\pi^2 - q^2}\right)\ln\left(1 + \frac{m_\pi^2 - q^2}{m_B^2}\right), \quad \Delta F_\pi = \frac{8\pi^2 f_B f_\pi}{N_c m_B \lambda_B}\left\langle \bar{u}^{-1}\right\rangle_\pi$$

Only one form factor $f_+(q^2)$ is required

Power-Suppressed Corrections in $B \to \pi \ell^+ \ell^-$ Decay

- Annihilation contributions are power suppressed
- Contains q^2-dependent first inverse moment of sub-leading B-meson LCDA

$$\lambda_{B,-}^{-1}(q^2) = \int_0^\infty \frac{\phi_-^B(\omega)\, d\omega}{\omega - q^2/M_B - i\epsilon}$$

- Specific feature: divergent at $q^2 \to 0$;
  result of small-ω behaviour $\phi_-^B(\omega)|_{\omega\to 0} \sim \mathrm{const}$
- Lower cut $q^2 \le m_\ell^2$ exists in $B \to \pi \ell^+ \ell^-$ decay
- Annihilation contributions of similar type enter in decay amplitudes of similar semileptonic decays $B \to V_{||} \ell^+ \ell^-$
- $V_{||}$ is longitudinally polarized light vector meson
- Decays with transversely polarized meson in final state are dependent on $\phi_+^B(\omega)$ to leading order
- Limit $q^2 = 0$ is realized in rare radiative decays $B \to V\gamma$

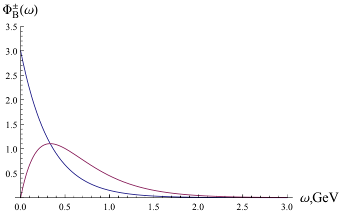
Models of the *B*-Meson Distribution Amplitudes

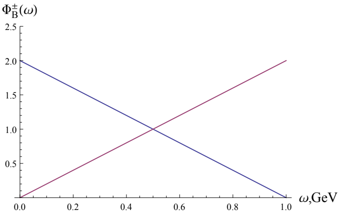
- Exponential model [A. Grozin & M. Neubert, PRD (1996)]

$$\phi_B^+(\omega) = \frac{\omega}{\omega_0^2}\, e^{-\omega/\omega_0}, \qquad \phi_B^-(\omega) = \frac{1}{\omega_0}\, e^{-\omega/\omega_0}, \qquad \omega_0 = \frac{2}{3}\,\bar{\Lambda}$$

- Light-meson-like model [H. Kawamura et al., PLB (2001)]

$$\phi_B^+(\omega) = \frac{\omega}{2\bar{\Lambda}^2}\,\theta(2\bar{\Lambda} - \omega), \qquad \phi_B^-(\omega) = \frac{2\bar{\Lambda} - \omega}{2\bar{\Lambda}^2}\,\theta(2\bar{\Lambda} - \omega)$$





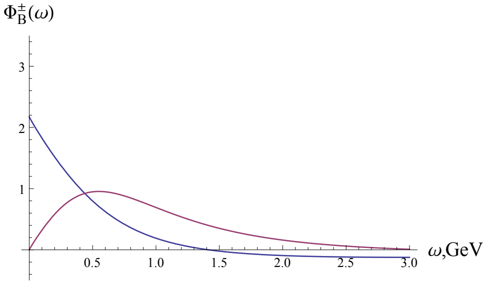
Models of the B-Meson Distribution Amplitudes

- BIK model [V. Braun, D. Ivanov & G. Korchemsky (2004)]

$$\phi_B^+(k,\mu_0) = \frac{4}{\pi\lambda_B}\frac{k}{k^2+1}\left[\frac{1}{k^2+1} - \frac{2(\sigma_B-1)}{\pi^2}\ln k\right]$$

$$\phi_B^-(k,\mu_0) = -\frac{2}{\pi\lambda_B}\left[\frac{k}{k^2+1} + \arctan k - \frac{4(\sigma_B-1)}{\pi^2}\{\ln k \arctan k - \mathrm{Im}\,\mathrm{Li}_2(ik)\}\right]$$

$$\mu_0 = 1\text{ GeV}, \quad \lambda_B^{-1}(\mu_0) = 2.15\text{ GeV}^{-1}, \quad \sigma_B(\mu_0) = 1.4$$



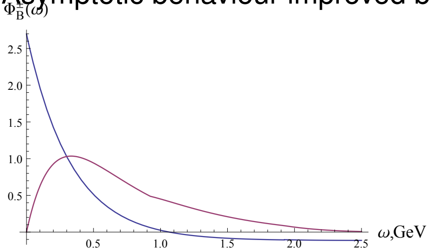
Models of the *B*-Meson Distribution Amplitudes

- Improved exponential model [S. Lee & M. Neubert (2005)]

$$\phi_B^+(\omega,\mu) = \frac{N\omega}{\omega_0^2}\, e^{-\omega/\omega_0} + \theta(\omega-\omega_t)\,\frac{4\alpha_{\rm st}}{3\pi\omega}\left[\left(\frac{1}{2}-\ln\frac{\omega}{\mu}\right)+\frac{4\bar\Lambda_{\rm DA}}{3\omega}\left(2-\ln\frac{\omega}{\mu}\right)+\ldots\right]$$

$$\phi_B^-(\omega,\mu) = \frac{N}{\omega_0}\, e^{-\omega/\omega_0} + \theta(\omega-\omega_t)\,\frac{4\alpha_{\rm st}}{3\pi\mu}\left\{-\frac{\mu}{2\omega}\left[1+2\ln\frac{\omega}{\mu}\right]+\frac{\bar\Lambda_{\rm DA}\mu}{3\omega^2}\left[3-2\ln\frac{\omega}{\mu}\right]\right\}$$
$$+\theta(\omega_t-\omega)\,\frac{4\alpha_{\rm st}}{3\pi\mu}\left\{-\frac{\mu}{2\omega_t}\left[1+2\ln\frac{\omega_t}{\mu}\right]+\frac{\bar\Lambda_{\rm DA}\mu}{3\omega_t^2}\left[3-2\ln\frac{\omega_t}{\mu}\right]\right\}$$

- Asymptotic behaviour improved by ”radiative tail“



$$\omega_0 = 0.417842 \text{ GeV}$$
$$\mu = 1 \text{ GeV}$$
$$\alpha_{\rm st}(1\,\text{GeV}) = 0.507$$
$$\bar\Lambda_{\rm DA} = 0.519 \text{ GeV}$$
$$\omega_t = 0.9243 \text{ GeV}$$
$$N = 0.937493$$

Distribution Amplitude Moments

- LCDAs are involved as convolutions into B-meson decay amplitudes
- First inverse moments are of great importance

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\, \phi_+^B(\omega;\mu), \qquad \frac{\sigma_{B,n}(\mu)}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\, \phi_+^B(\omega;\mu) \ln^n \frac{\mu}{\omega}$$

- To remember: B-meson-to-photon transition to leading order in $1/m_B$ and $1/(2E_\gamma)$ expansion

$$F^{(0)}_{B\to\gamma^*_\perp} = \frac{Q_u f_B m_B}{2E_\gamma}\, \lambda^{-1}_{B,+}(q^2), \qquad F^{(0)}_{B\to\gamma^*_\parallel} = \frac{Q_u f_B m_B}{2E_\gamma}\, \lambda^{-1}_{B,-}(q^2)$$

- Depend on q^2-dependent first inverse moments

$$\lambda^{-1}_{B,\pm}(q^2) = \int_0^\infty \frac{\phi_\pm^B(\omega)\, d\omega}{\omega - q^2/M_B - i\epsilon}$$

- $\lambda^{-1}_{B,-}(q^2)$ is logarithmically divergent in the limit $q^2 \to 0$

First Inverse Moments in Exponential Model

- Simple and logarithmic first inverse moments

$$\lambda_B(\mu) = \omega_0(\mu) = \frac{2}{3}\,\bar{\Lambda}; \quad \sigma_B(\mu) = \ln\frac{\mu}{\lambda_B(\mu)} + \gamma_E$$

- $\gamma_E \simeq 0.577$ is the Euler constant
- Numerically (b-quark mass is in the $\overline{\rm MS}$-scheme): $\lambda_B(1.0\ {\rm GeV}) \simeq 0.33$ GeV & $\sigma_B(1.0\ {\rm GeV}) \simeq 1.67$
- q^2-dependent first inverse moments

$$\lambda_{B,+}^{-1}(q^2) = \lambda_B^{-1} + \zeta\,\lambda_{B,-}(q^2), \quad \lambda_{B,-}^{-1}(q^2) = \lambda_B^{-1}\, e^{-\zeta}\,[-{\rm Ei}(\zeta) + i\pi]$$

- $\zeta = q^2/(m_B\lambda_B)$ — reduced momentum squared
- ${\rm Ei}(x)$ is the exponential integral function

First Inverse Moments in Light-Meson-Like Model

- Simple and logarithmic first inverse moments

$$\lambda_B(\mu) = \bar{\Lambda} = m_B - m_b; \quad \sigma_B(\mu) = \ln \frac{\mu}{2\lambda_B(\mu)} + 1$$

- Numerically (b-quark mass is in the $\overline{\mathrm{MS}}$-scheme):
  $\lambda_B(1.0\ \mathrm{GeV}) \simeq 0.5$ GeV & $\sigma_B(1.0\ \mathrm{GeV}) \simeq 1$
- q^2-dependent first inverse moments

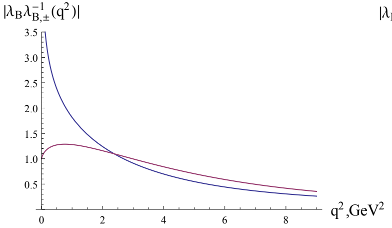
$$\lambda_{B,+}^{-1}(q^2) = \lambda_B^{-1}\left[\xi \ln|1/\xi - 1| + 1 + i\pi\xi\,\Theta\,(1-\xi)\right]$$

$$\lambda_{B,-}^{-1}(q^2) = \lambda_B^{-1}\left[(1-\xi)\ln|1/\xi - 1| - 1 + i\pi\,(1-\xi)\,\Theta\,(1-\xi)\right]$$

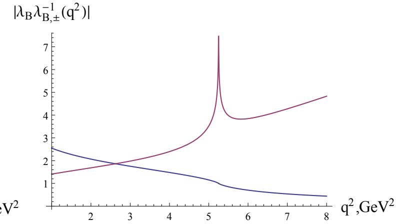
- $\xi = q^2/(2m_B\lambda_B)$ — reduced momentum squared
- $\Theta(x)$ is the unit-step function

First Inverse Moments of DA Models

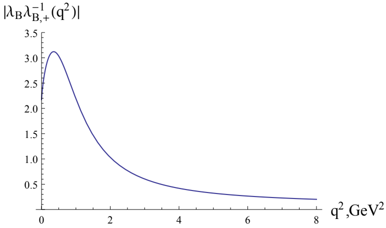
Exponential model



Light-meson-like model



BIK model



Improved exponential model

Under construction

Introduction

Ordinary baryons – colorless systems containing three quarks

Heavy baryons H_Q – one heavy quark $Q = c, b$

Doubly-heavy baryons $H_{Q_1 Q_2}$ – two heavy quarks Q_1 and Q_2

Triply-heavy baryons $H_{Q_1 Q_2 Q_3}$ – all three heavy quarks

Convenient framework: Heavy Quark Effective Theory (HQET)

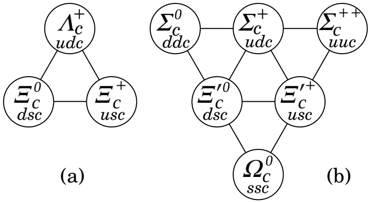
Features of the heavy-baryon system within HQET:

- heavy-quark spin S_Q is decoupled in the limit $m_Q \to \infty$
- classified by the total angular momentum j and parity p of the light-quark pair j^p

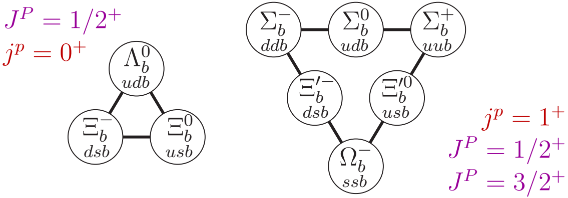
Similar features of the doubly-heavy-baryon system

Introduction

Ground-state ($\ell = 0$) charmed baryons



Ground-state ($\ell = 0$) bottom baryons



Introduction

Experimental measurements [PDG, 2015] and theoretical predictions based on HQET [X. Liu et al., 2008] and Lattice QCD [R. Lewis et al., 2009] for masses of ground-state bottom baryons (in units of MeV)

Baryon	$I(J^P)$	j^p	Experiment	HQET	Lattice QCD
Λ_b	$0(1/2^+)$	0^+	5619.51 ± 0.23	5637^{+68}_{-56}	$5641 \pm 21^{+15}_{-33}$
Σ_b^+	$1(1/2^+)$	1^+	5811.3 ± 1.9	5809^{+82}_{-76}	$5795 \pm 16^{+17}_{-26}$
Σ_b^-	$1(1/2^+)$	1^+	5815.5 ± 1.8	5809^{+82}_{-76}	$5795 \pm 16^{+17}_{-26}$
Σ_b^{*+}	$1(3/2^+)$	1^+	5832.1 ± 1.9	5835^{+82}_{-77}	$5842 \pm 26^{+20}_{-18}$
Σ_b^{*-}	$1(3/2^+)$	1^+	5835.1 ± 1.9	5835^{+82}_{-77}	$5842 \pm 26^{+20}_{-18}$
Ξ_b^-	$1/2(1/2^+)$	0^+	5794.4 ± 1.2	5780^{+73}_{-68}	$5781 \pm 17^{+17}_{-16}$
Ξ_b^0	$1/2(1/2^+)$	0^+	5791.8 ± 0.5	5780^{+73}_{-68}	$5781 \pm 17^{+17}_{-16}$
$\Xi_b^{\prime -}$	$1/2(1/2^+)$	1^+	5935.02 ± 0.05	5903^{+81}_{-79}	$5903 \pm 12^{+18}_{-19}$
Ξ_b^{*-}	$1/2(3/2^+)$	1^+	5955.33 ± 0.13	5903^{+81}_{-79}	$5950 \pm 21^{+19}_{-21}$
Ξ_b^{*0}	$1/2(3/2^+)$	1^+	5953.02 ± 0.55	5903^{+81}_{-79}	$5950 \pm 21^{+19}_{-21}$
Ω_b^-	$0(1/2^+)$	1^+	6048.0 ± 1.9	6036 ± 81	$6006 \pm 10^{+20}_{-19}$

Introduction

- Heavy baryons are copiously produced at the LHC
- Weak decays of bottom baryons induced by FCNC may give important information on physics beyond the SM
- LCDAs are the primary non-perturbative objects required for calculating decays into light particles based on the heavy quark expansion or within the method of Light-Cone Sum Rules (LCSRs)
- For a long time existing models for heavy baryons were motivated by the quark model
- Complete classification of the three-quark LCDAs of the Λ_b-baryon in QCD and main features of these LCDAs have been considered by V. Braun, P. Ball and E. Gardi (2008)
- Extension of this analysis for all ground-state bottom baryons have been done by A. Ali, C. Hambrock, A. P. and W. Wang (2013) and presented in this lecture

Light-Cone Distribution Amplitudes (LCDAs)

Light-cone distribution amplitudes of heavy baryons —
matrix elements of non-local light-ray operators
build off an effective heavy quark and two light quarks

- Similar in construction to *B*-meson LCDAs
- QCD description of nucleon LCDAs

Heavy Quark Symmetry $\Longrightarrow$ switch off the heavy-quark spin

$SU(3)_F$ antitriplet $\Longrightarrow$ scalar states with $J^P = j^p = 0^+$

$SU(3)_F$ sextets $\Longrightarrow$ axial-vector states with $J^P = j^p = 1^+$

LCDAs

$SU(3)_F$ antitriplet $J^P = j^p = 0^+$ scalar state

Non-local light-ray operators

$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1 n) C\gamma_5 \not{n} q_2^b(t_2 n)\right) h_v^c(0)|H(v)\rangle = f_H^{(2)} \Psi_2(t_1, t_2)$$
$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1 n) C\gamma_5 q_2^b(t_2 n)\right) h_v^c(0)|H(v)\rangle = f_H^{(1)} \Psi_3^s(t_1, t_2)$$
$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1 n) C\gamma_5 i\sigma_{\bar{n}n} q_2^b(t_2 n)\right) h_v^c(0)|H(v)\rangle = 2f_H^{(1)} \Psi_3^\sigma(t_1, t_2)$$
$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1 n) C\gamma_5 \not{\bar{n}} q_2^b(t_2 n)\right) h_v^c(0)|H(v)\rangle = f_H^{(2)} \Psi_4(t_1, t_2)$$

$q_i = u, d, s$ – light quark fields

C – charge conjugation matrix

$n^\mu,\ \bar{n}^\mu$ – two light-like vectors $(n\bar{n}) = 2$

Frame is adopted: $v^\mu = (n^\mu + \bar{n}^\mu)/2$

Light Quark Field

Light-quark fields living on the light cone
assumed to be multiplied by the Wilson lines

$$q(tn) = [0, tn]\, q(tn) = \mathrm{P} \exp\left\{ -ig_{\mathrm{st}}\, t \int_0^1 d\alpha\, n^\mu A_\mu(\alpha tn) \right\} q(tn)$$

Considered as generating function of formal expansion

$$q(tn) = \sum_{N=0}^{\infty} \frac{t^N}{N!} \left(n^\mu D_\mu\right)^N q(0)$$

The covariant derivative $D_\mu = \partial_\mu - ig_{\mathrm{st}} A_\mu$

Similar for the gluonic field

$$G_{\mu\nu}(tn) = [0, tn]\, G_{\mu\nu}(tn)$$

Heavy Quark Field

The heavy-quark field living on the light cone also includes the Wilson line but time-like [Korchemsky, Radushkin (1992)]

$$h_v(0) = \mathrm{P}\exp\left\{ i g_{\rm st} \int_{-\infty}^{0} d\alpha\, v^\mu A_\mu(\alpha v) \right\} \phi(-\infty)$$

Sterile field $\phi(-\infty)$ was introduced

LCDAs

Couplings $f_H^{(i)}$ are defined by local operators

$$\epsilon^{abc}\langle 0|\left(q_1^a(0)C\gamma_5 q_2^b(0)\right) h_v^c(0)|H(v)\rangle = f_H^{(1)}$$

$$\epsilon^{abc}\langle 0|\left(q_1^a(0)C\gamma_5 \not{v} q_2^b(0)\right) h_v^c(0)|H(v)\rangle = f_H^{(2)}$$

Scale dependence of the couplings (NLO order):

$$f_H^{(i)}(\mu) = f_H^{(i)}(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_1^{(i)}/\beta_0}\left[1 - \frac{\alpha_s(\mu_0)-\alpha_s(\mu)}{4\pi}\frac{\gamma_1^{(i)}}{\beta_0}\left(\frac{\gamma_2^{(i)}}{\gamma_1^{(i)}} - \frac{\beta_1}{\beta_0}\right)\right]$$

Example: Λ_b-baryon

NLO QCD sum rules [Groote et all., 1997]

$$f_{\Lambda_b}^{(1)}(\mu_0 = 1\ \mathrm{GeV}) \simeq f_{\Lambda_b}^{(2)}(\mu_0 = 1\ \mathrm{GeV}) \simeq 0.030 \pm 0.005\ \mathrm{GeV}^3$$

Supported by the non-relativistic constituent quark picture

LCDAs

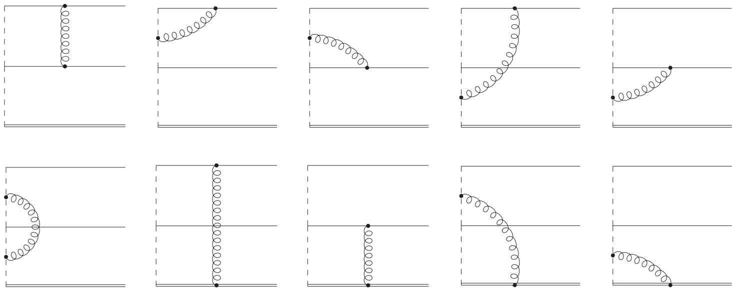
LCDAs $\Psi_i(t_1, t_2)$ are scale dependent

Fourier transrorm to the momentum space:

$$\begin{aligned}\Psi(t_1, t_2) &= \int_0^\infty d\omega_1 \int_0^\infty d\omega_2\, e^{-it_1\omega_1 - it_2\omega_2} \psi(\omega_1, \omega_2) \\ &= \int_0^\infty \omega\, d\omega \int_0^1 du\, e^{-i\omega(t_1 u + t_2 \bar{u})}\, \widetilde{\psi}(\omega, u)\end{aligned}$$

$\omega_1 = u\omega$, $\omega_2 = (1-u)\,\omega = \bar{u}\omega$ – energies of light quarks
LO evolution equation for $\psi_2(\omega_1, \omega_2; \mu)$: derived by identifying UV singularities of one-gluon-exchange diagrams

One-gluon exchange diagrams



LCDAs

Evolution equation is expressed in terms of two-particle kernels from evolution equations for B- and pseudoscalar mesons

$$\begin{aligned}\mu\frac{d}{d\mu}\,\psi_2(\omega_1,\omega_2;\mu) &= -\frac{\alpha_s(\mu)}{2\pi}\,\frac{4}{3}\left\{\int_0^\infty d\omega_1'\,\gamma^{\rm LN}(\omega_1',\omega_1;\mu)\,\psi_2(\omega_1',\omega_2;\mu)\right.\\ &+ \int_0^\infty d\omega_2'\,\gamma^{\rm LN}(\omega_2',\omega_2;\mu)\,\psi_2(\omega_1,\omega_2';\mu)\\ &- \left.\int_0^1 dv\,V(u,v)\,\psi_2(v\omega,\bar{v}\omega;\mu)+\frac{3}{2}\,\psi_2(\omega_1,\omega_2;\mu)\right\}\end{aligned}$$

Kernel $\gamma^{\rm LN}(\omega',\omega;\mu)$ controlling evolution of the B-meson LCDA

$V(u,v)$ is the ER-BL kernel

Term $3\psi_2/2$ results from $f_H^{(2)}$ renormalization subtraction

Evolution equation can be solved either numerically or semi-analytically [Braun et al., 2008]

LCDAs

$SU(3)_F$ sextet $\quad J^P = j^p = 1^+ \quad$ axial-vector state

Non-local light-ray operators (longitudinal polarization)

$$\epsilon^{abc}\langle 0| \left(q_1^a(t_1) C \not{n} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle = (\bar{v}\varepsilon)\, f_H^{(2)}\, \Psi_2^{\|}(t_1,t_2)$$

$$\epsilon^{abc}\langle 0| \left(q_1^a(t_1) C\, q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle = (\bar{v}\varepsilon)\, f_H^{(1)}\, \Psi_3^{\|s}(t_1,t_2)$$

$$\epsilon^{abc}\langle 0| \left(q_1^a(t_1) C\, i\sigma_{\bar{n}n} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle = 2\,(\bar{v}\varepsilon)\, f_H^{(1)}\, \Psi_3^{\|a}(t_1,t_2)$$

$$\epsilon^{abc}\langle 0| \left(q_1^a(t_1) C \not{\bar{n}} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle = -\,(\bar{v}\varepsilon)\, f_H^{(2)}\, \Psi_4^{\|}(t_1,t_2)$$

$$\bar{v}^\mu = (\bar{n}^\mu - n^\mu)/2 \quad (v\bar{v}) = 0 \quad (\bar{v}\bar{v}) = -1$$

$$\varepsilon^\mu = \varepsilon_\|^\mu + \varepsilon_\perp^\mu \qquad \varepsilon_\|^\mu = \eta\bar{v}^\mu \qquad \sigma_{\bar{n}n} = i\,(\not{\bar{n}}\not{n} - \not{n}\not{\bar{n}})/2$$

LCDAs

$SU(3)_F$ sixtet $J^P = j^p = 1^+$ axial-vector state

Non-local light-ray operators (transverse polarization)

$$\epsilon^{abc} \langle 0| \left(q_1^a(t_1) C \gamma_\perp^\mu \not{n} q_2^b(t_2) \right) h_v^c(0) |H(v,\varepsilon)\rangle = f_H^{(2)} \Psi_2^\perp(t_1,t_2) \varepsilon_\perp^\mu$$

$$\epsilon^{abc} \langle 0| \left(q_1^a(t_1) C \gamma_\perp^\mu q_2^b(t_2) \right) h_v^c(0) |H(v,\varepsilon)\rangle = f_H^{(1)} \Psi_3^{\perp s}(t_1,t_2) \varepsilon_\perp^\mu$$

$$\epsilon^{abc} \langle 0| \left(q_1^a(t_1) C \gamma_\perp^\mu i\sigma_{\bar{n}n} q_2^b(t_2) \right) h_v^c(0) |H(v,\varepsilon)\rangle = 2 f_H^{(1)} \Psi_3^{\perp a}(t_1,t_2) \varepsilon_\perp^\mu$$

$$\epsilon^{abc} \langle 0| \left(q_1^a(t_1) C \gamma_\perp^\mu \not{\bar{n}} q_2^b(t_2) \right) h_v^c(0) |H(v,\varepsilon)\rangle = f_H^{(2)} \Psi_4^\perp(t_1,t_2) \varepsilon_\perp^\mu$$

$$\gamma_\perp^\mu = \gamma^\mu - \left(\not{\bar{n}} \not{n} + \not{n} \not{\bar{n}} \right)/2$$

LCDAs

Switching on the heavy quark spin

r.h.s. of matrix elements of all non-local operators must be multiplied on the Dirac spinor $U(v)$ of the heavy quark h_v

$$\not{v}\, U(v) = U(v) \qquad \overline{U}(v)\, U(v) = 1$$

Scalar state: $J^P = j^p = 0^+ \Longrightarrow J^P = 1/2^+ : \quad H(v) \equiv U(v)$

Axial-vector state: $J^P = j^p = 1^+ \Longrightarrow J^P = 1/2^+, J^P = 3/2^+$

$$\begin{aligned} \varepsilon_\mu\, U(v) &= \left[\varepsilon_\mu\, U(v) - \frac{1}{3}\,(\gamma_\mu + v_\mu)\,\not{\varepsilon}\, U(v)\right] + \frac{1}{3}\,(\gamma_\mu + v_\mu)\,\not{\varepsilon}\, U(v) \\ &\equiv R_\mu^{3/2}(v) + \frac{1}{3}\,(\gamma_\mu + v_\mu)\, H(v) \end{aligned}$$

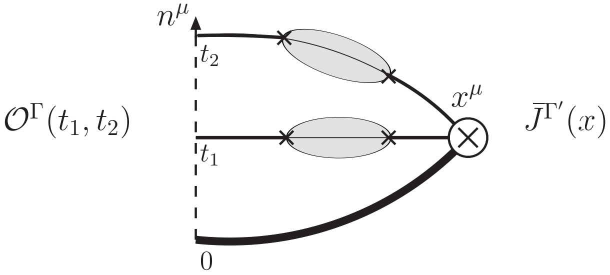
Rarita-Schwinger vector-spinor $R_\mu^{3/2}(v)$:

$$\not{v}\, R_\mu^{3/2}(v) = R_\mu^{3/2}(v), \quad v^\mu\, R_\mu^{3/2}(v) = 0, \quad \gamma^\mu\, R_\mu^{3/2}(v) = 0$$

QCD Sum Rules

Models for LCDAs can be obtained using QCD sum rules

Correlation functions involve the non-local light-ray operators and a suitable local current



QCD Sum Rules

Heavy-baryon local operators

$$\bar{J}^{\Gamma'}(x) = \epsilon^{abc}\left(\bar{q}_2^a(x)\,[A + B\,\not{v}]\,\Gamma' C^T \bar{q}_1^b(x)\right)\bar{h}_v^c(x)$$

Arbitrariness in the choice of local currents (variation in $A \in [0,1]$ and $B = 1 - A$) is adopted as an error estimate

Results are calculated for $A = B = 1/2$: supported by a constituent quark model picture [Braun et al., 2008]

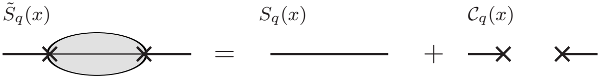
$$j^p = 0^+ \Longrightarrow \Gamma' = \gamma_5$$

$$j^p = 1^+ \Longrightarrow \Gamma' = \gamma_{\|}, \gamma_{\perp}$$

QCD Sum Rules

Propagators of the light quark fields $\tilde{S}_q(x)$ are not free

To take effects of the QCD background inside baryons into account, method of non-local condensates is used



$$S_q(x) = \frac{i\not{x}}{2\pi^2 x^4} - \frac{m}{4\pi^2 x^2} \qquad \mathcal{C}_q(x) = \frac{1}{12}\langle \bar{q}(x) q(0) \rangle$$

QCD sum rules

General parametrization [Mikhailov, Radyushkin, 1986, 1992]

$$\mathcal{C}_q(x) = \langle \bar{q} q \rangle \int_0^\infty d\nu \, e^{\nu x^2/4} \, f(\nu)$$

Non-local condensate shape is chosen according to the model [Braun et el., 1994; Braun et al., 2003]

$$f(\nu) = \frac{\lambda^{a-2}}{\Gamma(a-2)} \, \nu^{1-a} \, e^{-\lambda/\nu}, \qquad a = 3 + \frac{4\lambda}{m_0^2}$$

Parameters included:

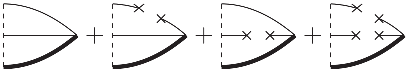
$\langle \bar{q} q \rangle$ — local quark condensate,
$\lambda = \langle \bar{q} D^2 q \rangle$ — correlation length,
$m_0^2 = \langle \bar{q} g_{\rm st} \sigma_{\mu\nu} G^{\mu\nu} q \rangle / \langle \bar{q} q \rangle$ — ratio of local mixed quark-gluon and quark condensates

QCD sum rules

Double Fourier transform of the correlation function

$$\Pi_{\Gamma\Gamma'}(\omega_1,\omega_2;E) = i\int_{-\infty}^{\infty}\frac{dt_1\,dt_2}{(2\pi)^2}\,e^{i(\omega_1 t_1+\omega_2 t_2)}\int d^4x\,e^{-iE(vx)}\,\langle 0|\mathcal{O}^{\Gamma}(t_1,t_2)\,\bar{J}^{\Gamma'}(x)|0\rangle$$

In momentum space, correlation function reads diagrammatically

$\Pi(\omega, u; E) =$ 

Heavy quark condensate term is suppressed by $1/m_Q$

Sum rule reads

$$|f_H|^2\,\psi^{\Gamma}(\omega,u)\,e^{-\bar{\Lambda}_H/\tau} = \mathbb{B}[\Pi](\omega,u;\tau,s_0)$$

$\mathbb{B}$ means the Borel-transform, $\bar{\Lambda}_H = m_H - m_Q$

s_0 – momentum cutoff from applying the quark-hadron duality

QCD sum rules

Analytic result for leading-twist transverse LCDA at $\mu_0 = 1$ GeV

$$f_H^{(2)} \left[A\, f_H^{(1)} + B\, f_H^{(2)} \right] \tilde{\psi}_2^{SR}(\omega, u)\, \mathrm{e}^{-\bar{\Lambda}/\tau} =$$

$$\frac{3\tau^4}{2\pi^4} \left[B\hat{\omega}^2\, u\bar{u} + A\,\hat{\omega}\,(\hat{m}_2 u + \hat{m}_1 \bar{u}) \right] E_1(2\hat{s}_\omega)\mathrm{e}^{-\hat{\omega}}$$

$$- \frac{\langle \bar{q}_1 q_1 \rangle \tau^3}{\pi^2} \left[A\hat{\omega}\bar{u} + B\hat{m}_2 \right] f(2\tau\omega u)\, E_{2-a}(2\hat{s}_\kappa)\, \mathrm{e}^{-\hat{\omega}}$$

$$- \frac{\langle \bar{q}_2 q_2 \rangle \tau^3}{\pi^2} \left[A\hat{\omega} u + B\hat{m}_1 \right] f(2\tau\omega\bar{u})\, E_{2-a}(2\hat{s}_{\bar{\kappa}})\, \mathrm{e}^{-\hat{\omega}}$$

$$+ \frac{2B}{3} \langle \bar{q}_1 q_1 \rangle \langle \bar{q}_2 q_2 \rangle\, \tau^2\, f(2\tau\omega u)\, f(2\tau\omega\bar{u})\, E_{3-2a}(2\hat{s}_{\kappa\bar{\kappa}})\, \mathrm{e}^{-\hat{\omega}},$$

The following function was introduced

$$E_a(x) = \frac{1}{\Gamma(a+1)} \int_0^x dt\, t^a \mathrm{e}^{-t} = 1 - \frac{\Gamma(a+1, x)}{\Gamma(a+1)}$$

$\bar{\Lambda} = m_H - m_b, \quad s_\omega = s_0 - \omega/2, \quad \kappa = \lambda/(2u\omega\tau), \quad \bar{\kappa} = \lambda/(2\bar{u}\omega\tau)$

$\hat{\omega} = \omega/(2\tau), \quad \hat{s}_\omega = s_\omega/(2\tau), \quad \hat{m}_{1,2} = m_{1,2}/(2\tau)$

$\hat{s}_\kappa = \hat{s}_\omega - \kappa/2, \quad \hat{s}_{\bar{\kappa}} = \hat{s}_\omega - \bar{\kappa}/2, \quad \hat{s}_{\kappa\bar{\kappa}} = \hat{s}_\omega - \kappa/2 - \bar{\kappa}/2$

QCD sum rules

Normalization of symmetric LCDAs ($t = 2, 3s, 4$)

$$\int_0^{2s_0} \omega d\omega \int_0^1 du \, \tilde{\psi}_t^{\rm SR}(\omega, u) \equiv 1$$

Normalization of antisymmetric LCDAs ($t = 3\sigma$) can be fixed by

$$\int_0^{2s_0} \omega d\omega \int_0^1 du \, C_1^{1/2}(2u-1) \, \tilde{\psi}_t^{\rm SR}(\omega, u) \equiv 1$$

Here, $C_n^m(x)$ are the Gegenbauer polinomials

QCD sum rules

QCD sum rules constrain certain moments

$$\langle f(\omega,u)\rangle_k \equiv \int_0^{2s_0} \omega d\omega \int_0^1 du\, f(\omega,u)\, \tilde{\psi}_t^{\rm SR}(\omega,u)$$

Numerical values of the parameters

$\bar\Lambda_{\Lambda_b}$	0.8 GeV	$s_0^{(\Lambda_b)}$	1.2 GeV
$\bar\Lambda_{\Xi_b}$	1.0 GeV	$s_0^{(\Xi_b)}$	1.3 GeV
τ	0.6 ± 0.2	m_s (1 GeV)	128 ± 21 MeV
$\langle\bar q q\rangle$ (1 GeV)	$-(242^{+28}_{-19})$ MeV^3	$\langle\bar s s\rangle/\langle\bar q q\rangle$	0.8 ± 0.3
m_0^2	0.8 ± 0.2 GeV^2	λ	0.16 GeV^2

Numerical analysis

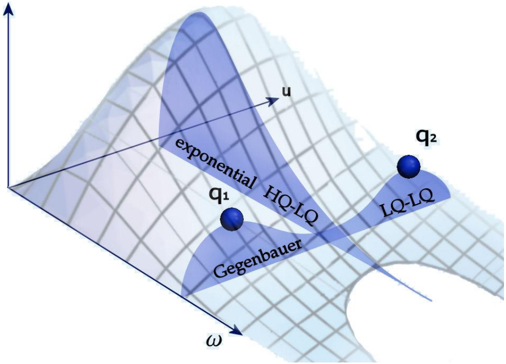
Numerical values of first several moments

H_Q	t	$\langle \omega^{-1} \rangle$	$\langle C_1^{3/2} \rangle$	$\langle \omega^{-1} C_1^{3/2} \rangle$	$\langle C_2^{3/2} \rangle$	$\langle \omega^{-1} C_2^{3/2} \rangle$
Λ_b	2	$1.65^{+0.91}_{-0.47}$	0	0	$1.00^{+0.54}_{-1.03}$	$0.61^{+0.76}_{-1.45}$
Ξ_b	2	$1.61^{+0.71}_{-0.42}$	$0.10^{+0.10}_{-0.06}$	$0.08^{+0.07}_{-0.04}$	$0.98^{+0.49}_{-0.82}$	$0.69^{+0.63}_{-1.07}$

H_Q	t	$\langle \omega^{-1} \rangle$	$\langle C_1^{1/2} \rangle$	$\langle \omega^{-1} C_1^{1/2} \rangle$	$\langle C_2^{1/2} \rangle$	$\langle \omega^{-1} C_2^{1/2} \rangle$
Λ_b	3s	$2.16^{+0.70}_{-0.36}$	0	0	$-0.032^{+0.022}_{-0.041}$	$-0.29^{+0.14}_{-0.27}$
	3σ	0	1	$1.54^{+0.14}_{-0.22}$	0	0
	4	$2.84^{+0.88}_{-0.46}$	0	0	$-0.108^{+0.035}_{-0.018}$	$-0.41^{+0.08}_{-0.15}$
Ξ_b	3s	$2.08^{+0.50}_{-0.29}$	$0.11^{+0.10}_{-0.06}$	$0.063^{+0.080}_{-0.047}$	$0.87^{+0.08}_{-0.14}$	$0.84^{+0.27}_{-0.45}$
	3σ	$0.00054^{+0.00033}_{-0.00054}$	1	$1.51^{+0.12}_{-0.19}$	$0.054^{+0.033}_{-0.054}$	$0.098^{+0.061}_{-0.098}$
	4	$2.73^{+0.61}_{-0.35}$	$0.12^{+0.09}_{-0.05}$	$0.05^{+0.09}_{-0.05}$	$0.55^{+0.18}_{-0.11}$	$0.99^{+0.16}_{-0.09}$

Numerical analysis

Model functions for the *b*-baryon LCDAs, composed of the exponential part for the heavy-light interaction and the Gegenbauer polynomials for the light-light interaction



Numerical analysis

Proposed simple models for LCDAs at the scale $\mu_0 = 1$ GeV

$$\tilde{\psi}_2(\omega, u) = \omega^2 u(1-u) \sum_{n=0}^{2} \frac{a_n^{(2)}}{\epsilon_n^{(2)\,4}} C_n^{3/2}(2u-1)\, \mathrm{e}^{-\omega/\epsilon_n^{(2)}},$$

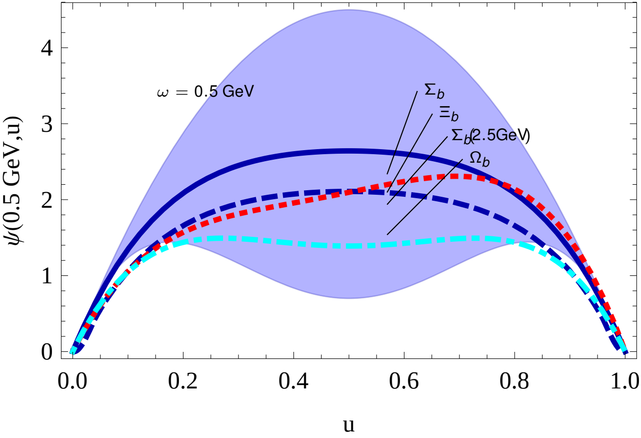
$$\tilde{\psi}_{3s}(\omega, u) = \frac{\omega}{2} \sum_{n=0}^{2} \frac{a_n^{(3)}}{\epsilon_n^{(3)\,3}} C_n^{1/2}(2u-1)\, \mathrm{e}^{-\omega/\epsilon_n^{(3)}},$$

$$\tilde{\psi}_{3\sigma}(\omega, u) = \frac{\omega}{2} \sum_{n=0}^{3} \frac{b_n^{(3)}}{\eta_n^{(3)\,3}} C_n^{1/2}(2u-1)\, \mathrm{e}^{-\omega/\eta_n^{(3)}},$$

$$\tilde{\psi}_4(\omega, u) = \sum_{n=0}^{2} \frac{a_n^{(4)}}{\epsilon_n^{(4)\,2}} C_n^{1/2}(2u-1)\, \mathrm{e}^{-\omega/\epsilon_n^{(4)}},$$

Numerical analysis

Twist-2 LCDAs of Σ (blue), Ξ (red) and Ω (cyan) baryons at the energy scales $\mu_0 = 1$ GeV estimated within the range $A \in [0, 1]$



Numerical analysis

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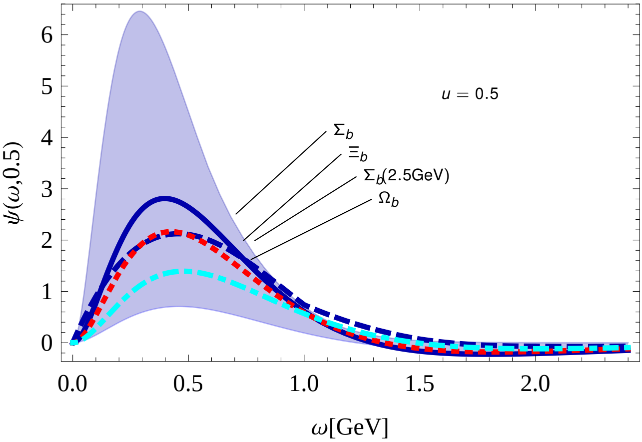


Operator Renormalization

Renormalization of heavy-light light-ray operators up to twist-three was performed by Knoedlseder, Offen (2011)

Corresponding evolution equations for the twist-three operators are written explicitly

Based on the $SL(2)$ symmetry of the evolution kernel exact analytic expressions for eigenfunctions and anomalous dimensions were calculated for the aligned hilicities of quarks Braun, Derkachev & Manashov (2014)

Evolution kernel of anti-aligned quark helicities, e. g. of Λ_b-baryon, contains an extra term which breaks integrability Braun, Derkachev & Manashov (2014)

Classification of the baryonic operators with three quarks and gluon was not presented

$\Lambda_b \to \Lambda \ell^+ \ell^-$ Decay in SQET

- Observed by CDF (Fermilab) and LHCb (CERN) Collab.
- Differential distribution in q^2 is measured
- Based on HQS and SCET, QCD factorization to leading order was worked out for the $\Lambda_b \to \Lambda$ transition [Feldmann & Yip (2012), Mannel & Wang (2011)]
- Double differential distribution in the helicity basis [Feldmann & Yip (2012)]

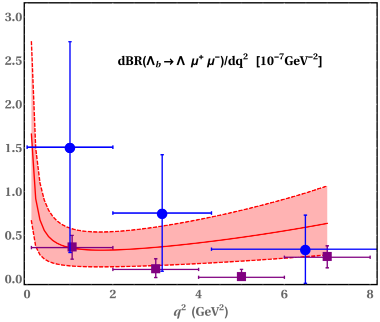
$$\frac{d^2\Gamma}{dq^2 d\cos\theta} \equiv \frac{3\alpha^2 G_F^2 |V_{ts} V_{tb}|^2}{64\pi^2} \left[\left(1 + \cos^2\theta\right) H_T(q^2) + 2\cos\theta\, H_A(q^2) + 2\left(1 - \cos^2\theta\right) H_L(q^2)\right]$$

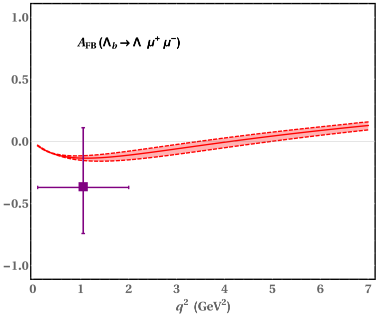
- In the SCET limit, form factors become simple

$$H_{T,A,L} \sim \left|\xi_\Lambda(n_+ p')\right|^2$$

$\Lambda_b \to \Lambda \ell^+ \ell^-$ Decay in SQET

- $\xi_\Lambda(n_+ p')$ is single universal "soft" form factor as a function of the light-cone momentum projection
- Light-Cone Sum Rules are worked out in SQET to NLO [Wang & Shen, JHEP (2016)]
- Numerical results (CDF and LHCb data)





Conclusions

- *B*-meson wave-function is known quite well
- Still some missing issues exist for experimental and theoretical study, especially in the B_s-meson field
- Study of bottom baryons is a more complicated task
- A lot of problems are under intensive investigations
- Heavy multiquark states are really a hot topic and attractive for both experimentalists and teoretitians