

Photon-meson transitions and axial anomaly

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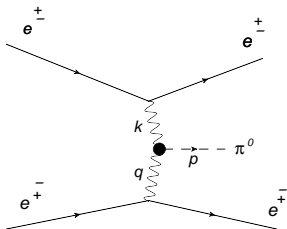
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Outline

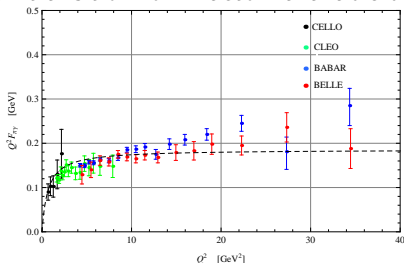
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Transition form factors



π^0 TFF: theoretical and experimental status

Pion transition form factor: available data



- The current experimental status of the pion transition form factor (TFF) $F_{\pi\gamma}$ is rather controversial.
- The measurements of the BABAR collaboration [Aubert et al. '09] show a steady rise of $Q^2 F_{\pi\gamma}$, surpassing the pQCD predicted asymptote $Q^2 F_{\pi\gamma} \rightarrow \sqrt{2} f_\pi$, $f_\pi = 130.7$ MeV at $Q^2 \simeq 10$ GeV² and questioning the collinear factorization.

Axial anomaly: real and virtual photons

- Axial anomaly determines the $\pi^0 \rightarrow \gamma\gamma$ decay width: a unique example of a low-energy process, precisely predicted from QCD.
- The dispersive approach to axial anomaly leads to the *anomaly sum rule* (ASR) providing a handy tool to study the meson transition form factors – $M \rightarrow \gamma\gamma^*$ (even beyond the factorization hypothesis).

$$\int_{4m^2}^{\infty} A_3(s, Q^2; m^2) ds = \frac{1}{2\pi} N_c C^{(a)} \quad (1)$$

- Holds for any Q^2 and any m^2 .
- It has neither α_s corrections (Adler-Bardeen theorem) nor non-perturbative corrections (t'Hooft's consistency principle).
- Exact nonperturbative relation – powerful tool.

Axial anomaly

In QCD, for a given flavor q , the divergence of the axial current $J_{\mu 5}^{(q)} = \bar{q} \gamma_\mu \gamma_5 q$ acquires both electromagnetic and gluonic anomalous terms:

$$\partial_\mu J_{\mu 5}^{(q)} = m_q \bar{q} \gamma_5 q + \frac{e^2}{8\pi^2} e_q^2 N_c F \tilde{F} + \frac{\alpha_s}{4\pi} N_c G \tilde{G}, \quad (2)$$

An octet of axial currents

$$J_{\mu 5}^{(a)} = \sum_q \bar{q} \gamma_5 \gamma_\mu \frac{\lambda^a}{\sqrt{2}} q$$

Singlet axial current $J_{\mu 5}^{(0)} = \frac{1}{\sqrt{3}} (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d + \bar{s} \gamma_\mu \gamma_5 s)$:

$$\partial^\mu J_{\mu 5}^{(0)} = \frac{1}{\sqrt{3}} (m_u \bar{u} \gamma_5 u + m_d \bar{d} \gamma_5 d + m_s \bar{s} \gamma_5 s) + \frac{\alpha_{em}}{2\pi} C^{(0)} N_c F \tilde{F} + \frac{\sqrt{3} \alpha_s}{4\pi} N_c G \tilde{G}, \quad (3)$$

The diagonal components of the octet of axial currents

$$J_{\mu 5}^{(3)} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d),$$

$$J_{\mu 5}^{(8)} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)$$

acquire an electromagnetic anomalous term only:

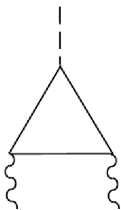
$$\partial^\mu J_{\mu 5}^{(3)} = \frac{1}{\sqrt{2}}(m_u \bar{u}\gamma_5 u - m_d \bar{d}\gamma_5 d) + \frac{\alpha_{em}}{2\pi} C^{(3)} N_c F \tilde{F}, \quad (4)$$

$$\partial^\mu J_{\mu 5}^{(8)} = \frac{1}{\sqrt{6}}(m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d - 2m_s \bar{s}\gamma_5 s) + \frac{\alpha_{em}}{2\pi} C^{(8)} N_c F \tilde{F}. \quad (5)$$

The electromagnetic charge factors $C^{(a)}$ are

$$\begin{aligned} C^{(3)} &= \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}}, \\ C^{(8)} &= \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \frac{1}{3\sqrt{6}}, \\ C^{(0)} &= \frac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2) = \frac{2}{3\sqrt{3}}. \end{aligned} \quad (6)$$

Anomaly sum rule



The matrix element for the transition of the axial current $J_{\alpha 5}$ with momentum $p = k + q$ into two real or virtual photons with momenta k and q is:

$$T_{\alpha\mu\nu}(k, q) = \int d^4x d^4y e^{(ikx+iqy)} \langle 0 | T \{ J_{\alpha 5}(0) J_{\mu}(x) J_{\nu}(y) \} | 0 \rangle; \quad (7)$$

Kinematics:

$$k^2 = 0, Q^2 = -q^2$$

The VVA triangle graph amplitude can be presented as a tensor decomposition

$$\begin{aligned}
 T_{\alpha\mu\nu}(k, q) &= F_1 \varepsilon_{\alpha\mu\nu\rho} k^\rho + F_2 \varepsilon_{\alpha\mu\nu\rho} q^\rho \\
 &+ F_3 k_\nu \varepsilon_{\alpha\mu\rho\sigma} k^\rho q^\sigma + F_4 q_\nu \varepsilon_{\alpha\mu\rho\sigma} k^\rho q^\sigma \\
 &+ F_5 k_\mu \varepsilon_{\alpha\nu\rho\sigma} k^\rho q^\sigma + F_6 q_\mu \varepsilon_{\alpha\nu\rho\sigma} k^\rho q^\sigma,
 \end{aligned} \tag{8}$$

$$F_j = F_j(p^2, k^2, q^2; m^2), \quad p = k + q.$$

Dispersive approach to axial anomaly leads to [\[Hořejší, Teryaev'95\]](#):

$$\int_{4m^2}^{\infty} A_3(s, Q^2; m^2) ds = \frac{1}{2\pi} N_c C^{(a)}, \tag{9}$$

$$A_3 \equiv \frac{1}{2} \text{Im}(F_3 - F_6), \quad N_c = 3;$$

$$\begin{aligned}
 C^{(3)} &= \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}}, \\
 C^{(8)} &= \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \frac{1}{3\sqrt{6}}, \\
 C^{(0)} &= \frac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2) = \frac{2}{3\sqrt{3}}.
 \end{aligned} \tag{10}$$

ASR and meson contributions

Saturating the l.h.s. of the 3-point correlation function (7) with the resonances and singling out their contributions to ASR (1) we get the (infinite) sum of resonances with appropriate quantum numbers:

$$\pi \sum f_M^a F_{M\gamma} = \int_{4m^2}^{\infty} A_3(s, Q^2; m^2) ds = \frac{1}{2\pi} N_c C^{(a)}, \quad (11)$$

where the coupling (decay) constants f_M^a :

$$\langle 0 | J_{\alpha 5}^{(a)}(0) | M(p) \rangle = ip_{\alpha} f_M^a, \quad (12)$$

and form factors $F_{M\gamma}$ of the transitions $\gamma\gamma^* \rightarrow M$ are:

$$\int d^4x e^{ikx} \langle M(p) | T \{ J_{\mu}(x) J_{\nu}(0) \} | 0 \rangle = \epsilon_{\mu\nu\rho\sigma} k^{\rho} q^{\sigma} F_{M\gamma} \quad (13)$$

- Sum of finite number of resonances decreasing $F_{M\gamma}^{\text{asympt}}(Q^2) \propto \frac{f_M}{Q^2}$ - infinite number of states are needed to saturate ASR (collective effect). [Y.K., A.Oganesian, O.Teryaev, PLB 695 (2011) 130]

Isovector channel: π^0

$$J_{\mu 5}^{(3)} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d), C^{(3)} = \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}}.$$

- π^0 + higher contributions ("continuum"):

$$\pi f_\pi F_{\pi\gamma}(Q^2) + \int_{s_0}^{\infty} A_3(s, Q^2) = \frac{1}{2\pi} N_c C^{(3)}. \quad (14)$$

The spectral density $A_3(s, Q^2)$ can be calculated from VVA triangle diagram:

$$A_3(s, Q^2) = \frac{1}{2\sqrt{2}\pi} \frac{Q^2}{(Q^2 + s)^2}. \quad (15)$$

The pion TFF:

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0 + Q^2}. \quad (16)$$

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0 + Q^2} \quad (17)$$

The limit $Q^2 \rightarrow \infty$ + pQCD prediction $Q^2 F_{\pi\gamma} = \sqrt{2} f_\pi$ gives

$$s_0 = 4\pi^2 f_\pi^2 = 0.67 \text{ GeV}^2$$

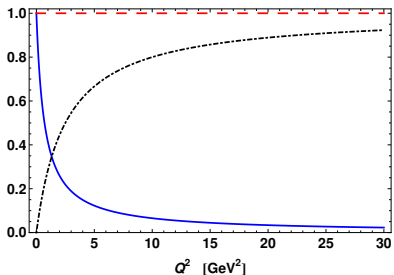
– fits perfectly the value extracted from SVZ (two-point) QCD sum rules

$s_0 = 0.7 \text{ GeV}^2$ [Shifman, Vainshtein, Zakharov'79].

– reproduces BL interpolation formula [Brodsky, Lepage'81]:

$$F_{\pi\gamma}^{\text{BL}}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{1}{1 + Q^2 / (4\pi^2 f_\pi^2)}. \quad (18)$$

Corrections interplay



- The full integral is exact

$$\frac{1}{2\pi} = \int_0^\infty A_3(s, Q^2) ds = I_\pi + I_{cont}$$

- The continuum contribution $I_{cont} = \int_{s_0}^\infty A_3(s, Q^2) ds$ may have perturbative as well as power corrections.
- $\delta I_\pi = -\delta I_{cont}$: small relative correction to continuum – due to exactness of ASR – **must** be compensated by large relative correction to the pion contribution!

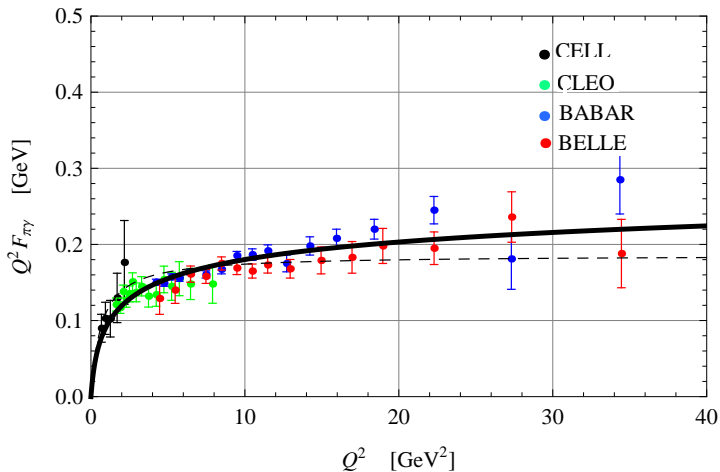
Possible corrections to A_3

- Perturbative two-loop corrections to spectral density A_3 are zero
[Jegerlehner&Tarasov'06]
- Nonperturbative corrections to A_3 are possible: vacuum condensates, instantons, short strings.
- General requirements for the correction $\delta I = \int_{s_0}^{\infty} \delta A_3(s, Q^2) ds$:
 $\delta I = 0$
 - at $s_0 \rightarrow \infty$ (the continuum contribution vanishes),
 - at $s_0 \rightarrow 0$ (the full integral has no corrections),
 - at $Q^2 \rightarrow \infty$ (the perturbative theory works at large Q^2),
 - at $Q^2 \rightarrow 0$ (anomaly perfectly describes pion decay width).

$$\delta I = \frac{1}{2\sqrt{2}\pi} \frac{\lambda s_0 Q^2}{(s_0 + Q^2)^2} \left(\ln \frac{Q^2}{s_0} + \sigma \right), \quad (19)$$

$$\delta F_{\pi\gamma} = \frac{1}{\pi f_\pi} \delta I_\pi = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{\lambda s_0 Q^2}{(s_0 + Q^2)^2} \left(\ln \frac{Q^2}{s_0} + \sigma \right). \quad (20)$$

Correction vs. experimental data



CELLO+CLEO+BABAR: $\lambda = 0.14$, $\sigma = -2.43$, $\chi^2/n.d.f. = 1.08$

Octet channel (η, η')

$$J_{\alpha 5}^{(8)} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\alpha\gamma_5 u + \bar{d}\gamma_\alpha\gamma_5 d - 2\bar{s}\gamma_\alpha\gamma_5 s),$$

$$\int_{4m^2}^{\infty} A_3(s, Q^2; m^2) ds = \frac{1}{2\pi} N_c C^{(8)}, \quad (21)$$

$$C^{(8)} \equiv \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \frac{1}{3\sqrt{6}}$$

ASR in the octet channel:

$$f_\eta^8 F_{\eta\gamma}(Q^2) + f_{\eta'}^8 F_{\eta'\gamma}(Q^2) = \frac{1}{2\sqrt{6}\pi^2} \frac{s_0}{s_0 + Q^2}. \quad (22)$$

- Significant mixing.
- η' decays into two real photons, so it should be taken into account explicitly along with η meson.

Large Q^2

$$Q^2 F_{\eta\gamma}^{as} = 2(C^{(8)} f_{\eta}^8 + C^{(0)} f_{\eta}^0) \int_0^1 \frac{\phi^{as}(x)}{x} dx, \quad (23)$$

$$Q^2 F_{\eta'\gamma}^{as} = 2(C^{(8)} f_{\eta'}^8 + C^{(0)} f_{\eta'}^0) \int_0^1 \frac{\phi^{as}(x)}{x} dx, \quad (24)$$

$\phi^{as}(x) = 6x(1-x)$. Then the ASR at $Q^2 \rightarrow \infty$:

$$4\pi^2((f_{\eta}^8)^2 + (f_{\eta'}^8)^2 + 2\sqrt{2}[f_{\eta}^8 f_{\eta'}^0 + f_{\eta'}^8 f_{\eta}^0]) = s_0 \quad (25)$$

$$Q^2 = 0$$

The ASR takes the form:

$$f_\eta^8 F_{\eta\gamma}(0) + f_{\eta'}^8 F_{\eta'\gamma}(0) = \frac{1}{2\sqrt{6}\pi^2}, \quad (26)$$

where

$$F_{M\gamma}(0) = \sqrt{\frac{4\Gamma_{M\rightarrow\gamma\gamma}}{\pi\alpha^2 m_M^3}}.$$

Additional constraint - $R_{J/\psi}$.

The radiative decays $J/\psi \rightarrow \eta(\eta')\gamma$ are dominated by non-perturbative gluonic matrix elements, and the ratio of the decay rates

$R_{J/\psi} = (\Gamma(J/\psi) \rightarrow \eta'\gamma)/(\Gamma(J/\psi) \rightarrow \eta\gamma)$ can be expressed as follows

[Novikov'79]:

$$R_{J/\psi} = \left| \frac{\langle 0 | G\tilde{G} | \eta' \rangle}{\langle 0 | G\tilde{G} | \eta \rangle} \right|^2 \left(\frac{p_{\eta'}}{p_{\eta}} \right)^3, \quad (27)$$

where $p_{\eta(\eta')} = M_{J/\psi}(1 - m_{\eta(\eta')}^2/M_{J/\psi}^2)/2$.

$$\partial_{\mu} J_{\mu 5}^8 = \frac{1}{\sqrt{6}}(m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d - 2m_s \bar{s}\gamma_5 s), \quad (28)$$

$$\partial_{\mu} J_{\mu 5}^0 = \frac{1}{\sqrt{3}}(m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d + m_s \bar{s}\gamma_5 s) + \frac{1}{2\sqrt{3}} \frac{3\alpha_s}{4\pi} G\tilde{G}. \quad (29)$$

$$R_{J/\psi} = \left(\frac{f_{\eta'}^8 + \sqrt{2}f_{\eta'}^0}{f_{\eta}^8 + \sqrt{2}f_{\eta}^0} \right)^2 \left(\frac{m_{\eta'}}{m_{\eta}} \right)^4 \left(\frac{p_{\eta'}}{p_{\eta}} \right)^3. \quad (30)$$

From experiment this ratio is: $R_{J/\psi} = 4.67 \pm 0.15$ [PDG 2015].

Mixing

Octet-singlet basis (of currents):

$$J_{\mu 5}^{(8)} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_{\mu}\gamma_5 u + \bar{d}\gamma_{\mu}\gamma_5 d - 2\bar{s}\gamma_{\mu}\gamma_5 s), \quad J_{\mu 5}^{(0)} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_{\mu}\gamma_5 u + \bar{d}\gamma_{\mu}\gamma_5 d + \bar{s}\gamma_{\mu}\gamma_5 s). \quad (31)$$

$$\langle 0 | J_{\alpha 5}^{(a)}(0) | M(p) \rangle = ip_{\alpha} f_M^a, \quad (32)$$

Matrix of decay constants (32)

$$\mathbf{F} = \begin{pmatrix} f_{\eta}^8 & f_{\eta'}^8 \\ f_{\eta}^0 & f_{\eta'}^0 \end{pmatrix} \quad (33)$$

- Octet-singlet ($SU(3)$) mixing scheme: $f_{\eta}^8 f_{\eta}^0 + f_{\eta'}^8 f_{\eta'}^0 = 0$.

$$\mathbf{F} = \begin{pmatrix} f_8 \cos \theta & f_8 \sin \theta \\ -f_0 \sin \theta & f_0 \cos \theta \end{pmatrix}. \quad (34)$$

For quark-flavour basis one explores the definitions of axial currents with decoupled light and strange quark composition:

$$J_{\mu 5}^q = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\alpha\gamma_5 u + \bar{d}\gamma_\alpha\gamma_5 d), \quad J_{\mu 5}^s = \bar{s}\gamma_\alpha\gamma_5 s, \quad (35)$$

$$\begin{pmatrix} J_{\mu 5}^8 \\ J_{\mu 5}^0 \end{pmatrix} = \mathbf{V}(\alpha) \begin{pmatrix} J_{\mu 5}^q \\ J_{\mu 5}^s \end{pmatrix}, \quad \mathbf{V}(\alpha) = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}, \quad (36)$$

where $\tan \alpha = \sqrt{2}$.

- *Quark-flavour mixing scheme:* [\[Feldmann,Kroll,Stech'97\]](#)

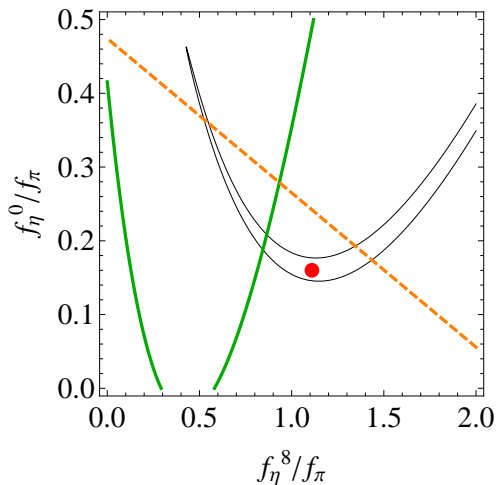
$$f_\eta^q f_\eta^s + f_{\eta'}^q f_{\eta'}^s = 0.$$

$$\mathbf{F}_{qs} = \begin{pmatrix} f_q \cos \phi & f_q \sin \phi \\ -f_s \sin \phi & f_s \cos \phi \end{pmatrix}. \quad (37)$$

$$4\pi^2((f_\eta^8)^2 + (f_{\eta'}^8)^2 + 2\sqrt{2}[f_\eta^8 f_\eta^0 + f_{\eta'}^8 f_{\eta'}^0]) = s_0 \quad (38)$$

$$f_\eta^8 F_{\eta\gamma}(0) + f_{\eta'}^8 F_{\eta'\gamma}(0) = \frac{1}{2\sqrt{6}\pi^2} \frac{s_0}{s_0 + Q^2} \quad (39)$$

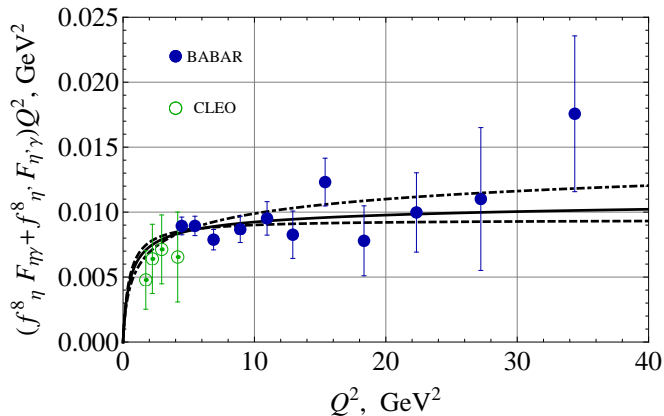
$$R_{J/\psi} = \left(\frac{f_{\eta'}^8 + \sqrt{2}f_{\eta'}^0}{f_\eta^8 + \sqrt{2}f_\eta^0} \right)^2 \left(\frac{m_{\eta'}}{m_\eta} \right)^4 \left(\frac{p_{\eta'}}{p_\eta} \right)^3. \quad (40)$$



Black curve: $\chi^2/n.d.f. = 1$,
 green curve: $f_\eta^8 f_\eta^0 + f_{\eta'}^8 f_{\eta'}^0 = 0$,
 orange curve: $f_\eta^q f_\eta^s + f_{\eta'}^q f_{\eta'}^s = 0$.

Octet-siglet scheme: mixing parameters

$$\begin{pmatrix} f_{\eta}^8 & f_{\eta'}^8 \\ f_{\eta}^0 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} 1.11 & -0.42 \\ 0.16 & 1.04 \end{pmatrix} f_{\pi}. \quad (41)$$

ASR in octet channel (space-like region, $Q^2 > 0$ ($q^2 < 0$))

η, η' TFFs

Add the hypothesis: the TFFs of the state $|q\rangle \equiv \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle)$ is related to the pion form factor as $F_{q\gamma}(Q^2) = (5/3)F_{\pi\gamma}(Q^2)$ (numerical factor comes from the quark charges $(e_u^2 + e_d^2)/(e_u^2 - e_d^2) = 5/3$). QF mixing scheme:

$$|q\rangle = \cos\phi|\eta\rangle + \sin\phi|\eta'\rangle, |s\rangle = -\sin\phi|\eta\rangle + \sin\phi|\eta'\rangle. \quad (42)$$

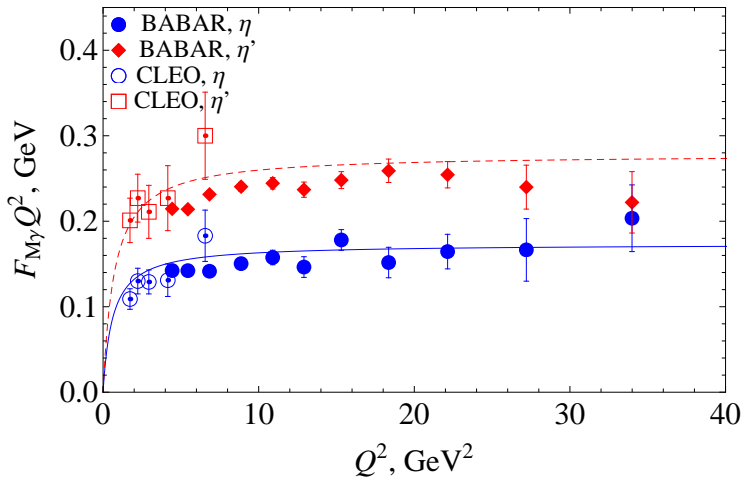
Then one can relate the form factors:

$$\frac{5}{3}F_{\pi\gamma} = F_{\eta\gamma} \cos\phi + F_{\eta'\gamma} \sin\phi. \quad (43)$$

$$F_{\eta\gamma}(Q^2) = \frac{5}{12\pi^2 f_s f_\pi} \frac{s_0^{(3)}(\sqrt{2}f_s \cos\phi - f_q \sin\phi)}{s_0^{(3)} + Q^2} + \frac{1}{4\pi^2 f_s} \frac{s_0^{(8)} \sin\phi}{s_0^{(8)} + Q^2}, \quad (44)$$

$$F_{\eta'\gamma}(Q^2) = \frac{5}{12\pi^2 f_s f_\pi} \frac{s_0^{(3)}(\sqrt{2}f_s \sin\phi + f_q \cos\phi)}{s_0^{(3)} + Q^2} - \frac{1}{4\pi^2 f_s} \frac{s_0^{(8)} \cos\phi}{s_0^{(8)} + Q^2}, \quad (45)$$

where $s_0^{(3)} = 4\pi^2 f_\pi^2$, $s_0^{(8)} = (4/3)\pi^2(5f_s^2 - 2f_\pi^2)$.

η, η' TFF in the space-like region ($Q^2 > 0$ ($q^2 < 0$))

Time-like region: $q^2 > 0 (Q^2 < 0)$ and VMD

The ASR for time-like q^2 is given by the double dispersive integral:

$$\int_0^\infty ds \int_0^\infty dy \frac{\rho^{(a)}(s, y)}{y - q^2 + i\epsilon} = N_c C^{(a)}, \quad a = 3, 8. \quad (46)$$

The real and imaginary parts of the ASR read:

$$p.v. \int_0^\infty ds \int_0^\infty dy \frac{\rho^{(a)}(s, y)}{y - q^2} = N_c C^{(a)}, \quad (47)$$

$$\int_0^\infty ds \rho^{(a)}(s, q^2) = 0, \quad a = 3, 8. \quad (48)$$

$$ReF_{\pi\gamma}(q^2) = \frac{N_c C^{(3)}}{2\pi^2 f_\pi} \left[p.v. \int_0^{s_3} ds \int_0^\infty dy \frac{\rho^{(a)}(s, y)}{y - q^2} \right] = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0 - q^2}.$$

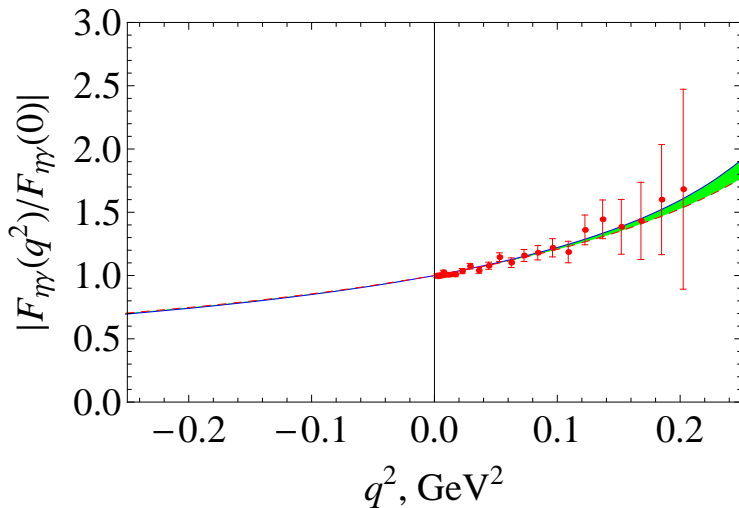
The TFF in the time-like region at $q^2 = s_3$ has a pole, which is numerically close to the ρ meson mass squared, $m_\rho^2 \simeq 0.59 \text{ GeV}^2$ —

VMD model.

$$F_{\eta\gamma}(q^2) = \frac{5}{12\pi^2 f_s f_\pi} \frac{s_3(\sqrt{2}f_s \cos \phi - f_q \sin \phi)}{s_3 - q^2} + \frac{1}{4\pi^2 f_s} \frac{s_8 \sin \phi}{s_8 - q^2}, \quad (49)$$

$$F_{\eta'\gamma}(q^2) = \frac{5}{12\pi^2 f_s f_\pi} \frac{s_3(\sqrt{2}f_s \sin \phi + f_q \cos \phi)}{s_3 - q^2} - \frac{1}{4\pi^2 f_s} \frac{s_8 \cos \phi}{s_8 - q^2}, \quad (50)$$

$$s_3 = 4\pi^2 f_\pi^2, \quad s_8 = (4/3)\pi^2(5f_q^2 - 2f_s^2).$$

η TFF in the time-like region vs. data

Summary

- Meson TFFs are unique quantities which link (seemingly different) important ideas: axial anomaly, mixing and VMD model.
- The ASR in the isovector channel gives the anomaly-based ground for the BL interpolation formula for the pion TFF in the space-like region, and for the VMD model in the time-like region.
- In order to describe the BABAR data on pion TFF, ASR requires a new nonperturbative correction to the spectral density. At the same time, the BELLE data can be described well without such a correction. This correction is absent in the local OPE and possibly originates from instantons or short strings.
- Mixing parameters of η and η' meson can be extracted from TFFs using the ASR.
- ASR in the time-like region of TFFs substantiate the VMD model. Anomaly relates axial and vector channels.

Thank you for your attention!