QCD vacuum as domain wall network

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Plan

1. Effective action
2. Ginzburg-Landau action
3. Domain wall network
4. HIC and vacuum
5. Chromomagnetic trap

Effective action

In Euclidean functional integral for YM theory one has to allow the gluon condensate to be nonzero:

$$Z = N \int_{\mathcal{F}_B} DA \int_{\Psi} D\psi D\bar{\psi} \exp\{-S[A,\psi,\bar{\psi}]\}$$

$$\mathcal{F}_B = \left\{ A : \lim_{V\to\infty} \frac{1}{V} \int_V d^4x g^2 F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) = B^2 \right\}.$$

L. D. Faddeev,
[arXiv:0911.1013 [math-ph]]

B.V. Galilo and S.N. Nedelko,
Phys. Rev. D84 (2011) 094017

H. Leutwyler,
Nucl. Phys. B 179 (1981) 129

Separation of the long range modes B^a_μ and local fluctuations Q^a_μ in the background B^a_μ, background gauge fixing condition $(D(B)Q = 0)$: $A^a_\mu = B^a_\mu + Q^a_\mu$

$$1 = \int_{\mathcal{B}} DB\Phi[A,B] \int_{\mathcal{Q}} DQ \int_{\Omega} D\omega \delta[A^\omega - Q^\omega - B^\omega]\delta[D(B^\omega)Q^\omega]$$

Q^a_μ – local (perturbative) fluctuations of gluon field with zero gluon condensate: $Q \in \mathcal{Q}$;
B^a_μ are long range field configurations with nonzero condensate: $B \in \mathcal{B}$.

$$Z = N' \int_{\mathcal{B}} DB \int_{\mathcal{Q}} DQ \int_{\Psi} D\psi D\bar{\psi} \det[D(B)D(B+Q)]\delta[D(B)Q] \exp\{-S[B+Q,\psi,\bar{\psi}]\}$$

Effective action

The character of long range fields has yet to be identified by the dynamics of fluctuations:

$$Z = N' \int_{\mathcal{B}} DB \int_{\Psi} D\psi D\bar{\psi} \int_{\mathcal{Q}} DQ \det[D(B)D(B+Q)]\delta[D(B)Q] \exp\{-S_{\mathrm{QCD}}[B+Q,\psi,\bar{\psi}]\}$$
$$= \int_{\mathcal{B}} DB \exp\{-S_{\mathrm{eff}}[B]\}$$

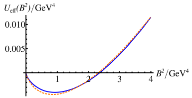
Global minima of $S_{\mathrm{eff}}[B]$ – field configurations that are dominant in the thermodynamic limit $V \to \infty$. Homogeneous Abelian (anti-)self-dual fields are of particular interest.

$$\langle F^2 \rangle : \quad A_\mu = -\frac{1}{2}\breve{n} F_{\mu\nu} x_\nu,\ \tilde{F}_{\mu\nu} = \pm F_{\mu\nu}$$
$$\breve{n} = T^3 \ \cos\xi + T^8 \ \sin\xi.$$

H. Pagels, and E. Tomboulis, Nucl. Phys. B **143** (1978) 485
H. Leutwyler, Nucl. Phys. B 179 (1981) 129

$$G\left(p^2\right) \sim \frac{1}{p^2}\left(1 - e^{-p^2/B_{\mathrm{vac}}}\right)$$

H. Leutwyler, Phys. Lett. B 96 (1980) 154



A. Eichhorn, H. Gies and J. M. Pawlowski, Phys. Rev. D 83, 045014 (2011)

Ginzburg-Landau action

Ginzburg-Landau effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4B^2}\left(D^{ab}_{\nu}F^{b}_{\rho\mu}D^{ac}_{\nu}F^{c}_{\rho\mu} + D^{ab}_{\mu}F^{b}_{\mu\nu}D^{ac}_{\rho}F^{c}_{\rho\nu}\right) - U_{\text{eff}}$$

$$U_{\text{eff}} = \frac{B^4}{12}\mathrm{Tr}\left(C_1F^2 + \frac{4}{3}C_2F^4 - \frac{16}{9}C_3F^6\right),$$

B.V. Galilo, S.N. Nedelko, Phys. Part. Nucl. Lett., 8 (2011) 67

D. P. George, A. Ram, J. E. Thompson and R. R. Volkas, Phys. Rev. D 87, 105009 (2013) [arXiv:1203.1048 [hep-th]]

where

$$D^{ab}_{\mu} = \delta^{ab}\partial_{\mu} - iA^{ab}_{\mu} = \partial_{\mu} - iA^{c}_{\mu}(T^{c})^{ab},$$

$$F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - if^{abc}A^{b}_{\mu}A^{c}_{\nu},$$

$$F_{\mu\nu} = F^{a}_{\mu\nu}T^{a}, \quad T^{a}_{bc} = -if^{abc}$$

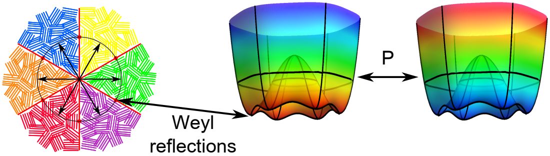
$$C_1 > 0,\ C_2 > 0,\ C_3 > 0.$$

U_{eff} possesses 12 degenerate discrete minima:

$$A_\mu = -\frac{1}{2} n_k F_{\mu\nu} x_\nu,\ \tilde{F}_{\mu\nu} = \pm F_{\mu\nu},$$

where the matrix $\breve{n}_k$ belongs to the Cartan subalgebra of $su(3)$

$$n_k = T^3\ \cos(\xi_k) + T^8\ \sin(\xi_k),$$
$$\xi_k = \frac{2k+1}{6}\pi,\ k = 0, 1, \ldots, 5.$$



Domain wall network

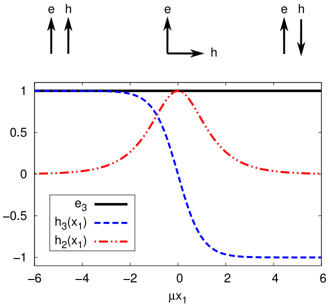
$$\mathcal{L}_{\rm eff} = -\frac{1}{2}\Lambda^2 b_{\rm vac}^2 \partial_\mu\omega\partial_\mu\omega - b_{\rm vac}^4\Lambda^4\left(C_2 + 3C_3 b_{\rm vac}^2\right)\sin^2\omega,$$

leads to sine-Gordon equation

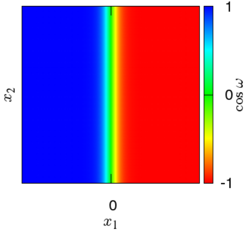
$$\partial^2\omega = m_\omega^2 \sin 2\omega, \quad m_\omega^2 = b_{\rm vac}^2\Lambda^2\left(C_2 + 3C_3 b_{\rm vac}^2\right),$$

and the standard kink solution

$$\omega(x_\nu) = 2\ \mathrm{arctg}\left(\exp(\mu x_\nu)\right), \quad \mu = \sqrt{2}m_\omega.$$

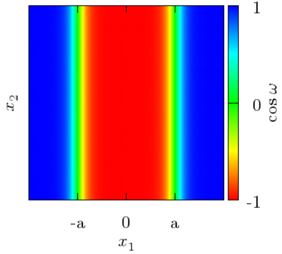


$$q(x) = g^2\tilde{F}F \propto \mathbf{eh} = \cos\omega$$

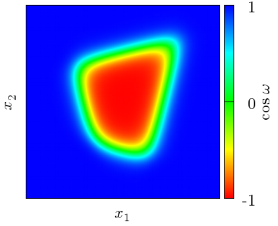


The general kink configuration can be parametrized as

$$\zeta(\mu_i, \eta^i_\nu x_\nu - q^i) = \frac{2}{\pi} \arctan \exp(\mu_i(\eta^i_\nu x_\nu - q^i)).$$



$$\omega(x_1) = \pi\zeta(\mu, -x_1 - a)\zeta(\mu, -x_1 + a)$$



$$\omega(x_1) = \pi \prod_{k=1}^{4} \zeta(\mu, \eta^i_\nu x_\nu - q_i)$$

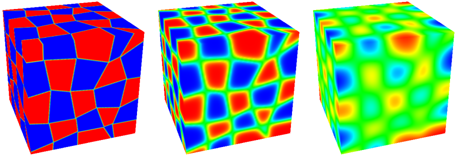
A single lump in two, three and four dimensions is given by

$$\omega(x) = \pi \prod_{i=1}^{k} \zeta(\mu_i, \eta^i_\nu x_\nu - q^i)$$

for $k = 4, 6, 8$, respectively. The general kink network is then given by the additive superposition of lumps

$$\omega = \pi \sum_{j=1}^{\infty} \prod_{i=1}^{k} \zeta(\mu_{ij}, \eta^{ij}_\nu x_\nu - q^{ij})$$

S.N. Nedelko, V.E. Voronin arXiv:1403.0415 [hep-ph]



$\langle F^2 \rangle = B^2$
$\langle |F\widetilde{F}| \rangle = B^2$

$\langle F^2 \rangle = B^2$
$\langle |F\widetilde{F}| \rangle \ll B^2$

Eigenmodes in Abelian (anti)-self-dual field

Eigenvalue problem for scalar field in $\mathbb{R}^4$:

$$-\left(\partial_\mu - i\breve{B}_\mu\right)^2 \Phi = \lambda\Phi,$$

$$B_\mu = B_{\mu\nu}x_\nu, \widetilde{B}_{\mu\nu} = \pm B_{\mu\nu}, B_{\mu\alpha}B_{\nu\alpha} = B^2\delta_{\mu\nu}.$$

It can be rewritten as

$$\left[\beta^+_\pm\beta_\pm + \gamma^+_+\gamma_+ + 1\right]\Phi = \frac{\lambda}{4B}\Phi,$$

$$\beta_\pm = \frac{1}{2}(\alpha_1 \mp i\alpha_2), \quad \gamma_\pm = \frac{1}{2}(\alpha_3 \mp i\alpha_4), \quad \alpha_\mu = \frac{1}{\sqrt{B}}x_\mu + \partial_\mu,$$

$$\beta^+_\pm = \frac{1}{2}(\alpha^+_1 \pm i\alpha^+_2), \quad \gamma^+_\pm = \frac{1}{2}(\alpha^+_3 \pm i\alpha^+_4), \quad \alpha^+_\mu = \frac{1}{\sqrt{B}}x_\mu - \partial_\mu.$$

The eigenfunctions and eigenvalues are

$$\Phi_{nmkl}(x) = \frac{1}{\pi^2\sqrt{n!m!k!l!}}\left(\beta^+_+\right)^k\left(\beta^+_-\right)^l\left(\gamma^+_+\right)^n\left(\gamma^+_-\right)^m\Phi_{0000}, \ \Phi_{0000} = e^{-\frac{1}{2}Bx^2}$$

$$\lambda_r = 4B(r+1), \ r = k+n \ \text{(self-dual field)}, \ r = l+n \ \text{(anti-self-dual field)}$$

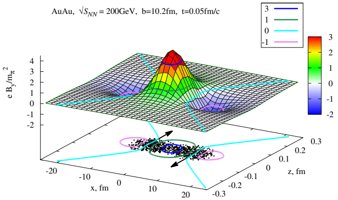
Propagator:

$$D^2(x)G(x,y) = -\delta(x-y), \quad G(x,y) = e^{ix\breve{B}y}H(x-y), \quad \tilde{H}(p^2) = \frac{1-e^{-p^2/B}}{p^2}.$$

Impact of electromagnetic fields on “QCD vacuum“

Strong electromagnetic fields are produced during relativistic heavy ion collisions

V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya, V. P. Konchakovski and S. A. Voloshin, Phys. Rev C 84 (2011)



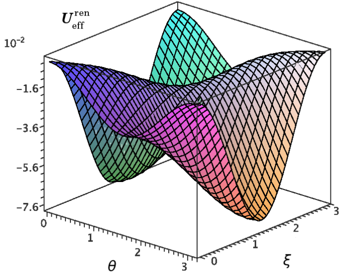
Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies!

B.V. Galilo and S.N. Nedelko, Phys. Rev. D84 (2011) 094017
M. D'Elia, M. Mariti and F. Negro, Phys. Rev. Lett. **110**, 082002 (2013)
G.S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP **1304**, 130 (2013)

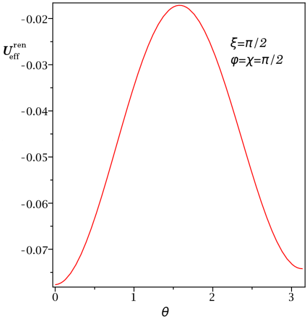
One-loop quark contribution to the effective potential in the presence of arbitrary homogeneous Abelian fields

$$U_{\rm eff} = -\frac{1}{V}\,\mathrm{Tr}\ln\frac{i\not{D}-m}{i\not{\partial}-m}$$

Effective potential (in units of $B^2/8\pi^2$)

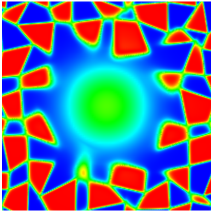


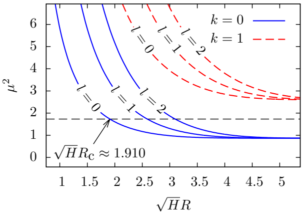
Effective potential as a function of angles θ and ξ for the pure magnetic field $H = 0.9B$ and $\phi = \chi$. The minimum is at $\theta = 0,\ \xi = \pi/2$.



Effective potential for $H = 0.9B,\ E = 0.5B$ and $\phi = \chi = \pi/2$. The minimum is at $\theta = 0,\ \xi = \pi/2$.

Chromomagnetic trap





$R_{\mathrm{crit}} \approx 0.5$ fm

Eigenvalue problem:

$$\left[-\breve{D}^2\delta_{\mu\nu} + 2i\breve{n}B_{\mu\nu}\right] Q_\nu = \lambda^2 Q_\mu,$$

$$\breve{n}Q_\mu(x) = 0, \quad x \in \mathcal{T} = \left\{x_1^2 + x_2^2 < R^2,\ (x_3, x_4) \in \mathbb{R}^2\right\}$$

Spectrum:

$$\lambda^2_{alk\nu} = p_4^2 + p_3^2 + \mu^2_{alk} + 2s_\nu\kappa_a v,$$
$$k = 0, 1, \ldots, \infty, \quad l \in \mathbb{Z},$$
$$s_1 = 1,\ s_2 = -1,\ s_3 = s_4 = 0,\ \kappa_a = \pm 1.$$

In Minkowski space-time the problem turns to the wave equation

$$-\left(\partial_\mu - i\breve{B}_\mu\right)^2 \phi(x) = 0.$$

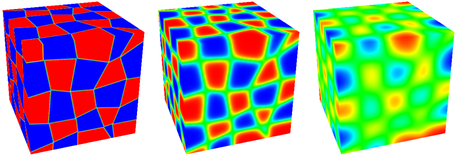
The solutions are

$$\phi^a(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[a^+_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3x_3} + b_{akl}(p_3) e^{-ix_0\omega_{akl} + ip_3x_3}\right] e^{il\vartheta} \phi_{alk}(r),$$

$$\phi^{a\dagger}(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[b^+_{akl}(p_3) e^{-ix_0\omega_{akl} + ip_3x_3} + a_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3x_3}\right] e^{-il\vartheta} \phi_{alk}(r),$$

$$p_0^2 = p_3^2 + \mu_{akl}^2, \quad p_0 = \pm\omega_{akl}(p_3), \quad \omega_{akl} = \sqrt{p_3^2 + \mu_{akl}^2}, \quad \text{quasiparticles}$$

$$k = 0, 1, \ldots, \infty, \quad l \in \mathbb{Z}.$$



$\langle F^2 \rangle = B^2$

$\langle |F\widetilde{F}| \rangle = B^2$

confinement

$\langle F^2 \rangle = B^2$

$\langle |F\widetilde{F}| \rangle \ll B^2$

deconfinement

N.K. Nielsen, P. Olesen, Phys. Lett. **B 79**, 304 (1978).
H.B. Nielsen, P. Olesen. Nucl. Phys. **B 160**, 380 (1979).
J. Ambjørn, N.K. Nielsen, P. Olesen Nucl. Phys. **B 152**, 75 (1979).
J. Ambjørn, P. Olesen Nucl. Phys. **B 170**, 265 (1980).