Search of chiral anomaly in kaon-photon reaction

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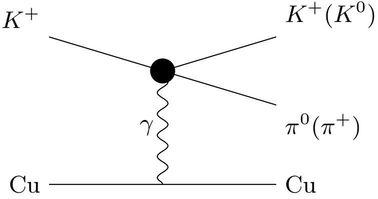
Institute for High-Energy Physics
Protvino (Serpukhov), Russia
OKA Detector

Current experiment
$E_K = 17.7$ GeV.



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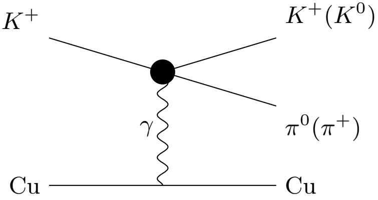
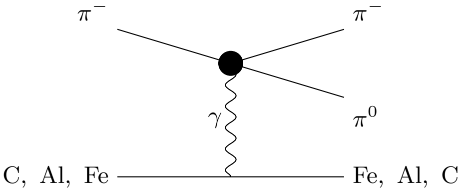
Current experiment
$E_K = 17.7$ GeV.

[Yu. M. Antipov *et. al.*, Phys. Rev. **D36**, 21 (1987)]
$E_\pi = 40$ GeV.





Massless QED Lagrangian:

$$\mathcal{L}_{\mathsf{QED}} = \bar{\psi}(ie\partial_\mu - eA)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

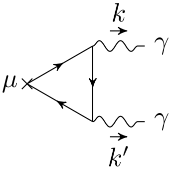
$U(1) \times U(1)$ symmetry transformations:

$$\psi \to \mathrm{e}^{i\alpha}\psi, \qquad \psi \to \mathrm{e}^{i\beta\gamma^5}\psi$$

Noether currents:

$$j^\mu = \bar{\psi}\gamma^\mu\psi, \qquad \partial_\mu j^\mu = 0,$$
$$j_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi, \quad \partial_\mu j_5^\mu = 0.$$

$$\langle\gamma(k), \gamma(k')|\partial_\mu j_5^\mu|0\rangle = \partial_\mu \langle\gamma(k), \gamma(k')|\bar{\psi}\gamma^\mu\gamma^5\psi|0\rangle$$

$= 2\partial_\mu$ 

$$\partial_\mu j_5^\mu = \frac{e^2}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \neq 0$$

QCD: $\pi^0 \to \gamma\gamma$

$$SU(3)_L \times SU(3)_R \to SU(3)_V$$

Goldstone bosons ϕ^a, $a = 1, \ldots, 8$:

$$\Sigma = \mathrm{e}^{\frac{2i}{F_\pi}\phi^a T^a},\ F_\pi = 92.2 \text{ MeV},\ \phi^a T^a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \overline{\pi^+} & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ \overline{K^+} & \overline{K^0} & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

Chiral perturbation theory:

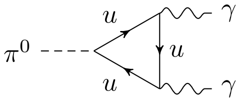
$$\mathcal{L}_{\text{ChPT}} = \frac{F_\pi^2}{8} \operatorname{tr}(D_\mu \Sigma D^\mu \Sigma^\dagger) + \mathcal{L}_{\text{WZ}}$$

$$D_\mu \Sigma = \partial_\mu \Sigma + ie[A_\mu, Q], \quad Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$$

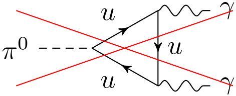
Wess-Zumino term (one of):

$$\mathcal{L}_{\text{WZ}} \supset \frac{1}{2\pi^2 F_\pi^5} \varepsilon^{\mu\nu\rho\sigma} \operatorname{tr}(\Sigma \partial_\mu \Sigma \partial_\nu \Sigma \partial_\rho \Sigma \partial_\sigma \Sigma) \supset \frac{e^2}{8\pi^2 F_\pi^5} \varepsilon^{\mu\nu\rho\sigma} \pi^0 F_{\mu\nu} F_{\rho\sigma}$$

$$\mathcal{L}_{\mathsf{ChPT}} = \frac{F_\pi^2}{8}\,\mathrm{tr}(D_\mu \Sigma D^\mu \Sigma^\dagger + \chi \Sigma^\dagger + \Sigma \chi^\dagger) + \mathcal{L}_{\mathsf{WZ}}$$



$$\mathcal{L}_{\text{ChPT}} = \frac{F_\pi^2}{8}\,\mathrm{tr}(D_\mu \Sigma D^\mu \Sigma^\dagger + \chi\Sigma^\dagger + \Sigma\chi^\dagger) + \mathcal{L}_{\text{WZ}}$$



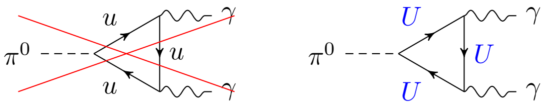
Wrong:

- No quarks in ChPT.
- No mesons in QCD.

$$\Delta\mathcal{L}_{\text{QCD}} = \bar{Q} i \not{D} Q + \frac{1}{\Lambda^2} (\bar{q}_R Q_L)(\bar{Q}_R q_L) + \text{h.c.},$$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \ Q = \begin{pmatrix} U \\ D \\ S \end{pmatrix}, \ \Lambda \to \infty$$

$$\mathcal{L}_{\text{ChPT}} = \frac{F_\pi^2}{8} \operatorname{tr}(D_\mu \Sigma D^\mu \Sigma^\dagger + \chi \Sigma^\dagger + \Sigma \chi^\dagger) + \bar{Q} i \not{D} Q$$
$$+ M \bar{Q}_R \Sigma Q_L + \text{h.c.} + \mathcal{L}_{\text{WZ}}, \quad M \to \infty$$



Wrong:

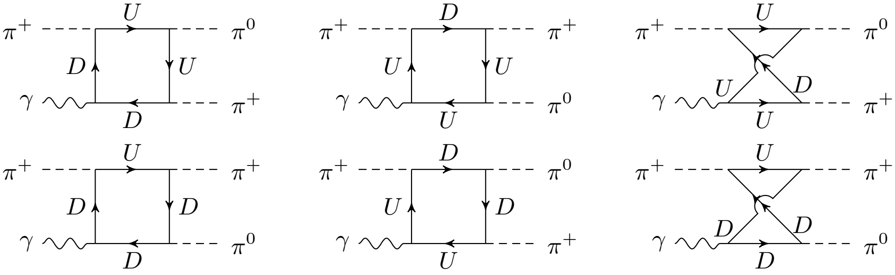
- No quarks in ChPT.
- No mesons in QCD.

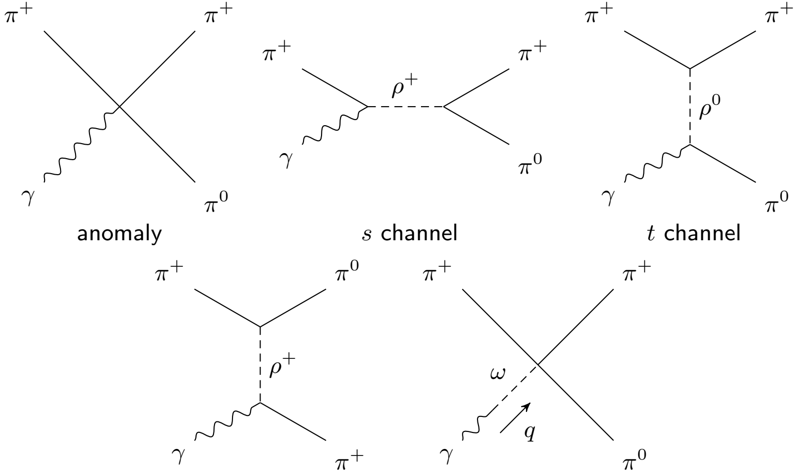
[H. Georgi, *"Weak Interactions"*, Ch. 6a.3]

$$\pi^+\gamma \to \pi^+\pi^0$$

Another Wess-Zumino term:

$$-\frac{ie}{4\pi^2 F_\pi^3}\varepsilon^{\mu\alpha\beta\gamma}A_\mu \partial_\alpha\pi^-\partial_\beta\pi^+\partial_\gamma\pi^0$$





anomaly, s channel, t channel, u channel, γ–ω conversion

$$A(\pi^+\gamma \to \pi^+\pi^0) = h(s,t,u)\cdot\varepsilon^{\mu\alpha\beta\gamma}A_\mu\partial_\alpha\pi^-\partial_\beta\pi^+\partial_\gamma\pi^0$$

$$h(s,t,u) = h(0)\left\{1 + \frac{2f_{\rho\pi\pi}f_{\rho\pi\gamma}}{m_\rho^2 h(0)}\left[\frac{s}{m_\rho^2 - s} + \frac{t}{m_\rho^2 - t} + \frac{u}{m_\rho^2 - u}\right] + \frac{f_{\omega\gamma}f_{\omega3\pi}}{m_\omega^2 h(0)}\frac{q^2}{m_\omega^2 - q^2}\right\}$$

$$h(0) = \frac{e}{4\pi^2F_\pi^3}$$

[Terent'ev, Phys. Lett. **38B**, 419 (1972)]

$$A(\pi^-\gamma \to \pi^-\pi^0) = h(s,t,u)\cdot \varepsilon^{\mu\alpha\beta\gamma} A_\mu \partial_\alpha \pi^- \partial_\beta \pi^+ \partial_\gamma \pi^0$$

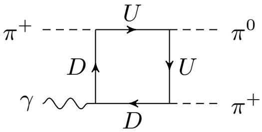
$$h(s,t,u) = h(0)\left\{1 + \frac{2 f_{\rho\pi\pi} f_{\rho\pi\gamma}}{m_\rho^2 h(0)}\left[\frac{s}{m_\rho^2 - s} + \frac{t}{m_\rho^2 - t} + \frac{u}{m_\rho^2 - u}\right] + \frac{f_{\omega\gamma} f_{\omega 3\pi}}{m_\omega^2 h(0)} \frac{q^2}{m_\omega^2 - q^2}\right\}$$

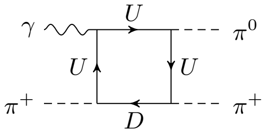
$h(0)$ values

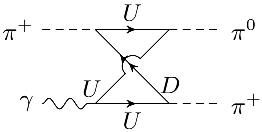
Theory	$\frac{e}{4\pi^2 F_\pi^3} = 9.8$ GeV^{-3}
Experiment at LO (1987)	12.9 ± 0.9 (stat.) $\pm\, 0.5$ (syst.) $\pm\, 1.0$ (sign) GeV^{-3}
Experiment at NNLO + EMC (2001)	10.7 ± 1.2 GeV^{-3}

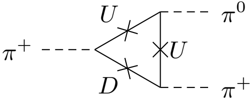
Update from the COMPASS Collaboration?

$\pi^+\gamma \to \pi^+\pi^0$



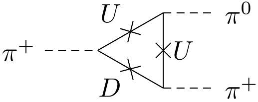




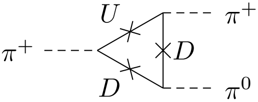


$$-\frac{1}{3} + \frac{2}{3} + \frac{2}{3} = 1$$

$\pi^+\gamma \to \pi^+\pi^0$

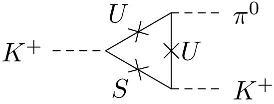


$-\frac{1}{3} + \frac{2}{3} + \frac{2}{3} = 1$



$-\frac{1}{3} + \frac{2}{3} - \frac{1}{3} = 0$

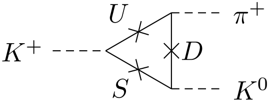
$K^+\gamma \to K^+\pi^0$



$-\frac{1}{3} + \frac{2}{3} + \frac{2}{3} = 1$

neutral pion production

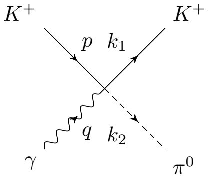
$K^+\gamma \to K^0\pi^+$



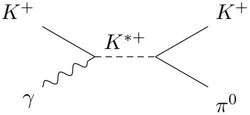
$-\frac{1}{3} + \frac{2}{3} - \frac{1}{3} = 0$

charged pion production

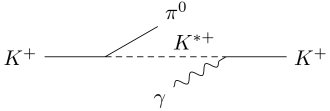
$K^+\gamma \to K^+\pi^0$



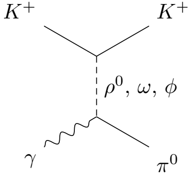
anomaly



s channel

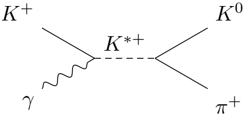


u channel

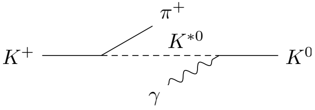


t channels

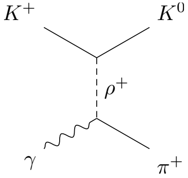
$K^+\gamma \to K^0\pi^+$



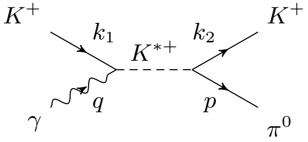
s channel



u channel



t channel



s-channel amplitude:

$$A_s^{(0)}(K^+\gamma \to K^+\pi^0) = -\frac{2f_{K^{*+}K^+\gamma} f_{K^{*+}K^+\pi^0}}{s - m_{K^{*+}}^2 + i\sqrt{s}\,\Gamma_{K^{*+}}(s)} \varepsilon^{\alpha\beta\gamma\delta}\epsilon_\alpha p_\beta k_{1\gamma} k_{2\delta}$$

$$A_s(K^+\gamma \to K^+\pi^0) = A_s^{(0)}(K^+\gamma \to K^+\pi^0) - A_s^{(0)}(K^+\gamma \to K^+\pi^0)|_{s=0}$$

Cross section:

$$\frac{d\sigma(K^+\gamma \to K^+\pi^0)}{dt} = \frac{1}{2^7\pi}\left(t + \frac{(st - m_{K^+}^2 m_{\pi^0}^2)(t - m_{\pi^0}^2)}{(s - m_{K^+}^2)^2}\right) \times \left| \frac{e}{4\pi^2 F_\pi^3} + \frac{2f_{K^{*+}K^+\gamma} f_{K^{*+}K^+\pi^0}}{m_{K^{*+}}^2 - s - i\sqrt{s}\,\Gamma_{K^{*+}}(s)} \cdot \frac{s}{m_{K^{*+}}^2} + \frac{2f_{K^{*+}K^+\gamma} f_{K^{*+}K^+\pi^0}}{m_{K^{*+}}^2 - u} \cdot \frac{u}{m_{K^{*+}}^2} + \frac{2f_{\rho^0\pi^0\gamma} f_{\rho^0 K^+K^+}}{m_{\rho^0}^2 - t} \cdot \frac{t}{m_{\rho^0}^2} + \frac{2f_{\omega\pi^0\gamma} f_{\omega K^+K^+}}{m_\omega^2 - t} \cdot \frac{t}{m_\omega^2} + \frac{2f_{\phi\pi^0\gamma} f_{\phi K^+K^+}}{m_\phi^2 - t} \cdot \frac{t}{m_\phi^2} \right|^2$$

$$
\begin{aligned}
f_{K^{*+}K^{+}\pi^{0}} &= 3.10 \\
f_{K^{*+}K^{0}\pi^{+}} &= 4.38 \\
f_{K^{*0}K^{+}\pi^{+}} &= 4.41 \\
f_{\rho^{0}K^{+}K^{+}} &= 3.16 \\
f_{\rho^{+}K^{+}K^{0}} &= -4.47 \\
f_{\omega K^{+}K^{+}} &= 3.16 \\
f_{\phi K^{+}K^{+}} &= -4.47 \\
f_{K^{*+}K^{+}\gamma} &= 0.240\ \text{GeV}^{-1} \\
f_{K^{*0}K^{0}\gamma} &= -0.385\ \text{GeV}^{-1} \\
f_{\rho^{0}\pi^{0}\gamma} &= 0.252\ \text{GeV}^{-1} \\
f_{\rho^{+}\pi^{+}\gamma} &= 0.219\ \text{GeV}^{-1} \\
f_{\omega\pi^{0}\gamma} &= 0.696\ \text{GeV}^{-1} \\
|f_{\phi\pi^{0}\gamma}| &= 0.040\ \text{GeV}^{-1}
\end{aligned}
$$

Decay widths:

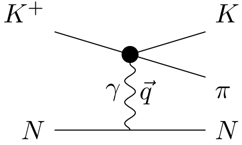
$$
\begin{aligned}
\Gamma(K^{*}\to K\pi) &\Longrightarrow |f_{K^{*}K\pi}| \\
\Gamma(K^{*}\to K\gamma) &\Longrightarrow |f_{K^{*}K\gamma}| \\
\Gamma(\phi\to K^{+}K^{-}) &\Longrightarrow |f_{\phi K^{+}K^{+}}| \\
\Gamma(\rho^{+}\to \pi^{+}\gamma) &\Longrightarrow |f_{\rho^{+}\pi^{+}\gamma}| \\
\Gamma(\rho^{0}\to \pi^{0}\gamma) &\Longrightarrow |f_{\rho^{0}\pi^{0}\gamma}| \\
\Gamma(\omega\to \pi^{0}\gamma) &\Longrightarrow |f_{\omega\pi^{0}\gamma}| \\
\Gamma(\phi\to \pi^{0}\gamma) &\Longrightarrow |f_{\phi\pi^{0}\gamma}|
\end{aligned}
$$

$SU(3)$ symmetry:

$$
\begin{aligned}
\sqrt{2}f_{K^{*+}K^{+}\pi^{0}} &= f_{K^{*+}K^{0}\pi^{+}} = f_{K^{*0}K^{+}\pi^{+}} = -f_{\rho^{+}K^{+}K^{0}} \\
&= \sqrt{2}f_{\rho^{0}K^{+}K^{+}} = \sqrt{2}f_{\omega K^{+}K^{+}} = -f_{\phi K^{+}K^{+}}
\end{aligned}
$$

$$
f_{K^{*+}K^{+}\gamma} = f_{\rho^{+}\pi^{+}\gamma} = f_{\rho^{0}\pi^{0}\gamma} = \frac{1}{3}f_{\omega\pi^{0}\gamma} = -\frac{1}{2}f_{K^{*0}K^{0}\gamma}
$$

The sign of the anomaly term is unknown.



Weizsacker-Williams equivalent photons approximation:

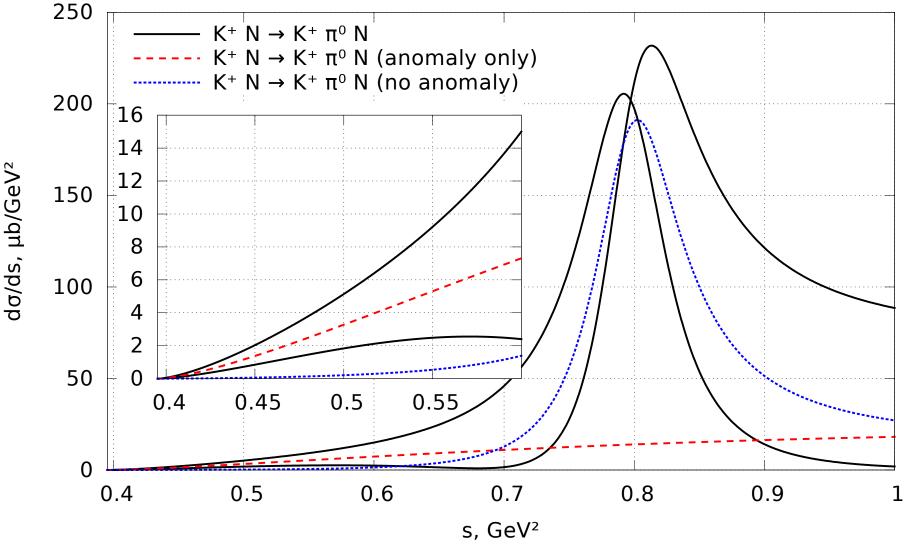
$$\frac{d\sigma(K^+N \to K\pi N)}{dt\ ds\ dq_\perp^2} = \frac{Z^2\alpha}{\pi(s-m_{K^+}^2)} \frac{q_\perp^2}{\left(q_\perp^2 + \left(\frac{s-m_{K^+}^2}{2E_K}\right)^2\right)^2} \frac{d\sigma(K^+\gamma \to K\pi)}{dt} |F(\vec{q}^{\,2})|^2$$

$$F(\vec{q}^{\,2}) = \exp\left(-\frac{\left\langle r^2 \right\rangle \vec{q}^{\,2}}{6}\right)$$

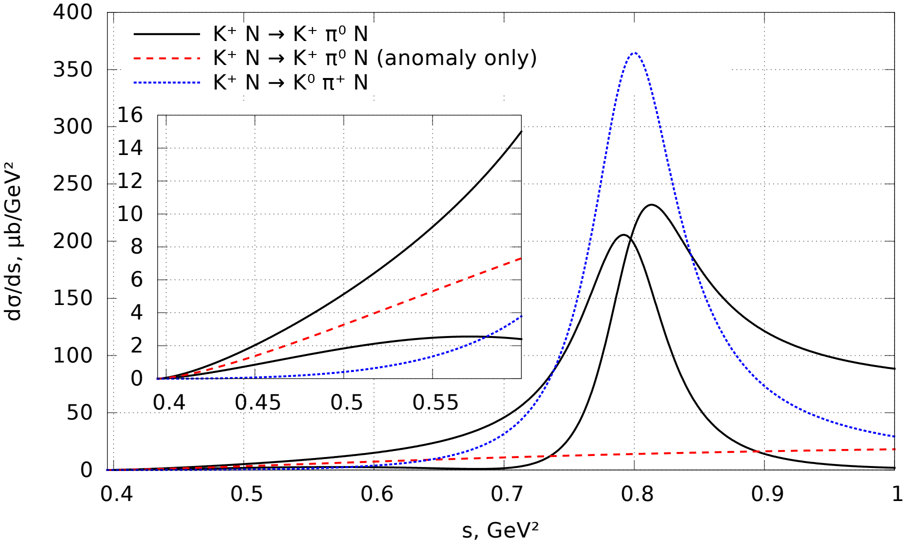
$$\frac{d\sigma(K^+N \to K\pi N)}{dt\ ds} = \frac{Z^2\alpha}{\pi} \frac{E_1(a)-1}{s-m_{K^+}^2} \frac{d\sigma(K^+\gamma \to K\pi)}{dt}$$

$$E_1(a) = \int\limits_a^\infty \frac{\mathrm{e}^{-z}}{z} dz,\ \ a = \frac{1}{3} r_0^2 A^{2/3} \left(\frac{s-m_{K^+}^2}{2E_K}\right)^2$$

[Berestetskiy, Lifshitz, Pitaevsky, *"Quantum Electrodynamics"*, §99]



K⁺ N → K⁺ π⁰ N
K⁺ N → K⁺ π⁰ N (anomaly only)
K⁺ N → K⁺ π⁰ N (no anomaly)
dσ/ds, μb/GeV²
s, GeV²
250
200
150
100
50
0
0.4
0.5
0.6
0.7
0.8
0.9
1
16
14
12
10
8
6
4
2
0
0.4
0.45
0.5
0.55



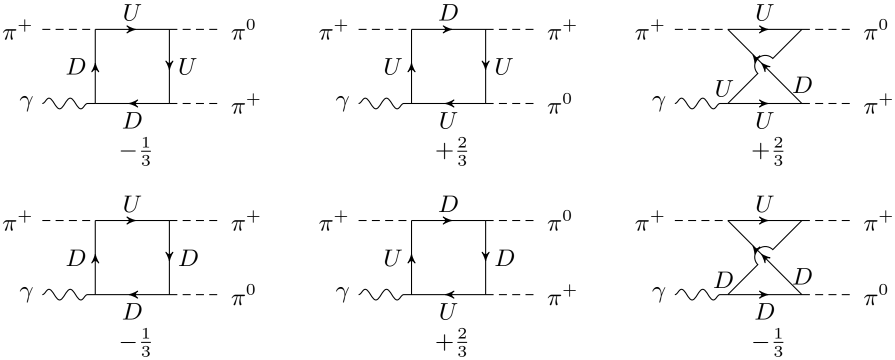
K⁺ N → K⁺ π⁰ N
K⁺ N → K⁺ π⁰ N (anomaly only)
K⁺ N → K⁰ π⁺ N
dσ/ds, μb/GeV²
s, GeV²
0
50
100
150
200
250
300
350
400
0.4
0.5
0.6
0.7
0.8
0.9
1
0
2
4
6
8
10
12
14
16
0.4
0.45
0.5
0.55

Conclusions

- A theoretical prediction has been made for the cross sections of $K^+\gamma \to K^+\pi^0$ and $K^+\gamma \to K^0\pi^+$ reactions at low energies. For the anomalous reaction, we predict two possible values depending on the a priori unknown sign of the interference term, which should be resolved by the experiment.
- It is possible to observe the chiral anomaly through comparison of cross section of $K^+\,\mathrm{Cu} \to K^+\pi^0\,\mathrm{Cu}$ reaction with that of $K^+\,\mathrm{Cu} \to K^0\pi^+\,\mathrm{Cu}$ reaction at $s \lesssim 0.6$ GeV^2. The point is that only the first one has the anomaly which manifests itself as an increase in the cross section at low s.
- Luminosity of $60\ \mu$b^{-1} at $0.4 < s < 0.6$ GeV^2 is planned to be collected in the Protvino experiment. In this case expected observations are ≈ 10 events of $K^0\pi^+$ production and either ≈ 20 or ≈ 70 events of $K^+\pi^0$ production, depending on the sign of the interference term.

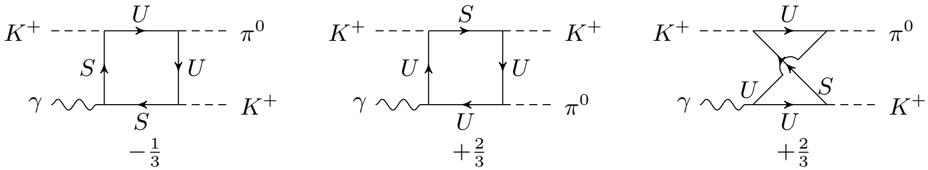
Thank you for your attention!

$$\pi^+\gamma \to \pi^+\pi^0$$



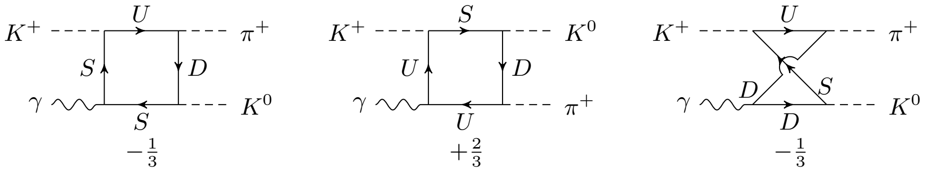
Total: 1

$$K^+\gamma \to K^+\pi^0$$



Total: 1

$$K^+\gamma \to K^0\pi^+$$



Total: 0