Heavy quarkonium production in the Parton Reggeization Approach

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Outline.

- **Introduction**
  - Motivation
  - NRQCD and NRQCD-factorization
  - State of the art
  - Short intorduction to the Parton Reggeization Approach. (See the lecture by V. A. Saleev tomorrow for better review!)
- **Review of the results for p_T-spectra of charmonia and bottomonia**
- $\psi(2S)$ **and** $\Upsilon(3S)$**:** p_T**-spectra and polarisation**
  - $\psi(2S)$ production at “moderate”-p_T, fragmentation
  - $\psi(2S)$ polarization, “polarization puzzle”
  - $\Upsilon(3S)$ polarization

Why to study heavy quarkonium production?

Oportunities:

- *Unique window to hadronization process.* Additional small parameter – heavy quark velocity $v^2 \sim 0.3$ for charmonia, $v^2 \sim 0.1$ for bottomonia.
- Interesting multiscale problem for pQCD. Resummations are required ($\log^2 p_T/M$, $\log p_T/M$, $\log 1/x$, ...).
- Probe for physics of Heavy Ion Collisions
- Signature for Multiple Parton Interactions

But for the last two, the SPS production in $pp(\bar{p})$ should be well understood!

Collinear factorization.

We are interested in the inclusive hard (hard scale $Q^2 \gg \Lambda^2_{QCD}$) processes in the inelastic proton-proton collisions at high energies:

$$p(P_1) + p(P_2) \to Y + X,$$

where $P^2_{1,2} = 0$, $2P_1 P_2 = S$.
Factorization formula of the Collinear Parton Model (CPM):

$$\boxed{d\sigma = \sum_{i,j} \int_0^1 dx_1 f_i(x_1, \mu_F^2) \int_0^1 dx_2 f_j(x_2, \mu_F^2) d\hat{\sigma}_{ij}(x_1, x_2, \mu_F^2, \mu_R^2).}$$

where $d\sigma$– inclusive hadronic cross-section, $d\hat{\sigma}$ – **hard scattering coefficient** or "partonic cross-section", because at LO it is the cross-section of the process:

$$i(q_1) + j(q_2) \to Y$$

where $i, j = q, \bar{q}, g$, $\boxed{q^\mu_{1,2} = x_{1,2} P^\mu_{1,2}} \Rightarrow \boxed{q^2_{1,2} = 0}$. Large logarithmic corrections $\sim \log \mu_F^2/Q^2$ are absorbed into the scale-dependence of ("Integrated") **Parton Distribution Functions** (PDFs) $f_i(x, \mu^2) \Rightarrow$ **DGLAP evolution equations**.

Nonrelativistic QCD

Estimate of heavy quark velocity:

$$\frac{m_Q v^2}{2} \sim \frac{\alpha_s(1/r)}{r},$$

mean radius and velocity are related $r \sim 1/(m_Q v) \Rightarrow$

$$v \sim \alpha_s(m_Q v),$$

Then for $m_Q = 1.5$ GeV $\Rightarrow v^2 \simeq 0.3$; $m_Q = 4.7$ GeV $\Rightarrow v^2 \simeq 0.1$.
NRQCD Lagrangian:[G. T. Bodwin, E. Braaten, G. P. Lepage, Phys. Rev. D**51** (1995) 1125]

$$L_{NRQCD} = L_{Heavy} + L_{Light} + \delta L$$

$$L_{Heavy} = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2m_Q} \right) \psi + \chi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2m_Q} \right) \chi$$

$$L_{Light} = -\frac{1}{2} Tr\left[F_{\mu\nu} F^{\mu\nu}\right] + \sum_f \bar{q}_f \; i\hat{D} \; q_f$$

Where $\hat{D} = \gamma^\mu D_\mu$, $D_\mu = \partial_\mu + ig_s A_\mu$

Nonrelativistic QCD. Velocity scaling.

L_{Heavy} is $O(M^4v^5)$, δL contains higher order corrections in v:

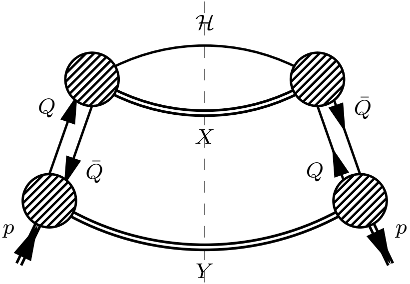
$$\delta L = \frac{c_1(\Lambda)}{8m_Q^3}\left[\psi^\dagger\left(\mathbf{D}\right)^2\psi - \chi^\dagger\left(\mathbf{D}\right)^2\chi\right] + \ldots$$

Velocity scaling rules:

Operator	Scaling
$\psi,\ \chi$	$(m_Q v)^{3/2}$
$\mathbf{D}$	$m_Q v$
$D_t, g_s A^0$	$m_Q v^2$
$g_s\mathbf{A},\ g_s\mathbf{E}/m_Q$	$m_Q v^3$
$g_s\mathbf{B}/m_Q$	$m_Q v^4$

$\langle H|\int d^3\mathbf{x}\psi^\dagger(x)\psi(x)|H\rangle \sim 1,\ V \sim r^3 \sim \frac{1}{(m_Q v)^3} \Rightarrow \psi^\dagger\psi \sim (m_Q v)^3$

NRQCD factorization.



NRQCD-factorization [Bodwin, Braaten, Lepage (1995)]:

$$d\hat{\sigma} = \sum_n d\hat{\sigma}\,[g+g\to n]\cdot\langle 0|\mathcal{O}^{\mathcal{H}}\,[n]\,|0\rangle$$

where $n = Q\bar{Q}\left[{}^{2S+1}L_J^{(1,8)}\right],\ Q\bar{Q}g,\ \dots$

Matrix elements of NRQCD-Operators $\mathcal{O}^{\mathcal{H}}\,[n] = \mathcal{O}_n^\dagger\left(a_{\mathcal{H}}^\dagger a_{\mathcal{H}}\right)\mathcal{O}_n$, **have definite** v**-scaling. LO** for the state $\mathcal{H}\left[{}^{2S+1}L_J\right]\Rightarrow Q\bar{Q}\left[{}^{2S+1}L_J^{(1)}\right]$. **NLO**:

$$Q\bar{Q}\left[{}^{2S+1}L_J^{(8)}\right],\ Q\bar{Q}\left[{}^{2S+1}(L\pm 1)_J^{(8)}\right], Q\bar{Q}\left[{}^{2(S\pm 1)+1}L_J^{(8)}\right].$$

Further support from pQCD-side [Kniehl, Butenshön (2011)]:

$$d\hat{\sigma}_{NLO}\left[{}^3P_J^{(1)}\right]\supset\frac{1}{\epsilon}\cdot d\hat{\sigma}_{LO}\left[{}^3S_1^{(8)}\right],$$

so we need $\left\langle\mathcal{O}^{\mathcal{H}}[{}^3S_1^{(8)}]\right\rangle$ to absorb this singularity.

Long-distance matrix elements (LDMEs, NMEs, ...).

Color-singlet NMEs:

$$\left\langle \mathcal{O}^{\mathcal{H}_J}\left[{}^3S_1^{(1)}\right]\right\rangle = 2N_c(2J+1)\frac{1}{4\pi}|R(0)|^2,$$
$$\left\langle \mathcal{O}^{\mathcal{H}_J}\left[{}^3P_J^{(1)}\right]\right\rangle = 2N_c(2J+1)\frac{3}{4\pi}|R'(0)|^2.$$

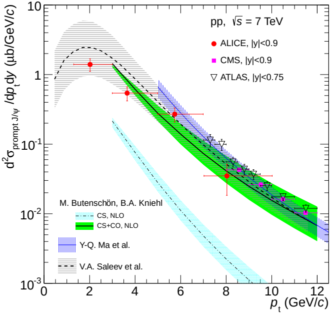
Radial wavefunction $R(0)$ or it's derivative in the origin $R'(0)$ is known from the potential models [Eichten, Quigg (1995)].
Multiplicative relations, proven in LO in v^2:

$$\left\langle \mathcal{O}^{\mathcal{H}}[{}^3P_J^{(1,8)}]\right\rangle = (2J+1)\left\langle \mathcal{O}^{\mathcal{H}}[{}^3P_0^{(1,8)}]\right\rangle,$$
$$\left\langle \mathcal{O}^{\mathcal{H}_J}[{}^3S_1^{(8)}]\right\rangle = (2J+1)\left\langle \mathcal{O}^{\mathcal{H}}[{}^3S_1^{(8)}]\right\rangle,$$

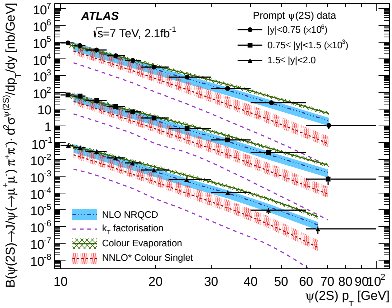
Color-octet NMEs may be obtained using nonperturbative techniques or by a fit.

State of the art. Fixed order.

Prompt J/ψ. Plot from [ALICE Collaboration, hep-ph/1205.5880]



Prompt $\psi(2S)$. Plot from [ATLAS Collaboration, hep-ph/1407.5532]



- Full NLO corrections in CPM are availible [B. A. Kniehl, M. Butenschoen, 2009-2011]. A few complete NLO fits of NMEs exist [B. A. Kniehl, M. Butenschoen; Y. Q. Ma et al.].
- NLO corrections *(2 additional partons+1-loop)* to the p_T-spectrum are large ($K > 2$), incomplete NNLO calculations (*3 additional partons+ IR cutoff*) [J. Landsberg et al., 2009] suggests that NNLO corrections to Color-Singlet channel are significant.

Multiscale process.

- Large NLO corrections to the p_T-spectrum in the Collinear Parton Model.
- $p_T < M$: Large-$(\alpha_s \log^2 p_T/M)^n$- corrections at n-th order. "Sudakov double-logs" [Dokshitzer, Diakonov, Troyan, 1978] regularize the power-divergence of p_T-spectrum at small p_T.

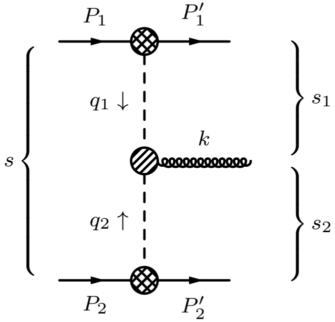
$$\frac{d\sigma}{dp_T^2} \sim \frac{1}{p_T^2} \exp\left[-N_c \frac{\alpha_s}{2\pi} \log^2 \left(\frac{p_T^2}{M^2} \right) \right] = \frac{1}{p_T^2} \left(\frac{p_T^2}{M^2} \right)^{-N_c \frac{\alpha_s}{2\pi} \log(p_T^2/M^2)}$$

- $p_T \gg M$: Large-$(\alpha_s \log p_T/M)^n$ corrections al n-th order. "Fragmentation logs". Can be resummed by DGLAP evolution of parton$\rightarrow$ hadron FF.
- $x \ll 1$ – small-x physics effects may be important ($\log 1/x$): BFKL-evolution, saturation of parton densities, Color-glass condensate e.t.c. (see e. g. [Y.-Q. Ma, R. Venugopalan, 2014])

$\Rightarrow$ **Fixed-order pQCD should work at some "moderate"** p_T.

Multi-Regge Kinematics.

At high energies, t-channel excange diagrams with Multi-Regge(MRK) or Quasi-Multi Regge(QMRK) Kinematics of the final-state dominate in the $2 \to 2 + n$ amplitude.



Sudakov (light-cone) decomposition:

$$k^\mu = \frac{1}{2}(k^+ n_-^\mu + k^- n_+^\mu) + k_T^\mu,$$

where $n_\pm^2 = 0$, $n_+ n_- = 2$, $n_\pm k_T = 0$, $k_pm = n_\pm k$. Rapidity $y = \log(k^+/k^-)/2$.
Double Regge limit (MRK):

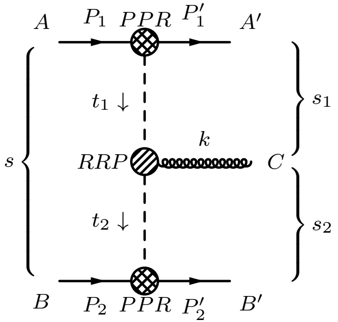
$$s_1 \gg -q_1^2,\; s_2 \gg -q_2^2,$$

momentum fractions $z_1 = q_1^+/P_1^+$, $z_2 = q_2^-/P_2^-$.
Properties of MRK:

- $y(P_1') \to +\infty$, $y(P_2') \to -\infty$, $y(k)$ – finite,
- $z_1 \sim z_2 \sim z \ll 1$, $|\mathbf{k}_T| \ll \sqrt{s}$ ("Small-z physics"),
- $q_1^+ \sim |\mathbf{q}_{T1}| \sim O(z) \gg q_1^- \sim O(z^2)$,
  $q_2^- \sim |\mathbf{q}_{T2}| \sim O(z) \gg q_2^+ \sim O(z^2)$.

Reggeization of amplitudes in QCD.

At high energies, t-channel excange diagrams with Multi-Regge(MRK) or Quasi-Multi Regge(QMRK) Kinematics of the final-state dominate in the $2 \to 2 + n$ amplitude.



In MRK asymptotics, $2 \to 3$-amplitude factorizes:

$$\mathcal{A}_{AB}^{A'B'C} = \gamma_{A'A}^{R_1} \cdot \left(\frac{s_1}{s_0}\right)^{\omega(t_1)} \frac{-i}{2t_1} \times$$
$$\Gamma_{R_1 R_2}^{C}(q_1, q_2) \cdot \frac{-i}{2t_2} \left(\frac{s_2}{s_0}\right)^{\omega(t_2)} \cdot \gamma_{B'B}^{R_2}$$

$\Gamma_{R_1 R_2}^{C}(q_1, q_2)$ – RRP production vertex,

$\gamma_{A'A}^{R}$ – PPR-scattering vertex,

$\omega(t)$ - Regge trajectory.

Two ways to obtain this asymptotics:

- BFKL-approach (Unitarity, renormalizability and gauge invariance), see. [Ioffe, Fadin, Lipatov, 2010].
- Effective action approach [Lipatov, 1995].

PRA. Factorization, Feynman rules.

Factorization formula [Gribov *et. al.* 1983; Collins *et. al.* 1991; Catani *et. al.* 1991]:

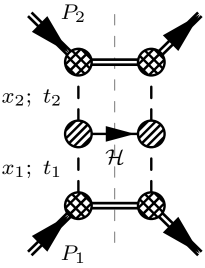
$$d\sigma = \int \frac{d^2\mathbf{q}_{T1}}{\pi} \int \frac{dx_1}{x_1} \Phi(x_1, t_1, \mu_F) \int \frac{d^2\mathbf{q}_{T2}}{\pi} \int \frac{dx_2}{x_2} \Phi(x_2, t_2, \mu_F) d\hat{\sigma}_{PRA}(q_1, q_2),$$

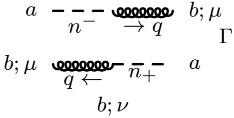
where $q^{\mu}_{1,2} = x_{1,2} P^{\mu}_{1,2} + q^{2}_{T1,2}$ $\Rightarrow$ $q^2_{1,2} = -\mathbf{q}^2_{T1,2} = -t_{1,2},$

and $\Phi(x, t, \mu^2)$ - **unintegrated (TMD) PDFs.**

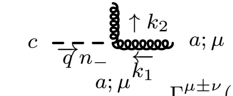
Feynman rules:

Factorization:

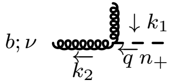




$$\Gamma^{\pm\mu}_{ab}(q) = i\delta^{ab} q^2 n^{\mu}_{\pm}$$



$$\Gamma^{\mu\pm\nu}_{acb}(k_1, q, k_2) = -g_s f^{abc} \frac{q^2}{k_1^{\pm}} n^{\mu}_{\pm} n^{\nu}_{\pm}$$



The Kimber-Martin-Ryskin unPDF.

The LO unPDF [Kimber, Martin, Ryskin, Watt, 2000]:

$$\Phi_i(x,t,\mu^2) = \frac{1}{t}\int\limits_0^{1-\Delta} dz\ T_i(t,\mu^2)\frac{\alpha_s(t)}{2\pi}P_{ij}(z)\frac{x}{z}f_j\left(\frac{x}{z},t\right),$$

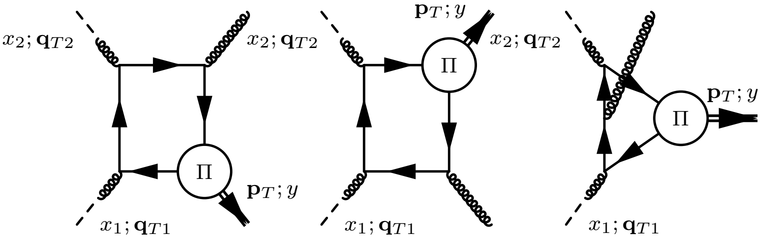
where $f_i(x,\mu^2)$ – collinear PDF and the Sudakov formfactor:

$$T_i(t,\mu^2) = \exp\left[-\sum_j \int\limits_t^{\mu^2}\frac{dk_T^2}{k_T^2}\frac{\alpha_s(k_T^2)}{2\pi}\int\limits_0^{1-\Delta} dz'\ z'\cdot P_{ij}(z')\right],$$

resums the Sudakov double-logs in LLA. The IR divergence is regularized by the KMR cutoff $\Delta = \frac{\sqrt{t}}{\mu+\sqrt{t}}$, which follows from the **rapidity ordering** between the last emission and the hard subprocess. The KMR unPDF has the normalization property:

$$\int\limits_0^{\mu^2} dt\Phi_i(x,t,\mu^2) = xf_i(x,\mu^2).$$

$R+R \to Q\bar{Q}\left[{}^3S_1^{(1)}\right] + g$ Amplitude.



Normalization factor ($\mathbf{q}_{1T}^2 = t_1,\ \mathbf{q}_{2T}^2 = t_2$):

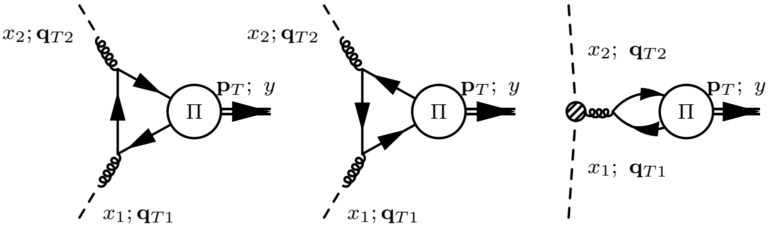
$$\mathcal{N} = \frac{(x_1 x_2 S)^2}{16 t_1 t_2}$$

Collinear limit:

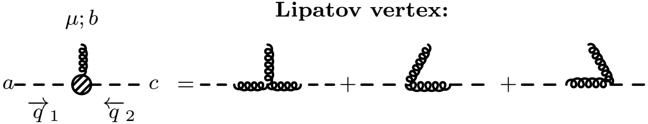
$$\lim_{t_{1,2}\to 0} \int_0^{2\pi} \frac{d\phi_1 d\phi_2}{(2\pi)^2} \left|\mathcal{A}_{PRA}\right|^2 = \overline{\left|\mathcal{A}_{CPM}\right|^2}$$

$2 \to 1$ Amplitudes.

Processes $R + R \to Q\bar{Q}\left[{}^3S_1^{(8)}, {}^1S_0^{(8)}, {}^3P_J^{(1,8)}\right]$ [Kniehl, Saleev, Vasin (2006)].



Lipatov vertex:



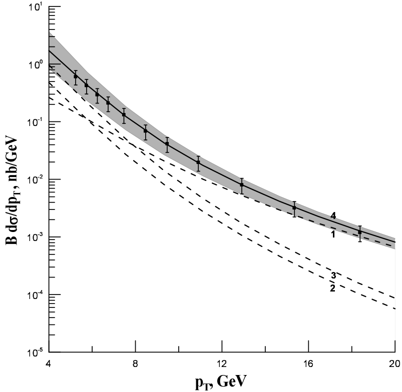
$$\Gamma^{-\mu+}_{abc}(q_1, q_2) = 2g_s f^{abc}\left[n^\mu_-\left(q_1^+ + \frac{q_1^2}{q_2^-}\right) - n^\mu_+\left(q_2^- + \frac{q_2^2}{q_1^+}\right) + (q_2 - q_1)^\mu\right]$$

Charmonium production. From Tevatron to LHC.

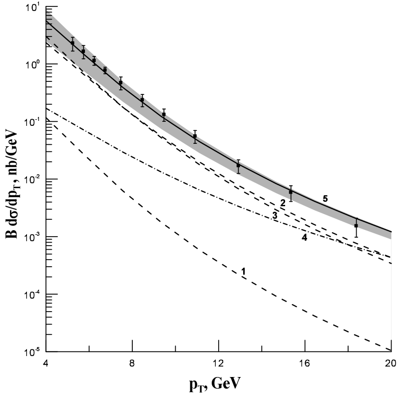
[V. A. Saleev, M. A. Nefedov, A. V. Shipilova, Phys. Rev. D**85** (2012) 074013]
NMEs for J/ψ, ψ', and χ_{cJ} mesons from fits of the CDF data in the NLO collinear parton model [B. A. Kniehl, M. Butenshoen] and in the parton Reggeization approach using the Blümlein, and KMR unintegrated gluon distribution functions.

NME	PM NLO	Fit B	Fit KMR
$\langle\mathcal{O}^{J/\psi}[^3S_1^{(1)}]\rangle/\text{GeV}^3$	1.3	1.3	1.3
$\langle\mathcal{O}^{J/\psi}[^3S_1^{(8)}]\rangle/\text{GeV}^3$	$(1.68 \pm 0.46) \times 10^{-3}$	$(1.89 \pm 0.27) \times 10^{-3}$	$(2.23 \pm 0.27) \times 10^{-3}$
$\langle\mathcal{O}^{J/\psi}[^1S_0^{(8)}]\rangle/\text{GeV}^3$	$(3.04 \pm 0.35) \times 10^{-2}$	$(1.80 \pm 0.25) \times 10^{-2}$	$(1.84 \pm 0.19) \times 10^{-2}$
$\langle\mathcal{O}^{J/\psi}[^3P_0^{(8)}]\rangle/\text{GeV}^5$	$(-9.08 \pm 1.61) \times 10^{-3}$	0	0
χ^2/d.o.f	—	1.0	1.0
$\langle\mathcal{O}^{\psi'}[^3S_1^{(1)}]\rangle/\text{GeV}^3$	6.5×10^{-1}	6.5×10^{-1}	6.5×10^{-1}
$\langle\mathcal{O}^{\psi'}[^3S_1^{(8)}]\rangle/\text{GeV}^3$	$(1.88 \pm 0.62) \times 10^{-3}$	$(6.72 \pm 1.15) \times 10^{-4}$	$(9.33 \pm 1.62) \times 10^{-4}$
$\langle\mathcal{O}^{\psi'}[^1S_0^{(8)}]\rangle/\text{GeV}^3$	$(7.01 \pm 4.75) \times 10^{-3}$	$(3.63 \pm 1.40) \times 10^{-3}$	$(3.27 \pm 1.44) \times 10^{-3}$
$\langle\mathcal{O}^{\psi'}[^3P_0^{(8)}]\rangle/\text{GeV}^5$	$(-2.08 \pm 2.28) \times 10^{-3}$	0	0
χ^2/d.o.f	—	0.033	0.051
$\langle\mathcal{O}^{\chi_{c0}}[^3P_0^{(1)}]\rangle/\text{GeV}^5$	8.9×10^{-2}	8.9×10^{-2}	8.9×10^{-2}
$\langle\mathcal{O}^{\chi_{c0}}[^3S_1^{(8)}]\rangle/\text{GeV}^3$	—	$(2.14 \pm 0.67) \times 10^{-4}$	$(1.69 \pm 0.9) \times 10^{-4}$
χ^2/d.o.f	—	0.89	0.41

$\psi(2S)$ and χ_{cJ} production at the Tevatron.

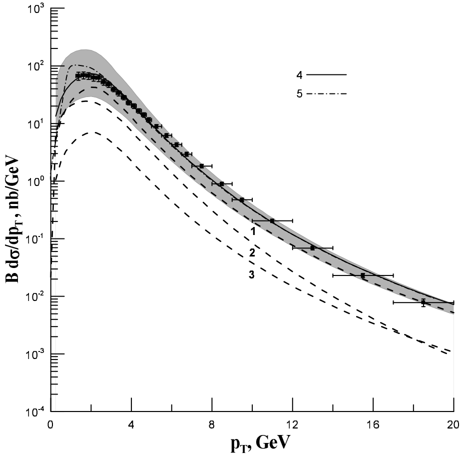


J/ψ transverse-momentum spectrum from ψ' decays from CDF Collaboration, $\sqrt{S} = 1.8$ TeV, $|\eta| < 0.6$, (1) – ${}^3S_1^{(8)}$ contribution, (2) – ${}^3S_1^{(1)}$, (3) – ${}^1S_0^{(8)}$, (4) sum of all contributions.



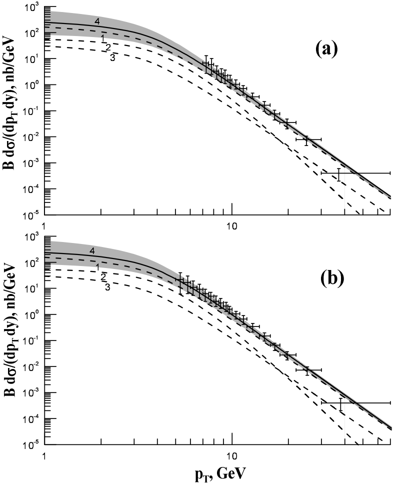
J/ψ transverse-momentum spectrum from χ_{cJ} decays from CDF Collaboration, $\sqrt{S} = 1.8$ TeV, $|\eta| < 0.6$, (1) – ${}^3P_0^{(1)}$, (2) – ${}^3P_1^{(1)}$, (3) – ${}^3P_2^{(1)}$, (4) – ${}^3S_1^{(8)}$, (5) sum of all contributions.

Prompt J/ψ production at the Tevatron.

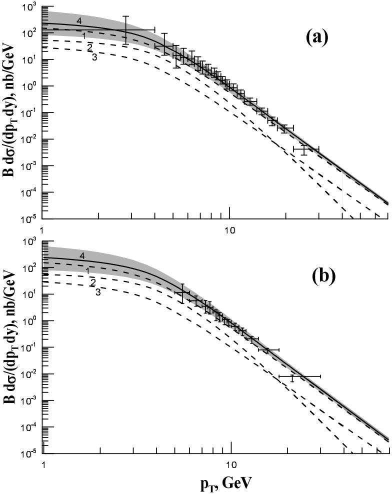


Prompt J/ψ transverse-momentum spectrum from CDF Collaboration, $\sqrt{S} = 1.96$ TeV, $|\eta| < 0.6$, (1) is the direct production, (2) from χ_{cJ} decays, (3) from ψ' decays, (4) sum of all contributions (KMR unPDF), (5) sum of all contributions (Blümlein unPDF).

Prompt J/ψ production at the LHC (ATLAS). $\sqrt{S} = 7$ TeV

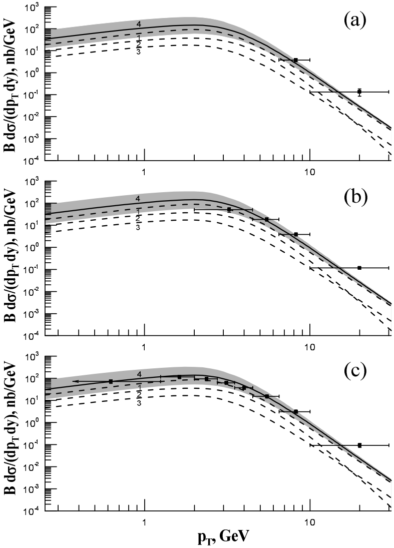


(a) $|y| < 0.75$, (b) $0.75 < |y| < 1.5$



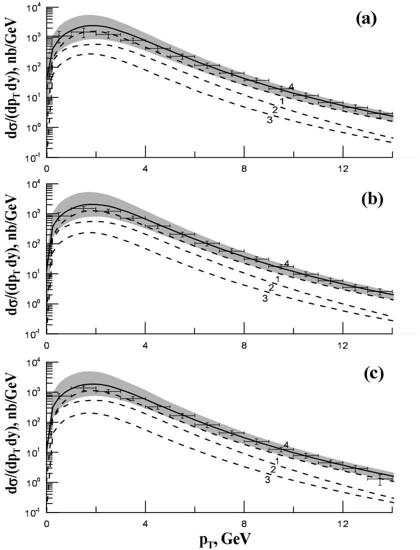
(a) $1.5 < |y| < 2.0$, (b) $2.0 < |y| < 2.4$

Prompt J/ψ production at the LHC (CMS). $\sqrt{S} = 7$ TeV

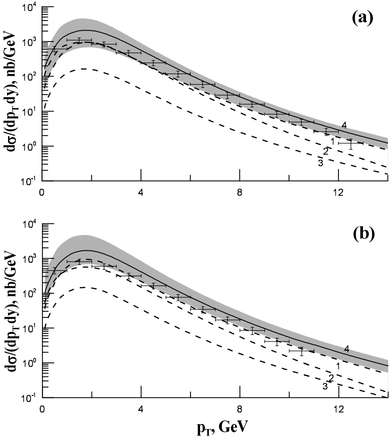


(a) $|y| < 1.2$, (b) $1.2 < |y| < 1.6$, (c) $1.6 < |y| < 2.4$

Prompt J/ψ production at the LHC (LHCb). $\sqrt{S} = 7$ TeV



(a) $2.0 < |y| < 2.5$, (b) $2.5 < |y| < 3.0$,
(c) $3.0 < |y| < 3.5$



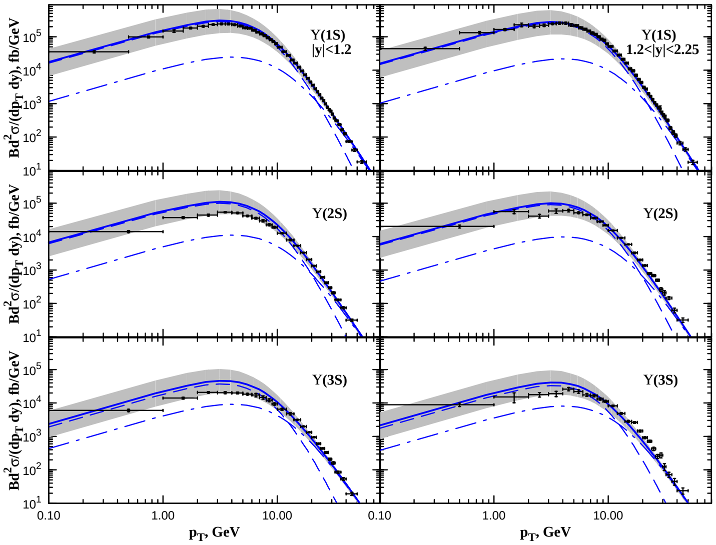
(a) $3.5 < |y| < 4$, (b) $4.0 < |y| < 4.5$

Bottomonium production at the LHC and Tevatron.

For the details see [M. A. Nefedov, V. A. Saleev, A. V. Shipilova, Phys. Rev. D88 (2013) 014003]

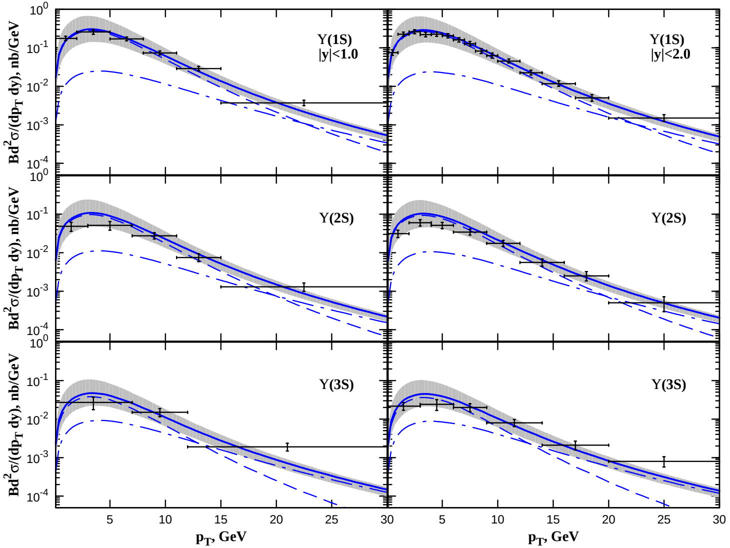
NME	Fit LO PRA
$\left\langle \mathcal{O}^{\Upsilon(1S)}\left[{}^3S_1^{(1)}\right]\right\rangle \times$ GeV^{-3}	9.28
$\left\langle \mathcal{O}^{\Upsilon(1S)}\left[{}^3S_1^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	2.31 ± 0.25
$\left\langle \mathcal{O}^{\Upsilon(1S)}\left[{}^1S_0^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	0.0 ± 0.05
$\left\langle \mathcal{O}^{\Upsilon(1S)}\left[{}^3P_0^{(8)}\right]\right\rangle \times 10^2$ GeV^{-5}	0.0 ± 0.38
$\left\langle \mathcal{O}^{\Upsilon(2S)}\left[{}^3S_1^{(1)}\right]\right\rangle \times$ GeV^{-3}	4.62
$\left\langle \mathcal{O}^{\Upsilon(2S)}\left[{}^3S_1^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	1.51 ± 0.17
$\left\langle \mathcal{O}^{\Upsilon(2S)}\left[{}^1S_0^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	0.0 ± 0.01
$\left\langle \mathcal{O}^{\Upsilon(2S)}\left[{}^3P_0^{(8)}\right]\right\rangle \times 10^2$ GeV^{-5}	0.0 ± 0.03
$\left\langle \mathcal{O}^{\Upsilon(3S)}\left[{}^3S_1^{(1)}\right]\right\rangle \times$ GeV^{-3}	3.54
$\left\langle \mathcal{O}^{\Upsilon(3S)}\left[{}^3S_1^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	1.24 ± 0.13
$\left\langle \mathcal{O}^{\Upsilon(3S)}\left[{}^1S_0^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	0.0 ± 0.01
$\left\langle \mathcal{O}^{\Upsilon(3S)}\left[{}^3P_0^{(8)}\right]\right\rangle \times 10^2$ GeV^{-5}	0.0 ± 0.02
$\left\langle \mathcal{O}^{\chi(1P)}\left[{}^3P_0^{(1)}\right]\right\rangle \times$ GeV^{-5}	2.03
$\left\langle \mathcal{O}^{\chi(1P)}\left[{}^3S_1^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	0.0
$\left\langle \mathcal{O}^{\chi(2P)}\left[{}^3P_0^{(1)}\right]\right\rangle \times$ GeV^{-5}	2.36
$\left\langle \mathcal{O}^{\chi(2P)}\left[{}^3S_1^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	0.0

Inclusive $\Upsilon(nS)$ production at the LHC (ATLAS). $\sqrt{S} = 7$ TeV.

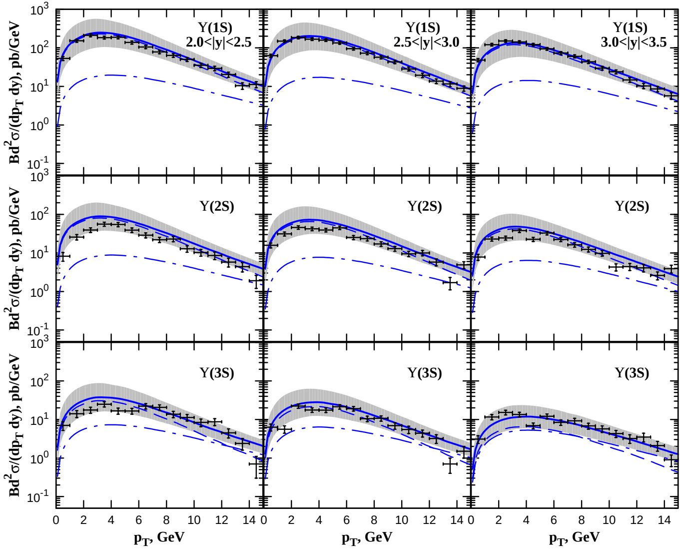


Dashed line – color-singlet contribution, dash-dotted line – color-octet contribution, solid line – sum of all contributions.

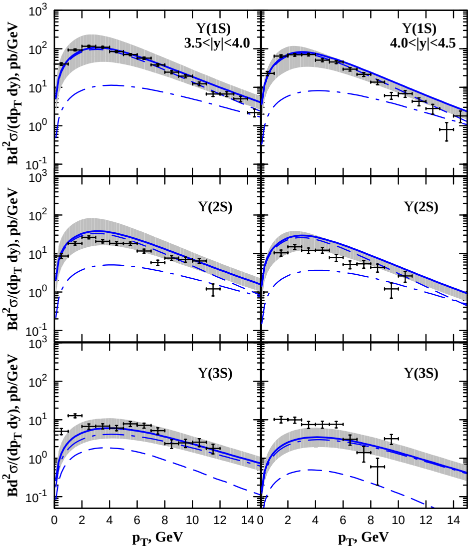
Inclusive $\Upsilon(nS)$ production at the LHC (CMS). $\sqrt{S} = 7$ TeV.

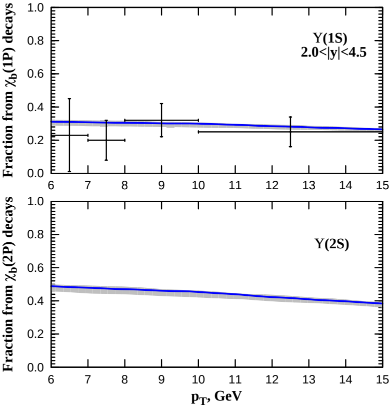


Inclusive $\Upsilon(nS)$ production at the LHC (LHCb). $\sqrt{S} = 7$ TeV.

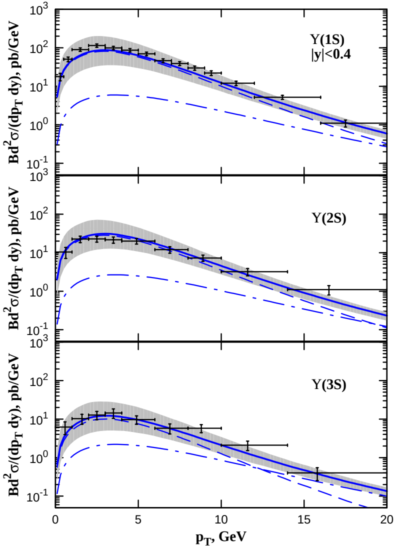


Inclusive $\Upsilon(nS)$ production at the LHC (LHCb). $\sqrt{S} = 7$ TeV.





Comparation with Tevatron data (CDF). $\sqrt{S} = 1.8$ TeV.



Motivation for the new study of $\psi(2S)$ and $\Upsilon(3S)$.

Recently, we have published the new study of p_T-spectra and polarization observables of $\psi(2S)$ and $\Upsilon(3S)$ [B. A. Kniehl, M. A. Nefedov, V. A. Saleev, hep-ph/1606.01079]

Motivation:

1. To consistently describe all data on $p_T(\psi(2S))$ and $p_T(J/\psi)$-spectra in $\psi(2S) \to J/\psi + X$-decays it is **absolutely necessary** to:
   - take into account the mass difference between states
     $$m_c = M(\psi(2S))/2,$$
   - take into account the momentum shift in the decay
     $$\langle p_T(J/\psi)\rangle = \frac{M_{J/\psi}}{M_{\psi(2S)}} p_T(\psi(2S)) + O\left(\frac{\Delta M^2}{M^2}, \frac{M}{p_T}\right).$$

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Motivation:

1. To consistently describe all data on $p_T(\psi(2S))$ and $p_T(J/\psi)$-spectra in $\psi(2S) \to J/\psi + X$-decays.
2. To study the onset of fragmentation with new (ATLAS, CMS) data for $p_T \leq 100$ GeV.

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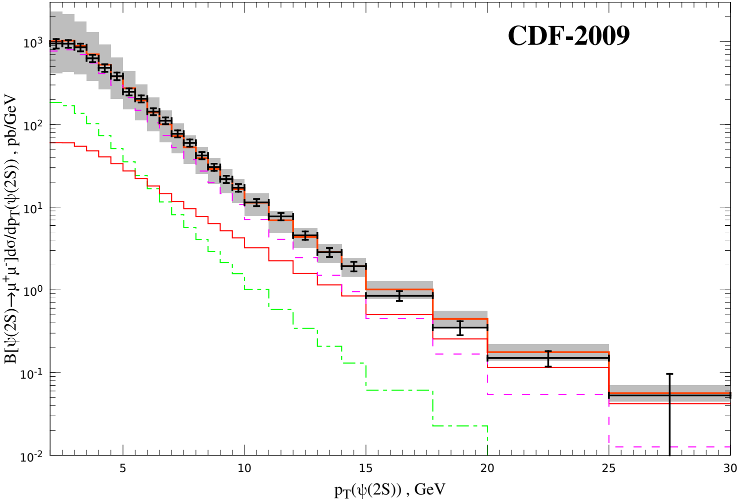
2. To study the onset of fragmentation with new (ATLAS, CMS) data for $p_T \leq 100$ GeV.
3. To present the LO PRA predictions on $\psi(2S)$ and $\Upsilon(3S)$ polarization observables.

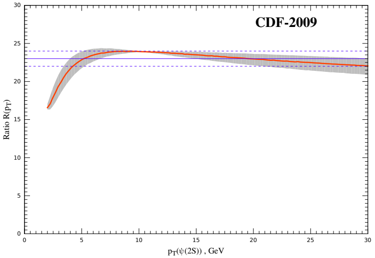
The last point is the reason for the choice of the states. The $\psi(2S)$ and $\Upsilon(3S)$ have *the smallest contribution of feeddown* from heavier charmonia and bottomonia below $D\bar{D}$ or $B\bar{B}$ thresholds. For $\psi(2S)$ there is no higher states below threshold, for $\Upsilon(3S)$ there is only $\chi_b(3P)$-state below threshold, but the **branchings are not known**.

CDF data, fit result.



The $p_T(\psi(2S))$ spectrum from CDF ($p\bar{p}$) at $\sqrt{S} = 1.96$ TeV and $|\eta| < 0.6$.
Contributions: ${}^3S_1^{(1)}$ – thin dash-dotted curve, ${}^3S_1^{(8)}$ – thin solid curve and ${}^3P_J^{(8)}$ – thin dashed curve.

The ${}^1S_0^{(8)} - {}^3P_J^{(8)}$ separation problem.



Dimensionless ratio of the hard-scattering cross-sections:

$$R_{\mathcal{H}}(p_T) = \frac{M_{\mathcal{H}}^2}{\frac{d\sigma}{dp_T}\left({}^1S_0^{(8)}\right)} \sum_{J=0}^{2} (2J+1) \frac{d\sigma}{dp_T}\left({}^3P_J^{(8)}\right),$$

approxiamtely constant (as in the NLO CPM):

$$R_{\psi(2S)} = 23.0 \pm 1.0,\ R_{\Upsilon(3S)} = 22.1 \pm 0.7.$$

$\Rightarrow$ Introducing the linear combination:

$$M_0^{\mathcal{H}} = \left\langle \mathcal{O}^{\mathcal{H}}\left[{}^1S_0^{(8)}\right]\right\rangle + R \frac{\left\langle \mathcal{O}^{\mathcal{H}}\left[{}^3P_0^{(8)}\right]\right\rangle}{M_{\mathcal{H}}^2}.$$

The fit result.

NMEs	Fusion PM (CDF)	Fragmentation PM ATLAS+CMS	NLO CPM [1]	NLO CPM [2]
$\left\langle \mathcal{O}^{\psi(2S)}\left[{}^3S_1^{(1)}\right]\right\rangle/\mathrm{GeV}^3$	0.65 ± 0.06	0.65 ± 0.06	0.76	0.65 ± 0.06
$\left\langle \mathcal{O}^{\psi(2S)}\left[{}^3S_1^{(8)}\right]\right\rangle/\mathrm{GeV}^3 \times 10^3$	1.84 ± 0.17	2.57 ± 0.1	1.2 ± 0.3	2.80 ± 0.49
$M_0^{\psi(2S)}/\mathrm{GeV}^3 \times 10^2$	3.1 ± 1.2	2.7 ± 1.1	2.0 ± 0.6	-3.8 ± 3.9
χ^2/d.o.f.	0.6	1.1	0.56	–
$\left\langle \mathcal{O}^{\Upsilon(3S)}\left[{}^3S_1^{(1)}\right]\right\rangle/\mathrm{GeV}^3$	3.54	–	3.54	–
$\left\langle \mathcal{O}^{\Upsilon(3S)}\left[{}^3S_1^{(8)}\right]\right\rangle/\mathrm{GeV}^3 \times 10^2$	2.50 ± 0.14	–	2.71 ± 0.13	–
$M_0^{\Upsilon(3S)}/\mathrm{GeV}^3 \times 10^2$	0.0 ± 0.17	–	1.083 ± 1.66	–
χ^2/d.o.f.	7.57	–	3.16	–

The CPM results [1]: Y. Q. Ma, et. al. (2015) , [2]: B. A. Kniehl, M. Butenshön , (2012) .

Towards high-p_T: fragmentation mechanism.

In the LO+LLA, only production of ${}^3S_1^{(8)}$-state aquire large log-corrections $\sim \log p_T/M$. For $p_T \gg M$, the factorization formula of fragmentation model is valid:

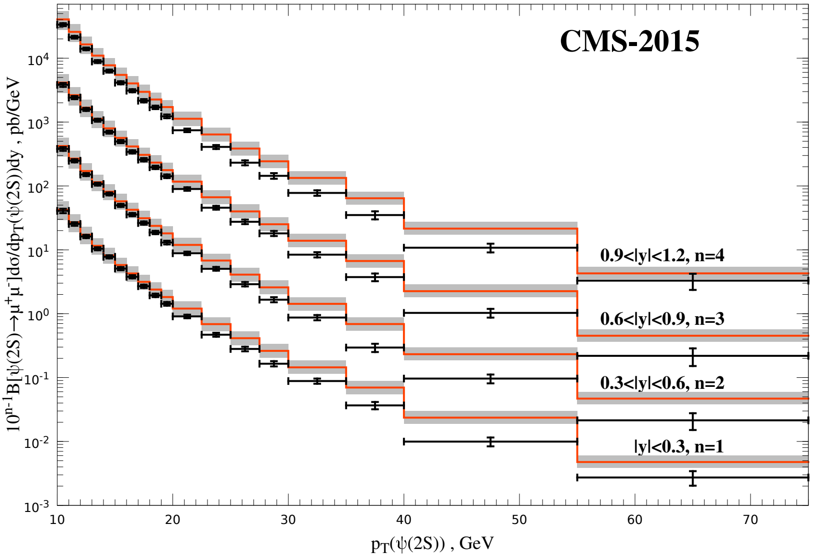
$$\frac{d\sigma}{dp_T(\mathcal{H})dy(\mathcal{H})} = \int\limits_0^1 dz \frac{d\sigma}{dp_T(g)dy(g)} \left(p_T(g) = \frac{p_T(\mathcal{H})}{z} \right) D_{g\to\mathcal{H}\left[{}^3S_1^{(8)}\right]}(z, \mu_F^2),$$

where the FF at the starting scale $\mu_{F0}^2 = M^2$ is perturbatively calculable using the NRQCD-factorization:

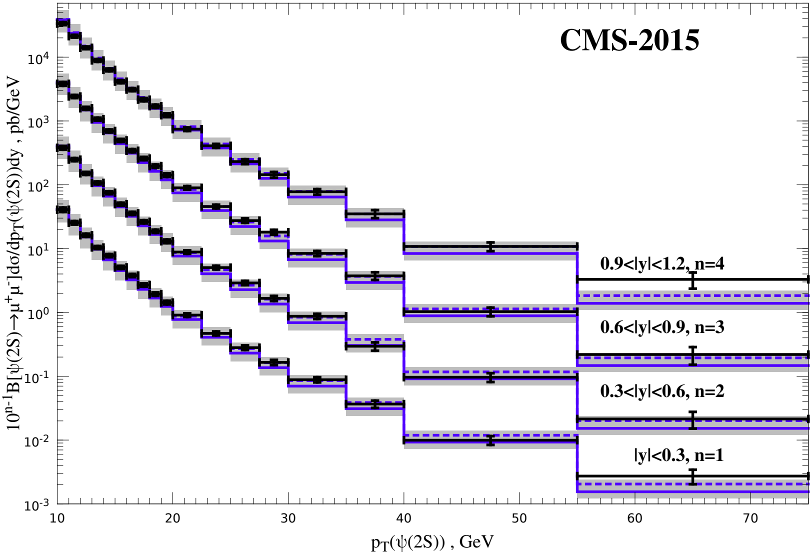
$$D_{g\to\mathcal{H}\left[{}^3S_1^{(8)}\right]}(z, \mu_{F0}^2) = \frac{\pi\alpha_s(\mu_{F0}^2)}{6M_{\mathcal{H}}^3} \left\langle \mathcal{O}^{\mathcal{H}} \left[{}^3S_1^{(8)}\right] \right\rangle \delta(1-z).$$

For higher scales, the FF is **DGLAP-evolved**, and the z-distribution is **softened**.

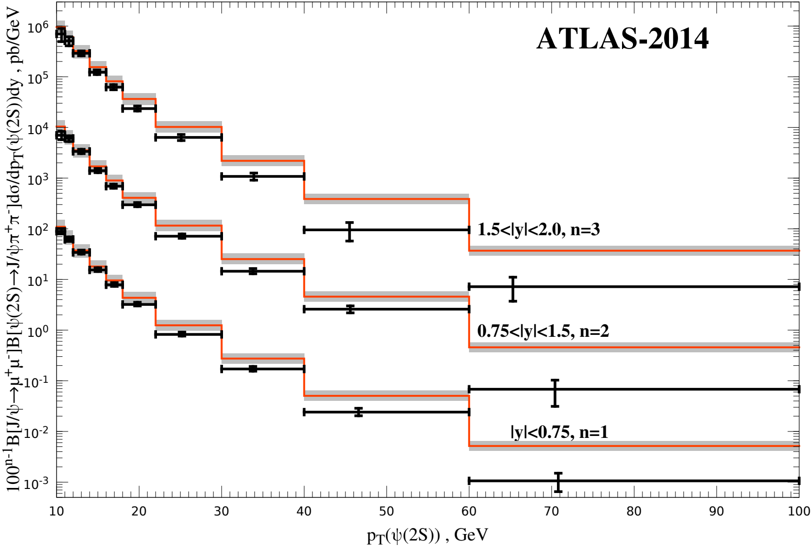
Prediction for CMS data ($\sqrt{S} = 7$ TeV) without fragmentation.



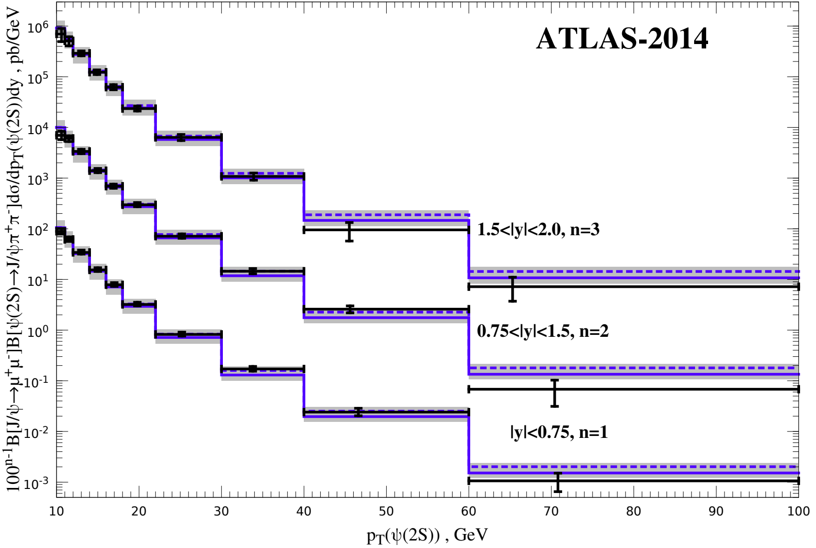
Prediction for CMS data ($\sqrt{S} = 7$ TeV) with fragmentation.



Prediction for ATLAS data ($\sqrt{S} = 7$ TeV) without fragmentation.



Prediction for ATLAS data ($\sqrt{S} = 7$ TeV) with fragmentation.



The fit result.

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Polarization observables.

Parametrization for angular distribution of lepton(μ^+) in the rest frame:

$$\frac{d\sigma}{d\Omega} \sim 1 + \lambda_\theta \cos^2(\theta) + \lambda_\varphi \sin^2(\theta)\cos(2\varphi) + \lambda_{\theta\varphi}\sin(2\theta)\cos(\varphi)\ ,$$

the choice of **coordinate system** in the rest frame is important. We use the **helicity-frame**. In this frame, the coefficient λ_θ:

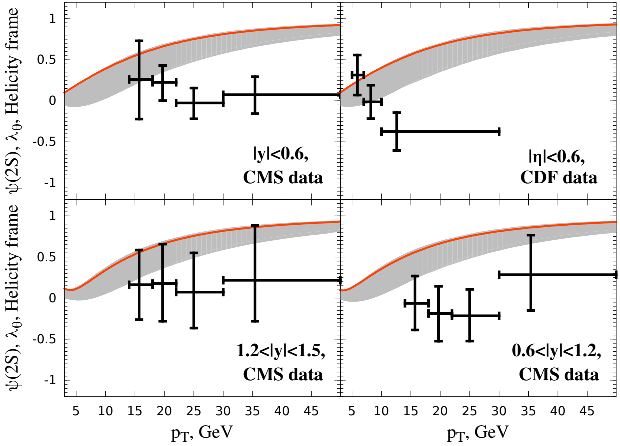
$$\lambda_\theta = \frac{\sigma^{\mathcal{H}} - 3\sigma_L^{\mathcal{H}}}{\sigma^{\mathcal{H}} + \sigma_L^{\mathcal{H}}}.$$

Assuming the chromoelectric-dipole transitions ($\Delta L = 1$, $\Delta S = 0$) for P-wave states and direct polarization transfer from ^3S-states we have:

$$\sigma_L^{\mathcal{H}} = \sigma_L^{\mathcal{H}}\left({}^3S_1^{(1)}\right) + \sigma_L^{\mathcal{H}}\left({}^3S_1^{(8)}\right) + \frac{1}{3}\left(\sigma^{\mathcal{H}}\left({}^1S_0^{(8)}\right) + \sigma^{\mathcal{H}}\left({}^3P_0^{(8)}\right)\right) + \frac{1}{2}\left(\sigma_T^{\mathcal{H}}\left({}^3P_1^{(8)}\right) + \sigma_1^{\mathcal{H}}\left({}^3P_2^{(8)}\right)\right) + \frac{2}{3}\sigma_0^{\mathcal{H}}\left({}^3P_2^{(8)}\right).$$

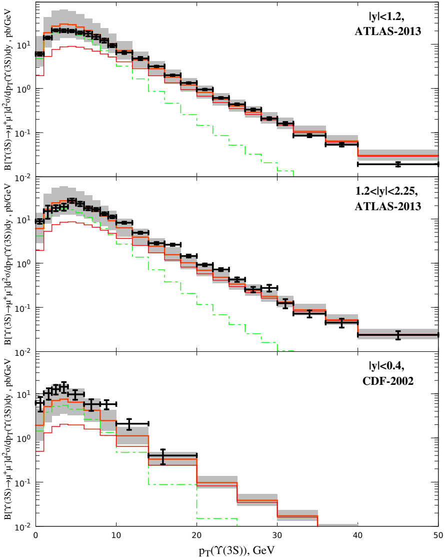
Result for $\psi(2S)$.

$\lambda_\theta = +1$ – transverse polarization, $\lambda_\theta = -1$ – longitudinal polarization, $\lambda_\theta = 0$ – unpolarized mixture.



The polarization at high-p_T is transverse, due to dominating ${}^3S_1^{(8)}$-state.

Fit result for $\Upsilon(3S)$.



The $p_T(\Upsilon(3S))$ spectrum from ATLAS (pp, $\sqrt{S} = 7$ TeV) CDF ($p\bar{p}$, $\sqrt{S} = 1.96$ TeV). Contributions: ${}^3S_1^{(1)}$ – thin dash-dotted curve, ${}^3S_1^{(8)}$ – thin solid curve and ${}^3P_J^{(8)}$ – thin dashed curve.

The fit result.

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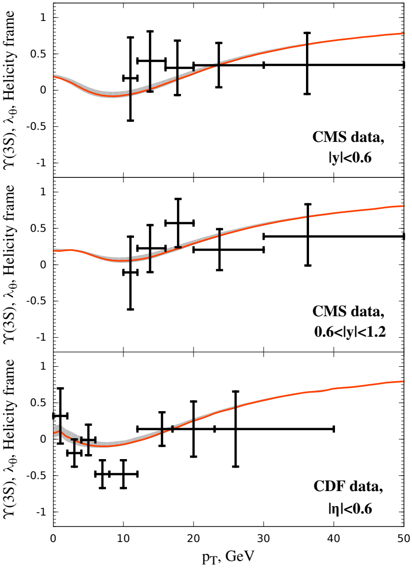
Prediction for λ_θ.



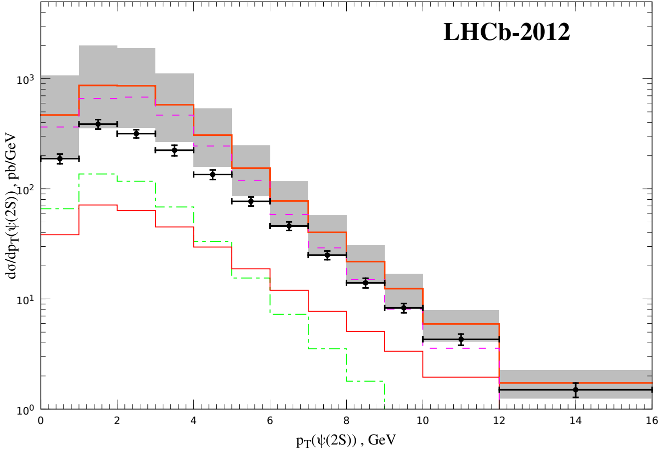
Conclusions.

- It is possible to describe all hadroproduction data on p_T-spectra in LO PRA with the same set of NMEs. But fragmentation corrections and correct decay kinematics should be included.
- The fit results for NMEs are rather close to the NLO CPM results.
- The polarization puzzle is still there. *What is the depolarization mechanism for $\psi(2S)$?*
- For $\Upsilon(3S)$ both p_T-spectra and polarization looks good, due to the smaller fraction of Color-Octet.

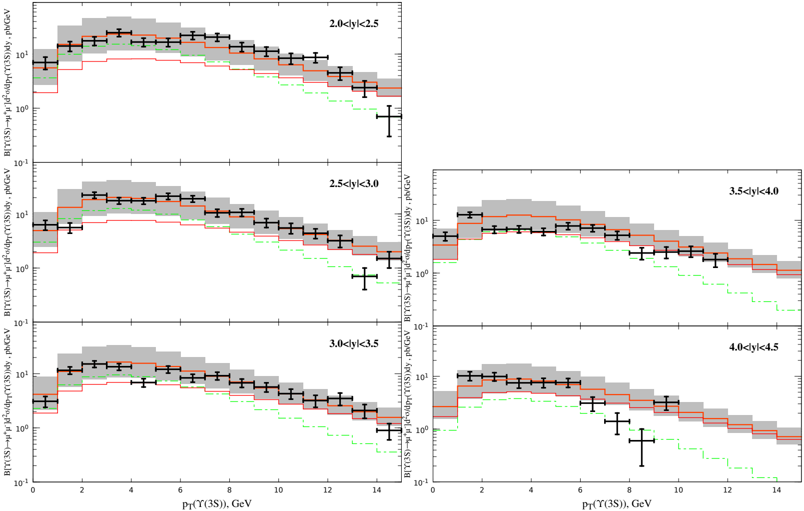
Thank you for your attention!

Backup slides.

The description of LHCb data on $\psi(2S)$.



The description of LHCb data on $\Upsilon(3S)$.



The description of $p_T(J/\psi)$- spectra.

The p_T-rescaling is used. The $p_T(\psi(2S))$ and J/ψ spectra differ by factor 2.