EXERCISES: "Optical probes of QVac. Nonlin."

-> If you have any questions / comments contact

felix.karbstein@uni-jena.de

1. Using $\{\gamma^\mu, \gamma_5\} = 0$ & $\gamma_5^2 = \mathbb{1}$

show that

$$S^1[A^\mu_{cl}] = -i\left\{\ln\det(-i\not{D}_{cl.}+m) - \ln\det(-i\not{\partial}+m)\right\}$$

can be rewritten as

$$S^1[A^\mu_{cl}] = -\frac{i}{2}\left\{\ln\det(\not{D}^2_{cl.}+m^2) - \ln\det(\not{\partial}^2+m^2)\right\}.$$

2. Show that (with $\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$)

$$\not{D}^2_{cl.} = -D^2_{cl.} - \frac{e}{2}\sigma^{\mu\nu}F^{cl.}_{\mu\nu}.$$

Recall $g^{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ and

$$F^{\mu\nu}_{cl.} = \left(\begin{array}{c|ccc} 0 & & \vec{E}^T & \\ \hline & 0 & B_3 & -B_2 \\ -\vec{E} & -B_3 & 0 & B_1 \\ & B_2 & -B_1 & 0 \end{array}\right).$$

3. Show that

$$\ln M - \ln M_0 = \lim_{\Lambda\to\infty}\left\{-\int_{1/\Lambda^2}^{\infty}\frac{dT}{T}\left(e^{-MT} - e^{-M_0 T}\right)\right\}$$

for $\mathrm{Re}\{M, M_0\} > 0$.

4. With the following convention of γ-matrices $\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}$, $\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$, with Pauli-matrices σ^i & $\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, determine $\frac{e}{2} F^{\mu\nu}_{cl} \sigma_{\mu\nu}$ explicitly for $A^\mu_{cl} = (0, 0, Bx, 0)$.

5. Determine the density of states $g(n)$ from the following condition (relevance will become clear in lecture on Wednesday).

$$\lim_{eB \to 0} \sum_{n=0}^{\infty} g(n) \overset{!}{=} \int \frac{dp_x}{\left(\frac{2\pi}{L}\right)} \int \frac{dp_y}{\left(\frac{2\pi}{L}\right)}$$

where $p_\perp^2 = p_x^2 + p_y^2$ for $eB = 0$

and $p_\perp^2 = 2eB\left(n + \frac{1}{2}\right)$ for finite eB.

6. Evaluate $\sum_{n=0}^{\infty} e^{-eBT(2n+1)}$

and $\mathrm{tr}_\gamma \, e^{\frac{e}{2}\sigma^{\mu\nu} F^{cl.}_{\mu\nu} T}$ for the above A^μ_{cl}. tr_γ denotes the trace over the Dirac 4×4 matrices structure.

7. Convince yourself that $(k^\mu = (\omega, \vec{k}))$

- $F^{\mu\nu}_{cl.} k_\nu = (\vec{k}.\vec{E}, \vec{k} \times \vec{B} + \omega \vec{E})$

- $\tilde{F}_{cl}^{\mu\nu} k_\nu = (\vec{k}.\vec{B}\,,\, -\vec{k}\times\vec{E} + \omega\vec{B})$

  where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$, $\epsilon^{0123} = 1$

  is the dual field-strength tensor

- $F^{\mu\alpha}F^{\nu}{}_{\alpha} - \tilde{F}^{\mu\alpha}\tilde{F}^{\nu}{}_{\alpha} = 2\mathcal{F}\, g^{\mu\nu}$

- $F^{\mu\alpha}\tilde{F}^{\nu}{}_{\alpha} = \tilde{F}^{\mu\alpha}F^{\nu}{}_{\alpha} = \mathcal{G}\, g^{\mu\nu}$

  with $\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(\vec{B}^2 - \vec{E}^2)$

  $\mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = -\vec{E}.\vec{B}$

We will discuss the solutions of these problems in the Str. F.-exer. on Wed, 20, $14^{\underline{30}} - 15^{\underline{30}}$!