1. D=10 (5.1)

IIA SUGRA

$$S = \int e^{-2\phi}\left[\frac{1}{2}\hat{R}*1 + 2d\hat{\phi}\wedge *d\phi - \frac{1}{4}\hat{H}_3\wedge *H_3\right] -$$

$$-\frac{1}{2}\left[\hat{F}_2\wedge *\hat{F}_2 + \hat{F}_4\wedge *\hat{F}_4\right] + \mathcal{L}_{top}$$

Fields:

$$\hat{H}_3 = d\hat{B}_2$$
$$\hat{F}_2 = dA_1$$
$$\hat{F}_4 = d\hat{C}_3 - d\hat{A}_1\wedge B_2$$

SUSY transf:

$$\delta\hat{\psi}_M = \nabla_M\epsilon + \dots$$
$$\delta\lambda = (\not\partial\hat{\Phi} + \frac{1}{2}\not{H}\Gamma_{11})\epsilon + \dots$$

$$\epsilon = \begin{bmatrix}\epsilon^1\\ \epsilon^2\end{bmatrix}\begin{matrix}+\\-\end{matrix} \quad \begin{matrix}\mathcal{N}=2\\ D=10\end{matrix}$$

Expected result: $M^{10} = M^4\times CY_3$

$\downarrow$

$\mathcal{N}=2$ in $D=4$ SUGRA

- n_V vectors, t^i
- n_H hypers, q^u

2. Decomposition of fields

IIA	Gravity	$(h_{1,1})$ vectors	$(2h_{2,1})$ hypers	1 hypers
$\hat{G}_{MN}$	$g_{\mu\nu}$	σ^i	$z^a, \tilde{z}^a$	x
$\hat{B}_{MN}$	x	b^i	x	a
$\hat{\Phi}$	x	x	x	ϕ
$\hat{A}_M$	A_μ	x	x	x
$\hat{C}_{MNK}$	x	$A_\mu{}^i$	$\xi^a, \tilde{\xi}^a$	$\xi^0, \tilde{\xi}^0$

vector: $|-1\rangle \to |-1/2\rangle \to |0\rangle$, $|-1\rangle \to |-1/2\rangle \to |0\rangle$ $\Rightarrow$ $(\underset{1}{A_\mu}, \underset{1/2}{\lambda^1}, \underset{1/2}{\lambda^2}, \underset{0}{z}, \underset{0}{z^*})$

hyper: $|-1/2\rangle \to |0\rangle \to |1/2\rangle$, $|-1/2\rangle \to |0\rangle \to |1/2\rangle$ $\Rightarrow$ $(\underset{1/2}{\lambda}, \underset{0}{z}, \underset{0}{z^*})$

Fields are decomposed w/ respect to harm. forms ξ
(cohomology classes)

1) Complex and Kähler structures:

$$h_{m\bar{n}} = \boxed{v^i} \omega_{i\, m\bar{n}}$$

$$h_{mn} = \Omega_{mkl}\, h^{k\bar{k}}\, h^{l\bar{l}}\, \chi_{a\, n\bar{k}\bar{l}} \boxed{z^a}$$

$$h_{\bar{m}\bar{n}} = \Omega_{\bar{m}\bar{k}\bar{l}}\, h^{k\bar{k}}\, h^{l\bar{l}}\, \bar{\chi}_{\bar{a}\, \bar{n} kl} \boxed{\tilde{z}^a}$$

2) Gauge fields:

$\hat{A}_1 = A^0$; $A_\mu(x)$

$\hat{B}_2 = B_2 + b^i \omega_i$; $B_{\mu\nu}(x)$; $B_{mn}(x,y) = b^i(x)\, \omega_{i\, m\bar{n}}(y)$

$\hat{C}_3 = C_3 + A^i \wedge \omega_i + \xi^A \alpha_A + \tilde{\xi}_B \beta^B$; (α_A, β^B) - real basis

C_3 ← non-dynamical in $D=4$

$B_2 \to a$ - 0-form. (axion) gauge → shift.

$dB_2 = H_3$; $d *_4 H_3 = 0$; $*_4 H_3 = G_1 = da$

$$H_3 \wedge * H_3 = * G_1 \wedge G_1 \Rightarrow \partial_\mu a\, \partial^\mu a$$

3. Supermultiplets:

$n_V = h_{1,1}$; (A^i, t^i) ; $t^i = b^i + i v^i$

$n_H = 2(h_{2,1}+1)$ $\quad (\underbrace{\xi^a, \tilde{\xi}^a}_{h_{2,1}}, \underbrace{z^a, \tilde{z}^a}_{h_{2,1}}, \underbrace{\xi^0, \tilde{\xi}^0}_{1}, \underbrace{a, \phi}_{1})$

space of scalar fields:

$$\cancel{\mathcal{M} = \mathcal{M}} \quad \cancel{\mathcal{K}^{sc} = \mathcal{K}^{n_V}} \quad \Upsilon^{sc} = \underset{\substack{\uparrow \\ \text{Kähler}}}{\Upsilon^{2n_V}} \times \underset{\substack{\uparrow \\ \text{quaternionic Kähler}}}{\Upsilon^{4(h_{2,1}+1)}}$$

4. Field strengths and Lagrangian

1) $\hat{H}_3 = d\hat{B}_2 = dB_2 + db^i \wedge \omega_i$

$*_{10}\hat{H}_3 = *_4 dB_2 \wedge *_6 \mathbb{1} + *_4 db^i \wedge *_6 \omega_i$

$$-\frac{1}{4}\int_{CY_3} H_3 \wedge *_{10} H_3 = -\frac{1}{4}\int_{CY} *_6 1 \cdot dB_2 \wedge * dB_2 -$$

$$- \frac{1}{4}\int_{CY} \omega_i \wedge * \omega_j \; db^i \wedge * db^j =$$

$$= -\frac{K}{4} dB_2 \wedge * dB_2 - K\, G_{ij}\, db^i \wedge * db^j$$

$K = \mathrm{Vol}(CY_3)$; G_{ij} – metric on the space of Kähler structures

2) $\hat{F}_2 = d\hat{A}_1 = dA_0$

$$-\frac{1}{2}\int_{CY} \hat{F}_2 \wedge * \hat{F}_2 = -\frac{K}{2} dA^0 \wedge * dA^0$$

3) $\hat{F}_4 = d\hat{C}_3 - d\hat{A}_1 \wedge \hat{B}_2 = dC_3 + dA^i \wedge \omega_i + d\xi^A \wedge \alpha_A +$

$+ d\tilde{\xi}_B \wedge \beta^B - dA^0 \wedge B_2 - dA^0 \wedge b^i \omega_i$

$*_{10}\hat{F}_4 = *_4 dC_3 \wedge *_6 \mathbb{1} + *_4 dA^i \wedge *_6 \omega_i + *_4 d\xi^A \wedge *_6 \alpha_A$

$+ *_4 d\tilde{\xi}_B \wedge *_6 \beta^B - *(dA^0 \wedge B_2) \wedge *_6 1 - *_4 dA^0 b^i \wedge *_6 \omega_i$

$$-\frac{1}{2}\int_{CY} \hat{F}_4 \wedge *_{10} \hat{F}_4 = -\frac{K}{2}(dC_3 - dA^0 \wedge B_2) \wedge *(dC_3 - dA^0 \wedge B_2)$$

$$- 2K\, G_{ij}(dA^i - dA^0 b^i) \wedge *(dA^j - dA^0 b^j) +$$

$$+ \frac{1}{2}(\mathrm{Im}\, M^{-1})^{AB}\left[d\tilde{\xi}_A + M_{AC}\, d\xi^C\right] \wedge * \left[d\tilde{\xi}_B + \overline{M}_{BC}\, d\xi^C\right]$$

5. Dualization of forms in D=4

~~1)~~ 3-form

$$\mathcal{L} = -\frac{K}{2}\left(dC_3 - dA^0\wedge B_2\right)\wedge *\left(dC_3 - dA^0\wedge B_2\right) + \frac{e_0}{\cancel{4}}\, dC_3$$

Lagrange multiplier:

EOM's for dC_3:

$$\underbrace{-K *\left(dC_3 - dA^0\wedge B_2\right)}_{= \text{const}} + \frac{e_0}{\cancel{4}} = 0$$ – solution of the EOM for C_3

e_0 – constant

$$\mathcal{L} = +\frac{1}{2K}\cdot\left(\frac{e_0}{\cancel{4}}\right)^2 *1 - \frac{e_0}{\cancel{4}} *\left(\frac{1}{K}\cdot\frac{e_0}{\cancel{4}}\right) + \frac{e_0}{\cancel{4}}\, dA^0\wedge B_2$$

$$= \boxed{-\frac{e_0^2}{2K} *1} + \frac{e_0}{\cancel{4}}\, dA^0\wedge B_2$$

potential for the field ~~k~~ combination K

2) ~~2-form~~

$$\cancel{\mathcal{L}_{H_3} = -\frac{1}{4}e^{-2\phi} H_3\wedge * H_3 + \overbrace{\frac{1}{2}H_3\wedge\left(\tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A\right) - e_0 A^0\wedge H_3}^{\mathcal{L}_{top}}}$$

~~EOM's~~

6. The resulting ~~potential~~ action

$$S = \int\Big[\frac{1}{2}R*1 - g_{i\bar{j}}\, dt^i\wedge * d\bar{t}^{\bar{j}} - h_{uv}\, Dq^u\wedge * Dq^v +$$

$$+\frac{1}{2}\,\mathrm{Im}\,\mathcal{N}_{IJ}F^I\wedge * F^J + \frac{1}{2}\,\mathrm{Re}\,\mathcal{N}_{IJ}F^I\wedge * F^J + V_E\Big]$$

$$\{q^u\} = \{\phi, a, z^a, \bar{z}^a, \xi^A, \tilde{\xi}_A\} \quad (2h_{2,1}+2)$$

Content of the resulting action:

$$h_{uv}\, Dq^u \wedge * Dq^v = d\phi \wedge * d\phi + g_{a\bar{b}}\, dz^a \wedge * d\bar{z}^b +$$

$$+ \frac{e^{4\phi}}{4}\left[Da + (\tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A)\right] \wedge * \left[Da + (\tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A)\right]$$

$$- \frac{e^{2\phi}}{2} (\mathrm{Im}\, \mathcal{M}^{-1})^{AB} \left[d\tilde{\xi}_A + \mathcal{M}_{AC}\, d\xi^C \right] \wedge * \left[d\tilde{\xi}_B + \bar{\mathcal{M}}_{BD}\, d\xi^D \right]$$

potential: $V_E = \frac{e^{4\phi}}{2K} e_0^2 * 1$; $\quad Da = da + 2 e_0 A^0$

Kähler potential

$$K = -\ln \frac{4}{3} \int J \wedge J \wedge J - \ln i \int \Omega \wedge \bar{\Omega} - \ln (S + \bar{S})$$

$\Downarrow$ $g_{a\bar{b}}$ $\qquad$ $\Downarrow$ $\mathcal{M}_{AB}$ $\qquad$ $S = \phi + i a$ axio-dilaton