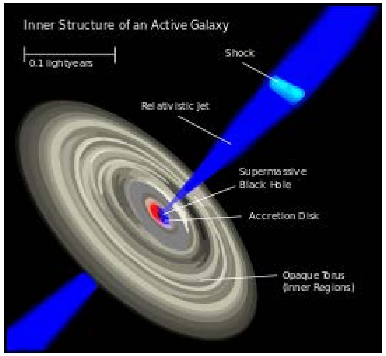

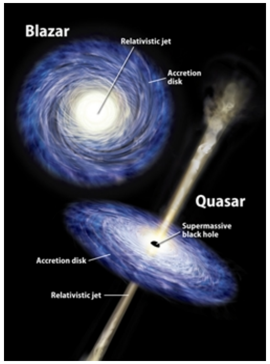

Advances in classical gravity II

Interacting massless particles

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Moscow State University

"Cosmology, Strings, New Physics"
BLTP, JINR , 07 September 2016

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- **Infrared photons and gravitons (Weinberg '65)**
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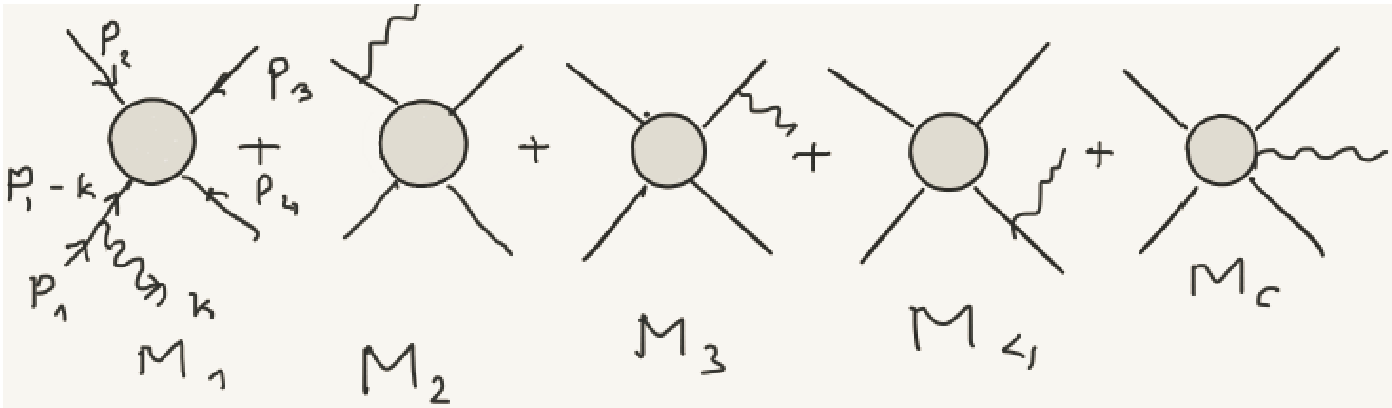
Weinberg 1965



Emission of graviton in the 2→ 2 particle scattering. In the infrared k → 0 limit the diagrams with emission from external legs (1-4) have IR divergence, since the particle propagator sits at a pole:

$$\frac{1}{(p_a-k)^2+m_a^2} = \frac{1}{\underbrace{p_a^2+m_a^2}_{=0} - 2(p_a\cdot k) + \underbrace{k^2}_{=0}} = -\frac{1}{2(p_a\cdot k)}$$

while the last diagram (bulk emission) remains finite. The total amplitude is

$$S_{fi} = -i\,\varepsilon_{\mu\nu}(k)\, M^{\mu\nu}(k)\, \delta\left(\sum_{a=1}^{4} p_a^{\mu} - k^{\mu}\right)$$

Extended soft factorization (Cachazo and Strominger)

$$\mathcal{M}_{n+1}(k_1, k_2, ..., \varepsilon k_{n+1}) = \left(\frac{1}{\varepsilon} S^{(0)} + S^{(1)} + \varepsilon S^{(2)} + \mathcal{O}(\varepsilon^2)\right) \mathcal{M}_n(k_1, k_2, ..., k_n)$$

$$S^{(0)} = \sum_{a=1}^{n} \frac{\epsilon_{n+1\,\mu\nu} k_a^\mu k_a^\nu}{k_{n+1} \cdot k_a}$$

$$S^{(1)} = \sum_{a=1}^{n} \frac{\epsilon_{n+1\,\mu\nu} k_a^\mu (k_{n+1\,\lambda} J_a^{\lambda\nu})}{k_{n+1} \cdot k_a}$$

$$S^{(2)} = \frac{1}{2} \sum_{a=1}^{n} \frac{\epsilon_{n+1\,\mu\nu} (k_{n+1\,\rho} J_a^{\rho\mu})(k_{n+1\,\lambda} J_a^{\lambda\nu})}{k_{n+1} \cdot k_a},$$

where $J_a^{\mu\nu}$ are angular momenta

$$M^{\mu\nu} = \sum_{a=1}^{4} M_a^{\mu\nu} + M_c^{\mu\nu}$$

where the external leg emission diagrams (with different couplings) give

$$M_a^{\mu\nu} \sim \frac{\varkappa_a p_a^\mu p_a^\nu + \text{longitudinal}\,(k^\mu \ldots)}{2(p_a \cdot k)}$$

Now consider the Ward identity following from the gauge invariance (transversality)

$$k_\mu M^{\mu\nu} = 0 \Rightarrow k_\mu M_c^{\mu\nu} + M \sum_a \frac{\varkappa_a (p_a \cdot k) p_a^\nu}{2(p_a \cdot k)}$$

In the limit k → 0 the contact term disappears, while the others give

$$\sum_{a=1}^{4} \varkappa_a p_a^\mu = 0 \quad \text{together with}$$

$$\sum_{a=1}^{4} p_a^\mu = 0 \quad \Rightarrow \quad \varkappa_1 = \varkappa_2 = \varkappa_3 = \varkappa_4 = \varkappa$$

Thus all coupling constants in the IR (classical) limit should be equal. This is nothing but the universality of gravitational interaction. Note that one of legs may be graviton itself, then this proves self-interaction

Similar considerations for emission of the spin three particle (trird rank symmetric tensor) lead to the identity

$$\sum_{a=1}^{4} \varkappa_a P_a^{\mu} P_a^{\nu} = 0$$

Together with the conservation of the momentum this implies vanishing of interaction in the classical limit

$$\varkappa_1 = \varkappa_2 = \varkappa_3 = \varkappa_4 = 0$$

Note that in quantum theory interaction of spins starting from 3 Involves in full tower of higher spins.

Finally, recall that Weinberg's argument for electromagnetic interaction leads to

$$\sum_{a=1}^{4} e_a = 0$$

which is just the conservation law for the electric charge

Gravitational memory (displacement)

Two particles in a free fall in gravitational field of a distant source of gravitational waves are relatively displaced via deviation equation:

$$\frac{d^2 \Delta x^j}{dt^2} = -DR_{titj}$$

where D is the initial distance between particles, R is the curvature tensor of the wave. Assuming wave generation via quadrupole formula, we get: where P is projection orthogonal to the radial direction (and substraction of the trace part)

$$R_{titj} = \frac{-1}{r} P \left[\frac{d^4 Q_{ij}}{dt^4} \right]$$

$$Q^{ij} = \int d^3x T^{00} x^i x^j$$

For the system of point particles

$$\frac{d^2}{dt^2} Q^{ij} = \sum_k m_k v_k^i v_k^j$$

Combining all this and integrating we get (Zeldovich and Polnarev 1974)

$$\Delta x^j = \frac{D}{r} P \left[\sum_k m_k v_k^i v_k^j (t = \infty) - \sum_k m_k v_k^i v_k^j (t = -\infty) \right]$$

Thus test particles memorize their displacement after the wave passed. Relation to Weinberg theorem is obvious. Non-linear → Christodoulou '91

Electromagnetic memory (velocity)

Consider similar system of charges

For test charges we have $m\frac{d^2\vec{x}}{dt^2} = q\vec{E}$ so the relative velocity will be $\Delta\vec{v} = \frac{q}{m}\int_{-\infty}^{\infty}\vec{E}dt$. In non-relativistic case $\vec{E} = \frac{1}{r}P\left[\frac{d^2\vec{p}}{dt^2}\right]$, where p is the dipole moment of the source. It follows that

$$\Delta\vec{v} = \frac{q}{mr}P\left[\frac{d}{dt}\vec{p}(t=\infty) - \frac{d}{dt}\vec{p}(t=-\infty)\right]$$

Like in the previous case, consider largely separated charges asymptotically free

$$\frac{d}{dt}\vec{p} = \sum_k q_k\vec{v}_k$$

Combining, we get the velocity kick (Bieri and Garfinkle 2013)

$$\Delta\vec{v} = \frac{q}{mr}P\left[\sum_k q_k\vec{v}_k(t=\infty) - \sum_k q_k\vec{v}_k(t=-\infty)\right]$$

Soft theorems from BMS (Strominger, …)

Future null infinity $$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z},$$

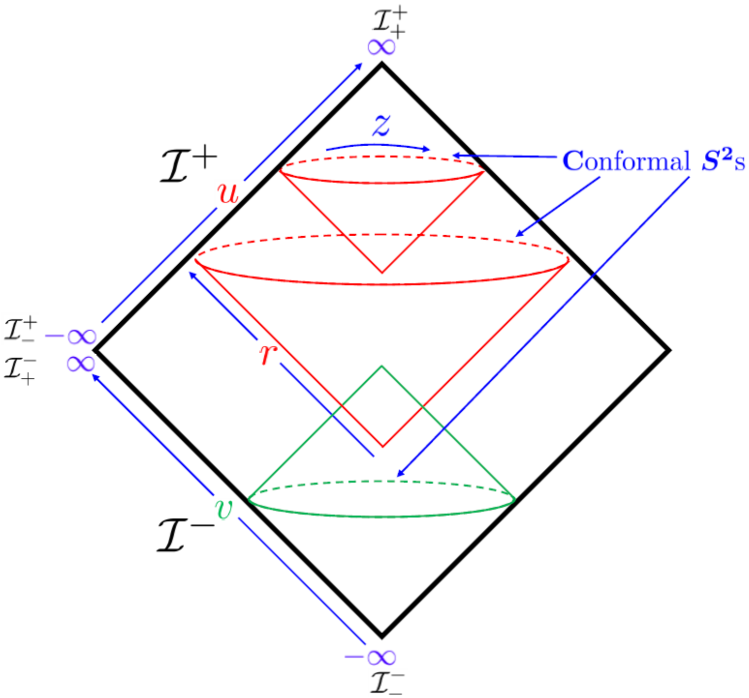
where $u = t - r$ and $\gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$ is complex metric on S^2

Supertranslations, arbitrary $f(z,\bar{z})$:

$$\zeta_f = f\partial_u - \frac{\gamma^{z\bar{z}}}{r}(\partial_z f\partial_{\bar{z}} + \partial_{\bar{z}} f\partial_z) + \ldots.$$

This is the leading asymptotic for AF solutions. BMS$^+$ transformations combine conformal transformations of the sphere with adjusting transformations of remaining coordinates leaving asymptotic expansions valid.

Similarly, BMS$^-$ transformations act on past null infinity. Certain subgroup of BMS$^+$ x BMS$^-$ is the symmetry of S-matrix. It generates Ward identity which reproduces soft photon and graviton theorems



Soft Hair on Black Holes

Stephen W. Hawking†, Malcolm J. Perry† and Andrew Strominger*

05 January 2016

We have reconsidered the black hole information paradox in light of recent insights into the infrared structure of quantum gravity. An explicit description has been given of a few of the pixels in the holographic plate at the future boundary of the horizon. Some information is accessibly stored on these pixels in the form of soft photons and gravitons. A complete description of the holographic plate and resolution of the information paradox remains an open challenge, which we have presented new and concrete tools to address.

1601.00921 *Phys. Rev.Lett.* 116 no. 23, (2016) 231301,

To date this paper has 69 citations, the recent one is arXiv 1609.01056, contains review

Horizon Supertranslation and Degenerate Black Hole Solution

Rong-Gen Cai[1,2] *, Shan-Ming Ruan[1] †, Yun-Long Zhang[3] ‡

Gravitational Black Hole Hair from Event Horizon Supertranslations

Artem Averin[a,b], Gia Dvali[a,b,c], Cesar Gomez[d], Dieter Lüst[a,b]

January 15, 2016

We discuss BMS supertranslations both at null-infinity BMS^- and on the horizon $BMS^{\mathcal{H}}$ for the case of the Schwarzschild black hole. We show that both kinds of supertranslations lead to infinetly many gapless physical excitations. On this basis we construct a quotient algebra $\mathcal{A} \equiv \mathcal{BMS}^{\mathcal{H}}/\mathcal{BMS}^-$ using suited superpositions of both kinds of transformations which cannot be compensated by an ordinary BMS-supertranslation and therefore are intrinsically due to the presence of an event horizon. We show that transformations in $\mathcal{A}$ are physical and generate gapless excitations on the horizon that can account for the gravitational hair as well as for the black hole entropy. We identify the physics of these modes as associated with Bogolioubov-Goldstone modes due to quantum criticality. Classically the number of these gapless modes is infinite. However, we show that due to quantum criticality the actual amount of information-carriers becomes finite and consistent with Bekenstein entropy. Although we only consider the case of Schwarzschild geometry, the arguments are extendable to arbitrary space-times containing event horizons.

Goldstone origin of black hole hair from supertranslations and criticality

Artem Averin[a,b], Gia Dvali[a,b,c], Cesar Gomez[d], Dieter Lüst[a,b]

June 21, 2016

Degrees of freedom that carry black hole entropy and hair can be described in the language of Goldstone phenomenon. They represent the pseudo-Goldstone bosons of certain supertranslations, called $\mathcal{A}$-transformations, that are spontaneously broken by the black hole metric. This breaking gives rise to a tower of Goldstone bosons created by the spontaneously-broken generators that can be labeled by spherical harmonics. Classically, the number of charges is infinite, they have vanishing VEVs and the corresponding Goldstone modes are gapless. The resulting hair and entropy are infinite, but unresolvable. In quantum theory the two things happen. The number of legitimate Goldstone modes restricted by requirement of weak-coupling, becomes finite and scales as black hole area in Planck units. The Goldstones generate a tiny gap, controlled by their gravitational coupling. The gap turns out to be equal to the inverse of black hole half-life, t_{BH}. Correspondingly, in quantum theory the charges are neither conserved nor vanish, but non-conservation time is set by t_{BH}. This picture nicely matches with the idea of a black hole as of critical system composed of many soft gravitons. The $\mathcal{A}$-Goldstones of geometric picture represent the near-gapless Bogoliubov-Goldstone modes of critical soft-graviton system.

Near Horizon Soft Hairs as Microstates of Three Dimensional Black Holes

H. Afshar,[1,*] D. Grumiller,[2,†] and M.M. Sheikh-Jabbari[1,‡]

We revisit the three dimensional Bañados–Teitelboim–Zanelli (BTZ) black holes in Einstein gravity with negative cosmological constant and the algebra of charges associated with nontrivial diffeomorphisms around their near horizon geometry (the near horizon "soft hair"). These soft hairs are arranged by the near horizon algebra which is the algebra of creation/annihilation operators of a two dimensional free boson theory. We show that the asymptotic conformal algebra is a specific subalgebra of the near horizon algebra. We propose that microstates of a generic BTZ black hole of a given mass and angular momentum, in a microcanonical description, are generic states in the Hilbert space of this near horizon algebra for which asymptotic Virasoro generators vanish. That is, microstates of a given BTZ black hole are not distinguishable by the asymptotic two dimensional conformal algebra. We count the microstates using the Hardy–Ramanujan formula for the number of partitions of a given integer into non-negative integers, recovering the Bekenstein–Hawking entropy of the BTZ black hole. We discuss possible extensions of our black hole microstate construction to astrophysical Kerr-type black holes.

arXiv: 1607.00009

Massless fields interacting with massless particles: consistent theory?

- **Photons, neutrino, gravitons in GR** +
- **Gluons in QCD** +
- **Massless charges (MC) in QED** -?

    Collinear divergences +

    Charge screening - +

    $e(\mu) \rightarrow 0$ **in the IR** - +
- **But: classical motion in external field** +
- **Massless QED in external magnetic field** +
- **MC exists, but unobservable: do not radiate** ?
- **Radiation from massless particles in GR** ?

Massless charges in electrodynamics

- **Whether Minkowski space QED with massless charged particles is non-contradictory? (Waks, Gribov, Smilga…)**
- **Can classical ED describe radiation from massless charges?**
- **Are ultrarelativistic limit and massless limit for radiating particle identical?**
- **Whether quantum radiation power from massless charge have classical limit?**

DG, Synchrotron radiation from massless charge, Physics Letters B 747, 400 (2015)
DG, Electromagnetic and gravitational radiation from massless particles, arXiv:1512.06826

Do massless particles radiate?

JOURNAL OF MATHEMATICAL PHYSICS **56**, 022901 (2015)

Electrodynamics of massless charged particles

Kurt Lechner[a)]

We derive the classical dynamics of massless charged particles in a rigorous way from first principles. Since due to ultraviolet divergences this dynamics does not follow from an action principle, we rely on (a) Maxwell's equations, (b) Lorentz- and reparameterization-invariance, and (c) local conservation of energy and momentum. Despite the presence of pronounced singularities of the electromagnetic field along Dirac-like strings, we give a constructive proof of the existence of a unique distribution-valued energy-momentum tensor. Its conservation requires the particles to obey standard Lorentz equations and they experience, hence, no radiation reaction. Correspondingly, the dynamics of interacting classical massless charged particles can be consistently defined, although they do not emit *bremsstrahlung* end experience no self-interaction. [http://dx.doi.org/10.1063/1.4906813]

B P Kosyakov **Massless interacting particles** Phys. A: Math. Theor. 41 (2008) 465401

We show that classical electrodynamics of massless charged particles and the Yang–Mills theory of massless quarks do not experience rearranging their initial degrees of freedom into dressed particles and radiation. Massless particles do not radiate. We propose a conformally invariant version of the direct interparticle action theory for these systems.

Larmor formula for radiation in classical ED in the massless limit

Power emitted by a classical charge in magnetic field

$$P_{\rm cl} = \frac{2e^4 H^2}{3m^2}\left(\frac{E}{m}\right)^2$$

diverges as m→0

Is this formula applicable to a «true» massless charge?

Lienard-Wiechert potential has singularity along the line parallel to the velocity

$$A^\mu = \left.\frac{ev^\mu}{R(1-\cos\theta)}\right|_{\rm ret}$$

reminiscent to collinear singularities in QFT.

Radical claims (Kosyakov, Lechner): **NO RADIATION**

BUT: spectral decomposition is correct,

Schott is still right!

Schott formula is valid for v=c

$$\frac{dP}{d\Omega} = \sum_{\nu=0}^{\infty} \frac{e^2\nu^2\omega_H^2}{2\pi}\left[\tan^2\theta J_\nu^2(\nu\beta\cos\theta) + \beta^2 J_\nu'^2(\nu\beta\cos\theta)\right] \qquad \beta \to 1$$

but the harmonic number $\omega = \nu\omega_H, \quad \omega_H = \frac{eH}{E}$ is no more bounded at high frequencies:

$$\nu \lesssim \nu_{\rm cr} \sim (1-\beta^2)^{-3/2} = (E/m)^3$$

so the total power diverges. But passing to continuous spectral distribution, integrating over angles, and taking the limit m → 0 one obtains the non-singular spectral distribution

$$\frac{dP}{d\omega} = \frac{e^2\omega_H 3^{1/6}\Gamma(2/3)}{\pi}\left(\frac{\omega}{\omega_H}\right)^{1/3}$$

which has the only problem to be non-integrable at high frequences, so a cutoff has to be introduced. This classical formula is relevant as low-frequency limit of en exact quantum formula.

Total power with quantum cutoff

Absence of classical cutoff corresponds to shrinking to zero of radiation formation length for massless charge. In quantum theory formation length can not be shorter than de Broglie length $\lambda_B = \hbar c/E$ therefore we cut on the quantum bound

$$\omega_{\max} = E/\hbar$$

obtaining

$$P_{\rm cut} = \frac{e^2\sqrt{3}\Gamma(2/3)}{4\pi\hbar^2}(3e\hbar HE)^{2/3}$$

This expression differs from the true quantum result only by numerical factor. It diverges as Planck's constant goes to zero as

$$\hbar^{-4/3}$$

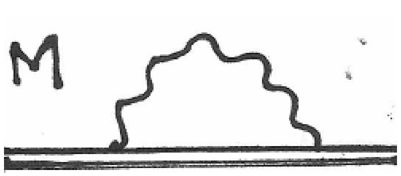
Non-analyticity in e and $\hbar$ indicates on non-perturbative nature of this result

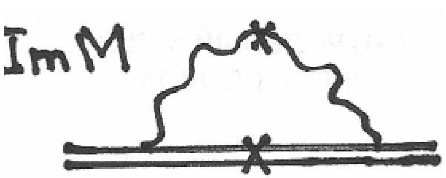
SR in quantum theory

- Start with exact solution of the Klein-Gordon (Dirac) equation in magnetic field producing the Landau spectrum (macroatom)

$$E = \sqrt{eH(2n+1) + p_z^2}$$

- Consider radiative transitions from n to n'
- Sum up over final states to get spectral-angular distribution and the total power
- For massive charges the detailed theory was developed by Sokolov, Klepikov, Ternov, Bagrov, Zhukovski, Borisov in 50--70-ies
- Later approaches: Schwinger, Baier, Ritus… -- using mass operator, summing over final quantum numbers implicitly





SR in massless QED: Schwinger approach

Exact calculation of one loop mass operator in massless scalar QED

The corresponding term in the action $-\frac{1}{2}\int \phi(x)M(x,x')\phi(x')dxdx'$

In Schwinger sympbolic notation the exact in H one-loop mass-operator of the massless charge reads

$$M = ie^2 \int \left[(\Pi - k)^\mu \frac{1}{k^2} \frac{1}{(\Pi - k)^2} (\Pi - k)_\mu \right] \frac{dk}{(2\pi)^4} \; - M_0$$

where $\Pi_\mu = -i\partial_\mu - eA_\mu,\;\; A_\mu$ stands for constant magnetic field H, and M_0 is the subtraction term. Exponentiating two propagators

$$\frac{1}{k^2}\frac{1}{(\Pi - k)^2} = -\int_0^\infty sds \int_0^1 \mathrm{e}^{-is\mathcal{H}}$$, with $$\mathcal{H} = (k - u\Pi)^2 - u(1-u)\Pi^2$$

one replaces integration over k is by averaging over states of the fictituos particle

$$M = ie^2 \int_0^\infty sds \int_0^1 du \langle \xi \big| (\Pi - k)^\mu \mathrm{e}^{-is\mathcal{H}} (\Pi - k)_\mu \big| \xi \rangle$$

treating $\mathcal{H}$ as Hamiltonian

Operator products are disentangled in Heisenberg representation for fictitious particle, and are taken on shell, i.e. $\phi(x) = \phi(\mathbf{r})\mathrm{e}^{-iEt}$ satisfying $\Pi^2\phi = 0$ with $E = \sqrt{eH(2n+1)}\,.$ The result reads

$$M = \frac{e^2}{4\pi}\int_0^1 du \int_0^\infty \frac{ds}{s}\left[\mathrm{e}^{-i\psi}\Delta^{-1/2}\left(E^2\Phi_1 + 4ieH\Phi_2 + i\Phi_3/s\right) - 2i/s\right]$$

$$\Phi_1 = 3 - 4u + u^2 - \frac{(1-u)^2}{\Delta}(4\cos 2x - 1) - \frac{u(1-u)}{x\,\Delta}\sin 2x\,,$$

$$\Phi_2 = \sin 2x - \frac{2u(1-u)\sin^2 x}{x\,\Delta}\cos 2x\,,$$

$$\Phi_3 = 1 + \frac{1-u}{\Delta}(2\cos 2x - 1) + \frac{u\sin 2x}{2x\,\Delta}(4\cos 2x - 3)\,,$$

where $x = eHsu$ and $\Delta = (1-u)^2 + u(1-u)\dfrac{\sin 2x}{x} + u^2\left(\dfrac{\sin x}{x}\right)^2$

$$\psi = (2n+1)[\beta - (1-u)x]\,,\quad \tan\beta = \left(\cot x + \frac{u}{x(1-u)}\right)^{-1}$$

This is true for all Landau levels n.

Quasiclassical motion n>>1

Simple analytical result can be obtained for high initial Landau levels n>>1. The imaginary part Im M (divided by -E) gives the total probability of radiation summed over all final n'. The integrals over x are computed in the leading approximation in $n^{-1/3}$ expanding the exponential and the rest of the integrand in powers of x. One gets

$$\Gamma = \frac{e^2}{4\pi E} \int_0^1 du \int_0^\infty \frac{dx}{x} \left(E^2 \Phi_1 \sin\psi \ + 2\frac{eHu}{x}(1-\cos\psi) \right)$$

where $\psi \approx (2n+1)\alpha x^3$. After x-integration one finds

$$\Gamma = \frac{e^2 \Gamma\left(2/3\right)\left(3eHE\right)^{2/3}}{8\pi\sqrt{3}E} \int_0^1 \frac{8 - 32u/3 + 19u^2/3 - 3u^3}{u^{2/3}(1-u)^{1/3}} du$$

and finally $$\Gamma = \frac{4e^2}{9E}\Gamma\left(2/3\right)\left(3eHE\right)^{2/3}$$ Decaying levels acquire the Imaginary parts of energy levels E - i Γ/2, so the spacing must be << Γ

This gives upper restriction $$n \ll \frac{81}{16\left[\Gamma\left(2/3\right)\right]^3}\frac{1}{\alpha^3} \approx 0.5 \cdot 10^7$$

Radiation spectrum

To get the spectral power of radiation one has to perform Fourrier decomposition inside the mass operator

$$P(\omega) = -\frac{\omega}{E}\,\mathrm{Im}\left(\int_{-\infty}^{\infty} \mathrm{e}^{i\omega\tau}\, M'\, \frac{d\tau}{2\pi}\right)$$

where the modified M is

$$M' = -ie^2\int_0^\infty s ds \int_0^1 du \langle\xi\big|(\Pi-k)^\mu \mathrm{e}^{-is\mathcal{H}}\mathrm{e}^{-ik^0\tau}(\Pi-k)_\mu\big|\xi\rangle$$

Calculations in the quasiclassical regime n>>1 give

$$P(\omega) = \frac{e^2 v}{4\pi E}\int_0^\infty \left(E^2(8-v^2)(1-v)^2 x\sin\psi\; + \frac{eHv}{x^2}(1-\cos\psi)\right)dx$$

where $v = \omega/E$. Integrating over x as before, one gets

$$P(\omega) = \frac{2e^2\,\Gamma(2/3)}{27\hbar E}\,(3e\hbar H E)^{2/3}\,\mathcal{P}(\hbar\omega/E)$$

where the normalized spectral distribution is introduced (red curve)

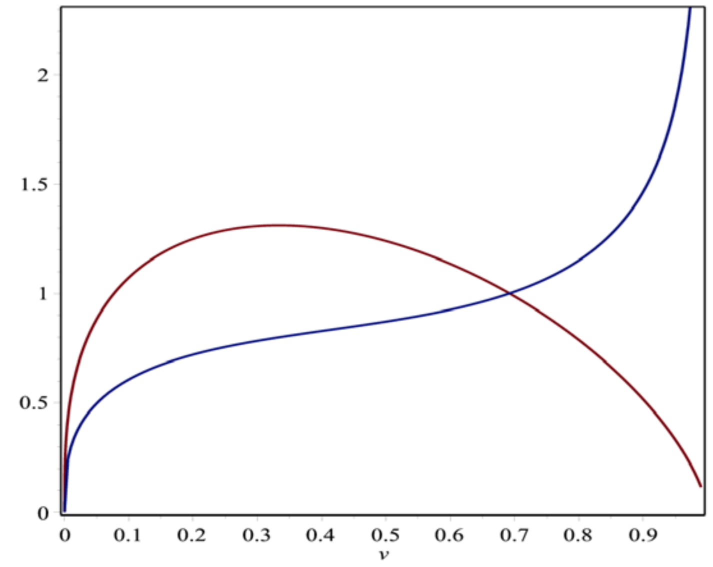
$$\mathcal{P}(v) = \frac{27}{2\pi\sqrt{3}}\,v^{1/3}(1-v)^{2/3}$$

The curve (red) has maximum $\hbar\omega_{\max} = \frac{1}{3}E$

The average photon energy

$$\langle\hbar\omega\rangle = E\int_0^1 v\mathcal{P}(v)\,dv = \frac{4}{9}E$$

For small frequencies the spectrum coincides with classical result.
The blue line shows the spectrum for spin 1/2



In the case of spin 1/2 calculations are essentially similar and lead to the following expression for the spectral power:

$$\mathcal{P}_{1/2}(v) = \frac{81\sqrt{3}}{64\pi}\, v^{1/3}(1-v)^{-1/3}(v^2-2v+2)\,. \qquad (14)$$

At the upper limit $v \to 1$ the spin 1/2 spectral power has an integrable divergence. The low-frequency limits are identical for both spins and coincide with the classical spectrum ($\hbar$ disappears):

$$P_{\rm cl}(\omega) = e^2 \frac{3^{1/6}}{\pi}\Gamma(2/3)\left(\frac{\omega}{\omega_H}\right)^{1/3}\omega_H\,, \qquad \omega_H = \frac{eH}{E}\,. \quad (15)$$

This power-low dependence exhibits ultraviolet catastrophe (no high-frequency cutoff), which is cured in quantum theory.

One can also investigate the case of vector massless particles, $s = 1$, but then the result is infinite: magnetized vector QED fails to describe radiation from massless vector charges. This could be expected in view of the results of Case and Gasiorovich,[5] who gave the arguments that electromagnetic interaction of massless charged particles with spin one and higher is controversial.

The total power is obtained integrating over the spectrum

$$P = \int_0^{E/\hbar} P(\omega) d\omega = \frac{2e^2\,\Gamma\,(2/3)}{27\hbar^2}\;(3e\hbar HE)^{2/3}$$

It has exactly the same functional form as the result of intergration of classical spectrum with quantum cut-off, differing only by the numerical factor of the order ½.

This quantity diverges for zero Planck's constant. Thus, synchrotron radiation form massless charge is essentially quantum, consisting of hard quanta of the order of particle energy. Remarkably, this does not depend on the value of magnetic field and the particle energy. Even in weak magnetic field of the Earth such particles would be observable by their radiation with universal spectrum.

No classical radiation reaction

- One consequence of quantum nature of SR from massless charge is that the radiation reaction problem becomes meaningless. Such an equation derived by Kazinski et al ('02) has strange features like non-lagrangian divergent terms and fifth derivative in the finite term. Meanwhile, massless limit in the usual Lorentz -Dirac equation diverges, like the Larmor formula. The reason is that quantum recoil makes the reaction problem stochastic.
- Moreover, in the sinchrotron radiation theory there is stronger restriction on the validity of classical radiation reaction equation due to excitation of the so-called betatron oscialltions. The threshold is

$$E_{\text{fluct}} \sim E_{1/5} = m \left(\frac{H_0}{H} \right)^{1/4}$$

- It is lower than the recoil treshold $E_{\text{recoil}} \sim m \frac{H_0}{H}$

SR emission of gravitons (flat space)

The charge also emits gravitons in Minkowski space in the framework of the linearized gravity $g_{\mu\nu} = \eta_{\mu\nu} + \varkappa h_{\mu\nu}$, with interaction

$$S_{\rm int} = -\frac{\varkappa}{2}\int h_{\mu\nu} T^{\mu\nu}$$

where $T^{\mu\nu} = T^{\mu\nu}_{\rm p} + T^{\mu\nu}_{F}$

Second term is Maxwell, it is needed to ensure conservation equation $\partial_\mu T^{\mu\nu} = 0$. The Maxwell field must be the sum of the external (magnetic) field and the retarded field of the charge

$$T^{\mu\nu}_{F} = \frac{1}{4\pi}\left(F^{\mu\lambda}\mathcal{F}_{\lambda}^{\ \nu{\rm ret}} + \mathcal{F}^{\mu\lambda{\rm ret}}F_{\lambda}^{\ \nu} - \frac{1}{2}\eta^{\mu\nu}F^{\kappa\lambda}\mathcal{F}^{\rm ret}_{\kappa\lambda}\right)$$

The retarded field has to be further split as $\mathcal{F}^{\rm ret}_{\mu\lambda} = \mathcal{F}^{\rm self}_{\mu\lambda} + \mathcal{F}^{\rm rad}_{\mu\lambda}$ to account for resonant transformation of EM wave to GW in the homogeneous magnetic field. One needs to keep only linear term in the retarded field of the charge (quadratic is self-energy like)

EM-GW tansformation (Gerzenshtein effect)

- Due to linearity of the Maxwell source term in the retarded field is splits as $T_F^{\mu\nu} = t^{\mu\nu} + S^{\mu\nu}$ where

$$t^{\mu\nu} = \frac{1}{4\pi}\left(F^{\mu\lambda}\mathcal{F}_{\lambda}^{\ \nu\mathrm{rad}} + \mathcal{F}^{\mu\lambda\mathrm{rad}}F_{\lambda}^{\ \nu} - \frac{1}{2}\eta^{\mu\nu}F^{\kappa\lambda}\mathcal{F}_{\kappa\lambda}^{\mathrm{rad}}\right)$$

$$S^{\mu\nu} = \frac{1}{4\pi}\left(F^{\mu\lambda}\mathcal{F}_{\lambda}^{\ \nu\mathrm{self}} + \mathcal{F}^{\mu\lambda\mathrm{self}}F_{\lambda}^{\ \nu} - \frac{1}{2}\eta^{\mu\nu}F^{\kappa\lambda}\mathcal{F}_{\kappa\lambda}^{\mathrm{self}}\right)$$

- The first is trivially conserved, while the second is conserved together with the mass term, i.e. S acts the non-local source of GW.
- The corresponding two GW amplitudes do not interfere and can be considered separately. The first source is EM-GW transformation
- In the magnetic field (Gertsenshtein effect),

$$\frac{dP_{\mathrm{res}}}{d\Omega} = GB^2R^2\,\frac{dP_{\mathrm{em}}}{d\Omega}$$

- so the same considerations apply to it: radiation is quantum.

Proper gravitational radiation

The second part of gravitational radiation (genuine GW) is generated by the sum of the sources $T^{\mu\nu} = T_{\mathrm{p}}^{\mu\nu} + S^{\mu\nu}$ has the spectrum falling with frequency

$$\frac{dP_{GW}}{d\omega} = \frac{\Gamma(1/3)\, 3^{5/6} G E^2 \omega_H}{2\pi} \left(\frac{\omega_H}{\omega}\right)^{1/3}$$

which is falling with frequency, but is still non-integrable, and quantum cut-off is thus needed.

“Quantum” spectrum

- The normalized graviton spectrum is

$$\mathcal{P}_{GW} = \frac{2}{3}\left(\frac{\hbar}{E}\right)^{2/3}\theta(E-\hbar\omega)\,\omega^{-1/3}$$

- and the average energy of emitted graviton

$$\langle\hbar\omega\rangle = \int_0^{E/\hbar}\hbar\omega\mathcal{P}_{GW}\,d\omega = \frac{2}{5}E$$

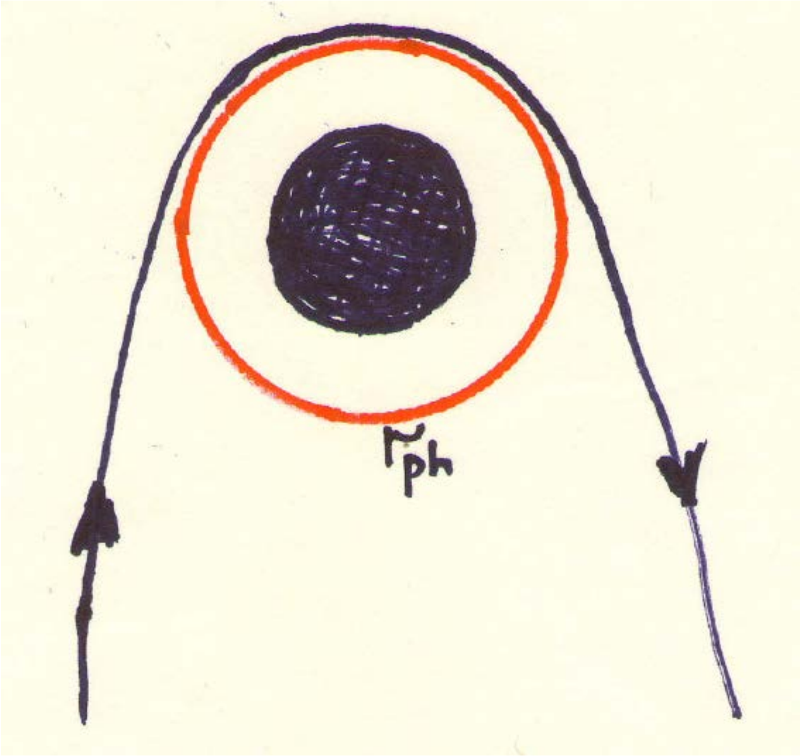
so radiation is again hard.

Thus, graviton emission by massless non-gravitating source computed within the linearized gravity in Minkowski space-time is essentially quantum process. Large recoil precludes possibility of classical description of back reaction a la Lorentz-Dirac.

Gravitational synchrotron radiation in non-linear GR

- One can imagine flows of photons scattered at large angle on black holes, their radiation can be estimated knowing power of GSR

- Gravitational radiation is described by the solution of the Teukolsky equation. Fluxes going to infinity and to black hole are the same, so we need to calculate only the Weyl Newman-Penrose scalar ψ_4



Relastivistic orbits near black holes

Time-like geodesics $$\left(\frac{dr}{d\tau}\right)^2 + U(r) = 0 \quad U(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{L^2}{r^2} + 1\right) - \gamma^2$$

$$\frac{d\phi}{d\tau} = \frac{L}{r^2}, \qquad \frac{dt}{d\tau} = \gamma\left(1 - \frac{2M}{r}\right)^{-1}$$

Circular orbits: $U(r_\mathrm{p}) = 0 = U'(r_\mathrm{p})$ lead to

$$\gamma = \left(1 - \frac{2M}{r_\mathrm{p}}\right)\left(1 - \frac{3M}{r_\mathrm{p}}\right)^{-1/2}, \quad \frac{L}{\gamma} = (Mr_\mathrm{p})^{1/2}\left(1 - \frac{2M}{r}\right)^{-1}$$ and

$$\omega_0 = \frac{d\phi}{dt} = \left(\frac{M}{r_\mathrm{p}}\right)^{1/2}, \quad \frac{dt}{d\tau} = \left(1 - \frac{3M}{r_\mathrm{p}}\right)^{-1/2}$$, orbits $3M < r_\mathrm{p} < 4M$

are unstable and jump to large angle scattering orbits with impact

$$b = \frac{L}{(\gamma^2 - 1)^{1/2}} = r_\mathrm{p}\left(\frac{4M}{r_\mathrm{p}} - 1\right)^{-1/2}$$

For $\gamma \gg 1$ unbound orbits close to $b = 3\sqrt{3}M$ make multiple revolutions

GSR from massive particles

- Null circular orbit is at $r_{\rm ph} = 3M$ (light ring)
- Massive ultrarelativistic orbit are close $r_{\rm p} = (3+\delta)M$
  so that $\gamma^2 = \dfrac{1}{3\delta}$
- Spectrum of GSR of different spins from massive ultrarelativistic particle was calculated Misner, Brill, Ruffini, Breuer, Chrzanowsky …) in 70-ies, it has a cut-off at the harmonic $m_{\rm cr} = \dfrac{12}{\pi}\gamma^2$ of the rotation frequency. The total power for spin two was computed in DG and Matiukhin, Sov J Nucl Phys v.45,555 ('87)

$$P_{GSR} = \frac{6{\rm e}^{-\pi/4}(r_{\rm p} - M)\,|\Gamma(1/4 + i/4)|^2}{\pi^{3/2} r_{\rm p}^2 (r_{\rm p} + 3M)} (\mu\gamma)^2 \ln(\gamma)$$

It diverges in the massless limit

Total radiation power from massless point particle at a circular orbit

$$P_{\rm GSR} = k\frac{E^2\omega_0}{M}\sum_1^{\infty}\frac{1}{m}$$

where the sum has no frequency cut-off and diverges

Introducing quantum cut-off $m_{\rm max} = \dfrac{E}{\hbar\omega_0}$, we get

$$P_{\rm GSR} = k\frac{E^2\omega_0}{M}\sum_1^{m_{\rm max}}\frac{1}{m} = k\frac{E^2\omega_0}{M}\ln\left(\frac{E}{\hbar\omega_0}\right)$$

Energy loss per revolution is $\Delta E_{\rm GSR} = k_1\dfrac{E^2}{M}\ln\left(\dfrac{E}{\hbar\omega_0}\right)$

and the radiation efficiency is $\epsilon = \dfrac{\Delta E}{E} = k_1\dfrac{E}{M}\ln\left(\dfrac{E}{\hbar\omega_0}\right)$

This is small in most astrophysical conditions, but without invoking

quantum theory, it would be infinite!

Conclusions

Famous soft photon/graviton theorems derived by Weinberg using perturbative Feynman diagrams are reproduced as Ward identities associated with Bondi-Metzner-Sachs symmetries of the world populated by massless particles

BMS supertranslations provide novel view on memory effects in gravitational and electromagnetic radiation

They also opens a new way to solve the black hole entropy/information problem introducing new (quantum) hair on black holes associated with BMS on event horizon looking as condensates of soft gravitons

Problem of radiation from massless particles is solved positively, showing that it non-zero and finite in quantum theory. Classical theory gives correct low-frequency spectrum but it diverges in the UV.
The same is true for massless particles emitting gravitational waves

It follows that pointlike massless particles do not move along null geodesics as assumed in General Relativity.