Exotic models of the very early Universe

Valery Rubakov

Institute for Nuclear Research
of the Russian Academy of Sciences,

Department of Particle Physics and Cosmology
Physics Faculty
Moscow State University

Outline

- Violating NEC: generalized Galileons (aka Horndeski)
- Towards Genesis
  and bouncing Universe
- “No-go”
- Ways to repair
  - Higher order terms
  - Modified Genesis
- Intermediate summary
- Inhomogeneous Universe
  - Toy model
  - General setting
  - Summary

Can the Null Energy Condition be violated in a simple and healthy way?

- Folklore until fairly recently: NO!

  Today: YES

Senatore' 2004;
V.R.' 2006;
Creminelli, Luty, Nicolis, Senatore' 2006

General properties of non-pathological
NEC-violating field theories:

Non-standard kinetic terms

Non-trivial background, instability of Minkowski space-time

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Example: scalar field, generalized Galileon $\pi(x^\mu)$,

$$L = F(X,\pi) + K(X,\pi)\cdot\Box\pi$$

$$\Box\pi \equiv \nabla_\mu\nabla^\mu\pi\ , \qquad X = (\partial_\mu\pi)^2$$

- Second order equations of motion (but L cannot be made first order by integration by parts)
- Generalization: Horndeski theory (1974) rediscovered several times

Fairlie, Govaerts, Morozov' 91;
Nicolis, Rattazzi, Trincherini' 09, ...

Minkowski:

$$L_n = K_n(X,\pi)\partial^{\mu_1}\partial_{[\mu_1}\pi\cdot\dots\cdot\partial^{\mu_n}\partial_{\mu_n]}\pi$$

Five Lagrangians in 4D, including $K_0 \equiv F$

Generalization to GR: L_0, L_1 trivial, $L_{n>1}$ non-trivial

Deffayet, Esposito-Farese, Vikman' 09

Simple playground

$$L = F(Y) \cdot e^{4\pi} + K(Y) \cdot \Box\pi \cdot e^{2\pi}$$

$$\Box\pi \equiv \nabla_\mu \nabla^\mu \pi \,, \qquad Y = e^{-2\pi} \cdot (\partial_\mu \pi)^2$$

Deffayet, Pujolas, Sawicki, Vikman' 2010
Kobayashi, Yamaguchi, Yokoyama' 2010

- Second order equations of motion
- Scale invariance: $\pi(x) \to \pi'(x) = \pi(\lambda x) + \ln\lambda$.

(technically convenient)

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Towards Genesis: homogeneous solution in Minkowski space (attractor)

$$e^{\pi_c} = \frac{1}{\sqrt{Y_*}(t_* - t)}$$

- $Y \equiv e^{-2\pi_c} \cdot (\partial_\mu \pi_c)^2 = Y_* =$ const, a solution to

$$Z(Y_*) \equiv -F + 2Y_*F' - 2Y_*K + 2Y_*^2K' = 0$$

$$' = d/dY \,.$$

Energy density

$$\rho = e^{4\pi_c} Z = 0$$

Effective pressure T_{11}:

$$p = e^{4\pi_c}\,(F - 2Y_*K)$$

Can be made negative by suitable choice of $F(Y)$ and $K(Y)$
$\Longrightarrow \rho + p < 0$, violation of the Null Energy Condition.

Turning on gravity

$$p = \mathrm{e}^{4\pi_c}(F - 2Y_* K) = -\frac{M^4}{Y_*^2 (t_* - t)^4}\,, \qquad \rho = 0$$

M: mass scale characteristic of π

- Use $\dot{H} = -4\pi G(p + \rho) \Longrightarrow$

$$H = \frac{4\pi}{3} \frac{M^4}{M_{Pl}^2 Y_*^2 (t_* - t)^3}$$

$$\rho \sim M_{Pl}^2 H^2 \sim \frac{1}{M_{Pl}^2 (t_* - t)^6}$$

Initial stage of Genesis

Early times $\Longrightarrow$ weak gravity, $\rho \ll p$.

Expansion, $H \neq 0$, is negligible for dynamics of π.

Perturbations about homogeneous Minkowski solution

$$\pi(x^\mu) = \pi_c(t) + \delta\pi(x^\mu)$$

- Quadratic Lagrangian for perturbations:

$$L^{(2)} = \mathrm{e}^{2\pi_c} Z' (\partial_t \delta\pi)^2 - B(\vec{\nabla}\delta\pi)^2 + W(\delta\pi)^2$$

$V = V[Y; F, K, F', K', K'']$. Absence of ghosts:

$$Z' \equiv dZ/dY > 0$$

No gradient instabilities and superluminal propagation

$$B > 0 \; ; \qquad B < \mathrm{e}^{2\pi_c} Z'$$

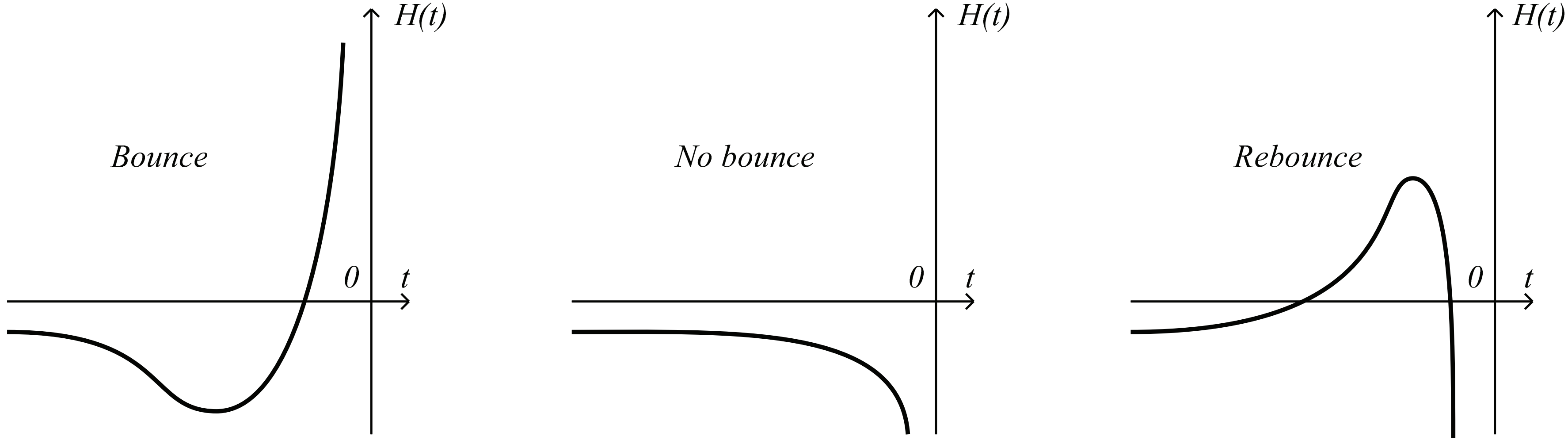
Can be arranged. Initial stage of Genesis healthy.

Bouncing Universe: marry ekpyrosis (e.g., scalar field with negative exponential potential) with Galileon.

Ekpyrotic field dominates at very early contraction epoch, then Galileon takes over, violates the NEC and produces bounce.

All this can happen in weak gravity regime.

Osipov, V.R. ’2013



Or simply make use of specially designed generalized Galileon

Ijjas, Steinhardt ’2016

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Further evolution

Taken at face value: Big Rip singularity

$$a(t) = e^{\frac{\epsilon}{M_{Pl}^2 (t_0 - t)^2}} \,, \quad \epsilon = \text{const} \gg 1 \,.$$

Known attempts to construct "complete" Genesis and bouncing cosmologies: either Big Rip singularity, $\pi = \infty$, $H = \infty$

Creminelli, Nicolis, Trincherini '2010

or gradient instability

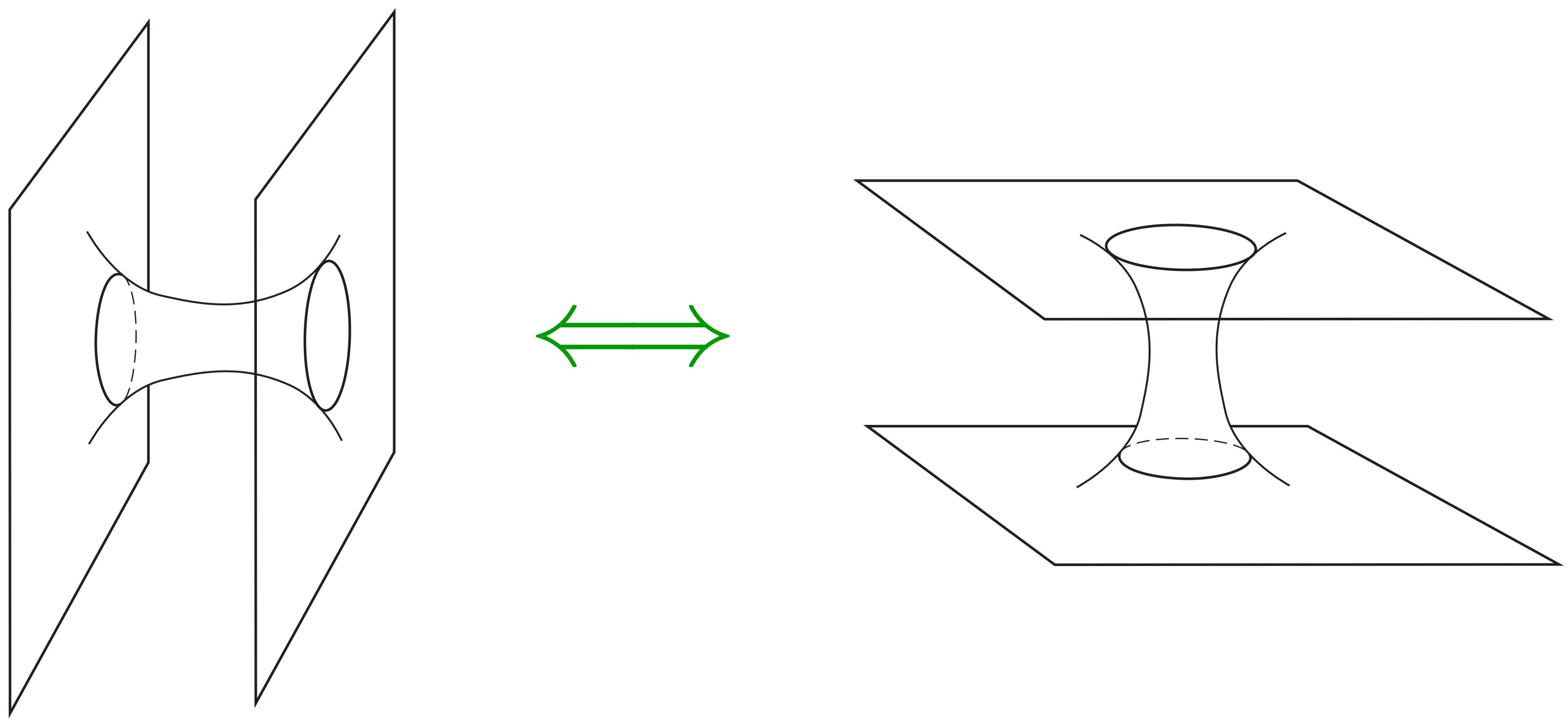
Cai, Easson, Brandenberger '2012;
Koehn, Lehners, Ovrut '2013;
Pirtskhalava, Santoni, Trincherini, Uttayarat '2014;
Qiu, Wang '2015;
Kobayashi, Yamaguchi, Yokoyama '2015;
Sosnovikov '2015

Is instability generic
or just a drawback of models constructed so far?

Can one construct healthy classical bounce and/or Genesis?

Another facet: Lorenzian traversable wormholes. Also require NEC violation.

Static wormhole $\Longleftrightarrow$ Bouncing Universe



Suspicious: wormhole $\Longrightarrow$ time machine.

No stable, static, spherically symmetric wormholes in generalized Galileon theories: always ghosts.

V.R. ’2016

Generalized Galileons

- Lagrangian

$$L = F(X,\pi) + K(X,\pi)\cdot\Box\pi$$

$$\Box\pi \equiv \nabla_\mu\nabla^\mu\pi\,, \qquad X = (\partial_\mu\pi)^2$$

- Galileon field equation

$$\left(-2F_X + 2K_\pi - 2K_{X\pi}\nabla_\mu\pi\nabla^\mu\pi - 2K_X\Box\pi\right)\Box\pi + (-4F_{XX} + 4K_{X\pi})\nabla^\mu\pi\nabla^\nu\pi\nabla_\mu\nabla_\nu\pi$$
$$-4K_{XX}\nabla^\mu\pi\nabla^\nu\pi\nabla_\mu\nabla_\nu\pi\Box\pi + 4K_{XX}\nabla^\nu\pi\nabla^\lambda\pi\nabla_\mu\nabla_\nu\pi\nabla^\mu\nabla_\lambda\pi + 2K_X\nabla^\mu\nabla^\nu\pi\nabla_\mu\nabla_\nu\pi$$
$$+2K_X R_{\mu\nu}\nabla^\mu\pi\nabla^\nu\pi + \ldots = 0\,;$$

where $F_\pi = \partial F/\partial\pi$, etc. Dots = less than two derivatives.

- Subtlety: Galileon E.O.M. involves second derivatives of metric. Likewise, Einstein eqs. involve second derivatives of Galileon. Same for linearized eqs. for perturbations.

- Quadratic effective action for Galileon perturbations not immediate even for high momentum/frequency modes.

- Trick: use Einstein equations to trade second derivatives of metric for second derivatives of Galileon.

Deffayet, Pujolas, Sawicki, Vikman '2010

equivalent to

Kobayashi, Yamaguchi, Yokoyama '2010

- Galileon perturbations χ about background π: resulting quadratic Lagrangian

$$
\begin{aligned}
L^{(2)} = & \left[F_X + K_X \Box\pi - K_\pi + \nabla_\nu (K_X \nabla^\nu \pi)\right] \nabla_\mu\chi\nabla^\mu\chi \\
& + \left[2(F_{XX} + K_{XX}\Box\pi)\nabla^\mu\pi\nabla^\nu\pi - 2(\nabla^\mu K_X)\nabla^\nu\pi - 2K_X\nabla^\mu\nabla^\nu\pi\right]\nabla_\mu\chi\nabla_\nu\chi \\
& - \kappa K_X^2 X^2 \nabla_\mu\chi\nabla^\mu\chi + 4\kappa K_X^2 X \nabla^\mu\pi\nabla^\nu\pi\nabla_\mu\chi\nabla_\nu\chi \, .
\end{aligned}
$$

$\kappa = 8\pi G$.

Specifying to spatially flat, homogeneous isotropic Universe,

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$$

- Energy-momentum

$$\rho = 2F_X X - F - K_\pi X + 6HK_X\dot{\pi}^3 ,$$
$$p = F - 2K_X X\ddot{\pi} - K_\pi X .$$

- Largangian for perturbations:

$$L^{(2)} = A\dot{\chi}^2 - \frac{1}{a^2}B(\partial_i\chi)^2 + \ldots$$

with

$$A = F_X + 2F_{XX}X - K_\pi - K_{X\pi}X + 6H\dot{\pi}(K_X + K_{XX}X) + 3\kappa K_X^2 X^2 ,$$
$$B = F_X - K_\pi + 2K_X\ddot{\pi} + K_{X\pi}X + 2K_{XX}X\ddot{\pi} + 4HK_X\dot{\pi} - \kappa K_X^2 X^2$$

No-go

Libanov, Mironov, V.R. 2016

No ghosts, gradient instabilities:

$$A > 0 , \quad B > 0$$

Use Friedmann equations to get

$$B\dot{\pi}^2 = \frac{d}{dt}\left(K_X\dot{\pi}^3 - \frac{1}{\kappa}H\right) - \kappa K_X\dot{\pi}^3\left(K_X\dot{\pi}^3 - \frac{1}{\kappa}H\right) .$$

Introduce combination

$$\mathscr{R} = a^{-1}\left(K_X\dot{\pi}^3 - \frac{1}{\kappa}H\right)$$

Then

$$\frac{B\dot{\pi}^2}{a} = \dot{\mathscr{R}} - \kappa a \mathscr{R}^2 .$$

$B > 0 \implies \dot{\mathscr{R}} - \kappa a \mathscr{R}^2 > 0.$ NB: one must have $\dot{\mathscr{R}} > 0.$

Integrate $\dot{\mathscr{R}}/\mathscr{R}^2 - \kappa a > 0$ from t_i to $t_f > t_i$:

$$\frac{1}{\mathscr{R}(t_i)} - \frac{1}{\mathscr{R}(t_f)} > \kappa \int_{t_i}^{t_f} dt \; a(t) \; .$$

Bouncing scenario, standard Genesis with usual expansion in the end:

$$\int_{-\infty}^{t_f} dt \; a(t) = \infty \, , \quad \int_{t_i}^{\infty} dt \; a(t) = \infty \; .$$

- Suppose $\mathscr{R}(t_i) > 0$. Then at $t > t_i$ one has $\mathscr{R}(t) > 0$ (remember $\dot{\mathscr{R}} > 0$!).

  $$\frac{1}{\mathscr{R}(t_f)} < \frac{1}{\mathscr{R}(t_i)} - \kappa \int_{t_i}^{t_f} dt \; a(t) \; .$$

  Right hand side changes sign at some $t_f \implies \mathscr{R}(t_f) = \infty$, singularity.

- Case $\mathscr{R}(t) < 0$:

$$\frac{1}{\mathscr{R}(t_i)} > \frac{1}{\mathscr{R}(t_f)} + \kappa \int_{t_i}^{t_f} dt\ a(t)\ .$$

$\Longrightarrow$ singularity in the past. Either singularity or gradient/ghost instability

  - Argument generalized to all Horndeski theories

    Kobayashi '2016

  - Argument intact in presence of matter (obeying NEC) which interacts with Galileon only gravitationally.

  - Extends to model with extra conventional scalar ϕ and

$$L = -\frac{1}{2\kappa}R + F(\pi, X, \phi) + K(\pi, X, \phi)\Box\pi + F_\phi(X_\phi, \phi)$$

    where $X_\phi = (\partial\phi)^2$.

    Kolevatov, Mironov '2016

  - Similar argument forbids wormholes (problem with A)

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Are there ways to repair?

- Bouncing scenario at 2-derivative level: not that I know of.

  Analogy to wormholes suggests that it should be hard, if at all possible.

  Way out: cure gradient instability by higher order terms in Galileon effective action $\Longleftrightarrow$ short duration of instability period, low cutoff scale of effective Galileon theory.

$$L = F(X,\pi) + K(X,\pi)\cdot\Box\pi - \frac{1}{\Lambda^2}(\Box\pi)^2$$

  Action for perturbations $\pi + \chi$ with gradient instability:

$$L^{(2)} = A\dot{\chi}^2 + \frac{1}{a^2}|B|(\partial_i\chi)^2 + \frac{1}{\Lambda^2}(\partial_i^2\chi)^2$$

  Valid for $|\mathbf{k}| \ll \Lambda$.

$$\omega^2 = \frac{1}{A}\left(-|B|\mathbf{k}^2 + \frac{1}{\Lambda^2}\mathbf{k}^4\right)$$

For $A, |B| \sim 1$ time scale of instability $t_{inst} \sim \Lambda^{-1}$. For small $|B|$ even shorter.

Arrange model in such a way that istability lasts less than t_{inst}.

Possible, though contrived

Pirtskhalava, Santoni, Trincherini, Uttayarat '2014; Koehn, Lehners, Ovrut '2015

Work in progress towards "complete" bouncing model.

- Genesis: way out

$$\int_{-\infty}^{t} dt\ a(t) = \text{ finite}$$

Elaborate. One still has

$$\int_{t}^{\infty} dt\ a(t) = \infty\ .$$

Thus, above argument requires $\mathscr{R} < 0$. Introduce

$$Q \equiv a(t)\mathscr{R} = K_X \dot{\pi}^3 - \frac{1}{\kappa} H < 0$$

Thus, we must have

$$H > \kappa K_X \dot{\pi}^3$$
$$\kappa K_X \dot{\pi}^3 |Q| > |\dot{Q}| \,.$$

Simple option: power law behavior of Q as $t \to -\infty \quad \Longrightarrow$

$$K_X \dot{\pi}^3 \propto |t|^{-1} \,, \quad H = \frac{h}{|t|} \Longrightarrow a = \frac{1}{|t|^h} \,, \; h > 1 \,.$$

Both energy density and pressure behave like $|t|^2$ as $t \to -\infty$.
Equation of state

$$p = \left(-1 - \frac{2}{3h}\right)\rho \,, \qquad w = -1 - \frac{2}{3h} \text{ instead of } w = -\infty \,.$$

Galileon energy-momentum, space-time curvature tend to zero as $t \to -\infty \Longrightarrow$ modified Genesis.

Modified Genesis

$$L = -\frac{1}{2\kappa}R - f^2(\partial\pi)^2 + \alpha_0 \mathrm{e}^{-2\pi}(\partial\pi)^4 + \beta_0 e^{-2\pi}(\partial\pi)^2 \cdot \Box\pi$$

Scaling of Galileon action under $\pi(x) \to \pi(\lambda x) + \ln\lambda$,
$g_{\mu\nu}(x) \to g_{\mu\nu}(\lambda x)$: $S \to \lambda^{-2}S$, same as Einstein–Hilbert.
[Cf. original Genesis

$$L = -\frac{1}{2\kappa}R - f^2\mathrm{e}^{2\pi}(\partial\pi)^2 + \alpha_*(\partial\pi)^4 + \beta_*(\partial\pi)^2 \cdot \Box\pi\,, \qquad S \to S \quad]$$

Solution at large negative t:

$$\mathrm{e}^{\pi} = \frac{1}{H_* \cdot |t|}\,, \qquad H = \frac{h}{|t|}$$

Constants H_*, h expressed through f, α_0, β_0.
Stability, consistency with putative further evolution $(Q < 0)$:

$$1 < \kappa\beta_0 H_*^2 < h \quad \Longleftrightarrow \quad \text{choice of } \; f, \alpha_0, \beta_0\,.$$

- Initial modified Genesis solution can be continued to "conventional" cosmology with π behaving as "normal" massless scalar field, $L = (\partial\pi)^2$.
- Reheating: similar to k-essence.
- Peculiarity: past geodesic incompleteness.

$$\int_{-\infty}^{t} a(t)\, dt < \infty$$

$\Longleftrightarrow$ backward geodesics reach $r = \infty$ in finite proper time

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Intermediate summary

Homogeneous and isotropic Universe:

- Bouncing cosmology particularly difficult
- Genesis is easier, but also pretty hard
- Exotic cosmology needs exotic fields that violate NEC.

  Interesting candidates: generalized Galileons.

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Inhomogeneous Universe

Primordial scalar perturbations

- Generated at some stage preceding the hot epoch
- Gaussian (or nearly Gaussian) random field $\zeta(\vec{x})$, with nearly flat (nearly Harrison–Zeldovich) power spectrum
  - Gaussianity suggests the origin: enhanced vacuum fluctuations of some (almost) free quantum field

Mukhanov's lectures

There must be some symmetry behind flatness of spectrum

- Inflation: symmetry of de Sitter space-time, $SO(4,1)$: spatial dilatations supplemented by time translations

$$\vec{x} \to \lambda \vec{x}\,, \quad t \to t - \frac{1}{2H} \log \lambda$$

+ nearly time-independent inflaton ϕ, $\dot{\phi}$

- Alternative: conformal symmetry $SO(4,2)$

Conformal group includes dilatations, $x^\mu \to \lambda x^\mu$.

$\Longrightarrow$ No scale, good chance for flatness of spectrum

First mentioned by Antoniadis, Mazur, Mottola' 97
Concrete models: V.R.' 09;
Creminelli, Nicolis, Trincherini' 10
General analysis: Libanov, Mironov, V.R.' 11;
Hinterbichler, Khoury' 11;
Hinterbichler, Joyce, Khoury' 12

What if our Universe started off from or passed through
an unstable conformal state
and then evolved to much less symmetric state we see today?

In line with developments in Quantum Field Theory

$N = 4$ super Yang–Mills; adS/CFT correspondence ...

Toy model: conformal rolling

V.R. '2009

- Main requirement: long evolution before the hot stage.

But otherwise insensitive to regime of cosmological evolution. Can work at inflation and its alternatives.

Model:

$$S = S_{G+M} + S_\phi$$

S_{G+M}: gravity plus dominating matter

S_ϕ: conformal complex scalar field ϕ with negative quartic potential. Spectator until late epoch.

$$S = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi + \frac{R}{6} |\phi|^2 - (-h^2 |\phi|^4) \right]$$

Conformal symmetry. Global symmetry $U(1)$. $\phi = 0$: unstable state with unbroken conformal symmetry. [Conformal symmetry explicitly broken at large fields. To be discussed later.]

Homogeneous and isotropic Universe, $ds^2 = a^2(\eta)[d\eta^2 - d\vec{x}^2]$:

In terms of the field $\chi(\eta,\vec{x}) = a(\eta)\phi(\eta,\vec{x}) = \chi_1 + i\chi_2$,
evolution is Minkowskian,

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\chi - 2h^2|\chi|^2\chi = 0$$

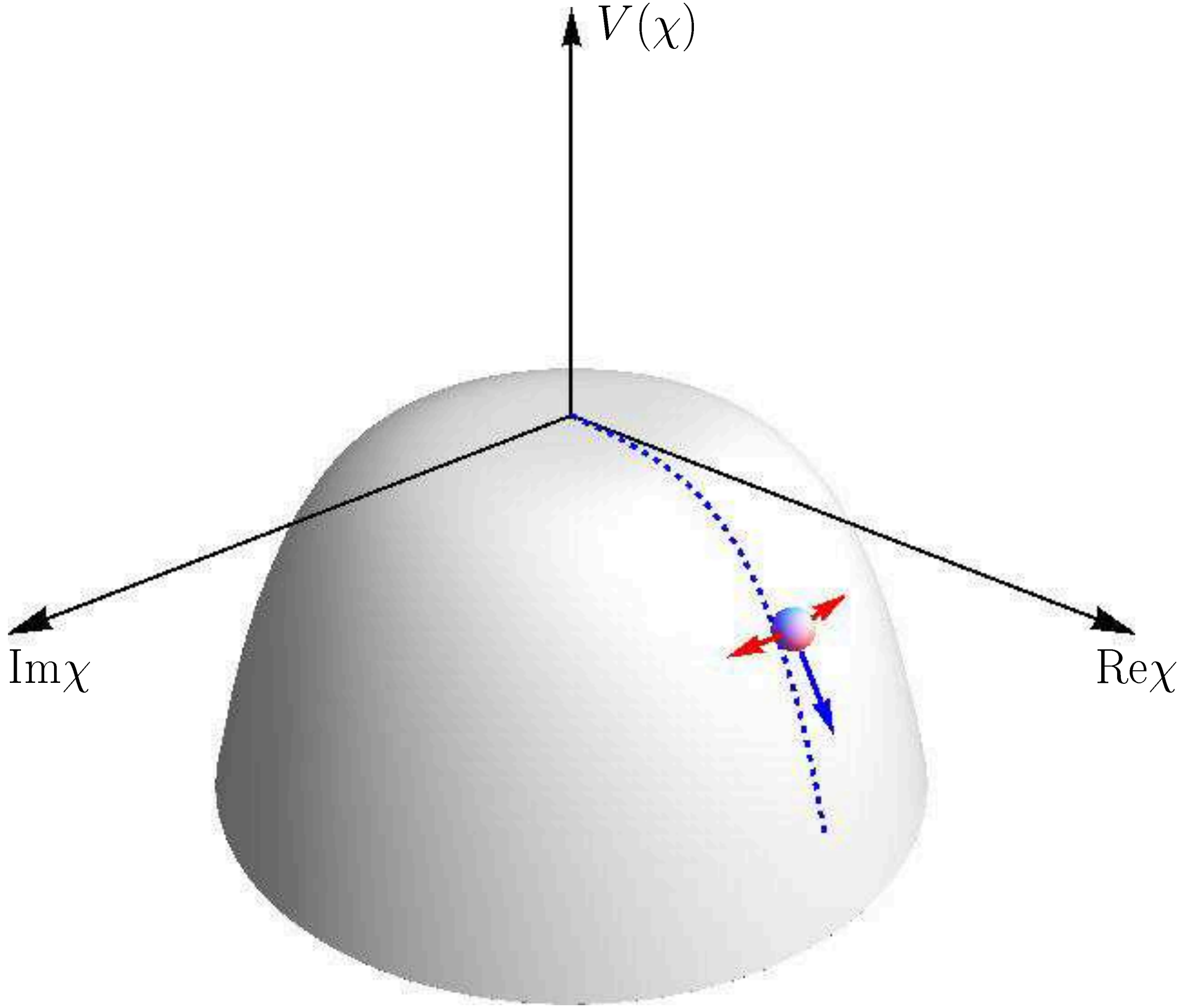
Homogeneous background solution, breaks $SO(4,2) \to SO(4,1)$

Attractor (real without loss of generality)

$$\chi_c(\eta) = \frac{1}{h(\eta_* - \eta)}$$

$\eta_* =$ constant of integration, end time of roll.

NB: Particular behavior $\chi_c \propto (\eta_* - \eta)^{-1}$
dictated by conformal symmetry.


$V(\chi)$
Imχ
Reχ

Fluctuations of Arg χ

automatically have flat power spectrum

Linearized equation for fluctuation $\delta\chi_2 \equiv \mathrm{Im}\chi$. Mode of 3-momentum k:

$$\frac{d^2}{d\eta^2}\delta\chi_2 + k^2\delta\chi_2 - 2h^2\chi_c^2\delta\chi_2 = 0$$

[recall $h\chi_c = 1/(\eta_* - \eta)$]. Very similar to massless scalar at inflaton.

Regimes of evolution:

Early times, $k \gg 1/(\eta_* - \eta)$, short wavelength regime,

χ_c negligible, free Minkowskian field

Late times, $k \ll 1/(\eta_* - \eta)$, long wavelength regime,

Phase $\delta\theta = \delta\chi_2/\chi_c$ freezes out with flat power spectrum:

$$\mathscr{P}_{\delta\theta} = \frac{h^2}{(2\pi)^2}$$

This is automatic consequence of global $U(1)$ and conformal symmetry

Comments:

- Mechanism requires long cosmological evolution: need

$$(\eta_* - \eta) \gg 1/k$$

  early times, short wavelength regime,
  well defined QFT vacuum

  - For $k \sim H_0$ this is precisely the requirement that the horizon problem is solved, at least formally.
  - This is a pre-requisite for most mechanisms that generate perturbations

- Small explicit breaking of conformal invariance $\Longrightarrow$ tilt of the spectrum

  Osipov, V.R. '2011

  Depends both on the way conformal invariance is broken and on the evolution of scale factor

- ϕ need not be spectator $\Longrightarrow$ Pseudo-conformal model with contracting phase of ekpyrotic type

  Hinterbichler, Khoury '2011

- adS/CFT interpretation of $V = -h^2\phi^4$ and $SO(4,2) \to SO(4,1)$ in terms of 3-brane in adS$_5$

  Hinterbichler, Stokes, Trodden '2014;
  Libanov, V.R., Sibiryakov '2014; Libanov, V.R. '2015

- Spectator scalar field in Galileon background has similar properties in Genesis models

  Creminelli, Nicolis, Trincherini '2010

Similarity is not an accident

Libanov, Mironov, V.R.
Hinterbichler, Joyce, Khoury

General setting:

- Effectively Minkowski space-time
- Conformally invariant theory
- Field ρ of conformal weight $\Delta \neq 0$
  - $\rho = \text{const} \cdot |\phi|$ in conformal rolling model
  - $\rho = \text{const} \cdot \mathrm{e}^{\pi}$ in Galilean Genesis; $\Delta = 1$ in both models.

Homogeneous classical solution

$$\rho_c(t) = \frac{1}{(t_* - t)^\Delta}$$

by conformal invariance.

- Another scalar field θ of effective conformal weight 0.
- Kinetic term dictated by conformal invariance (modulo field rescaling)

$$L_\theta = \rho^{2/\Delta}(\partial_\mu \theta)^2$$

- If potential terms negligible $\Longrightarrow$ θ develops perturbations with flat power spectrum. Automatic for Nambu–Goldstone field.

  Assume that conformal evolution ends up at some late time. Phase perturbations get converted into adiabatic perturbations by, e.g., modulated decay mechanism

Dvali, Gruzinov, Zaldarriaga' 03
Kofman' 03

or (pseudo-Goldstone) curvaton mechanism

Linde, Mukhanov' 97;
Enqvist, Sloth' 01; Lyth, Wands' 01; Moroi, Takahashi' 01;
K. Dimopoulos et. al.' 03

In either case

$$\zeta = \text{const} \cdot \delta\theta \ + \ \text{possible non-linear terms}$$

Adiabatic perturbations inherit shape of power spectrum and correlation properties from $\delta\theta$, plus possible additional non-Gaussianity.

Interaction between θ-perturbations and ρ-perturbations produce non-Gaussianities of specific forms (again dictated by conformal symmetry).

No tensor perturbations = primodial gravity waves

Summary on perturbations

- Observed Gaussianity of scalar perturbations suggests their origin: enhanced vacuum fluctuations of some (almost) free quantum field

- Flatness of scalar power spectrum may be a consequence of a symmetry: $SO(4,1)$ in inflationary theory or $SO(4,2) \to SO(4,1)$ in conformal models.

More options:

Matter bounce, Finelli, Brandenberger' 01.

Negative exponential potential, Lehners et. al.' 07;
Buchbinder, Khouri, Ovrut' 07; Creminelli, Senatore' 07.

Lifshitz scalar, Mukohyama' 09

- Only very basic things are known for the time being.
- To tell, we need to discover

more intricate properties of cosmological perturbations

- Primordial tensor modes = gravitational waves

  Sizeable amplitude, (almost) flat power spectrum predicted by simplest (and hence most plausible) inflationary models but not alternatives to inflation

  - Together with scalar and tensor tilts $\Longrightarrow$ properties of inflation

- Non-trivial correlation properties of density perturbations (non-Gaussianity) $\Longrightarrow$ potential discriminator between scenarios. Very small in single field inflation.

  - Shape of non-Gaussianity: three-point function of invariants $\vec{k}_1^2$, $\vec{k}_2^2$, $(\vec{k}_1 \cdot \vec{k}_2)$.

- Statistical anisotropy $\Longrightarrow$ anisotropic pre-hot epoch.

  - Shape of statistical anisotropy $\Longrightarrow$ specific anisotropic model

- Admixture of entropy perturbations $\Longrightarrow$ generation of dark matter and/or matter-antimatter asymmetry before the hot epoch.

At the eve of new physics

LHC $\Longleftrightarrow$ dedicated CMB polarization experiments,
data and theoretical understanding
of structure formation ...

chance to learn
what preceded the hot Big Bang epoch

Barring the possibility that Nature is dull