One loop partition function of six dimensional conformal gravity using heat kernel on AdS$_{2n}$

Iva Lovrekovic
Technische Universität Wien

lovrekovic@hep.itp.tuwien.ac.at

Content

lovrekovic@hep.itp.tuwien.ac.at

Introduction

- Partition function is the key quantity in computation of thermodynamic quantities and AdS/CFT
- In four dimensions imposing the right boundary conditions to conformal gravity leads to Einstein gravity, that was generalised to the six dimensional case [Pang, Pope, 2011]
- It arises from 7d gravitational effective action [Beccaria, Tseytlin, 2015]
- Plays important role in conformal supergravity [Beccaria, Tseytlin, 2015]
- It is relevant for the (0,2) theory [Skenderis, Henningson,1998] and it can be related to tensionless strings [Bailiou, Picco,Windy, green, 2001]

Introduction

- It has been studied in the framework of the ordinary derivative approach [Metsaev, 2011]

- and from the geometric aspect of the anomalies [Deser, Schwimmer, 1993]

- It consists of conformal anomaly of type A and B. They were considered in geometric description in arbitrary number of dimensions

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

$$I_1 = C_{\mu\nu\rho\sigma} C^{\mu\lambda\kappa\sigma} C_{\lambda}{}^{\nu\rho}{}_{\kappa}$$

$$I_2 = C_{\mu\nu\rho\sigma} C^{\rho\sigma\lambda\kappa} C_{\lambda\kappa}{}^{\mu\nu}$$

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

$$I_1 = C_{\mu\nu\rho\sigma} C^{\mu\lambda\kappa\sigma} C_{\lambda}{}^{\nu\rho}{}_{\kappa}$$

$$I_2 = \boxed{C_{\mu\nu\rho\sigma}} C^{\rho\sigma\lambda\kappa} C_{\lambda\kappa}{}^{\mu\nu}$$

$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - \frac{2}{n-2}\left(g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}\right) + \frac{2}{(n-1)(n-2)} R g_{\mu[\rho}g_{\sigma]\nu}$$

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

$$I_1 = C_{\mu\nu\rho\sigma} C^{\mu\lambda\kappa\sigma} C_{\lambda}{}^{\nu\rho}{}_{\kappa}$$

$$I_2 = C_{\mu\nu\rho\sigma} C^{\rho\sigma\lambda\kappa} C_{\lambda\kappa}{}^{\mu\nu}$$

$$I_3 = C_{\mu\nu\rho\sigma} \left(\delta^{\mu}_{\lambda} \Box + 4 R^{\mu}_{\lambda} - \frac{6}{5} R \delta^{\mu}_{\lambda} \right) C^{\lambda\nu\rho\sigma} + \nabla_{\mu} J^{\mu}$$

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

$$I_1 = C_{\mu\nu\rho\sigma} C^{\mu\lambda\kappa\sigma} C_{\lambda}{}^{\nu\rho}{}_{\kappa}$$

$$I_2 = C_{\mu\nu\rho\sigma} C^{\rho\sigma\lambda\kappa} C_{\lambda\kappa}{}^{\mu\nu}$$

$$I_3 = C_{\mu\nu\rho\sigma} \left(\delta^{\mu}_{\lambda} \Box + 4 R^{\mu}_{\lambda} - \frac{6}{5} R \delta^{\mu}_{\lambda} \right) C^{\lambda\nu\rho\sigma} + \nabla_{\mu} J^{\mu}$$

$$E_6 = \epsilon_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \epsilon^{\rho_1\sigma_1\rho_2\sigma_2\rho_3\sigma_3} R^{\mu_1\nu_1}{}_{\rho_1\sigma_1} R^{\mu_2\nu_2}{}_{\rho_2\sigma_2} R^{\mu_3\nu_3}{}_{\rho_3\sigma_3} \, ,$$

Conformal gravity in six dimensions

$$I_1 = C_{\mu\nu\rho\sigma} C^{\mu\lambda\kappa\sigma} C_{\lambda}{}^{\nu\rho}{}_{\kappa}$$

vanishes for
Einstein spaces

$$I_2 = C_{\mu\nu\rho\sigma} C^{\rho\sigma\lambda\kappa} C_{\lambda\kappa}{}^{\mu\nu}$$

$$I_3 = C_{\mu\nu\rho\sigma} \left(\delta^{\mu}_{\lambda} \Box + 4 R^{\mu}_{\lambda} - \frac{6}{5} R \delta^{\mu}_{\lambda} \right) C^{\lambda\nu\rho\sigma} + \boxed{\nabla_\mu J^\mu}$$

$$E_6 = \epsilon_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \epsilon^{\rho_1\sigma_1\rho_2\sigma_2\rho_3\sigma_3} R^{\mu_1\nu_1}{}_{\rho_1\sigma_1} R^{\mu_2\nu_2}{}_{\rho_2\sigma_2} R^{\mu_3\nu_3}{}_{\rho_3\sigma_3} \, ,$$

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

- Combination of invariants that is satisfied by Einstein's metric leads to action

$$\mathcal{S} = \kappa \int d^6x \sqrt{|g|} \left(4I_1 + I_2 - \frac{1}{3} I_3 - \frac{1}{24} E_6 \right)$$

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

- Combination of invariants that is satisfied by Einstein's metric leads to action

$$\mathcal{S} = \kappa \int d^6x \sqrt{|g|} \left(4I_1 + I_2 - \frac{1}{3} I_3 - \frac{1}{24} E_6 \right)$$

- We vary the action and split the metric

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

- Combination of invariants that is satisfied by Einstein’s metric leads to action

$$\mathcal{S} = \kappa \int d^6x \sqrt{|g|} \left(4I_1 + I_2 - \frac{1}{3} I_3 - \frac{1}{24} E_6 \right)$$

- We vary the action and split the metric

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

- Combination of invariants that is satisfied by Einstein's metric leads to action

$$\mathcal{S} = \kappa \int d^6x \sqrt{|g|} \left(4I_1 + I_2 - \frac{1}{3} I_3 - \frac{1}{24} E_6 \right)$$

- We vary the action and split the metric

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

background AdS metric

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

- Combination of invariants that is satisfied by Einstein's metric leads to action

$$\mathcal{S} = \kappa \int d^6x \sqrt{|g|} \left(4I_1 + I_2 - \frac{1}{3} I_3 - \frac{1}{24} E_6 \right)$$

- We vary the action and split the metric

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

background AdS metric

small perturbations

$$\delta g_{\mu\nu} = h_{\mu\nu}$$

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

$$\delta^{(1)}S = \int d^6x\sqrt{|g|}EOM\delta g_{\mu\nu}$$ +boundary terms

Conformal gravity in six dimensions

$$\delta^{(1)}S = \int d^6x\sqrt{|g|}EOM\delta g_{\mu\nu} \text{ +boundary terms}$$

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

$$\delta^{(1)}S = \int d^6x\sqrt{|g|}EOM\delta g_{\mu\nu}$$ +boundary terms

$$\delta\delta^{(1)}S \quad \Longrightarrow$$ 1. + 2. + 3.

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

$$\begin{tikzcd}
\delta^{(1)}S = \boxed{\int d^6x\sqrt{|g|}EOM\delta g_{\mu\nu}} \arrow[d, "\delta"'] & \text{+boundary terms} \\
\delta\delta^{(1)}S \Longrightarrow 1. \; + 2. + 3. &
\end{tikzcd}$$

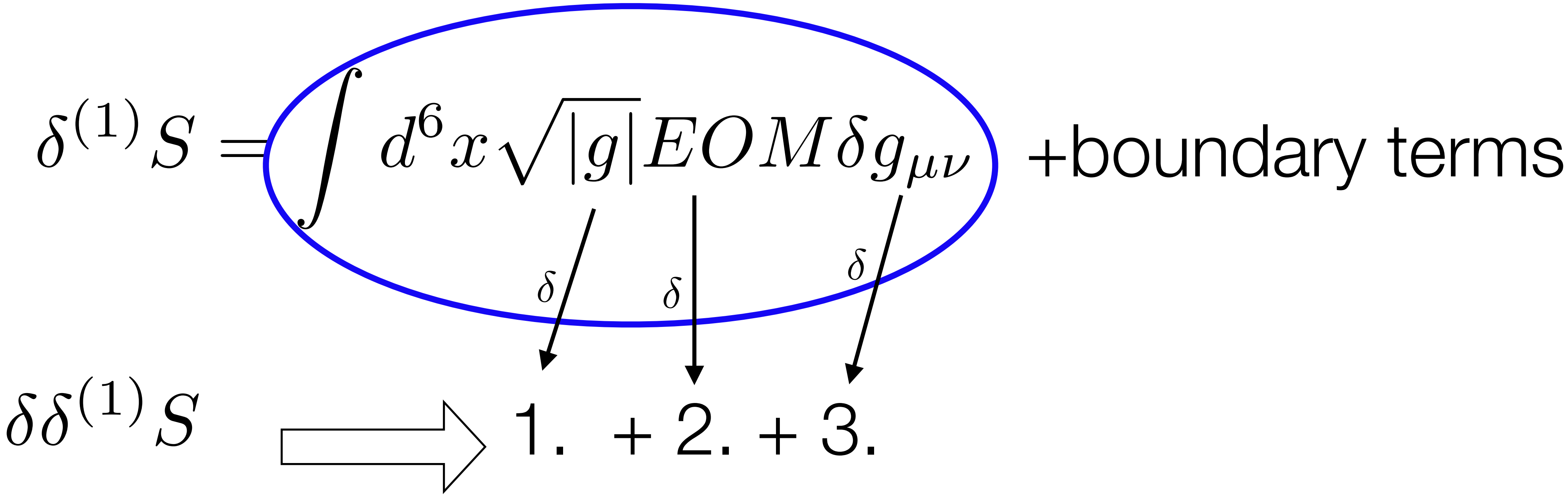
lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

$$\delta^{(1)}S = \boxed{\int d^6x\,\underset{\downarrow\,\delta}{\sqrt{|g|}}\,\underset{\downarrow\,\delta}{EOM}\,\delta g_{\mu\nu}} \text{ +boundary terms}$$

$$\delta\delta^{(1)}S \quad \Longrightarrow \quad 1. \ + 2. + 3.$$

Conformal gravity in six dimensions



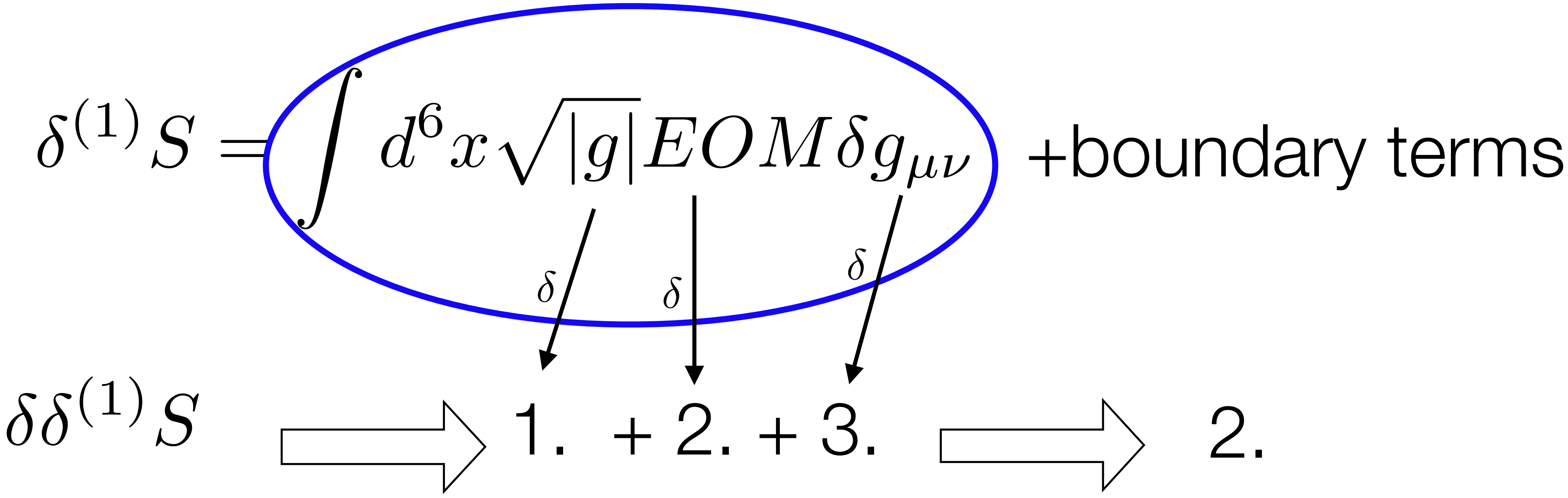
$$\delta^{(1)}S = \int d^6x\sqrt{|g|}EOM\delta g_{\mu\nu}$$ +boundary terms

δ δ δ

$$\delta\delta^{(1)}S \Rightarrow 1. + 2. + 3.$$

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

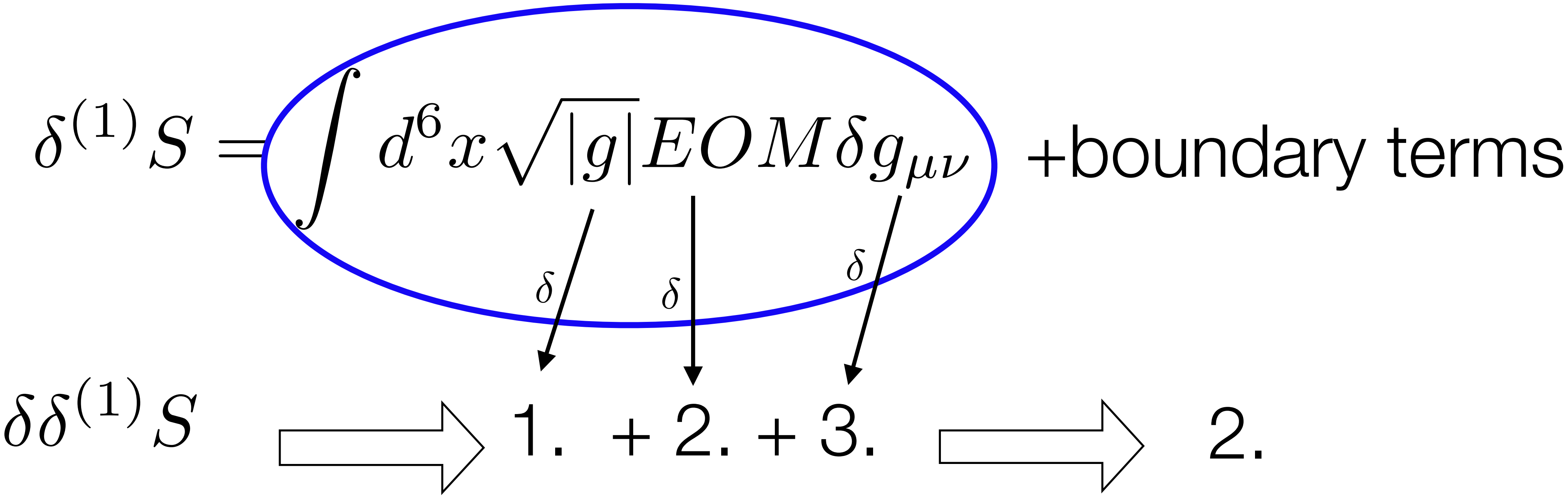


$$\delta^{(1)}S = \int d^6x\sqrt{|g|}EOM\delta g_{\mu\nu}$$ +boundary terms

$\delta\delta^{(1)}S \Rightarrow$ 1. + 2. + 3. $\Rightarrow$ 2.

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions



$$\delta^{(1)}S = \int d^6x\sqrt{|g|}EOM\delta g_{\mu\nu} \text{ +boundary terms}$$

$$\delta\delta^{(1)}S \Longrightarrow 1. + 2. + 3. \Longrightarrow 2.$$

$$\Longrightarrow \delta^{(2)}S = \int d^6x\sqrt{|g|}\delta EOM\delta g_{\mu\nu}.$$

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

$$I_1 = C_{\mu\nu\rho\sigma} C^{\mu\lambda\kappa\sigma} C_{\lambda}{}^{\nu\rho}{}_{\kappa}$$

$$I_2 = C_{\mu\nu\rho\sigma} C^{\rho\sigma\lambda\kappa} C_{\lambda\kappa}{}^{\mu\nu}$$

vanishes for Einstein spaces

$$\boxed{I_3 = C_{\mu\nu\rho\sigma}\left(\delta^{\mu}_{\lambda}\Box + 4R^{\mu}_{\lambda} - \frac{6}{5}R\delta^{\mu}_{\lambda}\right) C^{\lambda\nu\rho\sigma} + \boxed{\nabla_\mu J^\mu}}$$

$$E_6 = \epsilon_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\epsilon^{\rho_1\sigma_1\rho_2\sigma_2\rho_3\sigma_3} R^{\mu_1\nu_1}{}_{\rho_1\sigma_1} R^{\mu_2\nu_2}{}_{\rho_2\sigma_2} R^{\mu_3\nu_3}{}_{\rho_3\sigma_3}\,,$$

lovrekovic@hep.itp.tuwien.ac.at

Conformal gravity in six dimensions

$$\delta EOM^{\mu\nu} = \frac{1}{2}h_{\mu\nu}(-\nabla^2 + 2\lambda)(-\nabla^2 + 6\lambda)(-\nabla^2 + 8\lambda)h^{\mu\nu}$$

$\nabla^2 = \nabla_\mu \nabla^\mu$ covariant derivative w.r.t background metric

Transverse traceless gauge

$$\nabla^\mu h_{\mu\nu} = 0$$

$$h^\mu{}_\mu = 0$$

lovrekovic@hep.itp.tuwien.ac.at

1-loop partition function - determinants

$$Z_{CG} = \int \mathcal{D}\delta g_{\mu\nu} \times ghost \times exp(-\delta^{(2)}S)$$

lovrekovic@hep.itp.tuwien.ac.at

1-loop partition function - determinants

$$Z_{CG} = \int \mathcal{D}\delta g_{\mu\nu} \times ghost \times exp(-\delta^{(2)}S)$$

path integral over perturbations

lovrekovic@hep.itp.tuwien.ac.at

1-loop partition function - determinants

$$Z_{CG} = \int \mathcal{D}\delta g_{\mu\nu} \times ghost \times exp(-\delta^{(2)} S)$$

path integral over perturbations

determinants obtained upon eliminating gauge degrees of freedom

lovrekovic@hep.itp.tuwien.ac.at

1-loop partition function - determinants

$$Z_{CG} = \int \mathcal{D}\delta g_{\mu\nu} \times ghost \times exp(-\delta^{(2)} S)$$

path integral over perturbations

second variation
of action

determinants obtained upon eliminating
gauge degrees of freedom

lovrekovic@hep.itp.tuwien.ac.at

1-loop partition function - determinants

$$Z_{CG} = \int \mathcal{D}\delta g_{\mu\nu} \times ghost \times exp(-\delta^{(2)} S)$$

path integral over perturbations

second variation
of action

determinants obtained upon eliminating
gauge degrees of freedom

⇒ transverse traceless split of the metric

lovrekovic@hep.itp.tuwien.ac.at

1-loop partition function - determinants

$$Z_{CG} = \int \mathcal{D}\delta g_{\mu\nu} \times ghost \times exp(-\delta^{(2)} S)$$

path integral over perturbations

second variation of action

determinants obtained upon eliminating gauge degrees of freedom

⟹ transverse traceless split of the metric

$$Z_{CG}^{(6)} = Z_{gh} \int Dh_{\mu\nu}^{TT} Exp(-\delta^{(2)} S)$$

lovrekovic@hep.itp.tuwien.ac.at

1-loop partition function - determinants

$$Z_{CG}^{(6)} = \frac{[\det(-\nabla^2 - 5\lambda)_1]^{1/2}[\det(-\nabla^2 - 6\lambda)_0]^{1/2}}{[\det(-\nabla^2 + 2\lambda)_2]^{1/2}[\det(-\nabla^2 + 6\lambda)_2]^{1/2}[\det(-\nabla^2 + 8\lambda)_2]^{1/2}} \tag{1}$$

[Pang, 2012; Tseytlin, 2013]

1-loop partition function - determinants

$$Z^{(6)}_{CG} = \frac{\boxed{[\det(-\nabla^2 - 5\lambda)_1]^{1/2}}[\det(-\nabla^2 - 6\lambda)_0]^{1/2}}{\boxed{[\det(-\nabla^2 + 2\lambda)_2]^{1/2}}[\det(-\nabla^2 + 6\lambda)_2]^{1/2}[\det(-\nabla^2 + 8\lambda)_2]^{1/2}} \tag{1}$$

[Pang, 2012; Tseytlin, 2013]

$$Z^{(6)}_{EG} = \frac{[\det(-\nabla^2 - 5\lambda)_1]^{1/2}}{[\det(-\nabla^2 + 2\lambda)_2]^{1/2}}$$

Partition function of Einstein gravity

lovrekovic@hep.itp.tuwien.ac.at

1-loop partition function - determinants

$$Z_{CG}^{(6)} = \frac{[\det(-\nabla^2 - 5\lambda)_1]^{1/2}[\det(-\nabla^2 - 6\lambda)_0]^{1/2}}{[\det(-\nabla^2 + 2\lambda)_2]^{1/2}[\det(-\nabla^2 + 6\lambda)_2]^{1/2}[\det(-\nabla^2 + 8\lambda)_2]^{1/2}} \qquad (1)$$

[Pang, 2012; Tseytlin, 2013]

- conformal ghost $[\det(-\nabla^2 - 6\lambda)_0]^{1/2}$
- partially masses mode $[\det(-\nabla^2 + 6\lambda)_2]^{1/2}$
- massive mode $[\det(-\nabla^2 + 8\lambda)_2]^{1/2}$

lovrekovic@hep.itp.tuwien.ac.at

Heat kernel on a symmetric space

- To compute the determinant of the STT fields we have to evaluate corresponding heat kernel
- One could solve it on a manifold by direct evaluation and construction of the eigenvalues and eigenfunctions for the spin S Laplacian
- However, for manifold that is a homogeneous space, more convenient method is using group theoretic techniques as in Gopakumar et al., 2011

lovrekovic@hep.itp.tuwien.ac.at

Heat kernel on a symmetric space

Trace of a heat kernel is related to one loop partition function via

$$\log Z^{(S)} = \log\det(-\Delta_{(S)}) = Tr\log(-\Delta_{(S)}) = -\int\limits_0^\infty \frac{dt}{t} Tr e^{t\Delta_{(S)}}$$

partition function

Laplacian of
a spin-S field

proper time

trace of the
heat kernel

lovrekovic@hep.itp.tuwien.ac.at

Heat kernel on a symmetric space

Trace of a heat kernel is related to one loop partition function via

$$\log Z^{(S)} = \log\det(-\Delta_{(S)}) = Tr\log(-\Delta_{(S)}) = -\int\limits_0^\infty \frac{dt}{t} Tr e^{t\Delta_{(S)}}$$

partition function

Laplacian of
a spin-S field

proper time

trace of the
heat kernel

⟹ evaluate the trace of heat kernel

lovrekovic@hep.itp.tuwien.ac.at

Heat kernel on a symmetric space

Trace of a heat kernel is related to one loop partition function via

$$\log Z^{(S)} = \log\det(-\Delta_{(S)}) = Tr\log(-\Delta_{(S)}) = -\int\limits_0^\infty \frac{dt}{t} Tr e^{t\Delta_{(S)}}$$

partition function

Laplacian of
a spin-S field

proper time

trace of the
heat kernel

⇒ evaluate the trace of heat kernel

$$-\log\det(-\nabla^2_{(s)} + m_S^2) = \sum_{k\in\mathbb{Z}_+} \chi^{SO(2n-1)}_{(s,0)} \frac{2}{(1-e^{-k\beta})^{2n-1} e^{k\beta(n-1)e^{\frac{k\beta}{2}}} k} \frac{1}{k} e^{-k\beta\sqrt{\rho^2+s+m_s^2}}$$

lovrekovic@hep.itp.tuwien.ac.at

Heat kernel on a symmetric space

Trace of a heat kernel is related to one loop partition function via

$$\log Z^{(S)} = \log\det(-\Delta_{(S)}) = Tr\log(-\Delta_{(S)}) = -\int_0^\infty \frac{dt}{t} Tr e^{t\Delta_{(S)}}$$

partition function

Laplacian of
a spin-S field

proper time

trace of the
heat kernel

⟹ evaluate the trace of heat kernel

$$-\log\det(-\nabla^2_{(s)} + \boxed{m_S^2}) = \sum_{k\in\mathbb{Z}_+} \chi^{SO(2n-1)}_{(s,0)} \frac{2}{(1-e^{-k\beta})^{2n-1} e^{k\beta(n-1)e^{\frac{k\beta}{2}}} k} \frac{1}{k} e^{-k\beta\sqrt{\rho^2+s+m_s^2}}$$

mass of the field

lovrekovic@hep.itp.tuwien.ac.at

Heat kernel on a symmetric space

Trace of a heat kernel is related to one loop partition function via

$$\log Z^{(S)} = \log\det(-\Delta_{(S)}) = Tr\log(-\Delta_{(S)}) = -\int_0^\infty \frac{dt}{t} Tr e^{t\Delta_{(S)}}$$

partition function

Laplacian of
a spin-S field

proper time

trace of the
heat kernel

⇒ evaluate the trace of heat kernel

$$-\log\det(-\nabla^2_{(s)} + m_S^2) = \sum_{k\in\mathbb{Z}_+} \chi^{SO(2n-1)}_{(s,0)} \frac{2}{(1-e^{-k\beta})^{2n-1} e^{k\beta(n-1)e^{\frac{k\beta}{2}}} k} \frac{1}{k} e^{-k\beta\sqrt{\rho^2+s+m_s^2}}$$

character of the SO(2n-1) representation (s,0)

lovrekovic@hep.itp.tuwien.ac.at

Heat kernel on a symmetric space

Trace of a heat kernel is related to one loop partition function via

$$\log Z^{(S)} = \log\det(-\Delta_{(S)}) = Tr\log(-\Delta_{(S)}) = -\int\limits_0^\infty \frac{dt}{t} Tr e^{t\Delta_{(S)}}$$

partition function

Laplacian of
a spin-S field

proper time

trace of the
heat kernel

⇒ evaluate the trace of heat kernel

$$-\log\det(-\nabla^2_{(s)} + m_S^2) = \sum_{k\in\mathbb{Z}_+} \chi^{SO(2\boxed{n}-1)}_{(s,0)} \frac{2}{(1-e^{-k\beta})^{2n-1} e^{k\beta(n-1)e^{\frac{k\beta}{2}}}\, k} \frac{1}{k} e^{-k\beta\sqrt{\rho^2+s+m_s^2}}$$

dimension/2

lovrekovic@hep.itp.tuwien.ac.at

Heat kernel on a symmetric space

Trace of a heat kernel is related to one loop partition function via

$$\log Z^{(S)} = \log\det(-\Delta_{(S)}) = Tr\log(-\Delta_{(S)}) = -\int_0^\infty \frac{dt}{t} Tr e^{t\Delta_{(S)}}$$

partition function

Laplacian of
a spin-S field

proper time

trace of the
heat kernel

⇒ evaluate the trace of heat kernel

$$-\log\det(-\nabla^2_{(s)} + m_S^2) = \sum_{k\in\mathbb{Z}_+} \chi^{SO(2n-1)}_{(s,0)} \frac{2}{(1-e^{-k\beta})^{2n-1} e^{k\beta(n-1)e^{\frac{k\beta}{2}}} k} \frac{1}{k} e^{-k\boxed{\beta}\sqrt{\rho^2+s+m_s^2}}$$

inverse temperature

lovrekovic@hep.itp.tuwien.ac.at

One loop partition function

$$\log Z_6 = \sum_{k\in\mathcal{Z}_+} \frac{-e^{-\frac{5}{2}k\beta}}{k(1-e^{-k\beta})^5}(\chi^{SO(5)}_{(1,0)} e^{-\frac{7}{2}k\beta} + \chi^{SO(5)}_{(0,0)} e^{-\frac{7}{2}k\beta}$$

$$- \chi^{SO(5)}_{(2,0)} e^{-\frac{5}{2}k\beta} - \chi^{SO(5)}_{(2,0)} e^{-\frac{3}{2}k\beta} - \chi^{SO(5)}_{(2,0)} e^{-\frac{1}{2}k\beta})$$

$$q \equiv e^{-\beta}$$

$$\log Z_6 = \sum_{k\in\mathbb{Z}_+} \frac{-2q^{3k}}{k(1-q^k)^5}\left(3q^{3k} - 7q^{2k} - 7q - 7\right)$$

lovrekovic@hep.itp.tuwien.ac.at

Discussion

- Gravity analysis in lower dimensions with conformal gravity part bring analogous contributions of Einstein gravity, conformal ghost and partially massless mode [Gaberdiel, Grumiller, Vassilevich, 2010]

Conclusion and outlook

- The result for the heat kernel can be further used in evaluation of the Laplacians
- The result for the six dimensional conformal gravity can be used for the computations of thermodynamical quantities and for theories related to 6D CG

Thank you!

lovrekovic@hep.itp.tuwien.ac.at