$\cancel{d\xi = \partial_\mu \xi_\nu \, dx^\mu \wedge dx^\nu}$

Motivations for compactifications and moduli stabilization

1. $\mathcal{N}=1$, $D=10$ supergravity:

: g_{MN}, B_{MN}, ϕ ; NS-NS

IIA: C_M, C_{MNK}
IIB: C_{MN}, C_{MNKL} } R-R-sector

$$S = \int e^{-2\phi}\sqrt{-g}\left(\frac{1}{2}R + \frac{1}{4} g^{MN}\partial_M\phi\,\partial_N\phi - \frac{1}{12}H_{MNK}H^{MNK}\right) + \dots$$

$$H_{MNK} = \frac{1}{3}\partial_{[M}B_{NK]} \quad \text{– 3-form flux.}$$

2. need to go to $D=4$

the simplest case: ~~all~~ $\underset{10}{X^M} = (\underset{4}{x^\mu}, \underset{6}{y^a})$

- say: all fields $\Phi(X^M) = \Phi(x^\mu)$

$$\frac{\partial}{\partial y^a}\Phi(X^M) = 0 \qquad \text{~~KK compact~~}$$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + A_{\mu a}dx^\mu dy^a + \varphi_{ab}dy^a dy^b$$

need physical background for these (why fields do not scatter into non-trivial y^a's)

- KK-compact. Consider $X^M = (\underset{4}{x^\mu}, \underset{1}{x^4})$;

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + (dx^4)^2 \quad ; \quad x^4 \sim x^4 + 2\pi R$$

$$\Phi(X^M) = \sum_n \phi_n(x^\mu)\, e^{inx^4/R} \quad \text{– Fourier expansion.}$$

Klein-Gordon equation: $\nabla_M \nabla^M \Phi = 0 \Rightarrow$

$$\Rightarrow \left(\nabla_\mu \nabla^\mu - \frac{n^2}{R^2}\right)\phi_n = 0 \qquad m_n^2(\phi_n) = \frac{n^2}{R^2}$$

for $E \ll M_c \sim \frac{1}{R}$ – ignor the tower.

- Einstein gravity

$$ds^2 = G_{MN}\,dx^M dx^N = g_{\mu\nu}\,dx^\mu dx^\nu + g_{44}\left(dx^4 + A_\mu dx^\mu\right)^2$$

4-dim fields:

- $G_{44} = g_{44} =: e^{2\sigma}$ – scalar
- $G_{\mu 4} =: e^{2\sigma} A_\mu$ – vector
- $G_{\mu\nu} = g_{\mu\nu} + e^{2\sigma} A_\mu A_\nu$ – metric

according to their transf.

The action $\quad S = \int d^5x \sqrt{-G}\, R_5 = \int d^4x \sqrt{-g}\left(R_4 + F_{\mu\nu}^2 + (\partial_\mu \sigma)^2\right)$

σ – radion – massless

Toy model of moduli stabilization

• $S = \int d^6x \sqrt{-G}\left(M_6^4 R_G - M_6^2 F_{MN}F^{MN}\right) = S_R + S_F$

6d Planck mass

$6d = 4d + 2d$

all classified by genus g.

$ds^2 = g_{\mu\nu}\,dx^\mu dx^\nu + \Phi^2(x)\,\underbrace{\tilde{g}_{mn}}\,dy^m dy^n$;

moduli field

unit volume, genus g.

$$S_R = M_6^4 \int d^4x \sqrt{-g}\left(\underbrace{\left[\int d^2y \sqrt{\tilde{g}}\, R_2\right]}_{2-2g} + \Phi^2(x) R_g\right) + \dots$$

rescale $g \to h = \Phi^2 g$ to go to Einstein frame

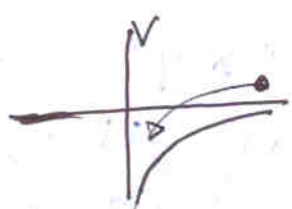
$$= M_6^4 \int d^4x \sqrt{-h}\left(R_h - V(\Phi)\right) + \dots$$

$$V(\Phi) \sim (2g-2)\frac{1}{\Phi^4} \quad \text{– potential}$$

Conclusions:

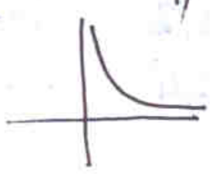
T^2 1) $g = 1 \Rightarrow V(\Phi) = 0 \rightarrow$ no source $\Rightarrow$ flat solution

S^2 2) $g = 0$, $V(\Phi) \sim -\frac{1}{\Phi^4}$



$\Phi \to 0$ singularity

3) $g > 1$ $V(\Phi) \sim +\frac{1}{\Phi^4}$



$\Phi \to \infty$ decompact.

• Including fluxes

$$\int_{M_g} F = n \quad - \text{ number of fluxes}$$

$$\int F_{MN} F^{MN} \longrightarrow \int \sqrt{-h}\, d^4x \cdot \left(-\frac{n^2}{\phi^6}\right)$$

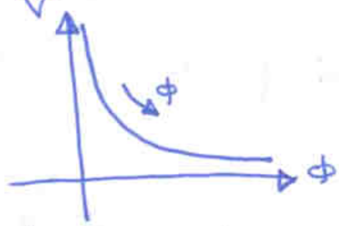
$$\boxed{V \sim (2g-2)\frac{1}{\phi^4} + \frac{n^2}{\phi^6}}$$ – flux corrected potential.

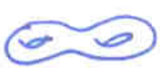
– $g = 0$; S^2

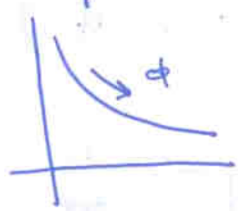
$$\phi_{min} = \frac{\sqrt{3}}{2} n$$

(Freund – Rubin solutions)

– $g = 1$; $\mathbb{T}^2$



– $g > 1$ 



spont. decomp. and break of the solution.

Supersymmetry

$\delta \Psi^{\pm}_M = \nabla_M \epsilon^{\pm}$; $\epsilon^{\pm}$ – 10-dim spinors of same or opposite chirality

$$\epsilon^+ = \varepsilon^+ \otimes \eta^+ + \varepsilon^- \otimes \eta^-$$

$\epsilon^A \to \{\epsilon^{a\alpha}, \epsilon^{\dot a \dot \alpha}\}$; $A = \overline{1,16}$; $a, \dot a = \overline{1,2}$ – 4d spinor

$\alpha, \dot\alpha = \overline{1,4}$ – 6d spinor

$\epsilon^{a\alpha} = \epsilon^{aI}\, \eta^{\alpha}_{I}$

$\epsilon^{\dot a \dot \alpha} = \epsilon^{\dot a I}\, \eta^{\dot\alpha}_{I}$

– basis 6d spinor on M_6

$$\varepsilon^I = \begin{bmatrix} \epsilon^{aI} \\ \epsilon^{\dot a I} \end{bmatrix}$$ – some number of spinors

p-brane:

$$S_p = Q_e \int_{W_{p+1}} C_{p+1}$$

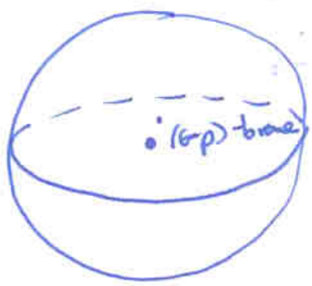
(6-p) -brane couples magnetically

$$C_{p+1} \to dC_{p+1} = F_{p+2}$$

$$*F_{p+2} = \tilde{F}_{8-p} = d\tilde{C}_{7-p}$$

$$10-(p+2) = 8-p.$$

$$Q_m = \int_{S^{p+2}} F_{p+2} \leftarrow \text{sphere around } (6\text{-}p)\text{-brane}.$$



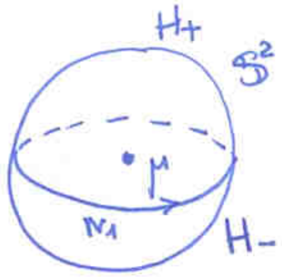
$p=0.$

$$d=4 \quad \int_{W_1} C_1 = S_0$$

magn: 0-brane

$$Q_m = \int_{S^2} F_2$$

e^{iS_p} — такой кусок в действии.



$$e^{i\int_{W_1} C_1} \qquad \frac{1}{2\pi}\int_{S^2} F_2 = n$$

$$Q_e \int_{W_1} C_1 = Q_e \oint_{\partial H_+} C_1^+ = Q_e \int_{H_+} dC_1^+ = \int_{H_+} F_2^+$$

$$= -\int_{\partial H_-} C_1^- = -\int_{H_-} F_2^-$$

$$\int_{H_+} F_2^+ + \int_{H_-} F_2^- = \int_{S^2} F_2 = Q_m$$

$e^{i\int_{W_1} C_1}$ — single-valued $\Rightarrow$ $Q_m Q_e = 2\pi n$

$$\frac{Q_m Q_e}{2\pi} = n$$

$F = \cancel{4}\cos^2(n\theta)\, d\theta$ $\qquad \int F = n$

$$\cancel{\int_0^{2\pi} \cos(n\theta)\, d\theta = \frac{1}{n}\sin(n\theta)}$$

$\oint$

$\{C^p_A, \alpha_p\}$ $\qquad \int_{C^A} \alpha_B = \delta^B_A \qquad F = n\,\alpha_{(2)}$

$$\int F \wedge * F = n^2 \int_{C^{(2)}} \alpha_{(2)} \wedge *_2 \alpha_{(2)} = \qquad \int F = n$$

$\phi^{-4} \quad \phi^{-2}$

$$\int \sqrt{-g}\, d^2x \to \overbrace{\phi^{-4}\,\phi^2}^{\phi^2}$$

$$= \oint \frac{n^2}{\phi^6} \qquad g^{\mu\mu} g^{\nu\nu} \to \phi^{-4}$$

$$= n^2 \int \alpha_{(2)} \wedge * \alpha_{(2)} = n^2 \int_{C^{(2)}} \alpha_{(2)} = n^2$$

X - 4-manifold; $\quad \int_X F_4 = \frac{2\pi n}{e}; \quad n \in \mathbb{Z}$

$F_4 = dA_3$ (locally)

action for membrane: $\quad S = e \int_W A_3$

$$\int_{\Sigma_{(2)}} F_{(2)} = \int_\Sigma F_{\mu\nu}\, dx^\mu \wedge dx^\nu = \int_\Sigma F_{\mu\nu}\, d\Sigma^{\mu\nu} =$$

$$= \int_\Sigma F_{ij}\, dx^i \wedge dx^j = \int \epsilon_{ijk} F_{ij}\, d\Sigma^k =$$

$$= \int B_k\, d\Sigma^k$$ – поток магн. поля