Grand Unification & High-Scale Supersymmetry - II

- Structure of Standard Model of Particle Physics points towards "grand unification" of strong and electroweak interactions (quark and lepton content, gauge group, "unification" of gauge couplings, small neutrino masses ...) GUT groups: SU(5), SO(10), ...

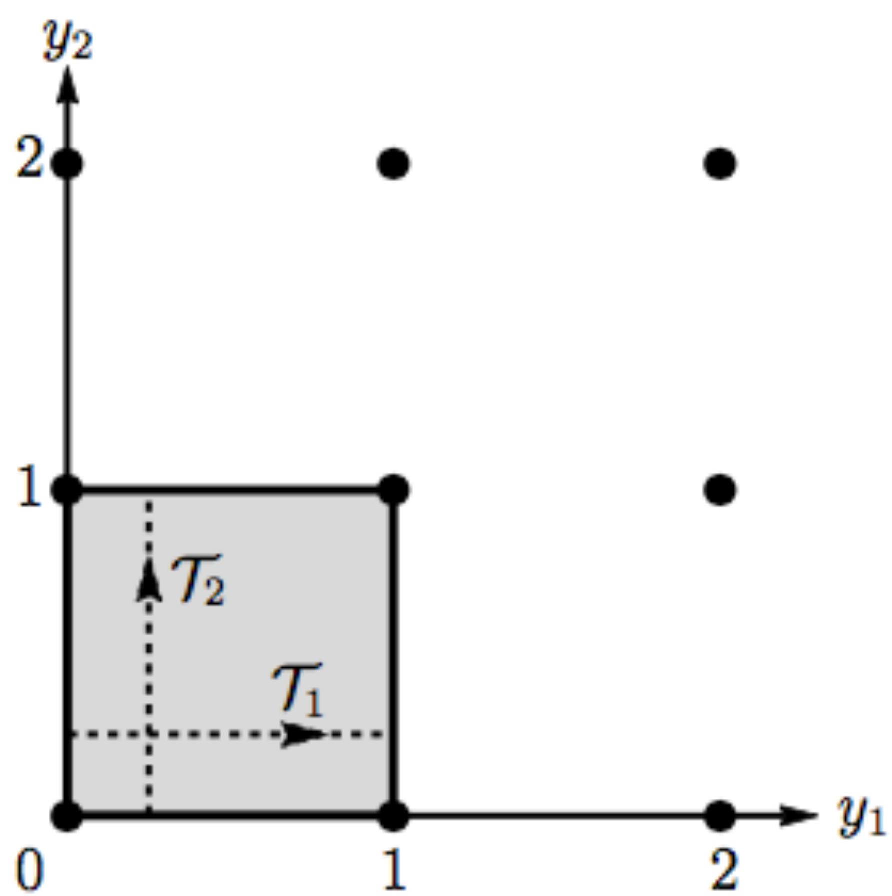
- Strong theoretical arguments for supersymmetry at "high" energy scales (extra dimensions, strings; SM and gravity)

- Energy scale of grand unification: $\Lambda_{\rm GUT} \simeq 10^{15} \dots 10^{16}\ \rm GeV$
  energy scale of supersymmetry breaking: $\Lambda_{\rm SB} \simeq$??
  Electroweak hierarchy problem? This talk: $\Lambda_{\rm SB} \sim \Lambda_{\rm GUT}$

Supersymmetric Unification in 6d

WB, Dierigl, Ruehle, Schweizer arXiv: 1506.05771, 1507.06819, 1603.00654, 1606.05653

Supersymmetric GUTs strongly suggest extra space dimensions ("Orbifold GUTs",... [Kawamura '00; Hall, Nomura '01;... Hebecker, Trapletti '04;...]). Start from toy model: 6d supergravity with $U(1)$ gauge field



compactification on **torus**, metric:

$$(g_6)_{MN} = \begin{pmatrix} r^{-2}(g_4)_{\mu\nu} & 0 \\ 0 & r^2(g_2)_{mn} \end{pmatrix}$$

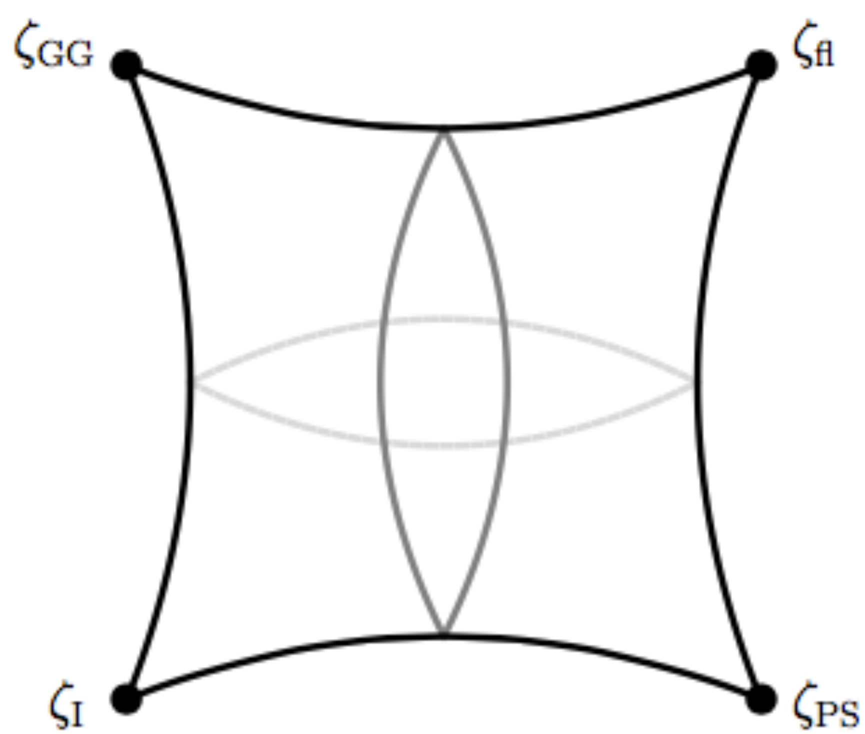
Wilson lines and flux (bulk matter):

$$W = \mathcal{P}\exp\left[iq\int_{\mathcal{T}} A\right]$$

$$\frac{q}{2\pi}\int_{T^2}\langle F\rangle = \frac{qf}{2\pi} \equiv M \in \mathbb{Z}$$

Split symmetries

Consider SO(10) GUT group in 6d, broken at orbifold fixed points to standard SU(5)xU(1), Pati-Salam SU(4)xSU(2)xSU(2) and flipped SU(5)xU(1), with Standard Model group as intersection; bulk fields 45, 16, 16*, 10's [Asaka, WB, Covi '02,'03]; full 6d gauge symmetry:

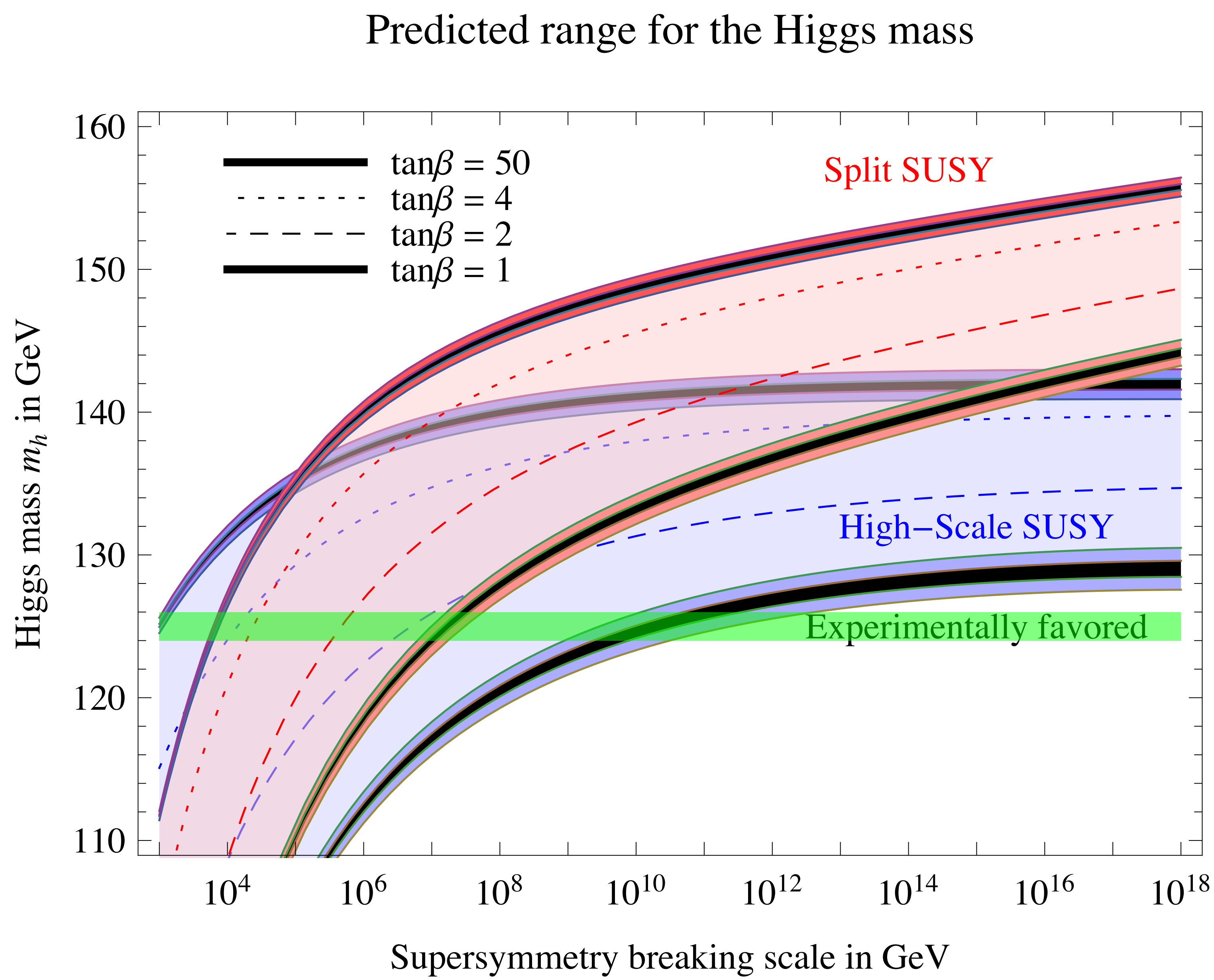


$$SO(10) \times U(1)_A$$

N 16's, i.e. quark-lepton generations, from flux (N quanta; Witten '84, ...):

$$\mathbf{16}\ [SO(10)] \sim \mathbf{5}^* + \mathbf{10} + \mathbf{1}\ [SU(5)] \sim \mathbf{q}, \mathbf{l}, \mathbf{u^c}, \mathbf{e^c}, \mathbf{d^c}, \nu^\mathbf{c}\ [G_{\rm SM}]$$

Vacuum stability & SUSY at high scales



[Degrassi et al '12]

Matching of SM Higgs coupling to MSSM at SUSY breaking scale for `Split SUSY' (one Higgs doublet, higgsinos and gauginos light) and `High-scale SUSY' (one Higgs doublet light). Is the SUSY breaking scale necessarily much below the GUT scale?

Extrapolating the THDM to the GUT scale

Bagnaschi, Brummer, WB, Voigt, Weiglein arXiv: 1512.07761

Is SUSY breaking at the GUT scale consistent with RG running of couplings and vacuum stability? Excluded for one light Higgs doublet! 6d GUT model suggests to consider **2 light Higgs doublets** [Gunion, Haber '03... Lee, Wagner '15])

$$V = m_1^2\, H_1^\dagger H_1 + m_2^2\, H_2^\dagger H_2 - \left(m_{12}^2\, H_1^\dagger H_2 + \text{h.c.} \right) + V_4\,,$$

$$V_4 = \frac{\lambda_1}{2}(H_1^\dagger H_1)^2 + \frac{\lambda_2}{2}(H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 |H_1^\dagger H_2|^2$$
$$+ \left(\frac{\lambda_5}{2}(H_1^\dagger H_2)^2 + \lambda_6 (H_1^\dagger H_2)(H_1^\dagger H_1) + \lambda_7 (H_1^\dagger H_2)(H_2^\dagger H_2) + \text{h.c.} \right)$$

Matching conditions at SUSY breaking scale determine quartic couplings:

$$\lambda_1 = \frac{1}{4}\left(g^2 + {g'}^2\right)\,, \quad \lambda_2 = \frac{1}{4}\left(g^2 + {g'}^2\right)$$
$$\lambda_3 = \frac{1}{4}\left(g^2 - {g'}^2\right)\,, \quad \lambda_4 = -\frac{1}{2}g^2\,, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0$$

Yukawa couplings in THDM:

$$\mathcal{L}_{\rm Yuk} = h_u \bar{q}_L H_2 u_R + h_d \bar{d}_R H_1 q_L + h_e \bar{e}_R H_1 l_L + \text{c.c.}$$

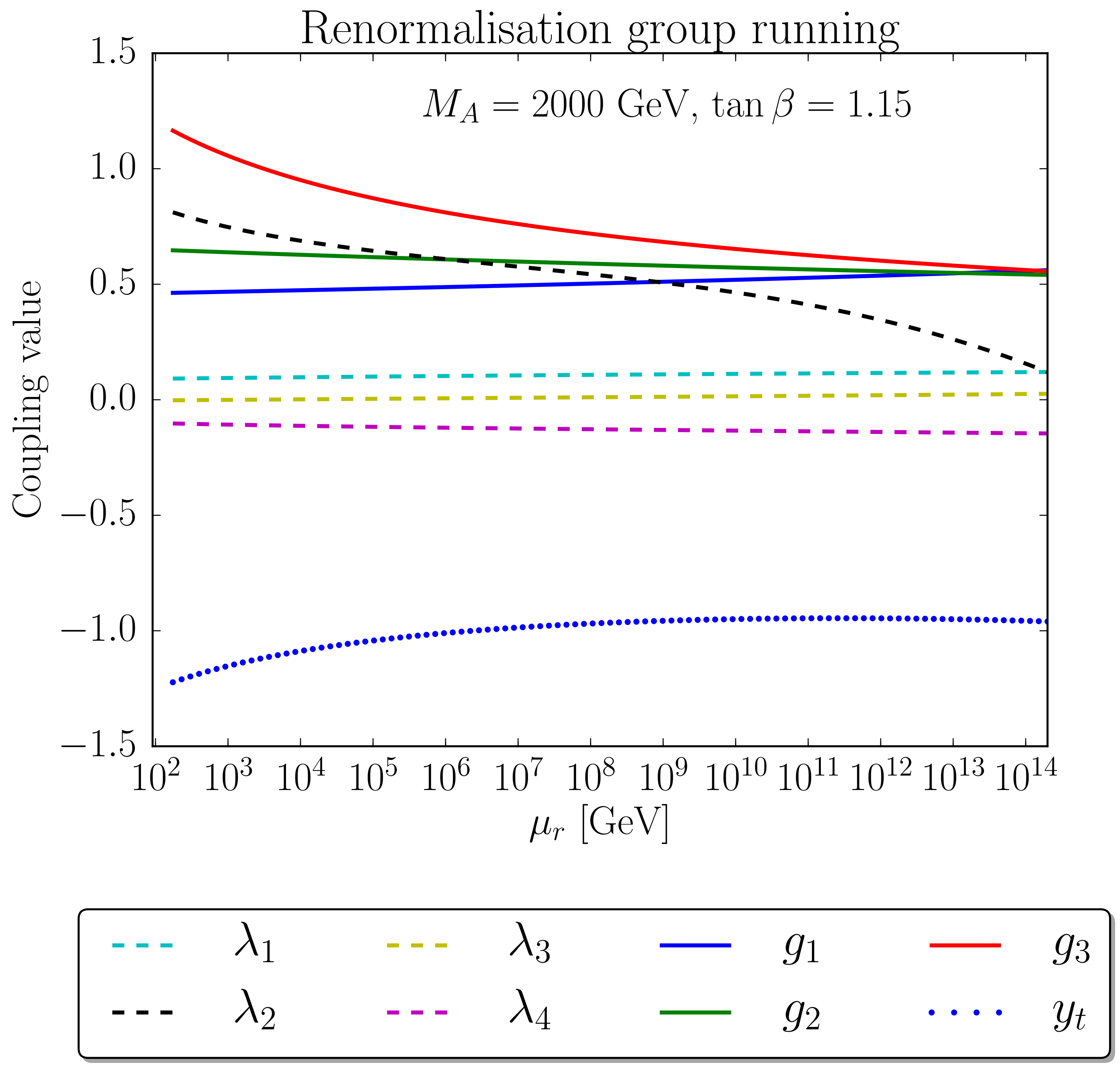
structure of THDM in "decoupling limit":

$$v_{1,2} = \langle H_{1,2} \rangle \,, \quad \tan\beta = \frac{v_2}{v_1}$$

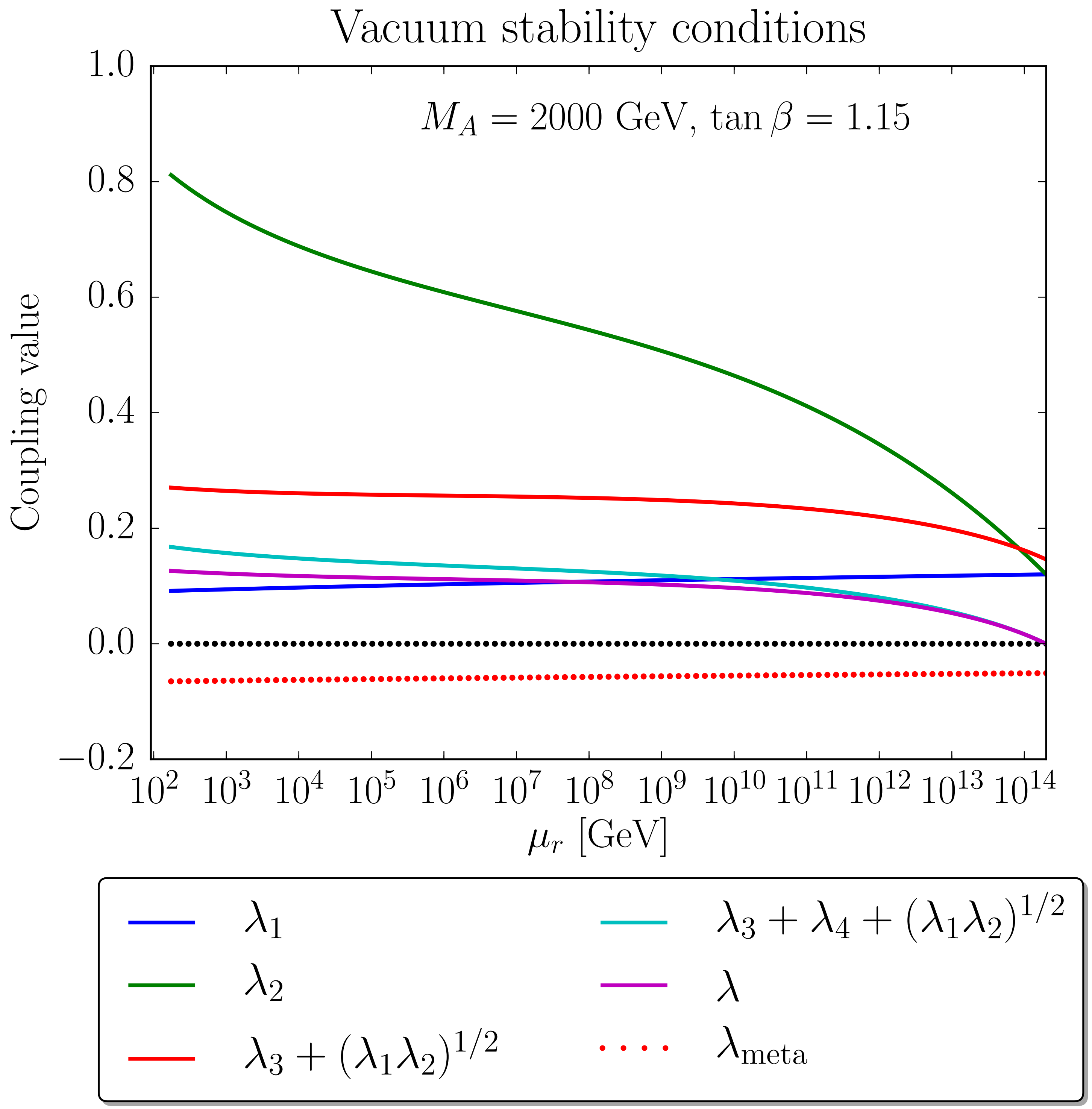
$$m_A^2 = \frac{2}{\sin 2\beta} m_{12}^2 \,, \quad m_h = m_h(m_i^2, \lambda_j, m_A^2, \tan\beta)$$

$$m_H, m_H^\pm = m_A + \mathcal{O}\left(\frac{v^2}{m_A^2}\right) \,, \; m_A^2 \gg v^2$$

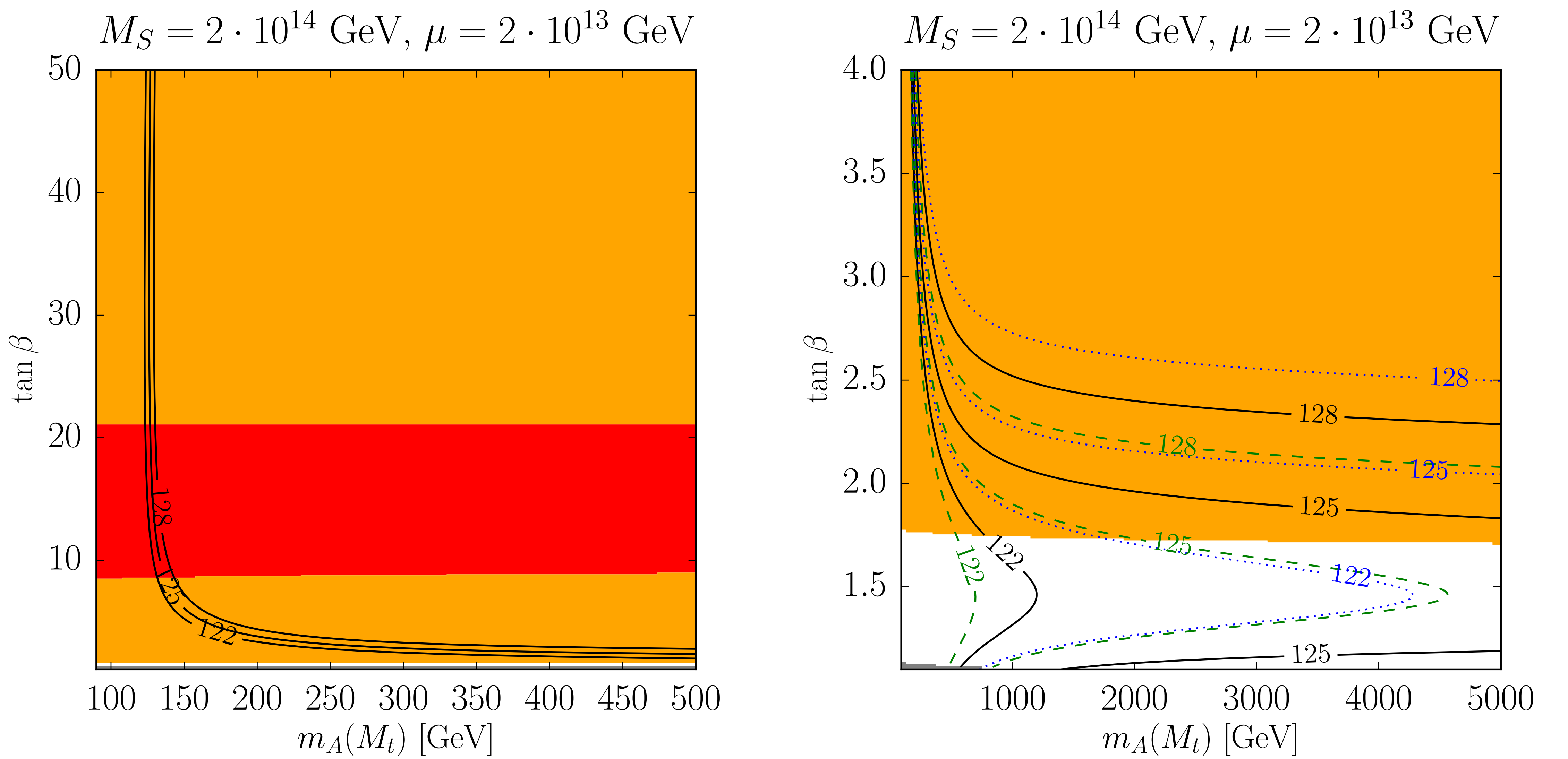
strong constraints on Higgs masses and tanβ from matching to SUSY!



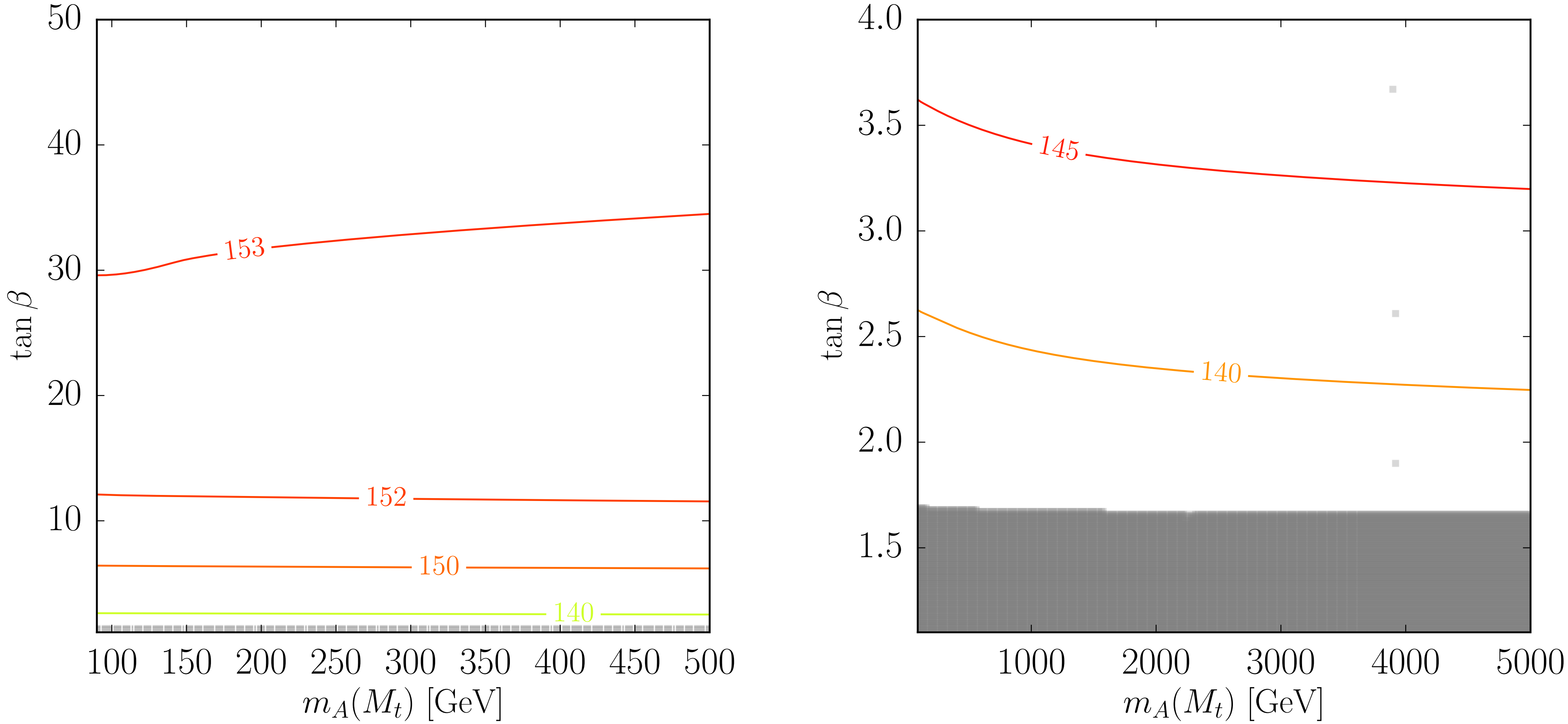
example of RG running of gauge, Yukawa and quartic couplings; THDM, gauge coupling unification, small tanβ



vacuum stability conditions are fulfilled; small tanβ, additional Higgs bosons are heavy!



THDM, result of parameter scan; red: excluded, vacuum unstable; orange: metastable vacuum; white: stable vacuum; large tanβ excluded; small tanβ allowed with heavy Higgses, $M_A > 1 \text{ TeV}$; same results for light higgsino, mass essentially unconstrained



THDM with light higgsinos and gauginos (similar to split SUSY),result of parameter scan; white: vacuum stable; grey: nonperturbative region; vacuum always stable, but light Higgs too heavy! Split SUSY inconsistent!

Summary: THDM & high-scale SUSY

- Contrary to SM, THDM consistent with high-scale SUSY; quartic couplings can be matched to gauge couplings, as specified in MSSM, yields very predictive framework:

$$\tan\beta \lesssim 2\,, \quad m_A, m_H, m_{H^\pm} \gtrsim 1\text{TeV}$$

- Discovery of additional heavy Higgs bosons with small tanβ at LHC challenging

- Most promising signature at LHC for high-scale SUSY: discovery of "light" higgsinos (neutralinos and charginos, also challenging):

$$m_{\tilde{h}^0,\tilde{h}^\pm} = 100\ \text{GeV} \ldots 1\ \text{TeV}$$

Open theoretical questions

- Consistent compactification: SUSY breaking, Minkowski/ de Sitter vacua?

- Quantum corrections to Higgs potential mass parameters: enormous fine tuning?? Important effect of Landau level structure of KK tower and partial SUSY!

- 6d supergravity interesting intermediate step towards embedding into string theory (intersecting D-brane models [Blumenhagen, Braun, Kors, Lust '02]; F-theory [Taylor '11])

Stabilizing the compact dimensions

Consider bosonic part of 6d supergravity action [Nishino, Sezgin '86, ... , Aghababai, Burgess, Parameswaran, Quevedo '03,... , Lee, Nilles, Zucker '04, ..., Braun, Hebecker, Trapletti '07,...], with dilaton, vector and tensor fields, and bulk matter field:

$$S_B = \int \left(\frac{M_6^4}{2} \left(R - d\phi \wedge * d\phi \right) - \frac{1}{4 M_6^4 g_6^4} e^{2\phi} H \wedge * H - \frac{1}{2 g_6^2} e^{\phi} F \wedge * F \right) ,$$

$$F = dA \,, \quad H = dB - X_3^0 \,, \quad X_3^0 = -\omega_{3G} = -A \wedge F$$

crucial: cancellation of **all** anomalies (bulk, fixed-point and flux zero-modes of matter field) by Green-Schwarz mechanism:

$$S_{\mathrm{GS}}[A,B] = -\int \left(\frac{\beta}{2} A \wedge F + \alpha \delta_O A \wedge v_2 \right) \wedge dB$$

with $\beta = -q^4/(2\pi)^3, \ \alpha = q^3/(2\pi)^2$, local anomaly:

$$\delta_O(y) = \frac{1}{4} \sum_{i=1}^{4} \delta(y - \zeta_i)$$

metric of compact space with shape moduli $\tau_{1,2}$,

$$(g_2)_{mn} = \frac{1}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & \tau_1^2 + \tau_2^2 \end{pmatrix}$$

Further moduli, combinations of dilaton and radion,

$$t = r^2 e^{-\phi}, \quad s = r^2 e^{\phi}$$

4d Planck mass and 4d gauge coupling ($x_{5,6} = L\ y_{1,2}$),

$$M_4^2 = \frac{L^2}{2} M_6^4, \quad \frac{1}{g_4^2} = \frac{L^2}{2 g_6^2}$$

effective, dimensionless length scale

$$\ell = g_4 M_4 \langle r \rangle L$$

Computation yields 4d effective bosonic action:

$$
\begin{aligned}
S_{\mathrm{B}}^{(4)} = \int_M \Big\{ & \frac{M_{\mathrm{P}}^2}{2}\Big(R_4 - \frac{1}{2t^2} dt \wedge * dt - \frac{1}{2s^2} ds \wedge * ds - \frac{1}{2\tau_2^2} d\tau \wedge * d\overline{\tau}\Big) \\
& - \frac{1}{2g^2}\Big(sF \wedge * F + (c + g^2 \beta \ell^2 b) F \wedge F\Big) - \frac{g^2 M_{\mathrm{P}}^4}{2st^2}\frac{f^2}{\ell^4} \\
& - \frac{M_{\mathrm{P}}^2}{4t^2}\Big(db + \frac{2f}{\ell^2}A\Big) \wedge * \Big(db + \frac{2f}{\ell^2}A\Big) \\
& - \frac{M_{\mathrm{P}}^2}{4s^2}\Big(dc + g^2(2\alpha + \beta f)A) \wedge * (dc + g^2(2\alpha + \beta f)A\Big) \\
& -(d + iqA)\phi_+ \wedge * (d - iqA)\bar{\phi}_+ - m_+^2 |\phi_+|^2 - \frac{g^2 q^2}{2s}|\phi_+|^4 \Big\}
\end{aligned}
$$

b and c originate from tensor field, linear combination makes vector boson massive; also charged bulk matter field added; mass of lowest KK state:

$$
m_+^2 = -g^2 M_{\mathrm{P}}^2 \frac{qf}{st\ell^2} = \frac{qf}{2stV_2}
$$

Effective low energy theory contains massless chiral fermions, axion, massive moduli and massive vector boson, with mass due to **classical flux** and **quantum anomaly** (Stueckelberg mechanism):

$$S_a = \int_M \Big(-\frac{1}{2} s_0 \hat{F} \wedge *\hat{F} - \frac{f^2}{2t_0^2 s_0} - \frac{s_0}{2} m_{\hat{A}}^2 \hat{A} \wedge *\hat{A} - \frac{\kappa}{2} da \wedge *da + \lambda a \hat{F} \wedge \hat{F} \Big) \,,$$

$$m_{\hat{A}}^2 = \frac{4f^2}{s_0 t_0^2} + \frac{1}{s_0^3} \alpha^2 (1+N)^2 \,,$$

$$\kappa = \frac{1}{4f^2 s_0^2 + \alpha^2 (1+N)^2 t_0^2} \,, \quad \lambda = \frac{2f s_0^2 - \alpha\beta(1+N) t_0^2}{4f^2 s_0^2 + \alpha^2 (1+N)^2 t_0^2}$$

with couplings determined by charge of matter field:

$$\alpha = \frac{q^3}{(2\pi)^2} \,, \quad \beta = -\frac{q^4}{(2\pi)^3}$$

Supersymmetric low-energy effective Lagrangian, given in terms of Kahler potential, gauge kinetic function and superpotential (defined at orbifold fixed points):

$$K = -\ln(S + \bar{S} + iX^S V) - \ln(T + \bar{T} + iX^T V) - \ln(U + \bar{U}) \,,$$
$$S = \frac{1}{2}(s + ic) \,, \quad T = \frac{1}{2}(t + ib) \,,$$
$$X^T = -i\frac{f}{\ell^2} \,, \quad X^S = -i\frac{N+1}{(2\pi)^2}$$

U is shape modulus; Killing vectors due to quantized flux and Green-Schwarz term, note opposite signs! Gauge kinetic function [cf. Ibanez, Nilles '87]:

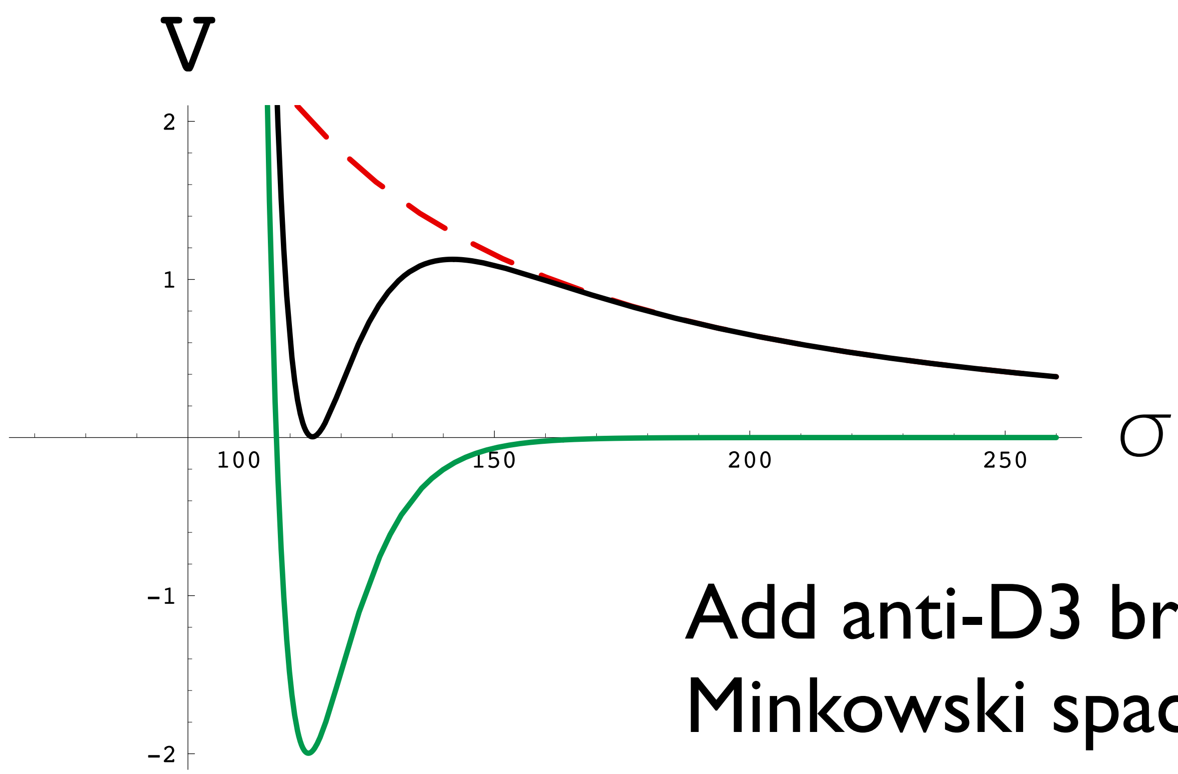
$$H = h_S S + h_T T \,, \quad h_S = 2 \,, \quad h_T = -\frac{2\ell^2}{(2\pi)^3}$$

Note **opposite sign** of the two contributions! Result: no scale model with gauged shift symmetry, involving S and T!

KKLT "uplift problem"

Nonperturbative potential to stabilize volume modulus in KKLT scenario yields AdS ground state (see Kallosh, Linde '04):

$$K = -3\log(\bar{T} + T)\,, \quad W = W_0 + W_1 e^{-aT}$$



$$V_{\rm AdS} = -3e^K|W|^2$$

Add anti-D3 brane, Polonyi field, D-term... to obtain Minkowski space:

$$V_{\rm KKLT}(\sigma_0) = V_F + V_D = |F|^2 - 3m_{3/2}^2 + \frac{1}{2}D^2 \approx 0\,,$$

$$3m_{3/2}^2 \approx \frac{1}{2}D^2 + |F|^2$$

Use KKLT-type superpotential at fixed points as “down lift”:

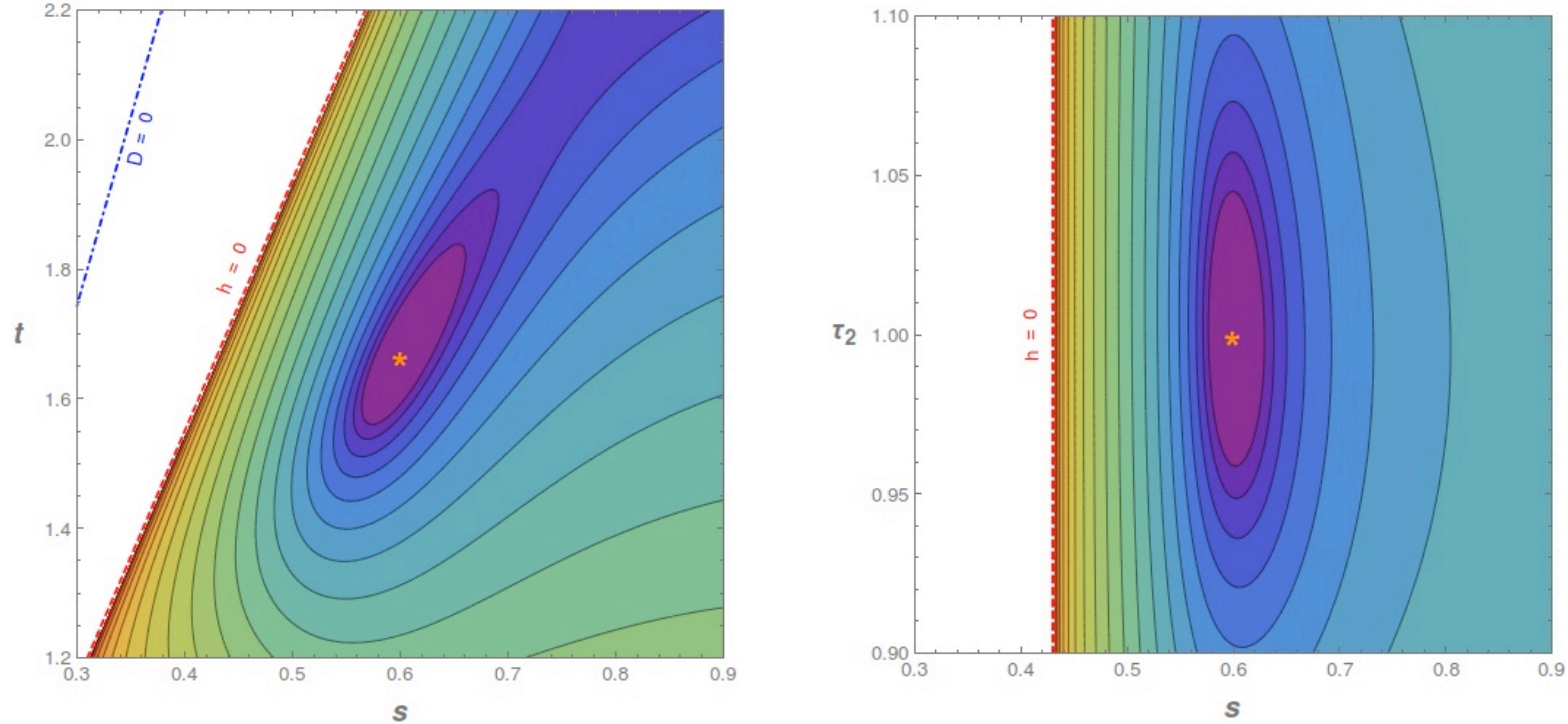
$$W = W(Z,U) = W_0 + W_1\, e^{-aZ} + W_2\, e^{-\tilde{a}U}\,, \quad Z = -iX^T S + iX^S T$$

Scalar potential involving F- and D-terms:

$$V = V_F + V_D = e^K (K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2) + \frac{1}{2h} D^2\,,$$

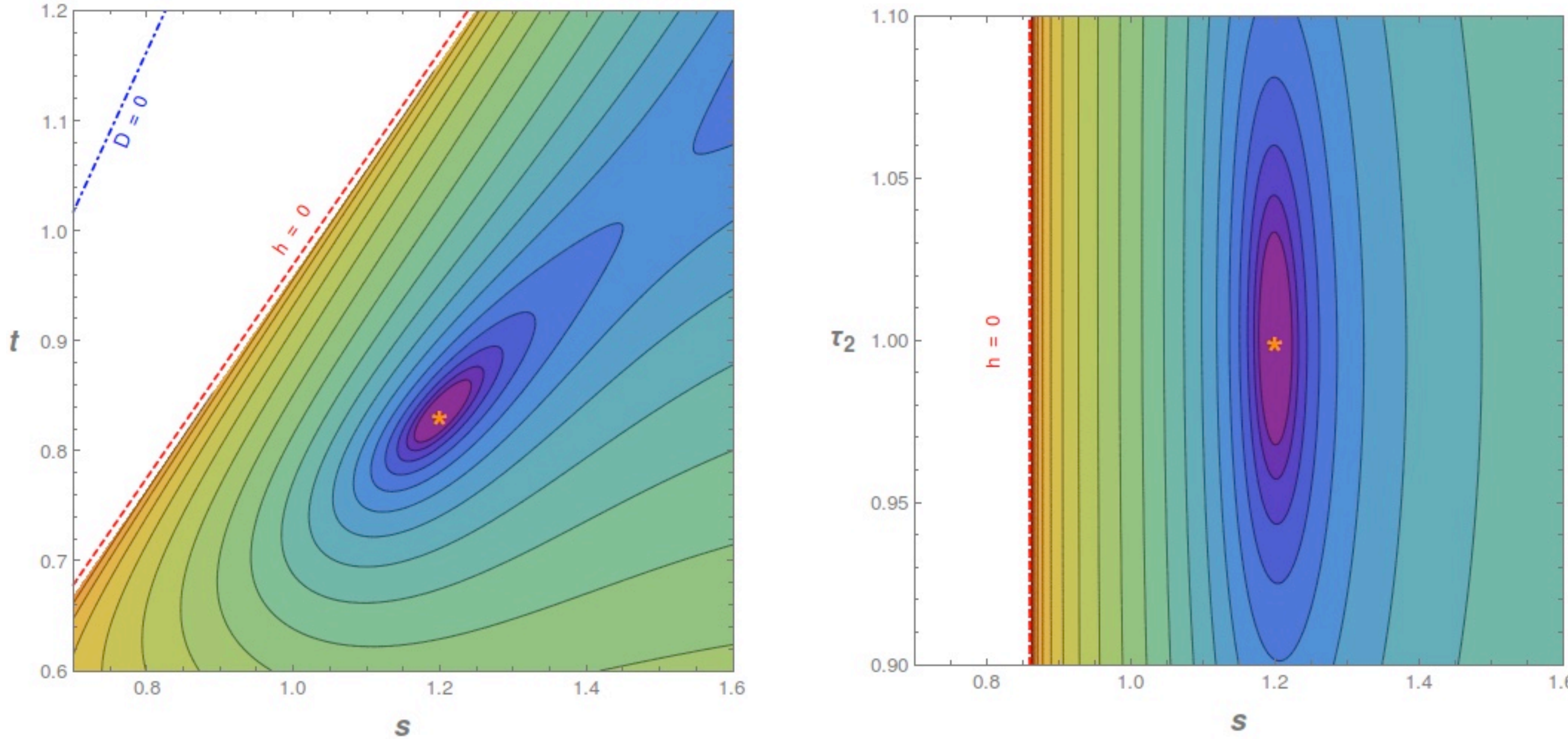
$$D = iK_i X^i = -\frac{i}{s} X^S - \frac{i}{t} X^T\,.$$

Due to flux AND quantum corrections to gauge kinetic function and Killing vectors, **de Sitter vacua exist** without further F-term uplift (e.g. Polonyi)! Size of extra dimensions determined by parameters of superpotential; example: $W_0 \sim W_1 \sim 10^{-3}\,, a \sim 1 \rightarrow r\ell \sim 10^2$, i.e. GUT scale extra dimensions. Hence most basic ingredients of 6d compactifications sufficient to obtain de Sitter vacua and moduli stabilization!



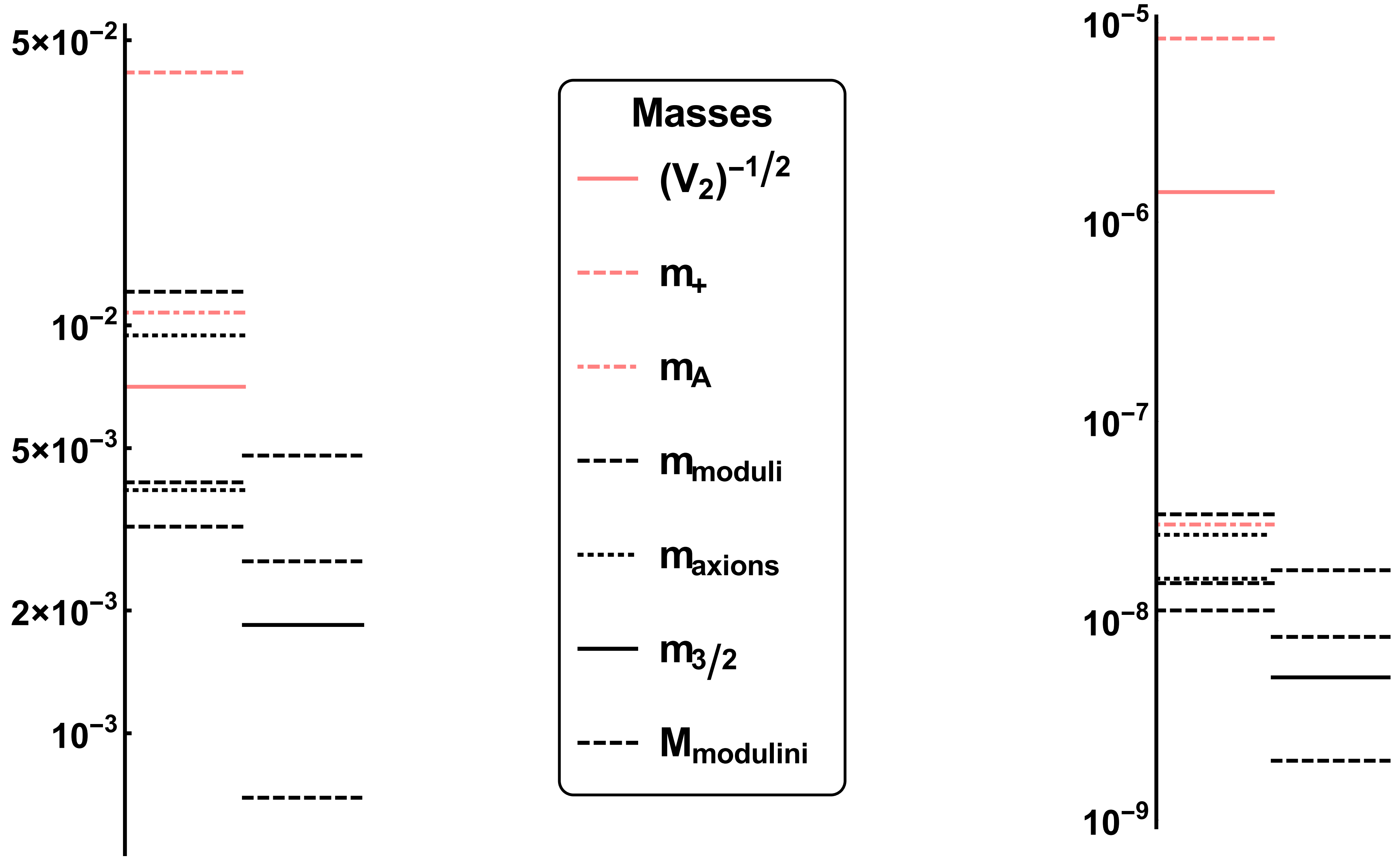
de Sitter (Minkowski) metastable minimum with GUT scale extra dimensions:

$$g = 0.2\,, \quad L = 200\,, \quad W \sim 10^{-2}$$



de Sitter (Minkowski) metastable minimum with intermediate scale extra dimensions:

$$g = 4 \cdot 10^{-3}\,, \quad L = 10^{6}\,, \quad W \sim 10^{-8}$$



boson and fermion masses from GUT scale down to "large" extra dimensions:

$$m_A^2 \propto L^{-3}\,, \quad m^2_{\text{moduli}} \propto L^{-3}\,, \quad m^2_{\text{axions}} \propto L^{-3}$$

$$m_{3/2} \propto L^{-3/2}\,, \quad M_{\text{modulini}} \propto L^{-3/2}$$

Conclusions

- Supersymmetric extensions of Standard Model strongly motivated, but what is the scale of SUSY breaking??

- Higher-dimensional GUT model with flux suggests GUT scale for SUSY breaking; emerging low energy spectrum reminiscent of `spread' SUSY (2HDM + higgsino + ...)

- Extrapolations of THDMs to GUT scale consistent with RG running and vacuum stability

- Anomalous U(1) allows de Sitter vacua

- Possible discoveries at LHC: Higgs bosons, light higgsinos