Scaling laws in inclusive production of cumulative and high-momentum particles in proton nucleus collisions at high energies

Alexey Aparin

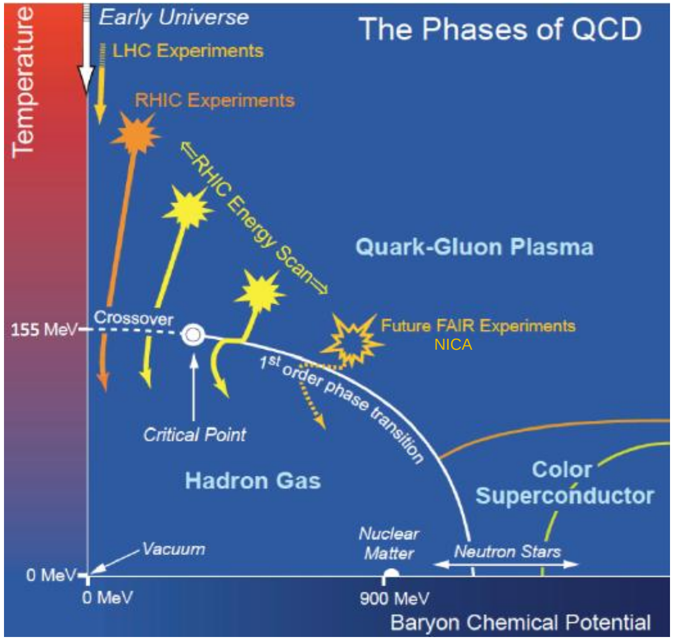
V&B Laboratory of high energy physics

Joint institute for nuclear research

Alushta-2016

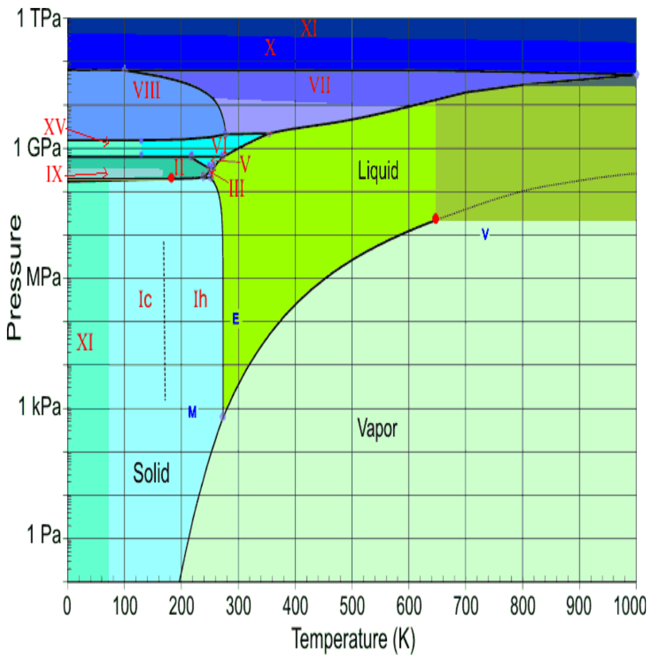
Phase diagram of matter

The phase diagram of strongly interacting matter requires further study



- Phases - 2?
- Phase boundaries -?
- Phase transitions - ?
- Triple Point - ?
- Critical Point - ?

Features of this phase diagram are well known from thermodynamics



- Phases (18)
- Phase boundaries
- Phase transitions
- Triple Point (16)
- Critical Point (2)

Self-similarity in physics

- Self-similarity means that a pattern is similar to a part of itself.
- Universal description using self-similarity variables constructed as suitable combinations of physical quantities.

Self-similarity variables Π (Re, π, M,…)

Hydrodynamics

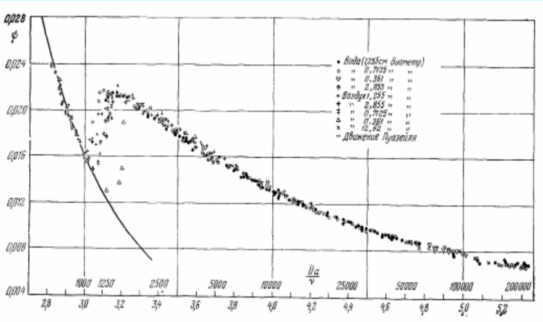
Re=dVρ/η

d – tube diameter

V – fluid velocity

ρ – fluid density

η – fluid viscosity

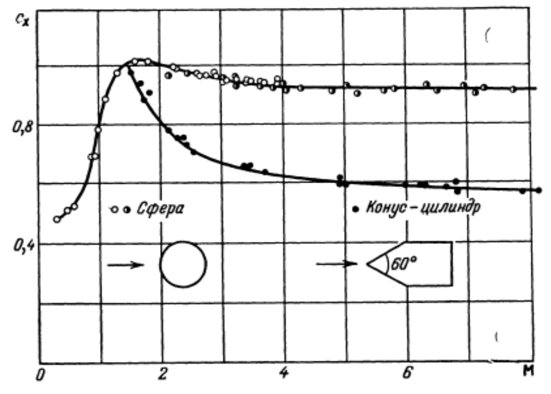


Aerodynamics

M=v/c

v – flow velocity

c – velocity of sound in the medium



Dimensionless function & self-similarity variable

Motivation

Main task is to find clear signatures of new physics (phase transition, critical point) in compressed nuclear matter. Phenomenological approach of z-scaling was used.

Analysis of experimental data on inclusive spectra of cumulative hadron production in pA collisions to verify properties of z-scaling and search for critical effects.

- pA is a reference frame for pp and AA
- in cumulative process enhancement of nuclear matter compression
- particle formation is sensitive to state of matter
- search for indications of phase transition & CP

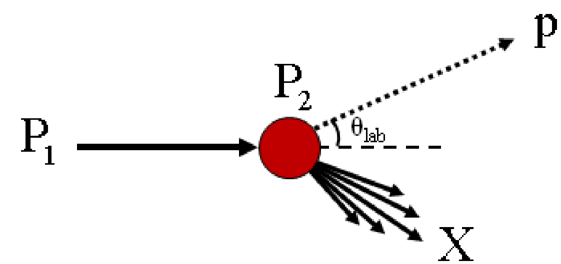
Cumulative processes

A.M.Baldin & V.S.Stavinsky (1971)

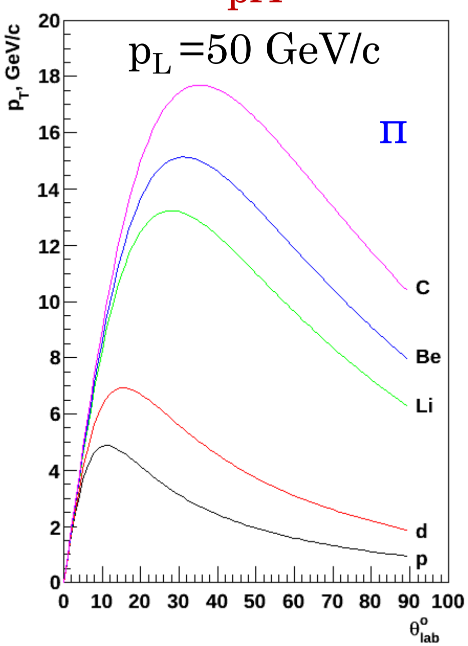
The cumulative particle is a particle produced
in the region forbidden for free nucleon kinematics:

$$P_1 + P_2 \rightarrow p + X$$

$$(P_1 + P_2 - p)^2 = M_X^2 \Rightarrow p_{max}^A > p_{max}^p$$

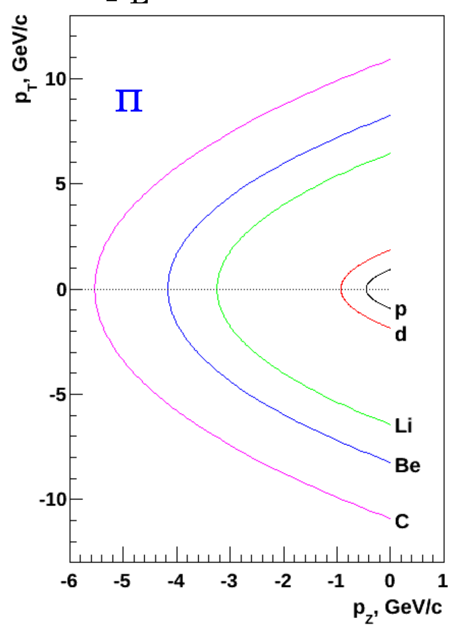


pA



pA

p_L =400GeV/c

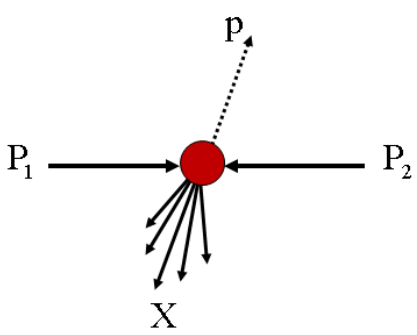


Conservation laws:

- 4-momentum
- electric charge
- baryon number
- flavors (u,d,s,c,b)

z-scaling

Basic principles: locality, self-similarity, fractality



Locality: collisions of hadrons and nuclei are expressed via interactions of their constituents (partons, quarks and gluons,...).
Self-similarity: interactions of the constituents are mutually similar.
Fractality: the self-similarity over a wide scale range.

Main hypothesis of z-scaling

$s^{1/2}$, p_T, θ_{cms}

$Ed^3\sigma/dp^3$

Inclusive particle distributions can be described in terms of constituent subprocesses and parameters characterizing bulk properties of the system.
Scaled inclusive cross section of particles depends in a self-similar way on a single scaling variable z.

x_1,x_2

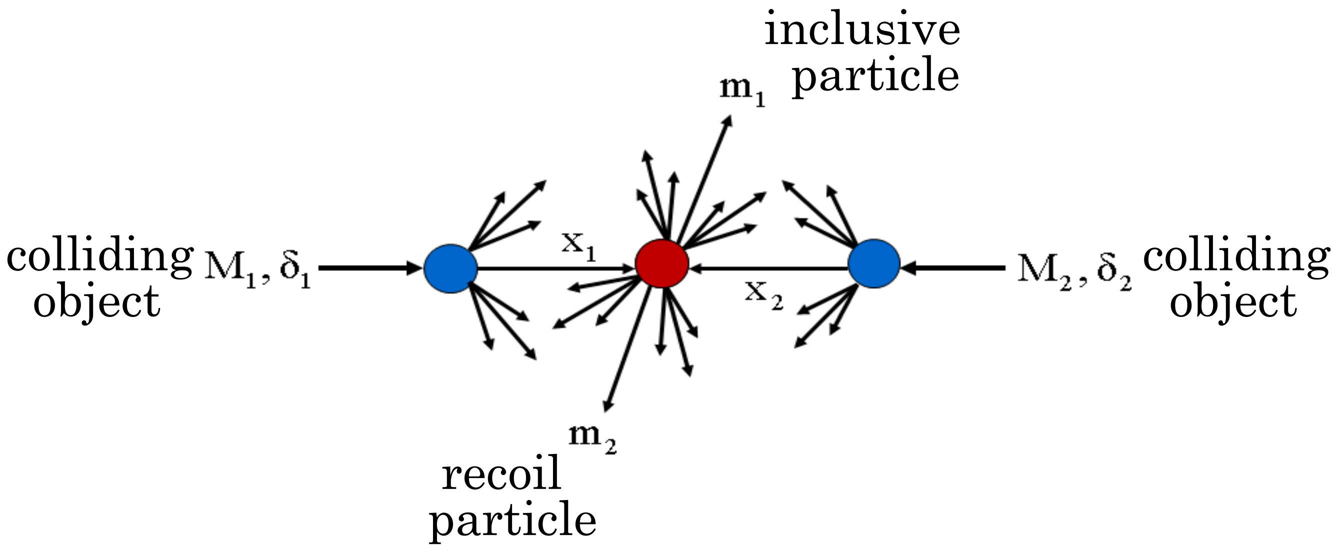
δ_1,δ_2

$\Psi(z)$

I.Zborovsky, M.Tokarev, Yu.Panebratsev, G.Skoro
Int.J.Mod.Phys. A16 (2001) 1281

Locality of hadron interactions

Interaction picture: binary collisions



V.S.Stavinsky
JINR Rapid Comm.
18-86, Dubna (1986)

A.A.Baldin
JINR Rapid Comm.
54-92, Dubna (1992)

M.Tokarev, I.Zborovský
Yu.Panebratsev, G.Skoro
Phys.Rev.D54 5548 (1996)
Int.J.Mod.Phys.A16 1281 (2001)

Constituent subprocess:

$$(x_1 M_1) + (x_2 M_2) \Rightarrow (m_1) + (x_1 M_1 + x_2 M_2 + m_2)$$

Kinematical condition (4-momentum conservation law):

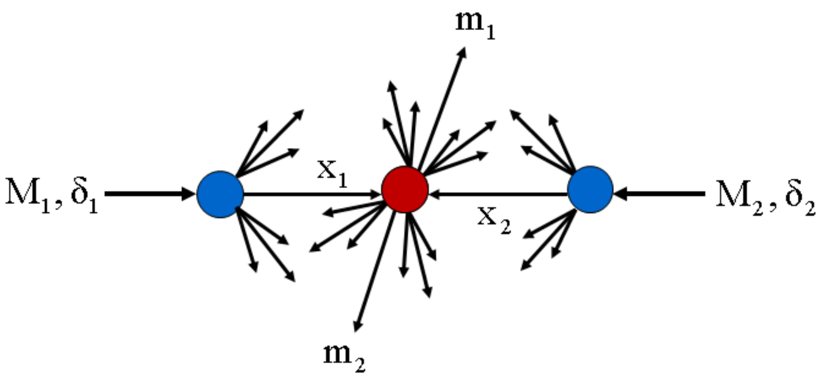
$$(x_1 P_1 + x_2 P_2 - p)^2 = M_X^2$$

Recoil mass: $M_X = x_1 M_1 + x_2 M_2 + m_2$

Self-similar variable z

$$z = z_0 \, \Omega^{-1}$$

$$z_0 = \frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)\, m}$$



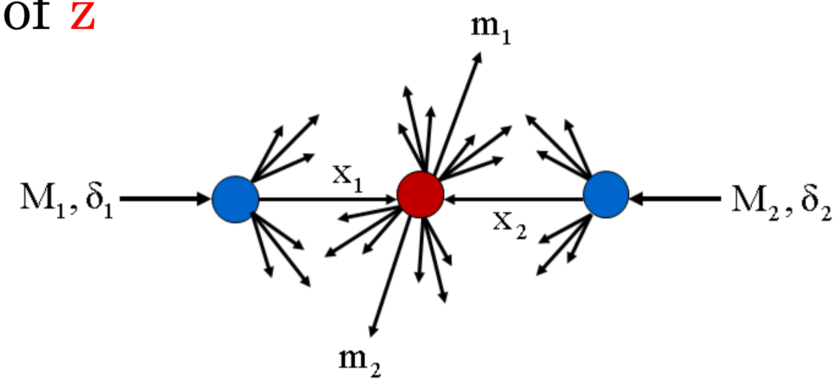
- $\sqrt{s_\perp}$ is the transverse kinetic energy of the subprocess consumed on production of m_1 and m_2
- $dN_{ch}/d\eta|_0$ is the multiplicity density of charged particles at $\eta = 0$
- m is an arbitrary constant (fixed at the value of nucleon mass)
- Ω^{-1} is the minimal resolution at which a constituent subprocess can be singled out of the inclusive reaction

Fractal measure z

Fractality is reflected in definition of z

$$z = z_0 \, \Omega^{-1}$$

$$\Omega = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}$$



Ω is relative number of configurations containing a sub-process with fractions x_1, x_2 of the corresponding 4-momenta
δ_1, δ_2 are parameters characterizing structure of the colliding objects
Ω^{-1} (x_1, x_2) characterizes resolution at which a constituent subprocess can be singled out of the inclusive reaction

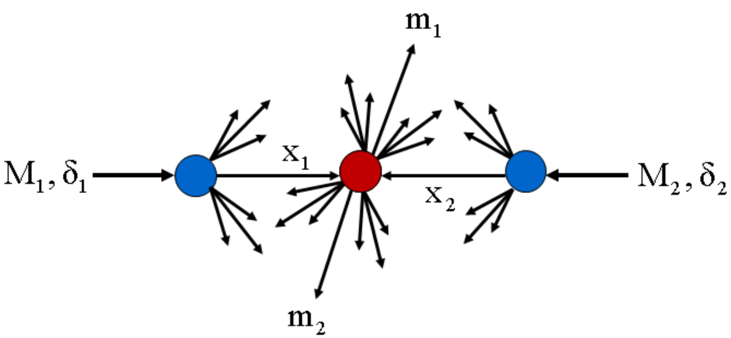
$$z(\Omega)\,|_{\Omega^{-1} \to \infty} \to \infty$$

Fractal measure z diverges as the resolution Ω^{-1} increases.

Scaling function $\Psi(z)$

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\cdot\sigma_{inel}} \cdot J^{-1} \cdot E\frac{d^3\sigma}{dp^3} \quad \Longleftrightarrow \quad \int E\frac{d^3\sigma}{dp^3} dy d^2 p_{\perp} = \sigma_{inel} \cdot N$$

$$\int_0^\infty \Psi(z)dz = 1$$

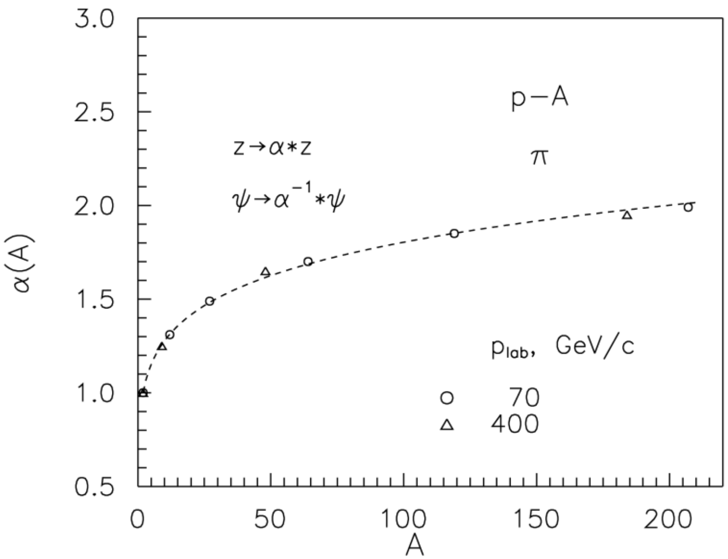


- σ_{in} – inelastic cross section
- N – average multiplicity of the corresponding hadron species
- $dN/d\eta$ – pseudorapidity multiplicity density at angle θ (η)
- $J(z,\eta;p_T^2,y)$ – Transition jacobian
- $Ed^3\sigma/dp^3$ – inclusive cross section

The scaling function $\Psi(z)$ is probability density to produce the inclusive particle with the corresponding z.

A-dependence of z-scaling

Taking into account mass dependence of z and Ψ(z) allow us to compare data from different experiments



$z \to \alpha(A) \cdot z$

$\Psi(z) \to \alpha^{-1}(A) \cdot \Psi(z)$

$\alpha(A) \approx 0.9\, A^{0.15}$

Self-similarity of nuclear modification of constituent interactions and hadron formation.

$$z = z_0\, \Omega^{-1} \qquad \Omega = (1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}$$

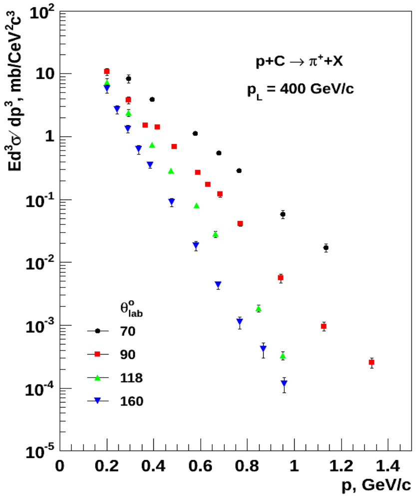
$$\delta_1 = A_1\delta,\ \delta_2 = A_2\delta$$

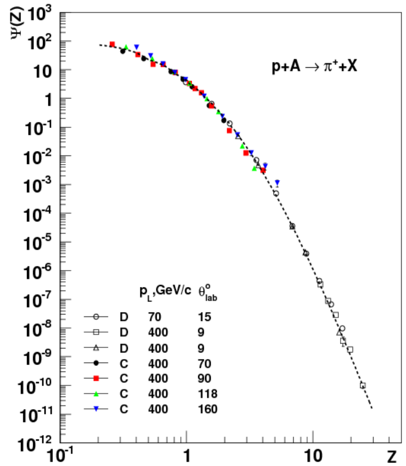
Selfsimilarity parameter z
"Critical exponents" δ_1, δ_2
allow one to check hot medium conditions
Discontinuities in δ_1, δ_2 will be strong signature of new physics effects

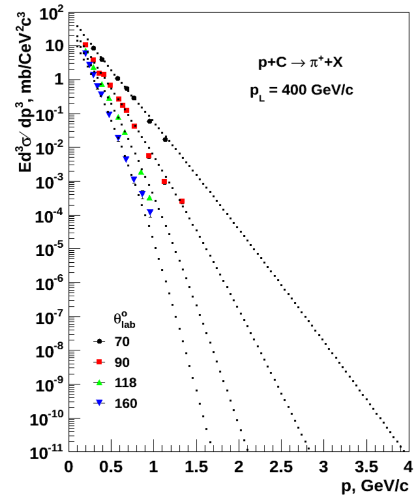
M.Tokarev, Yu.Panebratsev, I.Zborovsky, G.Skoro
JINR E2-99-113; Int. J. Mod. Phys. A16 1281 (2001).

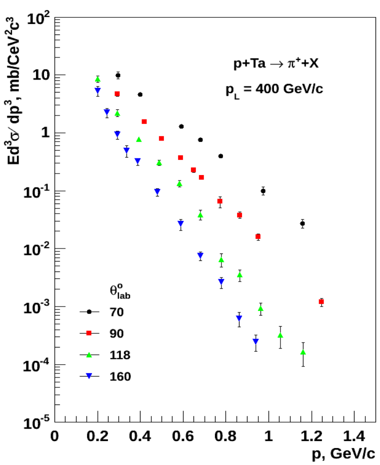
Cumulative pion spectra in pA at FNAL

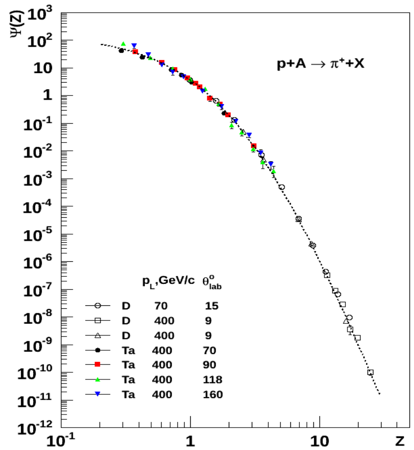
G.Leksin et al. p_L = 400 GeV/c, A = Li,Be,C,Al,Cu,Ta θ_{lab} = 70,90,118,160 deg.

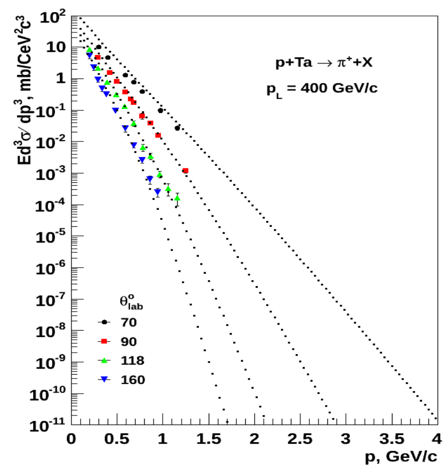






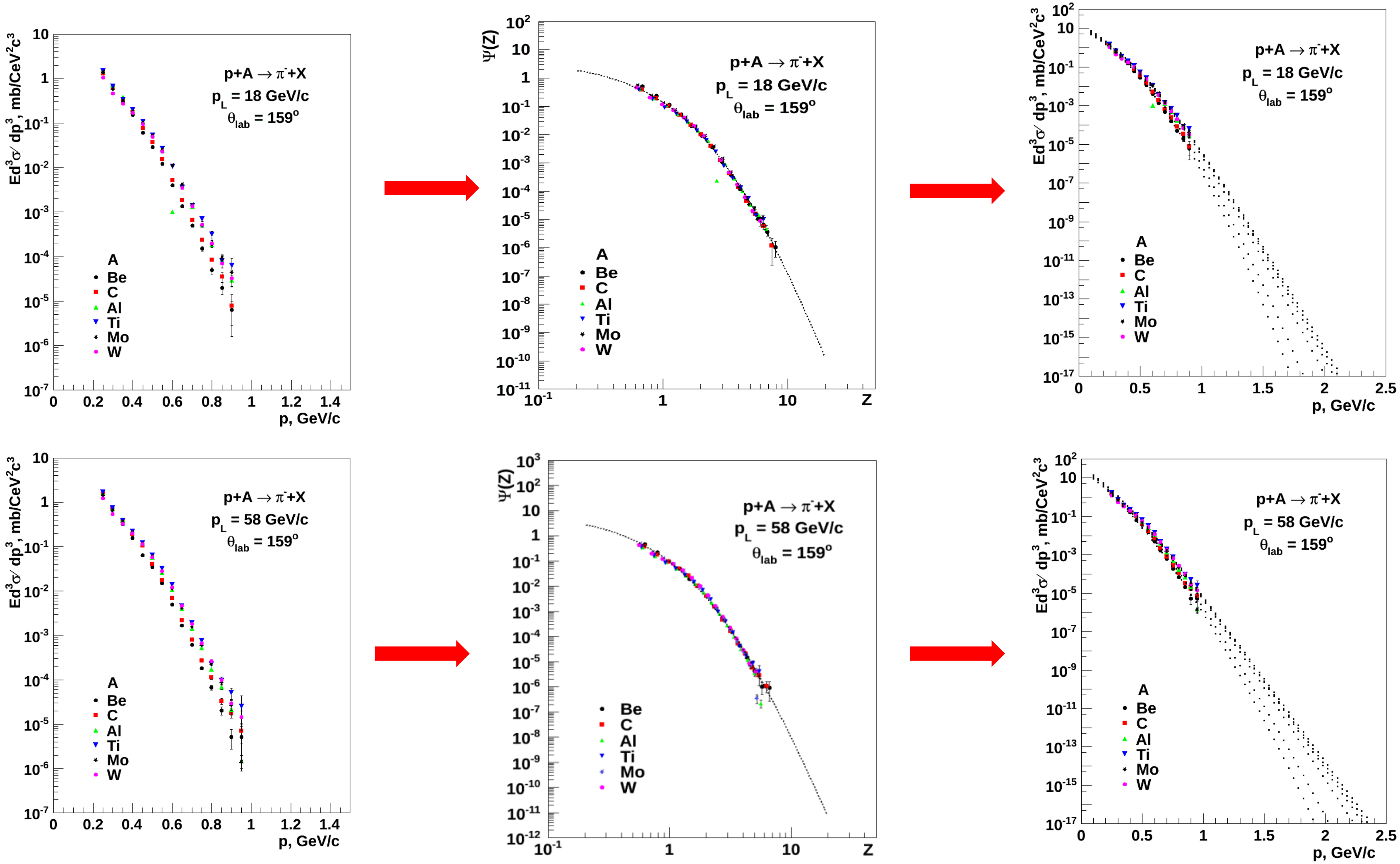






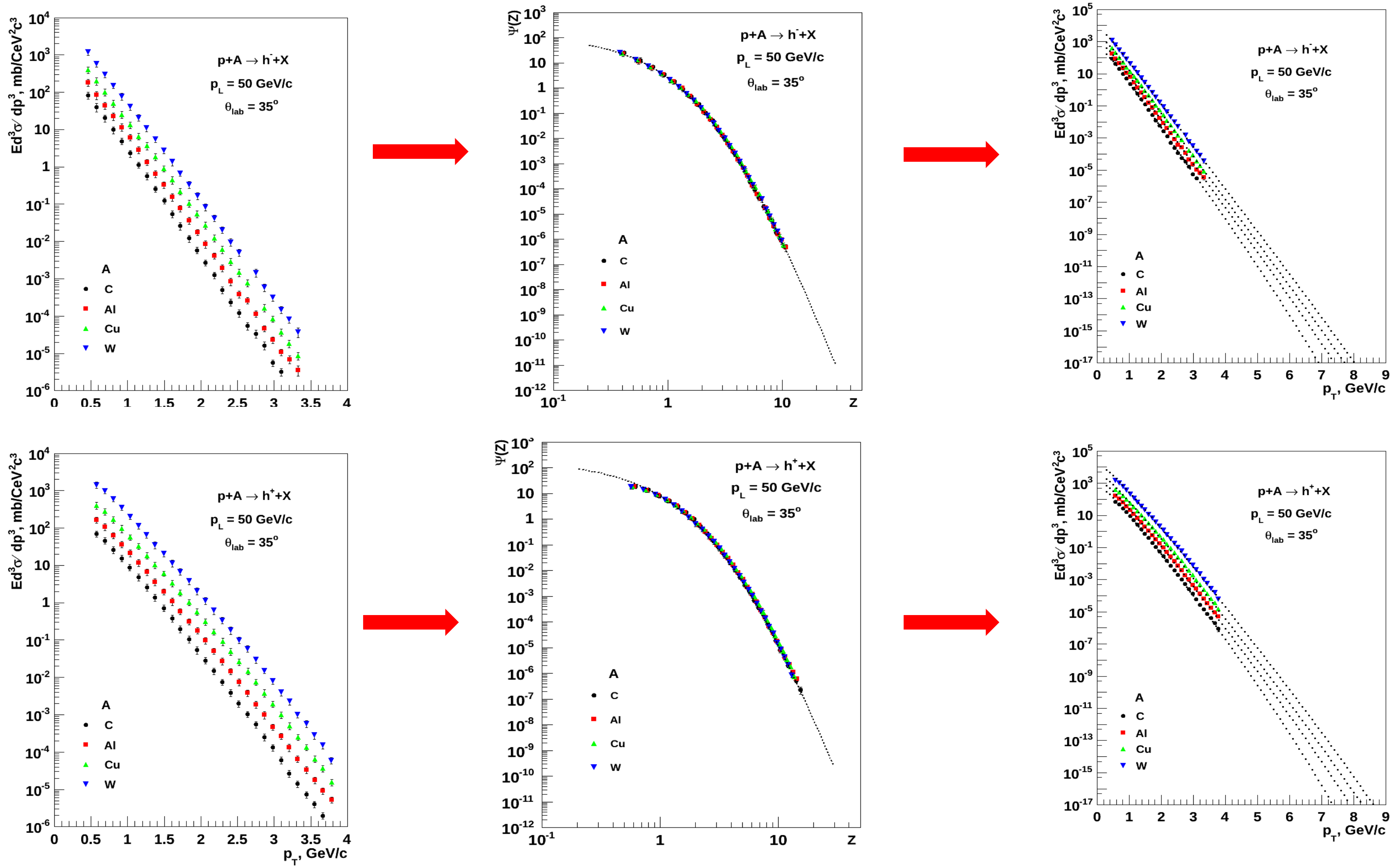
Low-p_T cumulative pion spectra in pA at U70

U70 (L.Zolin et al.) p_L = 18, 58 GeV/c, A = Be,C,Al,Ti,Mo,W θ_{lab} = 159 deg.



High-p_T cumulative hadron spectra in pA at U70

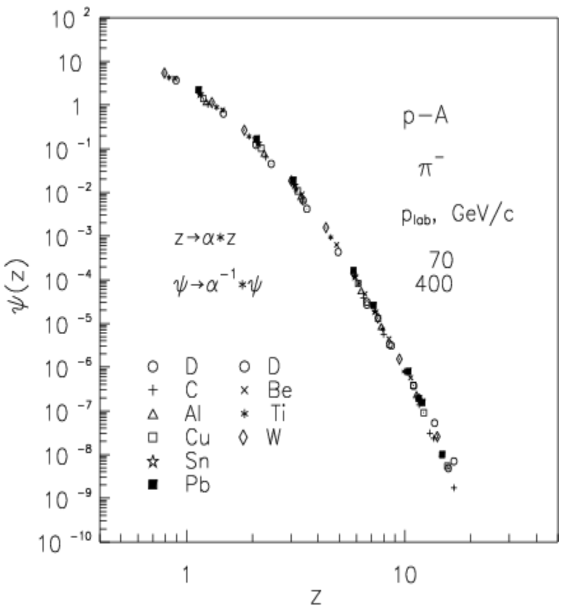
V.Gapienko et al. p_L = 50 GeV/c, A = C,Al,Cu,W θ_{lab} = 35 deg.



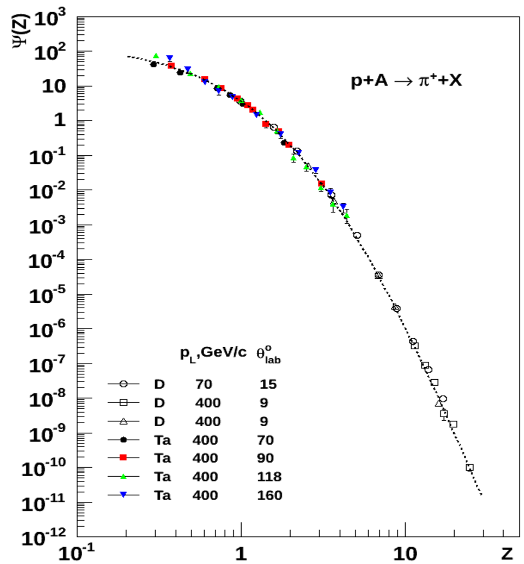
Scaling features in p-A collisions

FNAL (J.Cronin, G.Leksin, D.Jaffe) & U70 (R.Sulyaev, V.Gapienko)

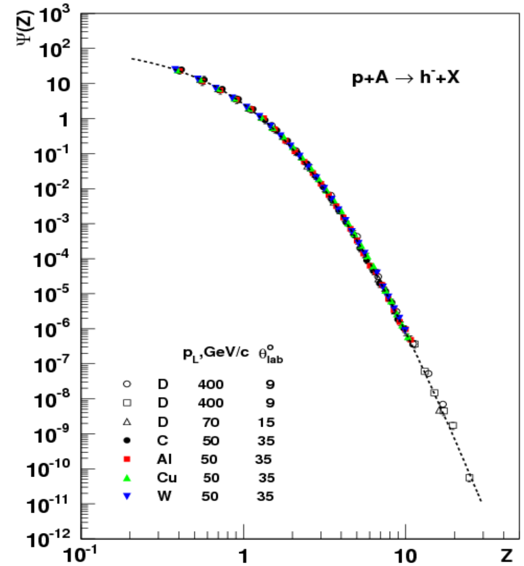
high-p_T & noncumulative



low-p_T & cumulative



high-p_T & cumulative



- Beam Energy Scan in pA
- Spectra of cumulative identified particles
- Multiplicity density $dN_{ch}/d\eta$ vs. $\sqrt{s}$ and η

Spectra form is parameterized by:

$$\Psi(z) = C \cdot \left(1 + (q-1) \cdot z / T\right)^{1/(1-q)}$$

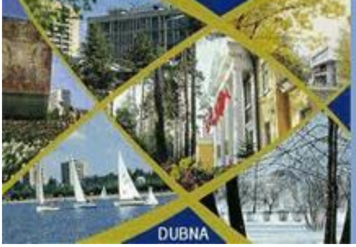
Search for phase transition & CP ⟺ Search for violation of z-scaling

Conclusions

- Indication on self-similarity of pion production in pA collisions at high energies in the cumulative region were found.
- The universality of the shape of $\Psi(z)$ in non-cumulative and cumulative pion production in pA was confirmed.
- The shape of the scaling function $\Psi(z)$ is well described by the Tsallis-like distribution.
- Search for the scaling violation in the deep-cumulative region is suggested.

The results can be used to develop a program to search for new physics phenomena in pA collisions at U70, RHIC, LHC & NICA, FAIR

A lot of work is ahead!!!



Thank you for your attention!

Backup slides

Самоподобие в физике частиц

Скейлинг Бъеркена:
В определенной области кинематических переменных протонные структурные функции не зависят от переданного импульса $q^2 = -Q^2$, а являются функциями только безразмерной переменной $x = Q^2/2M\nu$

$$\lim_{\nu\to\infty, q^2\to\infty} 2MW_1(\nu, q^2) \to F_1(\omega)$$

$$\lim_{\nu\to\infty, q^2\to\infty} \nu W_2(\nu, q^2) \to F_2(\omega)$$

$$\omega = 2M\nu / q^2$$

Скейлинг Фейнмана, предельная фрагментация Янга:
Вероятность рождения инклюзивной частицы с определенным значением продольного импульса p_L или p_T, при разных энергиях столкновения является универсальной функцией от переменной $x_F = p_L/p_{L\,max}$

$$Ed^3\sigma / dp^3 = f(x_F, p_\perp)$$

$$\lim_{s\to\infty, x\to 0} f'(s, x, p_\perp) \to f(p_\perp)$$

P-KNO скейлинг:
вероятность рождения n частиц $P_n(s)$ пропорциональна функции $\Psi(n/\langle n\rangle)$, зависящей от отношения $n/\langle n\rangle$ и удовлетворяющей условию нормировки

$$P_n(s) = \frac{1}{\langle n\rangle}\psi\left(\frac{n}{\langle n\rangle}\right), \quad \int_0^\infty \psi(Z)dZ = 1$$

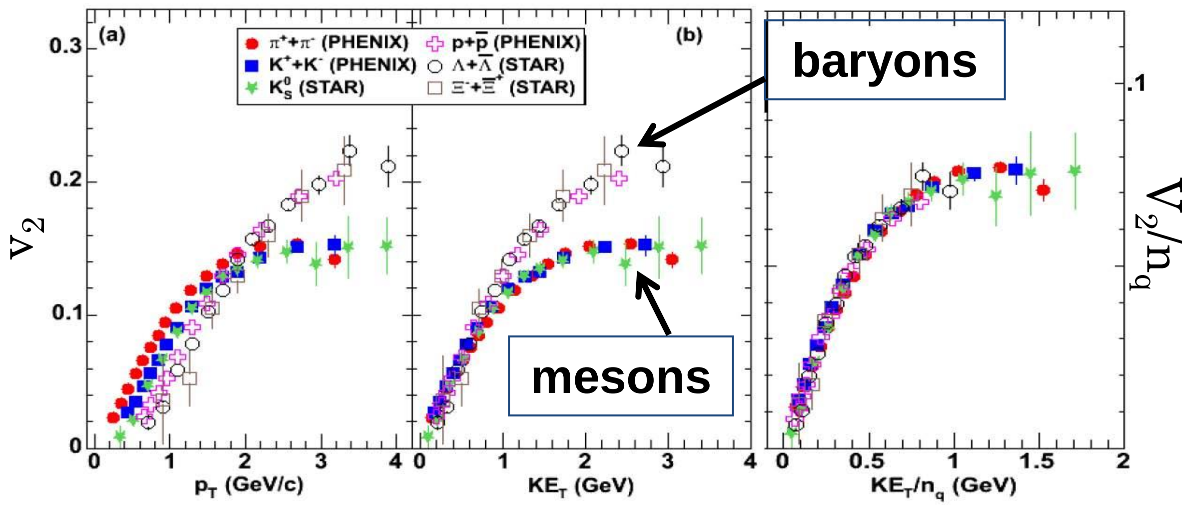
$$P_n = \int_{nZ_0}^{(n+1)Z_0} \psi(Z)dZ$$

Самоподобие в физике частиц

Правила кваркового счета
Матвеев-Мурадян-Тавхелидзе, Бродский-Фарад:
определяют энергетическую зависимость дифференциальных сечений рассеяния на большие углы при высоких энергиях столкновений и фиксированном угле ϑ, а также поведение формфакторов адронов $F_a(t)$ при больших передачах импульса $t = q^2$

$$\frac{d\sigma}{dt}(a+b\to c+d)\sim\frac{f(\vartheta)}{s^{n_a+n_b+n_c+n_d-2}},\quad F_a(t)\sim\frac{1}{|t|^{n_a-1}}\qquad \begin{array}{c} s,t\to\infty \\ s/t\to const\end{array}$$

NCQ-скейлинг в эллиптическом потоке:



Richard Seto
For the PHENIX Collaboration
June 17, 2014

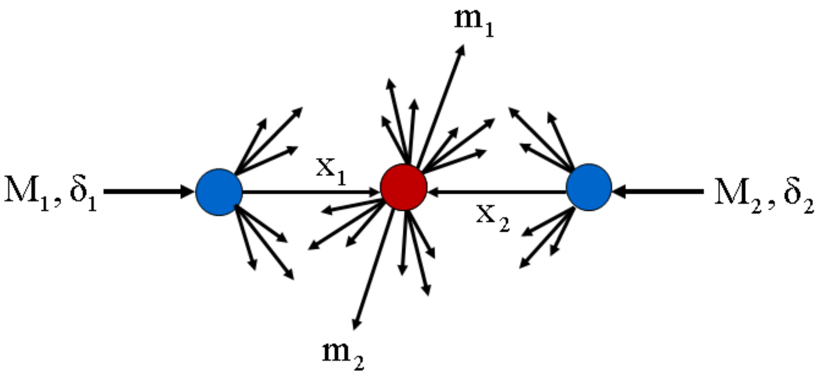
Доли импульса x_1, x_2

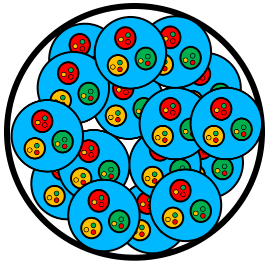
Принцип минимального разрешения: доли импульса x_1, x_2 определяются так, чтобы минимизировать разрешение Ω^{-1} фрактальной меры z для всех конституентных подпроцессов не нарушая законы сохранения.

$$\Omega = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}$$

$$\partial\Omega / \partial x_1 \big|_{x_2 = x_2(x_1)} = 0$$

$$(x_1 P_1 + x_2 P_2 - p)^2 = M_x^2$$





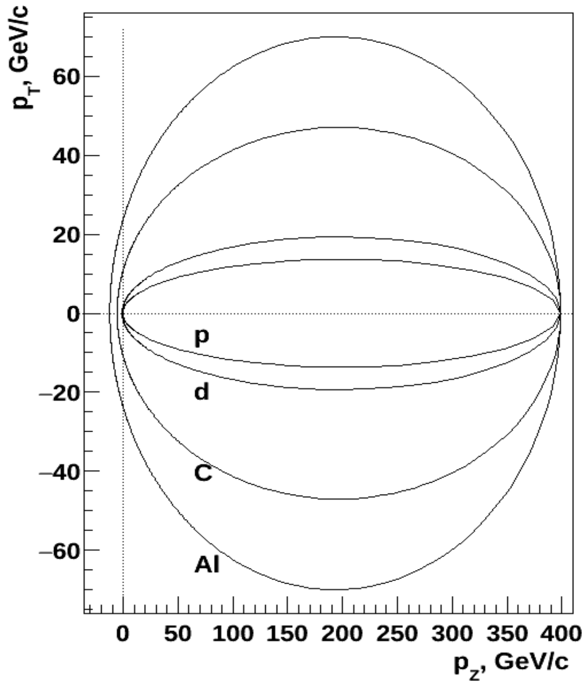
Масса системы отдачи:

$$M_X = x_1 M_1 + x_2 M_2 + m_2$$

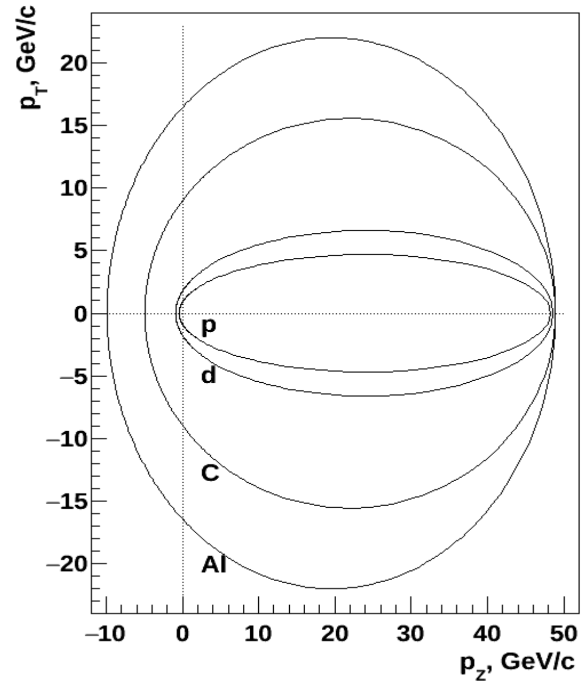
Кинематика pA столкновений

реакция p + A –> π^+ + X

p_L = 50 GeV/c



p_L = 400 GeV/c



В несимметричных адрон-ядерных столкновениях выбор кинематической области исследования существенно влияет на постановку эксперимента. Преимущества большей статистики в передней полусфере нивелируется повышенным фоном фрагментов ядра.

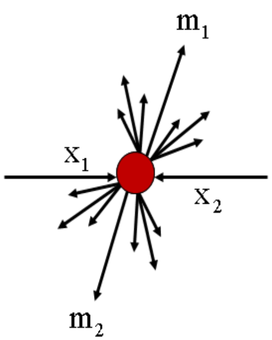
Transverse kinetic energy $\sqrt{s_\perp}$

$$s_\perp^{1/2} = (s_\lambda^{1/2} - M_1\lambda_1 - M_2\lambda_2) - m_1 + (s_\chi^{1/2} - M_1\chi_1 - M_2\chi_2) - m_2$$

energy consumed for the inclusive particle m_1

energy consumed for the recoil particle m_2

Fraction decomposition:



$$\lambda_{1,2} = \kappa_{1,2} + \nu_{1,2}$$

$$\kappa_{1,2} = \frac{(P_{2,1}p)}{(P_2P_1) - M_1M_2}, \quad \nu_{1,2} = \frac{M_{2,1}m_2}{(P_2P_1) - M_1M_2}$$

$$\chi_{1,2} = (\mu_{1,2}^2 + \omega_{1,2}^2)^{1/2} \mp \omega_{1,2}$$

$$\mu_{1,2}^2 = \alpha^{\pm 1}(\lambda_1\lambda_2 + \lambda_0)\frac{1-\lambda_{1,2}}{1-\lambda_{2,1}}$$

$$\omega_{1,2} = \mu_{1,2}U, \quad U = \frac{\alpha - 1}{2\sqrt{\alpha}}\xi, \quad \alpha = \frac{\delta_2}{\delta_1}$$

$$\lambda_0 = \bar{\nu}_0 - \nu_0$$

$$\xi^2 = (\lambda_1\lambda_2 + \lambda_0)/[(1-\lambda_1)(1-\lambda_2)]$$

$$\bar{\nu}_0 = \frac{0.5m_2^2}{(P_1P_2) - M_1M_2}, \quad \nu_0 = \frac{0.5m_1^2}{(P_1P_2) - M_1M_2}$$

$$s_\lambda = (\lambda_1P_1 + \lambda_2P_2)^2$$

$$s_\chi = (\chi_1P_1 + \chi_2P_2)^2$$

The scaling variable z and scaling function $\Psi(z)$ are expressed via relativistic invariants.

z-скейлинг и самоподобие в рождении частиц

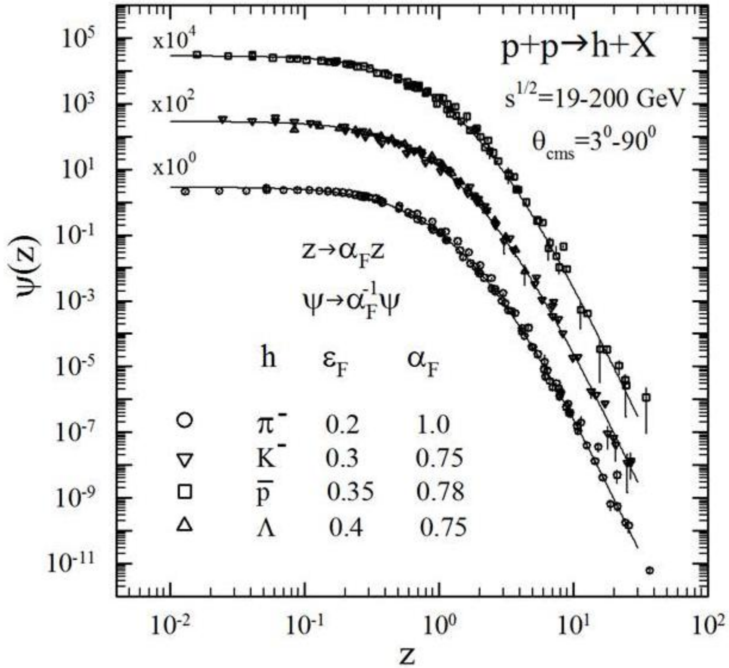
π^-, K^-, $\bar{p}$, Λ
in pp collisions

FNAL:
PRD 75 (1979) 764

ISR:
NPB 100 (1975) 237
PLB 64 (1976) 111
NPB 116 (1976) 77
(low p_T)
NPB 56 (1973) 333
(small angles)

STAR:
PLB 616 (2005) 8
PLB 637 (2006) 161
PRC 75 (2007) 064901

Saturation at low z



Power law at high z

Energy scan of spectra
at U70, ISR, Sp$\bar{p}$S, SPS, HERA, FNAL(fixed target), Tevatron, RHIC, LHC

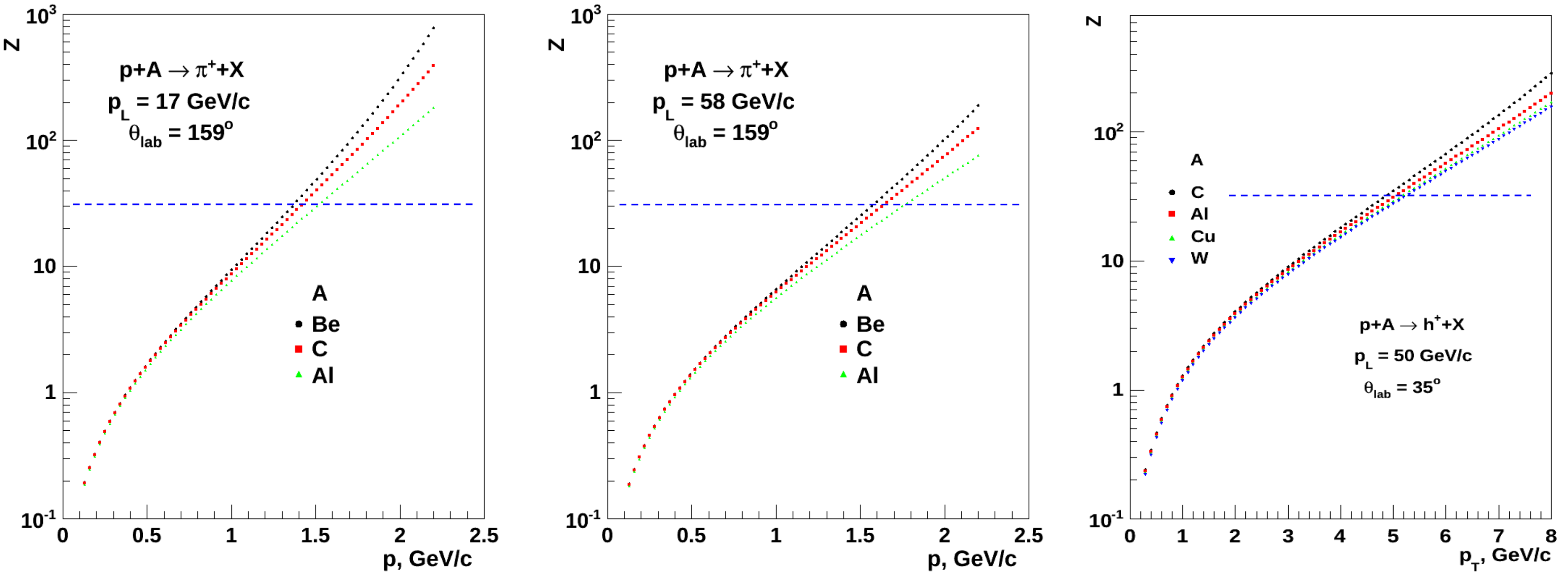
M.T. & I.Zborovsky
T.Dedovich
Phys.Rev.D75,094008(2007)
Int.J.Mod.Phys.A24,1417(2009)
J. Phys.G: Nucl.Part.Phys. 37,085008(2010)
Int.J.Mod.Phys.A27,1250115(2012)
J.Mod.Phys.3,815(2012)

- $\psi(z) \sim z^{-\beta}$ at high z
- ε_F, α_F independent of p_T, $s^{1/2}$, θ_{cms}

Скейлинг – “коллапс” экспериментальных точек на одну кривую.
Безразмерная функция Ψ и переменная подобия z.

$z\text{–}p_T$ & $z\text{–}p$ plots

Most suitable kinematic region to search for new physics in pA



$z > 30$

- Deep-cumulative region
- Power law, $\Psi(z) \sim z^{-\beta}$
- Universality of the shape of $\Psi(z)$
- Discontinuity of δ_A

Tsallis function

Boltzmann-Gibbs entropy

$$S_{BG} = -k\sum_{i=1}^{W} p_i \ln p_i$$

Additivity:

S(A+B)=S(A)+S(B)

Tsallis entropy

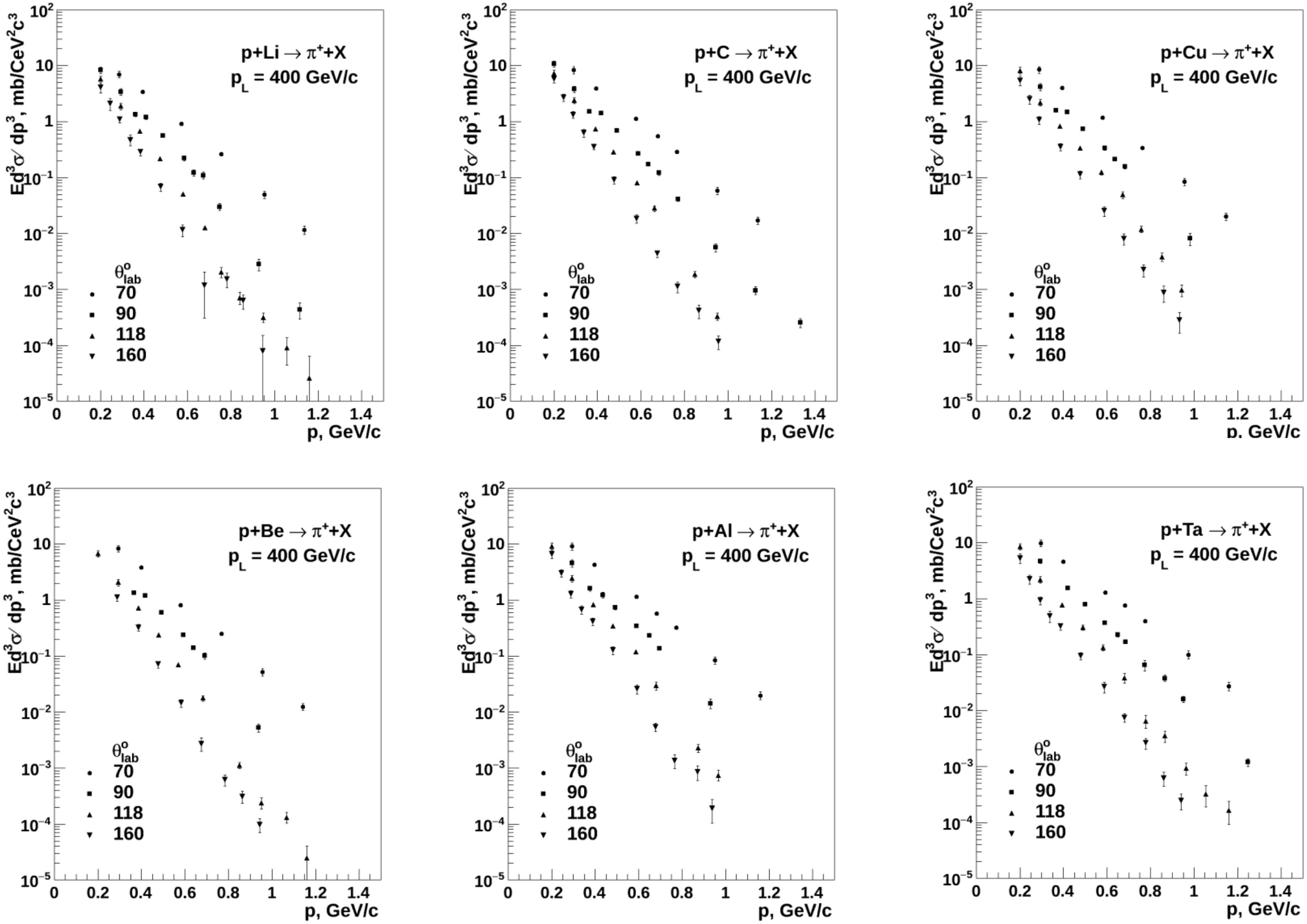
$$S_q = k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} = k\sum_{i=1}^{W} p_i \ln_q (1/p_i) = -k\sum_{i=1}^{W} p_i^q \ln_q p_i^q \xrightarrow[q\to 1]{} S_{BG}$$

Nonadditivity:

S(A+B)=S(A)+S(B)+(1-q)S(A)S(B), except for q =1

Спектры пионов

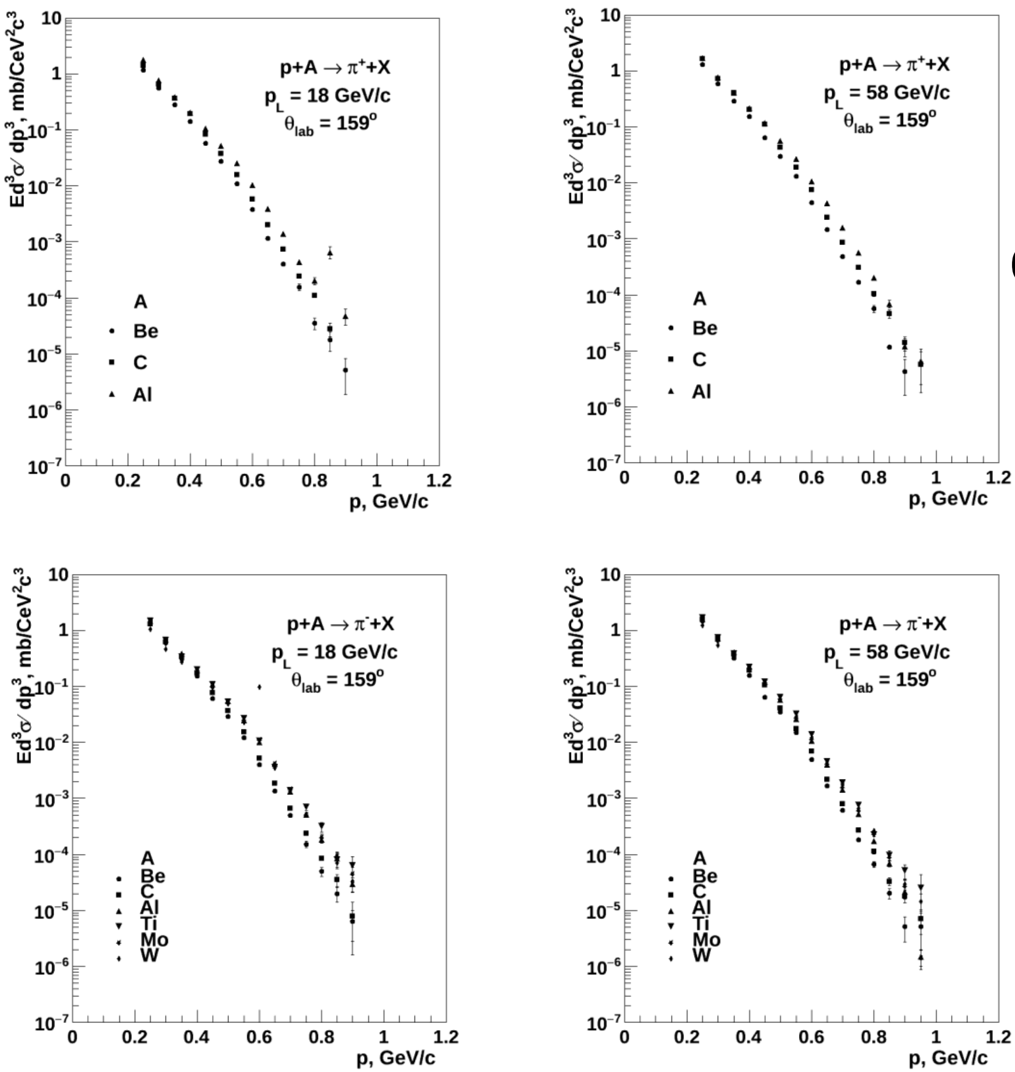
p_L = 400 GeV/c, A = Li,Be,C,Al,Cu,Ta θ_{lab} = 70,90,118,160 deg.



N.A. Nikiforov et al., Phys.Rev.C22 (1980)700.

Спектры пионов

p_L = 18, 58 GeV/c, A = Be,C,Al,Ti,Mo,W θ_{lab} = 159 deg.



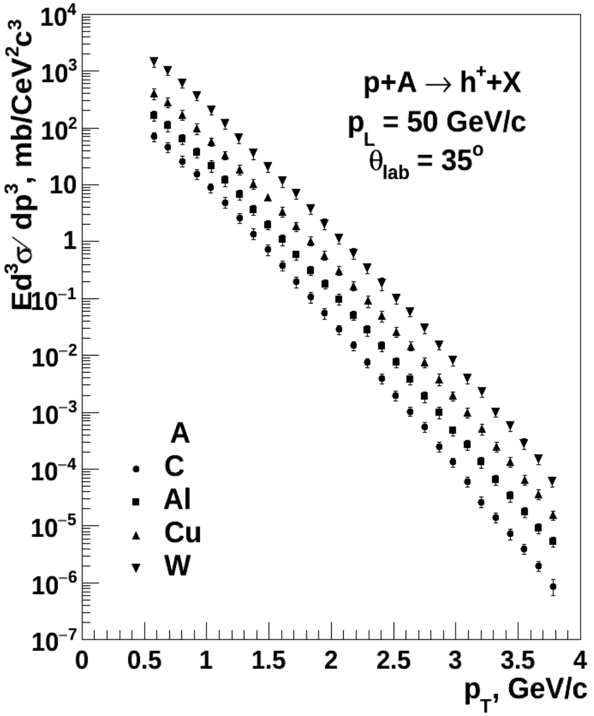
p_L (GeV/c)	p	C	Ti	W	
18	0.43	4.1	9.56	14.1	p^{π}_{max} (GeV/c)
58	0.46	5.2	16.3	34.5	

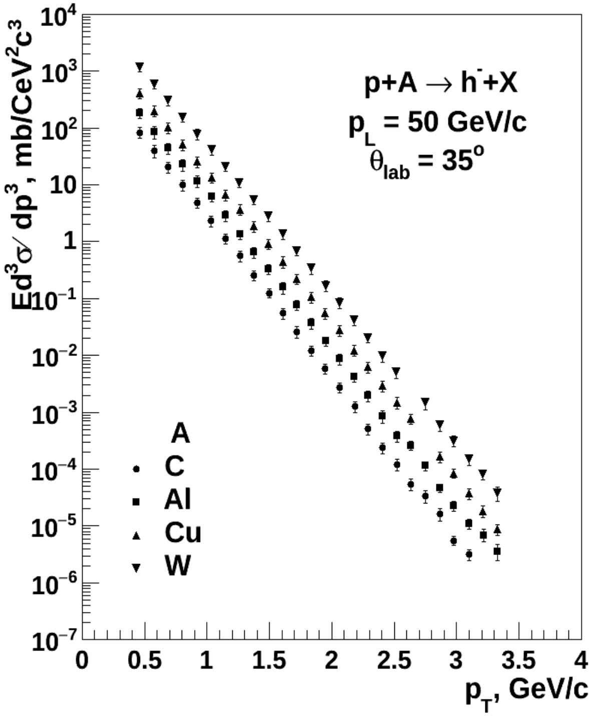
в таблице представлены максимально допустимые значения поперечного импульса инклюзивного пиона

O.P.Gavrishchuk et al., Nucl. Phys. A523 (1991) 589.

Спектры адронов

p_L = 50 GeV/c,   A = C,Al,Cu,W, θ_{lab} = 35 deg.





в таблице представлены максимально допустимые значения поперечного импульса инклюзивного пиона

V.V.Ammosov et al., Yad.Fiz. 76 (2013) 1276.

	p	C	Al	Cu	W
$p^{\pi}_{T\,max}$ (GeV/c)	2.62	15.6	20.7	24.4	26.7