Three-loop numerical calculation of critical exponents of the directed percolation process

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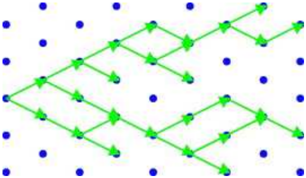
Directed Percolation

Directed bond percolation (DP)

The open bonds can be passed of an agent only from one of the two connecting sites, whence the allowed passage direction globally defines a preferred direction in space.



Chemical reactions:

$$A + A \xrightarrow{\kappa} A \qquad A \xrightarrow{\sigma} A + A \qquad A \xrightarrow{\mu} \varnothing$$

Directed Percolation

- Absorbing and active phase
- Non-equlibrium second order phase transition
- Mean field equation

$$\partial_t n(t) = (\sigma - \mu) n(t) - \kappa n(t)^2$$

- Absorbing state: $(\sigma < \mu) \qquad n(\infty) = 0$
- Active state: $(\sigma > \mu) \qquad n(\infty) = (\sigma - \mu)/\kappa$
- Chemical reactions:

$$A + A \xrightarrow{\kappa} A \qquad A \xrightarrow{\sigma} A + A \qquad A \xrightarrow{\mu} \varnothing$$

Directed Percolation

- Stochastic approach – Langevin equation

$$\partial_t \psi = D_0 \left[(\nabla^2 - \tau_0)\psi + \lambda_0 \psi^2 \right] + \zeta$$
$$\langle \zeta(t, \mathbf{x}) \zeta(t', \mathbf{x}') \rangle = D_0 \lambda_0 \psi(t, \mathbf{x}) \delta(t - t') \delta(\mathbf{x} - \mathbf{x}')$$

- Stochastic problem is equivalent to the field theoretic model of the doubled set of fields with action functional

$$S(\psi^\dagger, \psi) = \psi^\dagger(-\partial_t + D_0 \nabla^2 - D_0 \tau_0)\psi + \frac{D_0 \lambda_0 \mu^{\epsilon/2}}{2} \left((\psi^\dagger)^2 \psi - \psi^\dagger \psi^2 \right)$$

and the integration over the arguments of the fields is implied, for instance

$$\psi^\dagger \partial_t \psi = \int dt \int d\mathbf{x} \psi^\dagger(t, \mathbf{x}) \partial_t \psi(t, \mathbf{x})$$

Renormalization group

- The basic RG differential equation for the renormalized Greens function Γ_R

$$\left(\mu\partial_\mu + \beta_\lambda\partial_\lambda - \tau\gamma_\tau\partial_\tau - D\gamma_D\partial_D - n_\psi\gamma_\psi - n_{\psi^\dagger}\gamma_{\psi^\dagger}\right)\Gamma_R = 0$$

- β and γ functions

$$\gamma_x = \mu\partial_\mu \ln Z_x, \qquad \beta_x = \mu\partial_\mu x$$

- Analytical calculation using the renormalization group method and ϵ – expansion encountered considerable problems.
- Renormalization procedure in terms of the R operation

$$\Gamma_R = R\Gamma = (1-K)R'\Gamma$$

- The choice of K is ambiguous - Null-momentum substraction scheme

Renormalization group

- Using R-operation let us define the following functions[1]

$$f_i = R[-\tilde{\tau}\partial_{\tilde{\tau}}\bar{\Gamma}_i(\tilde{\tau})]|_{\tilde{\tau}=1}, \qquad \tilde{\tau} = \frac{\tau}{\mu^2}$$

- RG functions using diagrams of one ireducible functions reduce to convergent integrals
- Normalized Green function

$$\bar{\Gamma}_1 = \partial_{i\omega}\Gamma_{\psi^\dagger\psi}\Big|_{p=0,\omega=0} \qquad \bar{\Gamma}_3 = -\frac{\Gamma_{\psi^\dagger\psi} - \Gamma_{\psi^\dagger\psi}\Big|_{\tau=0}}{D\tau}\Bigg|_{p=0,\omega=0}$$

$$\bar{\Gamma}_2 = -\frac{1}{2D}\partial_p^2\Gamma_{\psi^\dagger\psi}\Big|_{p=0,\omega=0} \qquad \bar{\Gamma}_4 = \frac{\Gamma_{\psi^\dagger\psi^\dagger\psi} - \Gamma_{\psi^\dagger\psi\psi}}{D\lambda\mu^\epsilon}\Bigg|_{p=0,\omega=0}$$

satisfying the conditions

$$\bar{\Gamma}_i|_{\tau=\mu^2} = 1, \quad i = 1, 2, 3, 4$$

[1] L. Ts. Adzhemyan and M. V. Kompaniets. In: *Theor. Math. Phys.* 169.1 (2011), L. Ts. Adzhemyan et al. In: *Theor. Math. Phys.* 175.3 (2013).

Numerical Calculation

- Numerical Calculation Γ_i

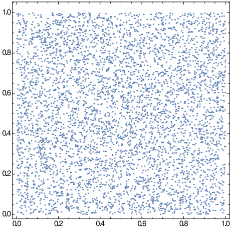
	1 - loop	2 - loop	3 - loop
$\Gamma_{\psi^\dagger\psi}$	1	2	17
$\Gamma_{\psi^\dagger\psi\psi}$	1	12	150

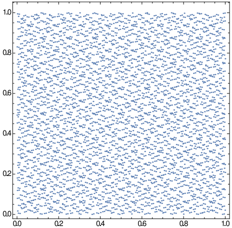
- Python 2.7 - library Graphine, GraphState
- Cuba[2] is a library for multidimensional numerical integration. (Vegas, Suave, Divonne and Cuhre)
- Numcal - numericla calculation

[2] R. Kreckel. In: *Comput. Phys. Commun* 106 (1997), pp. 258–266.

Numcal

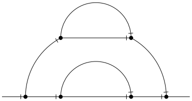
- program for numerical calculation
- GiNaC : from Graphine, GraphState to GiNaC archive file (.gar)
- Numcal : is interface between cuba and ginac
- Vegas[3] - Monte Carlo algorithm that use s importance sampling as a variance-reduction technique and Sobol quasi-random sample are used as basic integration mathod.



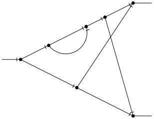


[3] T. Hahn. In: *Comput. Phys. Commun* 168 (2005).

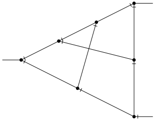
Calculation



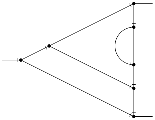
$= -1.270732.10^{-03} \pm 1.998392.10^{-07}$, 481.61s



$= -7.130491.10^{-04} \pm 1.090229.10^{-05}$, 717.05s



$= -8.553174.10^{-03} \pm 4.7621614.10^{-07}$, 544.19s



$= 1.599815.10^{-03} \pm 5.1470342.10^{-06}$, 417.26s

Table 1: Sample of diagrams for three-point Green function, which are calculated numerically (10^9 iterations) using Vegas.

Results

- Deviation from the critical space dimension $\epsilon = 4 - d$
- Critical exponents

$$z = 2 - \gamma_D^* = 2 - \frac{\varepsilon}{6} - 0.116824\varepsilon^2 + O(\epsilon^3)$$
$$\eta = -\frac{\varepsilon}{3} - 0.27228\varepsilon^2 + O(\epsilon^3)$$

- The calculation accurary for integrals was 10^{-4}
- Critical exponents with data from the analytic calculation[4]

$$z = 2 - \frac{\epsilon}{6}\left[1 + \left(\frac{67}{144} + \frac{59}{72}\ln\frac{4}{3}\right)\epsilon + O(\epsilon^2)\right] = 2 - \frac{\epsilon}{6} - 0.116836\epsilon^2 + O(\epsilon^3)$$
$$\eta = -\frac{\epsilon}{3}\left[1 + \left(\frac{25}{144} + \frac{161}{72}\ln\frac{4}{3}\right)\epsilon + O(\epsilon^2)\right] = -\frac{\epsilon}{3} - 0.272316\epsilon^2 + O(\epsilon^3)$$

[4] H. K. Janssen and U. C. Tauber. In: *Ann. Phys.* 315.147192 (2004).

Thank you for your attention.