Longitudinal form factor of the weak vector current in pion β-decay

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Content:
1. gWTI for broken SU(2) isospin symmetry
2. f_- near the mass shell from gWTI
3. Numerical estimates of the pion radius $\langle r^2 \rangle$ and f_- using dispersion techniques

New Trends in High-Energy Physics
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Introduction

ON MASS SHELL ➡ **2 form factors per nucleon**

Electromagnetic current of nucleons:

$$J^{\mu} = \bar{u}(p',s')\left(\gamma^{\mu}\left(\frac{1}{2}F_{1s} + \frac{\tau_3}{2}F_{1v}\right) + i\sigma^{\mu\nu}q_{\nu}\left(\frac{1}{2}F_{2s} + \frac{\tau_3}{2}F_{2v}\right)\right)u(p,s)$$

Isospin rotation of the isovector component gives

$$J^{\pm\mu}{}_{W} = \frac{G_F}{\sqrt{2}}\bar{u}(p',s')\left(\gamma^{\mu}F_1 + i\sigma^{\mu\nu}q_{\nu}F_{2v}\right)\tau^{\pm}u(p,s)$$

F_1 – universality of G_F

F_{2v} – weak magnetism - contributes to β-decays

Gerstein and Zeldovich (1955)

Introduction

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Gerstein and Zeldovich (1955)

OFF MASS SHELL → **12 form factors**

Introduction

OFF MASS SHELL → **12 form factors per nucleon**

$$\Gamma_\mu(p',p) = \sum_{\kappa,\kappa'=\pm 1} \Lambda_{\kappa'}(p')(\gamma_\mu \mathcal{F}_1^{\kappa'\kappa} + i\sigma_{\mu\nu}q_\nu \mathcal{F}_2^{\kappa'\kappa} + q_\mu(p'^2 - p^2)\mathcal{F}_3^{\kappa'\kappa})\Lambda_\kappa(p)$$

- **Why don't rotate 10 other form factors?**

Answer: **ON THE MASS SHELL AT TREE LEVEL**

Exact isotopic symmetry → **only 2 ones contribute**

Broken isotopic symmetry → **3 ones contribute**

ISOSPIN ROTATION OF F_3 IS NOT SUFFICIENT

Introduction

The most general expansion of the off-shell vector vertex:

$$\Gamma_\mu(p',p) = \sum_{\kappa'\kappa} \Lambda_{\kappa'}(p')(\gamma_\mu \mathcal{F}_1^{\kappa'\kappa} + i\sigma_{\mu\nu}q_\nu \mathcal{F}_2^{\kappa'\kappa} + q_\mu(p'^2 - p^2)\mathcal{F}_3^{\kappa'\kappa})\Lambda_\kappa(p)$$

where $\kappa, \kappa' = \pm 1$ $\quad , q = p' - p$

$$\mathcal{F}_\alpha^{\kappa'\kappa} = \mathcal{F}_\alpha^{\kappa'\kappa}(p'^2, p^2, q^2)$$

functions of 3 variables

$$\Lambda_\kappa(p) = \frac{\kappa\hat{p} + M}{2M} \qquad M = \sqrt{p^2}$$

$$\hat{p}\Lambda_\kappa(p) = \kappa M \Lambda_\kappa(p)$$

Negative C-parity implies:

$$C^T\left(\Gamma_\mu(p',p)\right)^T C = -\Gamma_\mu(p',p),$$

$$(p',p)^T = (-p,-p').$$

$$C_L^T \begin{pmatrix} 1 \\ i\gamma_5 \\ \gamma^\mu \\ \gamma_5\gamma^\mu \\ \sigma_{\mu\nu} \\ i\gamma_5\sigma_{\mu\nu} \end{pmatrix}^T C_L = \begin{pmatrix} 1 \\ i\gamma_5 \\ -\gamma^\mu \\ \gamma_5\gamma^\mu \\ -\sigma_{\mu\nu} \\ -i\gamma_5\sigma_{\mu\nu} \end{pmatrix}$$

$$\rightarrow \boxed{\mathcal{F}_\alpha^{\kappa'\kappa}(p'^2, p^2, q^2) = \mathcal{F}_\alpha^{\kappa\kappa'}(p^2, p'^2, q^2)}$$

Introduction

- **In this context, we start from the form factor $f_- \equiv \mathcal{F}_2$ of pion β-decay:**

$$\Gamma_\mu\left(p', p\right) = \left(p' + p\right)_\mu \mathscr{F}_1 + q_\mu\left(p'^2 - p^2\right)\mathscr{F}_2$$

Conclusions are based on

1. Generalized Ward identity for broken isospin SU(2) symmetry ($f_- \equiv \mathcal{F}_2$ on and off mass shell).

2. Similar to the standard analysis of $K_{\ell 3}$ decays (f_- on mass shell).

3. Numerical estimates use dispersion techniques

π_{e3} Form Factor f_- Near the Mass Shell

CVC hypothesis: $J^3_{WEAK} \leftrightarrow J_{EM}$ **by isospin rotation**

1. On the mass shell + exact SU(2): $\leftarrow$ **CVC**

$$\partial_\mu J^\mu{}_{EM} = 0 \leftrightarrow \partial_\mu J^{a\mu}{}_{WEAK} = 0$$

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$$\partial_\mu J^\mu{}_{EM} = 0 \leftrightarrow \partial_\mu J^{a\mu}{}_{WEAK} = 0$$

2. **Off the mass shell + exact SU(2):**

$$(p'-p)_\mu \Gamma^{a\mu}(p',p) = \Delta^{-1}(p')T^a - T^a \Delta^{-1}(p)$$ ← **WTI**

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3. On the mass shell + broken SU(2): ← **partial CVC**

$$\partial_\mu J^\mu{}_{EM} = 0 \leftrightarrow \partial_\mu J^{3\mu}{}_{WEAK} = 0, \quad \partial_\mu J^{\pm\mu}{}_{WEAK} \neq 0$$

π_{e3} Form Factor f_- Near the Mass Shell

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$$(p' - p)_\mu \Gamma^{a\mu}(p', p) = \Delta^{-1}(p')T^a - T^a \Delta^{-1}(p)$$ ← WTI

3. **On the mass shell + broken SU(2):** ← partial CVC

$$\partial_\mu J^\mu{}_{EM} = 0 \leftrightarrow \partial_\mu J^{3\mu}{}_{WEAK} = 0, \quad \partial_\mu J^{\pm\mu}{}_{WEAK} \neq 0$$

4. **Off the mass shell + broken SU(2):**

$$(p' - p)_\mu \Gamma^{a\mu}(p', p) =$$ **?** ← gWTI

$U(1)$ Vector Vertex e.g. pion electromagnetic current

The most general vertex (J = 0):

$$\Gamma_\mu(p',p) = (p'+p)_\mu \mathscr{F}_1 + q_\mu (p'^2 - p^2) \mathscr{F}_2$$

WTI: $$\Delta^{-1}(p') - \Delta^{-1}(p) = q_\mu \Gamma_\mu(p',p),$$

where $$\Delta^{-1}(p) = p^2 - m^2 - \Sigma(p^2, m)$$

$$\Sigma(m^2, m) = 0,$$

$$\frac{\partial}{\partial p^2}\Sigma(p^2, m)\Big|_{p^2=m^2} = 0.$$

⇒

$$q^2 \mathscr{F}_2(p'^2, p^2, q^2) = \frac{\Delta^{-1}(p') - \Delta^{-1}(p)}{p'^2 - p^2} - \mathscr{F}_1(p'^2, p^2, q^2).$$

$U(1)$ **Vector Vertex** e.g. pion electromagnetic current

The most general vertex (J = 0):

$$\Gamma_\mu(p',p) = (p'+p)_\mu \mathcal{F}_1 + q_\mu (p'^2 - p^2) \mathcal{F}_2$$

In the limit $p'^2 = p^2 = m^2$, we obtain

$$\mathcal{F}_2(m^2, m^2, q^2) = \frac{1 - \mathcal{F}_1(m^2, m^2, q^2)}{q^2}. \quad \leftarrow \textbf{WTI}$$

In the vicinity of $q^2 = 0$, the form factor $\mathcal{F}_1$ can be expanded to give

$$\mathcal{F}_2(m^2, m^2, 0) = -\frac{1}{6}\langle r^2 \rangle_v, \qquad (6)$$

where $\langle r^2 \rangle_v$ is the vector charge radius.

Generalized Ward-Takahashi Identity for Broken Isotopic Symmetry

THE IDEA BEHIND:

For the bare vertices and propagators

$$\frac{1}{\hat{p}+\hat{q}-m_f^{[0]}}\hat{q}\frac{1}{\hat{p}-m_i^{[0]}} = -\frac{1}{\hat{p}+\hat{q}-m_f^{[0]}} + \frac{1}{\hat{p}-m_i^{[0]}} + \frac{1}{\hat{p}+\hat{q}-m_f^{[0]}}\delta m_{fi}^{[0]}\frac{1}{\hat{p}-m_i^{[0]}}$$

$$\delta m_{fi}^{[0]} = m_f^{[0]} - m_i^{[0]}$$

Generalized WTI (gWTI):

$$q_\mu \Gamma^\mu_{fi}(p',p) = S_f^{-1}(p') - S_i^{-1}(p) + \delta m_{fi}\Theta_{fi}(p',p)$$

Vector vertex **Scalar vertex**

Generalized Ward-Takahashi Identity for Broken Isotopic Symmetry

Variation of the propagator:

$$\varphi \longrightarrow \varphi' = e^{-i\chi}\varphi,$$

$$\chi = \sum_a \chi^a T^a$$

$$\mathrm{Tr}(T^a T^b) = 2\delta^{ab}.$$

$$\chi \rightarrow 0$$

$$i\Delta^{\alpha\beta}(x',x) = \langle 0 | T\varphi^\alpha(x')\varphi^\beta(x) | 0 \rangle$$

$$i\delta\Delta(x',x) = \chi(x')\Delta(x',x) - \Delta(x',x)\chi(x)$$

In matrix notation, $\Delta^{-1} i\delta\Delta\Delta^{-1} = -i\delta\Delta^{-1} = [\Delta^{-1}, \chi]$

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{eff}}\left((D_\mu\varphi)^\dagger, D_\mu\varphi, \varphi^\dagger, \varphi\right)$$

$$iD_\mu = i\partial_\mu - eA_\mu - B_\mu$$

$$e = T^3 \ \& \ B_\mu = B_\mu^a T^a$$

CVC for elementary vertices

Isospin symmetry is violated by the mass term & EM interactions

$$\varphi^\dagger m^2 \varphi$$

with $[m^2, T^a] \neq 0$

Generalized Ward-Takahashi Identity for Broken Isotopic Symmetry

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$$i\delta\Delta(x', x) = \chi(x')\Delta(x', x) - \Delta(x', x)\chi(x)$$

$$\varphi \longrightarrow \varphi' = e^{i\chi}\varphi,$$

Compensating transformation generates the field transformation in the Lagrangian:

$$m^2 \longrightarrow m'^2 = m^2 + [m^2, i\chi],$$

$$eA_\mu \longrightarrow eA'_\mu = eA_\mu + [e, i\chi]A_\mu,$$

$$B_\mu \longrightarrow B'_\mu = B_\mu + \partial_\mu\chi + [B_\mu, i\chi].$$

Generalized Ward-Takahashi Identity for Broken Isotopic Symmetry

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$$\chi \rightarrow 0$$

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$$i\delta\Delta(x', x) = \chi(x')\Delta(x', x) - \Delta(x', x)\chi(x)$$

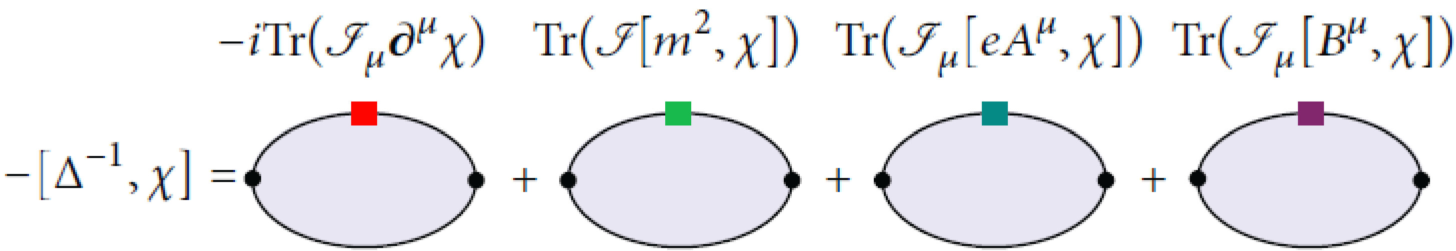
$$\varphi \longrightarrow \varphi' = e^{i\chi}\varphi,$$

Variation of the Lagrangian:

$$\delta\mathscr{L}_{\mathrm{eff}} = -\mathrm{Tr}\left(\mathscr{J}_\mu\left(\partial^\mu\chi + [e, i\chi]A^\mu + [B^\mu, i\chi]\right)\right) - \mathrm{Tr}\left(\mathscr{J}\left[m^2, i\chi\right]\right),$$

$$i\delta\Delta(x', x) = \left\langle 0 \left| T\varphi(x')\tilde{\varphi}(x)\, i\int d^4y\,\delta\mathscr{L}_{\mathrm{eff}}(y) \right| 0 \right\rangle$$

Generalized Ward-Takahashi Identity for Broken Isotopic Symmetry



4. Off the mass shell + broken SU(2):

$$(p' - p)_\mu \Gamma^{a\mu}(p', p) = \quad ? \quad \leftarrow \text{gWTI}$$

$$(p' - p)_\mu \Gamma^{a\mu}(p', p) = \Delta^{-1}(p')T^a - T^a \Delta^{-1}(p) + \Theta^a(p', p) + \Omega^a(p', p). \quad \leftarrow \text{ANSWER}$$

For pions: $\left(T^a\right)_{ij} = -\varepsilon_{aij} \rightarrow [T^a T^b] = \varepsilon_{abc} T^c$

$SU(2)$ **Vector Vertex**

Results

Vertex (off mass shell):

$$\Gamma^a_\mu(p',p) = (p'+p)_\mu \underbrace{\left(\mathcal{F}^a_{1-} + \left(p'^2 - p^2\right)\mathcal{F}^a_{1+}\right)}_{f_+} + q_\mu \underbrace{\left(\left(p'^2 - p^2\right)\mathcal{F}^a_{2-} + \mathcal{F}^a_{2+}\right)}_{f_-}$$

In the standard notations (on mass shell):

$$\left\langle \pi^0(p') \left| \bar{d}\gamma_\mu(1-\gamma_5)u \right| \pi^+(p) \right\rangle = \sqrt{2}\left((p'+p)_\mu f_+ + q_\mu f_-\right)$$

isospin factor

$SU(2)$ Vector Vertex

In the standard notations:

$$\langle \pi^0(p') | \bar{d}\gamma_\mu (1-\gamma_5) u | \pi^+(p) \rangle = \sqrt{2}\left((p'+p)_\mu f_+ + q_\mu f_- \right)$$

1. On the mass shell + exact SU(2):

$$\partial_\mu J^\mu{}_{EM} = 0 \leftrightarrow \partial_\mu J^{a\mu}{}_{WEAK} = 0$$

$$f_- = 0.$$

$SU(2)$ Vector Vertex

In the standard notations:

$$\left\langle \pi^0\left(p'\right)\left|\bar{d}\gamma_\mu\left(1-\gamma_5\right)u\right|\pi^+\left(p\right)\right\rangle = \sqrt{2}\left(\left(p'+p\right)_\mu f_+ + q_\mu f_-\right)$$

2. Off the mass shell + exact SU(2):

$$(p'-p)_\mu \Gamma^{a\mu}(p',p) = \Delta^{-1}(p')T^a - T^a\Delta^{-1}(p)$$

$$f_- = -\frac{p'^2-p^2}{6}\left\langle r^2\right\rangle_v^{T=1}$$

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In the standard notations:

$$\langle \pi^0 (p') | \bar{d} \gamma_\mu (1 - \gamma_5) u | \pi^+ (p) \rangle = \sqrt{2} \left((p' + p)_\mu f_+ + q_\mu f_- \right)$$

3. On the mass shell + broken SU(2):

$$\partial_\mu J^\mu{}_{EM} = 0 \leftrightarrow \partial_\mu J^{3\mu}{}_{WEAK} = 0, \quad \partial_\mu J^{\pm\mu}{}_{WEAK} \neq 0$$

$$f_- = \left(m_{\pi^0}^2 - m_{\pi^+}^2 \right) \left(\mathcal{F}_{2-} \left(\mu^2, \mu^2, 0 \right) + \mathcal{F}_{2+} \left(\mu^2, \mu^2, 0 \right) \right)$$

$$= \frac{m_{\pi^+}^2 - m_{\pi^0}^2}{6} \left(\langle r^2 \rangle_v^{T=1} - \langle r^2 \rangle_s^{T=2} \right).$$

$SU(2)$ Vector Vertex

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← **gWTI**

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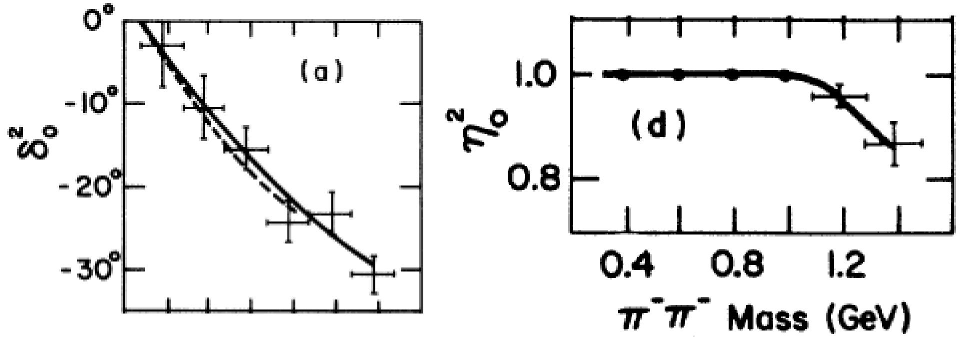
← gWTI

$$f_- = -\frac{p'^2 - p^2}{6}\langle r^2 \rangle_v^{T=1} + \frac{m_{\pi^0}^2 - m_{\pi^+}^2}{6}\langle r^2 \rangle_s^{T=2}$$

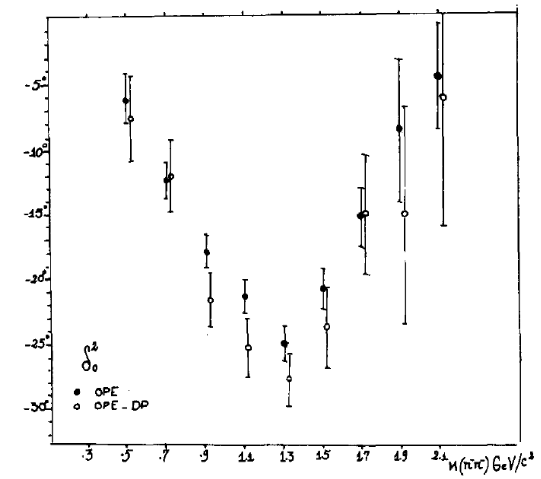
Numerical Estimates

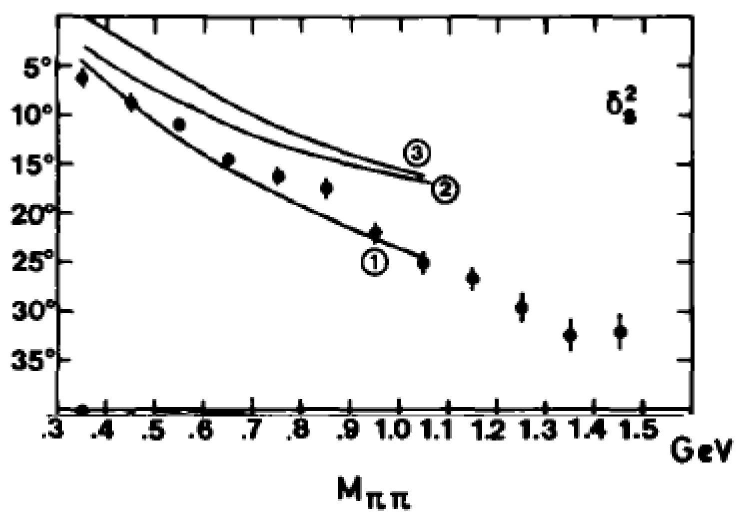
$$\left\langle r^2 \right\rangle_s^T = \frac{6}{\pi} \int_{4\mu^2}^{+\infty} \frac{\delta^T(s')}{s'^2} ds'.$$

$\delta^{T=2}_{J=0}$ experimental data



D. Cohen, T. Ferbel, P. Slattery, and B. Werner, "Study of $\pi\pi$ scattering in the isotopic-spin-2 channel," *Physical Review D*, vol. 7, no. 3, pp. 661–668, 1973.





N. B. Durusoy, M. Baubillier, R. George, M. Goldberg, and A. M. Touchard, "Study of the $I = 2\pi\pi$ scattering from the reaction $\pi^- d \rightarrow \pi^-\pi^- p_s p$ at 9.0 GeV/c," *Physics Letters B*, vol. 45, no. 5, pp. 517–520, 1973.

W. Hoogland, S. Peters, G. Grayer et al., "Measurement and analysis of the $\pi^+\pi^+$ system produced at small momentum transfer in the reaction $\pi^+ p \rightarrow \pi^+\pi^+ n$ at 12.5 GeV," *Nuclear Physics B*, vol. 126, no. 1, pp. 109–123, 1977.

Numerical Estimates

$$\langle r^2\rangle_v^{T=1} = (0.672 \pm 0.008\,\mathrm{fm})^2$$ = from ππ scattering data

$$\langle r^2\rangle_s^{T=2} = -0.10 \pm 0.03 \pm 0.03\,\mathrm{fm}^2$$ = new estimate from dispersion theory

$$f_- = (2.97 \pm 0.25) \times 10^{-3}$$

NB: two times higher than the Jaus (1999) quark model predictions

W. Jaus, "Covariant analysis of the light-front quark model," *Physical Review D*, vol. 60, Article ID 054026, 1999.

POSSIBLE APPLICATIONS (on shell)

$$f_- = (2.97 \pm 0.25) \times 10^{-3}$$

- $\pi^+ \longrightarrow \pi^0 e^+ \nu_e$

  $q^2 \sim \Delta m^2_\pi$

$$(\Delta B/B)^{th} = -0.94 \times 10^{-3} f_-$$

$$B^{exp} = (1.036 \pm 0.006) \times 10^{-8}$$

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- $\tau^- \longrightarrow \pi^- \pi^0 \nu_\tau$

  $q^2 \sim 4m^2_\pi$

  (enhanced f_- effect)

  $(\Delta B/B)^{th} = \ldots f_-$ (?)

  $B^{exp} = (25.52 \pm 0.09)\%$

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  $(\Delta B/B)^{th} = \ldots f_-(?)$

  $B^{exp} = (25.52 \pm 0.09)\%$

- $\tau^- \longrightarrow K^- K^0 \nu_\tau$

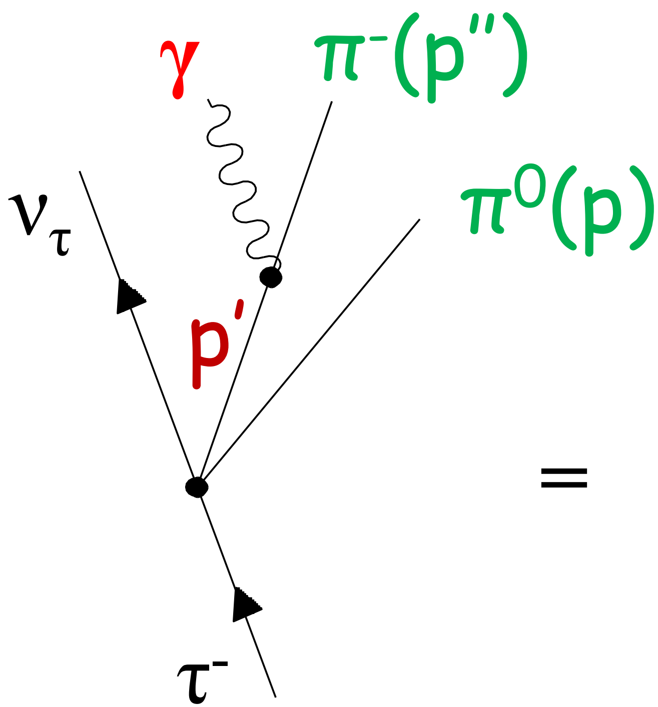
  $q^2 \sim 4m^2_K$

  $f_- = ?$ & $(\Delta B/B)^{th} = \ldots f_-(?)$

  $B^{exp} = (1.49 \pm 0.05) \times 10^{-3}$

POSSIBLE APPLICATIONS (off shell)

$\tau^- \to \pi^-\pi^0\nu_\tau\gamma$ with soft γ



$= \quad \dots$

$p^2=m_\pi^{\;2}$

$$f_- = -\frac{p'^2 - p^2}{6}\left\langle r^2\right\rangle_v^{T=1} + \frac{m_{\pi^0}^2 - m_{\pi^+}^2}{6}\left\langle r^2\right\rangle_s^{T=2}$$

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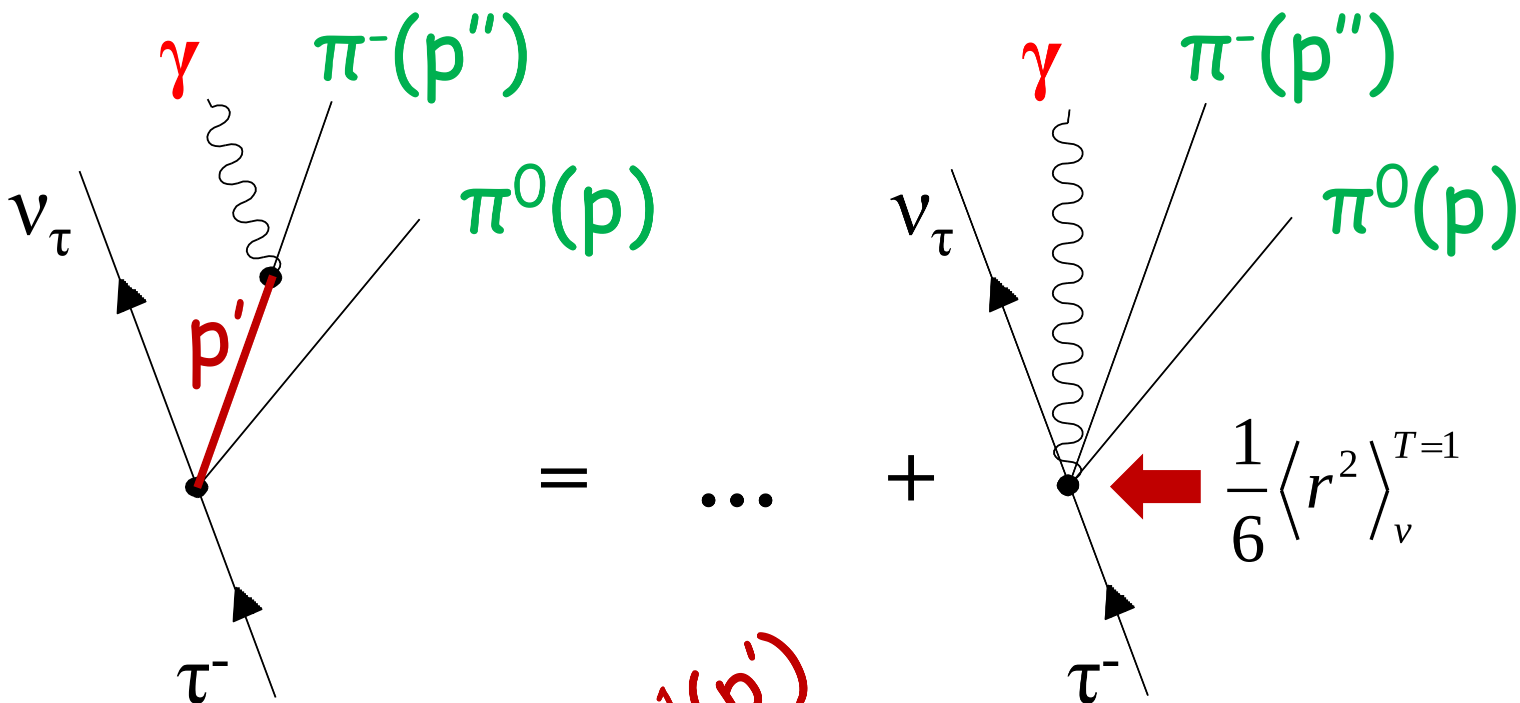

$p^2=m_\pi^2$

$=\Delta^{-1}(p')$

$$f_- = -\frac{p'^2 - p^2}{6}\langle r^2\rangle_v^{T=1} + \frac{m_{\pi^0}^2 - m_{\pi^+}^2}{6}\langle r^2\rangle_s^{T=2}$$

CONCLUSIONS: β–decay of pion

$$\langle \pi^0(p') | \bar{d}\gamma_\mu (1-\gamma_5) u | \pi^+(p) \rangle = \sqrt{2}\left((p'+p)_\mu f_+ + q_\mu f_- \right)$$

- As distinct from the weak magnetism, isotopic rotation of longitudinal weak vector form factor $f_- \equiv \mathcal{F}_2$ is not sufficient.

  There exists scalar contribution.

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- gWTI for broken SU(2) derived
- Extending Equivalence Theorem: Green's functions AND SOME OF THEIR DERIVATIVES are unique on shell.

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$$\langle \pi^0(p') | \bar{d}\gamma_\mu (1-\gamma_5) u | \pi^+(p) \rangle = \sqrt{2}\left((p'+p)_\mu f_+ + q_\mu f_- \right)$$

- **4 versions CVC -> 4 predictions f_:**

$$f_- \overset{1}{=} 0$$ CVC + on shell

$$\overset{2}{=} -\frac{p'^2 - p^2}{6} \langle r^2 \rangle_v^{T=1}$$ WTI CVC + off shell

$$\overset{3}{=} \frac{m_{\pi^+}^2 - m_{\pi^0}^2}{6} \left(\langle r^2 \rangle_v^{T=1} - \langle r^2 \rangle_s^{T=2} \right)$$ pCVC + on shell

$$\overset{4}{=} -\frac{p'^2 - p^2}{6} \langle r^2 \rangle_v^{T=1} + \frac{m_{\pi^0}^2 - m_{\pi^+}^2}{6} \langle r^2 \rangle_s^{T=2}$$ pCVC + off shell

in agreement with $K_{\ell 3}$ analysis

$$f_- = (2.97 \pm 0.25) \times 10^{-3}$$