Linear optics

Case study, Module 3

Magnetic Rigidity

- Bending radius in uniform magnetic field

$$\frac{1}{\rho} = \frac{eB}{pc}$$

- The magnetic rigidity definition

$$B\rho = \frac{pc}{e} = \frac{\beta E}{e}$$

- Practical formula

$$B\rho[\mathrm{T \cdot m}] = \frac{10}{2.998}\frac{\beta}{Q}E[\mathrm{GeV}]$$

Matrix formalism

Any (linear) magnetic element could be presented with it’s transfer matrix.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = T \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

2x2 matrix examples:

$$T_{drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}, \quad T_{quad} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

One-turn matrix can be composed of elements matrices:

$$M = T_n T_{n-1} \ldots T_2 T_1.$$

Twiss parametrization

One-turn matrix M can be presented in the following parametrization:

$$M = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

Where $\alpha(s),\beta(s),\gamma(s)$ – Twiss functions, $\mu=2\pi\nu$ – betatron one-turn phase advance.

Dispersion

Equation is a Hill's equation with rhs:

$$D'' + k_x(s)D = \frac{1}{r_0(s)}$$

With constant k, r

$$\begin{pmatrix} D \\ D' \\ 1 \end{pmatrix}_{out} = \begin{pmatrix} \cos\sqrt{k}s & \frac{1}{\sqrt{k}}\sin\sqrt{k}s & \frac{1}{kr_0}\left(1-\cos\sqrt{k}s\right) \\ -\sqrt{k}\sin\sqrt{k}s & \cos\sqrt{k}s & \frac{1}{\sqrt{kr_0}}\sin\sqrt{k}s \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} D \\ D' \\ 1 \end{pmatrix}_{in}$$

For pure dipole,

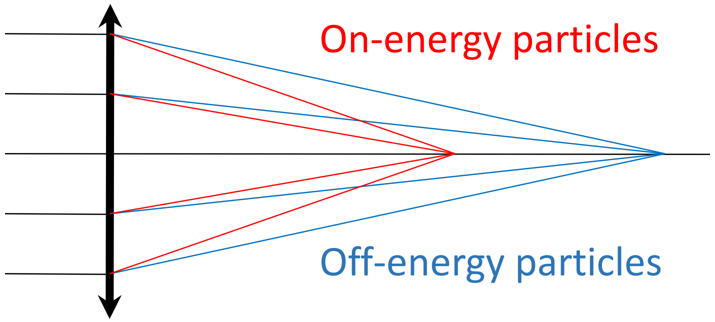
G=0, $s/r_0=\theta$

$$\begin{pmatrix} \cos\theta & r_0\sin\theta & r_0\left(1-\cos\theta\right) \\ -\frac{1}{r_0}\sin\theta & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow[\substack{\theta\to 0 \\ r\to\infty \\ r\theta\to L}]{} \begin{pmatrix} 1 & L & \frac{L\theta}{2} \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$

Chromaticity

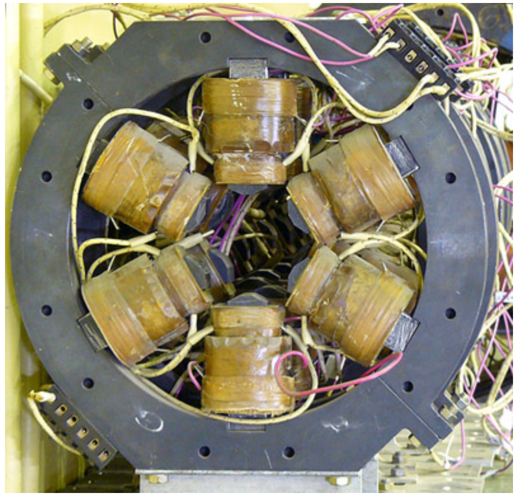
$$k_x = \frac{eG}{pc} \approx \frac{eG}{p_0 c}\left(1 - \frac{\Delta p}{p_0}\right) = k_{x0}\left(1 - \frac{\Delta p}{p_0}\right)$$

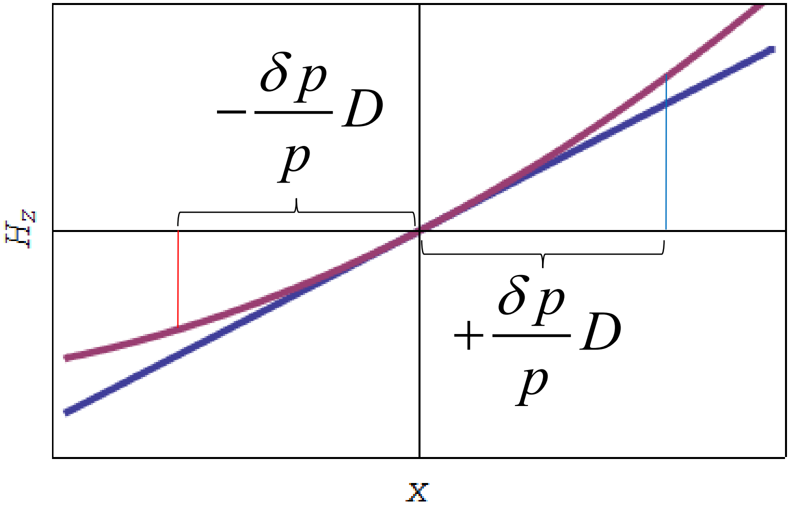
Chromatic aberrations, tune chromaticity:



$$\Delta \nu_{x,y} \approx \frac{\beta_{x,y}\Delta P}{4\pi} = \frac{1}{4\pi}\int \beta_{x,y}\Delta k\, ds = \mp\frac{\Delta p}{p_0}\frac{1}{4\pi}\int \frac{G}{B\rho}\beta_{x,y}\, ds < 0$$

Note: Expression is not full! Only main term for strong focusing machines is here.





Chromaticity correction

$$\frac{\partial \nu_{x,y}}{\partial \delta} = \pm\frac{1}{4\pi}\int \frac{SD}{B\rho}\beta_{x,y}\, ds$$

where $S = \frac{\partial^2 B_y}{\partial x^2}$

Problems

- Problem 1. Estimate the dipole's field of RHIC collider, operating with fully stripped gold ions at 100 GeV/u energy. Ring perimeter 3.834km.
- Problem 2. Derive stability condition for FODO lattice. Assume strong focusing thin lens approximation.
- Problem 3. Assume FODO lattice with lenses of equal absolute strength P. Find min and max values of β-function.
- Problem 4. Find the dispersion function in the arc with regular FODO lattice. Assume small bending angle per one cell.
- Problem 5. Calculate natural tune chromaticity for FODO lattice with quads of equal absolute strength P. What sextupole strength is needed to cure chromaticity?