

Tools for Lattice Design

Mark J. Boland

Canadian Light Source
and
University of Saskatchewan

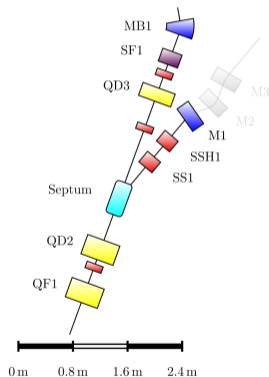
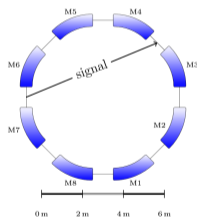
1 November 2019

Outline

- 1 The Goal of Lattice Design
- 2 Basic Physics of Lattice Design
- 3 Practical Tools
- 4 Specific Codes and References

Introduction

- What do we mean by lattice?
- An arrangement of magnets to transport a beam of charged particles.



Drawing Lattices

- \LaTeX package `tikz-palattice`.
- Nice way to visualise a lattice concept (if you are into \LaTeX).



The force on a particle with charge q by electric field \mathbf{E} and magnetic field \mathbf{B} is the sum of the forces from these fields

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

For highly relativistic particles $v \sim c$ so the electric and magnetic fields have the same magnitude for $\mathbf{E} = c\mathbf{B}$. However, for $B = 1$ T which can easily be achieved technically, this equates to $E = 8 \times 10^8$ Vm^{-1} , which is beyond present engineering capabilities. Therefore we will concentrate on steering and focusing of charged particles in magnetic fields and analogous with light is called **beam optics**.

Transverse motion

Since magnetic fields produce forces perpendicular to the motion of a charged particle they cause transverse motion, i.e. in the horizontal and vertical planes. We can define our coordinate system as

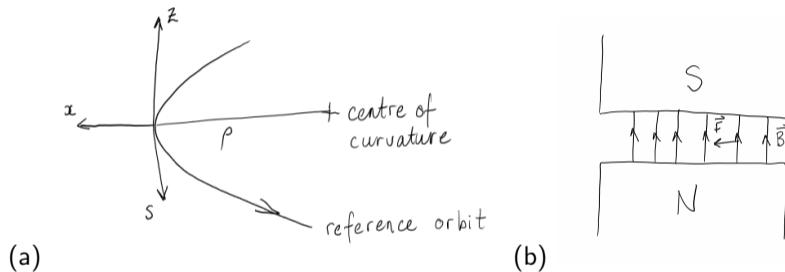


Figure 1: (a) Coordinate system for charged particle in motion in a magnetic field. (b) End view of a dipole magnet.

The coordinates are s which is tangent to the motion and follows along with the particle, x which is positive outwards along the radius of curvature and z which is positive vertically upwards.

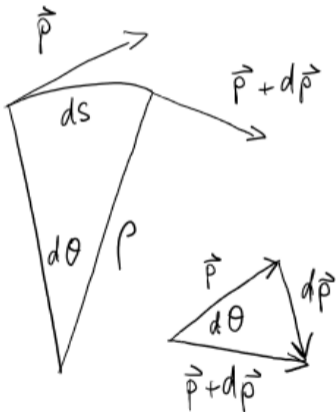
Divergence

A beam consists of a bundle of trajectories which move about the reference orbit and have a certain divergence from the orbit s in each plane given by

$$x' = \frac{dx}{ds} \text{ and } z' = \frac{dz}{ds}.$$

In each plane a particle will have the coordinates (x, x', z, z') to describe their transverse motion. We use magnetic fields to change the position x and direction x' as they move in an accelerator. We shall look at bending that force a particle in a horizontal (x) circular orbit, like in a storage ring, and at focusing magnets that contain the divergence x' and z' of the beam so the trajectories don't escape the machine.

Magnetic rigidity



For relativistic momentum \mathbf{p} a magnetic field \mathbf{B} causes a change in momentum $d\mathbf{p}$ given by

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = e\mathbf{v} \times \mathbf{B}$$

$$\frac{d|\mathbf{p}|}{dt} = |\mathbf{p}| \frac{d\theta}{dt} = \frac{|\mathbf{p}|}{\rho} \frac{ds}{dt}.$$

We can also rewrite magnitude of the Lorentz force as

$$e|\mathbf{v} \times \mathbf{B}| = e|\mathbf{B}| \frac{ds}{dt}$$

Magnetic rigidity

Combining the two equations we get what is known as the *magnetic rigidity*

$$B\rho = \frac{p}{e}.$$

If we work in units where the momentum is expressed as pc which are units of energy rather than units of momentum, then we get the formula

$$B\rho [\text{Tm}] = 3.34 [\text{GeV}/c]$$

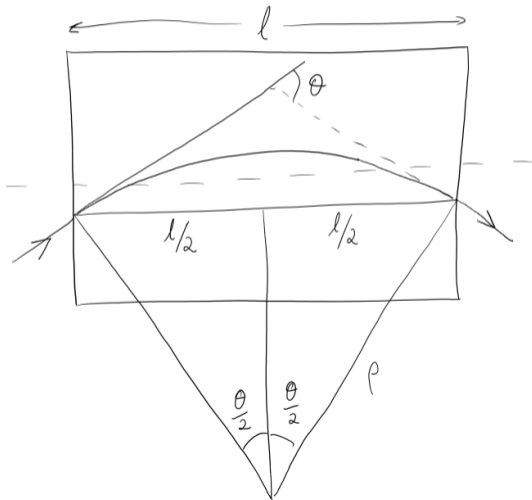
For the CLS we get $B\rho \sim 9.7 \text{ Tm}$.

Bending magnet

Now if we want to bend a beam by a certain angle θ we have the following:

From the geometry we get for small angles

$$\sin \frac{\theta}{2} = \frac{(l/2)}{\rho} = \frac{lB}{2(B\rho)}$$
$$\theta \approx \frac{lB}{B\rho}$$



CLS bend magnet

For example, the CLS with the 2.9 GeV beam energy and the magnetic rigidity of 9.7 Tm, we have dipole magnets that are 1.873 m long with a field of 1.354 T, so we get a bend angle from one diopole of

$$\theta_{bend} = \frac{\ell B}{2(B\rho)} = \frac{1.87 \cdot 1.35}{9.7} = 0.26 \text{ rad.}$$

Now we want the beam to go a full circle of 2π , so we need a number of these dipole in the CLS

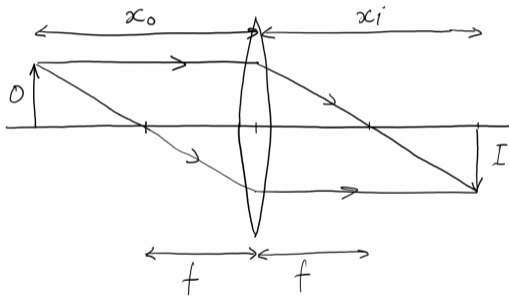
$$N_{bend} = \frac{2\pi}{\theta_{bend}} = 24.$$

The bend magnets are arranged in pairs across 12 symmetrical cells to form a 24 sided polygon referred to as a ring for short hand.

Focusing

In visible light optics we are familiar with a focusing lens to magnify or concentrate light.

In geometric optics, light rays passing through a thin lens are deflected through an angle towards a focal point. Rays coming into the lens further from the horizontal axis are bent with a larger angle.



How do we construct such a lens to focus charged particles?

We need to construct a magnetic field that increases with distance from the reference orbit.

Quadrupole magnet

Instead of using two parallel poles to produce a uniform field, we use four poles with hyperbolic curve shapes on the poles to produce a field gradient. The field is cancelled at the centre of the quadrupole and increases linearly along the positive x -direction.

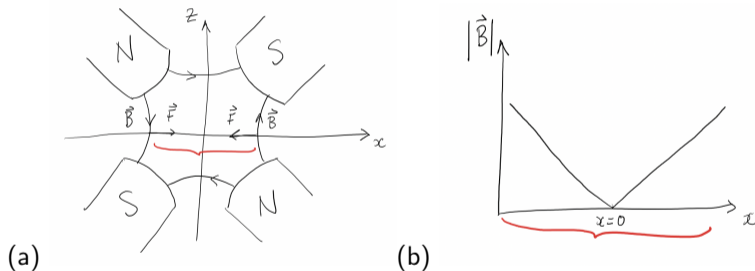


Figure 2: (a) End view of a quadrupole magnet. (b) Field of a quadrupole in the horizontal plane.

About the central point $x = 0$ the magnitude of the field increases in both directions, the opposite direction of the \mathbf{B} field ensures the force is always pointing to the centre of the quadrupole.

Quadrupole strength

The strength of the quadrupole can also be given in terms of the magnetic rigidity

$$k = \frac{1}{B\rho} \frac{dB_z}{dx}.$$

Then the angle of deflection of a particle at point x in a magnet of length l is

$$\Delta x' = \theta = \frac{l(dB_z/dx)x}{B\rho} = lkx.$$

Comparing to a focussing optical lens where the deflection is given by

$$\Delta x' = -\frac{x}{f},$$

we can define the focal length of a horizontally focusing quadrupole as

$$f = -\frac{1}{kl}$$

Mechanical analogy

If we place a series of these dipole and quadrupole magnets together we get bending and focusing forces analogous to a marble running down a gutter.

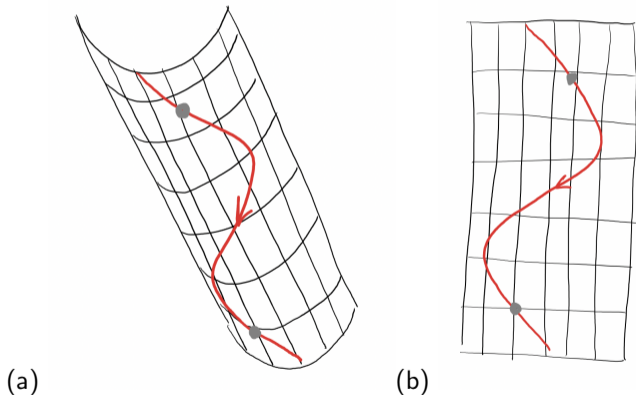


Figure 3: A marble running through a gutter (a) angle, (b) top view, as an analogy to a particle in a circular accelerator.

Phase space

When we observe the position and angle of a particle at one point each time it circulates, it carves out an ellipse in $x - x'$ phase space.

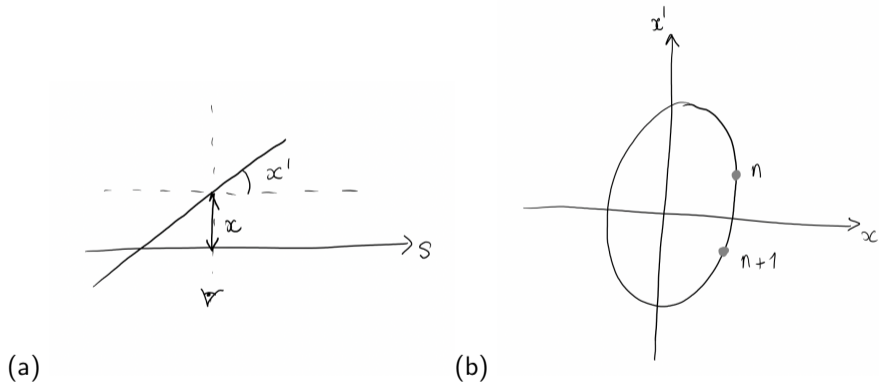


Figure 4: (a) Horizontal position and angle at a point in s . (b) Locus of points traces out by a particle circulating in a storage ring.

Hills Equation

particle dynamics equation of motion

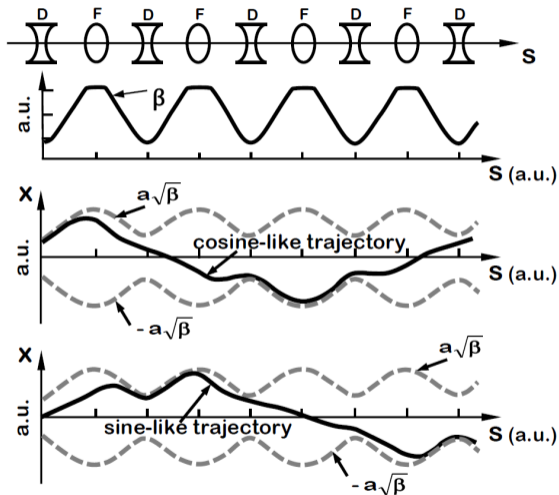
$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad \frac{d\mathcal{E}}{dt} = \mathbf{F} \cdot \mathbf{v},$$

where we use the relativistic momentum and energy

$$\mathbf{p} = \gamma m\mathbf{v}, \quad \mathcal{E} = \gamma mc^2.$$

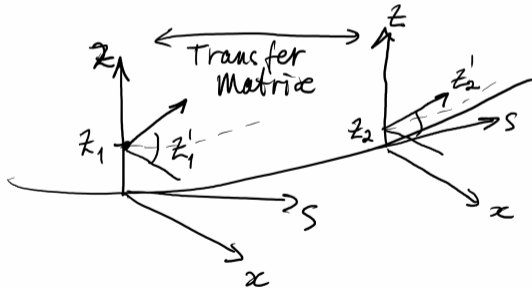
Hills equation for linear motion in magnetic fields:

$$\frac{d^2y}{ds^2} + Ky(s)y = 0$$



Matrix formalism

- Goal: transport particle coordinates (x_1, x'_1) to another point in the accelerator lattice (x_2, x'_2) .
- Require matrices that describe the function of field free *drifts*, dipoles, quadrupoles, etc. as determined from the equations of motion.
- Mainly focus on circular machines or transport lines with alternating gradients and strong focusing.



Example matrix – drift

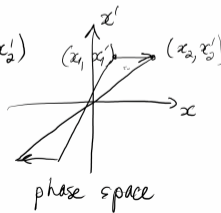
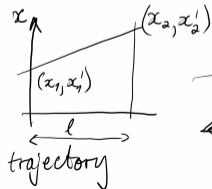
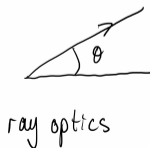
Consider a particle with an initial position and angle (x_1, x'_1) passing through a field-free drift space of length ℓ

$$\begin{pmatrix} x_2 \\ x'_2 \end{pmatrix} = \begin{pmatrix} 1 & \ell \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x'_1 \end{pmatrix}.$$

We see the angle does not change but there is an additional position offset dependent on the initial angle.

$$x_2 = x_1 + \ell x'_1$$

$$x'_2 = x'_1$$

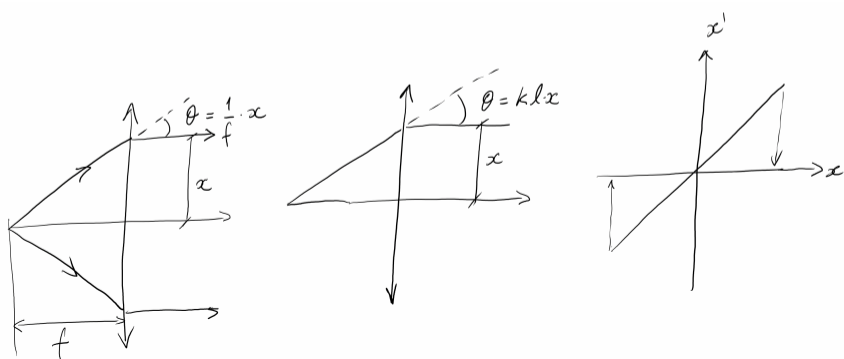


Example matrix – quad

Consider a particle with an initial position and angle (x_1, x_1') passing through a thin quadrupole of strength k

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}.$$

We see the position does not change but there is an additional angle offset dependent on the initial position $x_2 = x_1$ and $x_2' = -\frac{x_1}{f} + x_1'$.



Recap of Hill's equation

Hill's equation (linear periodic coefficients), where y is used to denote either the x or z transverse coordinate.

$$\frac{d^2y}{ds^2} + k(s)y = 0,$$

where a restoring force at the quadrupoles is given by

$$k = -\frac{1}{B\rho} \frac{dB_z}{dx}.$$

A solution in the horizontal plane

$$x = \sqrt{\beta(x)}\epsilon \cos(\phi(s) + \phi_0).$$

Condition

$$\phi = \int \frac{ds}{\beta(s)}.$$

Property of a particular machine

$$\sqrt{\beta(s)}$$

Property of a particle beam ϵ . Physical meanings:

- Beam envelope $\sqrt{\beta(s)}\epsilon$.
- Maximum particle excursions $y = \sqrt{\beta(s)}\epsilon$ and $y' = \sqrt{\beta(s)}/\epsilon$.

Check Hill's equation

Differentiate $y = \sqrt{\beta(s)}\epsilon \cos(\phi(s) + \phi_0)$ and substitute $w = \sqrt{\beta}$ and $\phi = \phi(s) + \phi_0$ we get

$$y' = \sqrt{\epsilon} \left[w'(s) \cos \phi - \frac{d\phi}{ds} w(s) \sin \phi \right].$$

For a solution to this equation we need $\frac{d\phi}{ds} = \frac{1}{\beta(s)} = \frac{1}{w^2(s)}$. Substitute and differentiate again we get

$$y' = \sqrt{\epsilon} \left[w'(s) \cos \phi - \frac{1}{w(s)} \sin \phi \right]$$
$$y'' = \sqrt{\epsilon} \left[w''(s) \cos \phi - \frac{w'(s)}{w^2(s)} \sin \phi + \frac{w'(s)}{w^2(s)} \sin \phi - \frac{1}{w^3} \cos \phi \right].$$

Now substitute into $y'' + ky = 0$

Check Hills' equation cont.

$$w'' - \frac{1}{w^3} + kw = 0.$$

This has no general analytical solution but if we put it back in terms of β we have the constraint

$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + k\beta^2 = 1.$$

The beam trajectory in the horizontal plane is then is described by the so called **beta function** $\beta(s)$ and the constant ϵ with the form

$$x(s) = \sqrt{\epsilon}\sqrt{\beta(s)} \cos(\phi(s) + \phi_0),$$

with

$$\phi(s) = \int_0^s \frac{d\sigma}{\beta(\sigma)}.$$

Liouville's theorem

We can re-write the constraint from Hill's equation as

$$\gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s) = \epsilon,$$

where we define $\alpha(s) \equiv -\frac{1}{2}\beta'(s)$, $\beta(s) \equiv w(s)^2$ and $\gamma(s) \equiv \frac{1+\alpha^2(s)}{\beta(s)}$. This is the definition of the $x - x'$ phase ellipse from the equations of motion and Liouville's theorem states that the area of this phase ellipse remains constant in time. This theorem and the equations of motion do not strictly hold during acceleration for particles synchrotron radiation damping, the phase ellipse can shrink in these situations. The area of an ellipse with major and minor axes a, b has an area $A = ab\pi$, so we can find that the area of the phase space motion of a particle in an accelerator is $A = \pi\epsilon$. ϵ is also known as the Courant-Snyder invariant for a single particle. We can then draw the ellipse which is then defined for every point along the path s in an accelerator.

Phase ellipse

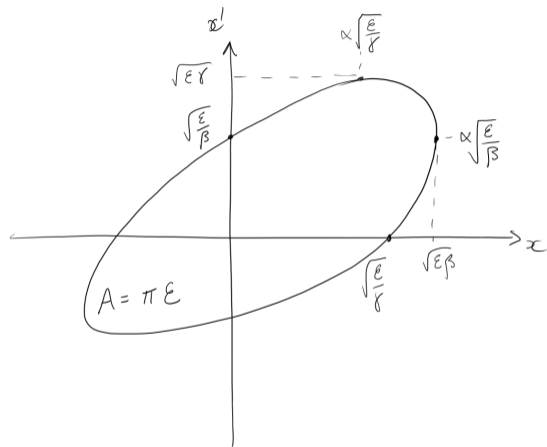


Figure 5: Phase ellipse of Courant-Snyder invariant in $x - x'$ at a single point s .

Twiss Matrix

Now let us construct a matrix that takes us from point 1 to point 2 in the machine

$$\begin{pmatrix} y(s_2) \\ y'(s_2) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} y(s_1) \\ y'(s_1) \end{pmatrix} = \mathbf{M}_{12} \begin{pmatrix} y(s_1) \\ y'(s_1) \end{pmatrix}$$

We use $w = \sqrt{\beta}$ and $\phi = \frac{1}{\beta} = \frac{1}{w^2}$ and have

$$y = \sqrt{\epsilon} w \cos(\phi + \phi_0)$$

$$y' = \sqrt{\epsilon} w' \cos(\phi + \phi_0) - \frac{\sqrt{\epsilon}}{w} \sin(\phi + \phi_0)$$

We can construct two solutions, one with initial condition $\phi_0 = 0$ the so-called *cosine solution* $C(s)$, and one $\phi_0 = \pi/2$ the *sine solution* $S(s)$. We then can place these in the matrix equation and solve for a, b, c, d to find the matrix elements.

Twiss Matrix cont.

Writing $\phi = \phi_2 - \phi_1$ the matrix elements are

$$\mathbf{M}_{12} = \begin{pmatrix} \frac{w_2}{w_1} \cos \phi - w_2 w_1' \sin \phi & w_1 w_2 \sin \phi \\ -\frac{1+w_1 w_1' w_2 w_2'}{w_1 w_2} \sin \phi - \left(\frac{w_1'}{w_2} - \frac{w_2'}{w_1} \right) \cos \phi & \frac{w_1}{w_2} \cos \phi + w_1 w_2' \sin \phi \end{pmatrix}$$

If we now simplify the situation to where system is periodic from 1 \rightarrow 2, then $w_1 = w_2 = w$, $w_1' = w_2' = w'$, $\mu = \phi_2 - \phi_1$, the matrix becomes

$$\mathbf{M} = \begin{pmatrix} \cos \mu - w w' \sin \mu & w^2 \sin \mu \\ -\frac{1+w^2 w'^2}{w^2} \sin \mu & \cos \mu + w w' \sin \mu \end{pmatrix}$$

Twiss Matrix cont.

Using the definition of the Twiss parameters from before we get α, β, γ (not to be confused with the relativistic β, γ !)

$$\begin{aligned}\alpha &= -ww' = -\frac{\beta'}{2}, \\ \beta &= w^2, \\ \gamma &= \frac{1 + (ww')^2}{w^2} = \frac{1 + \alpha^2}{\beta}.\end{aligned}$$

The Twiss matrix can be simplified further to

$$\mathbf{M} = \begin{pmatrix} \cos \mu + \alpha \sin \mu & \beta \sin \mu \\ -\gamma \sin \mu & \cos \mu - \alpha \sin \mu \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

We can now construct a lattice of a series of elements with a matrix defined for each element. This is practically how accelerators and transfer lines are designed and their properties calculated.

Lattice

Practically the solutions to the matrix equations are calculated with computers.

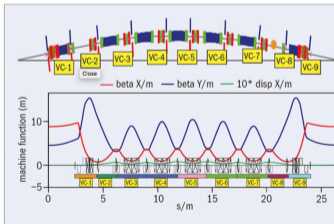
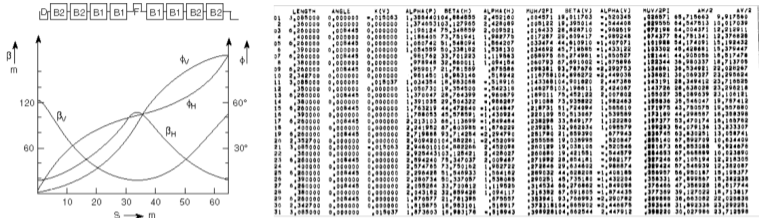
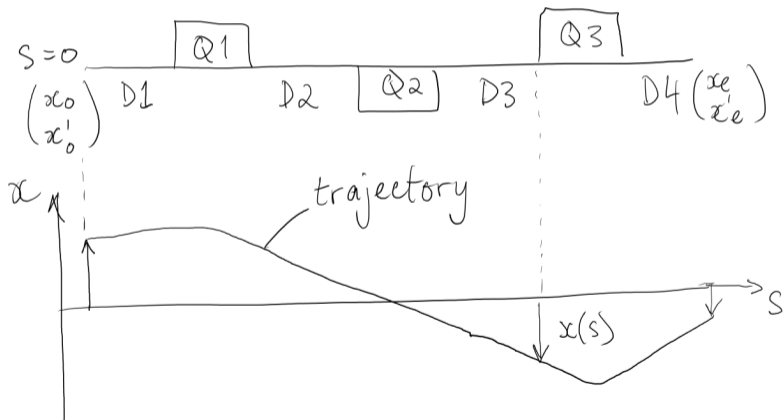


Figure 6: Twiss parameters as a function of s .

Combining elements

To calculate a trajectory through a series of elements \mathbf{M}_i we just multiply the matrices together to get for example

$$\mathbf{X}_E = \mathbf{M}_{D4} \cdot \mathbf{M}_{Q3} \cdot \mathbf{M}_{D3} \cdot \mathbf{M}_{Q2} \cdot \mathbf{M}_{D2} \cdot \mathbf{M}_{Q1} \cdot \mathbf{M}_{D1} \cdot \mathbf{X}_0$$



Focusing doublet

For a pair of thin lenses with focusing power f_1, f_2 with a drift space length L in between we get

$$\mathbf{M}_x^{1 \rightarrow 2} = \begin{pmatrix} 1 & 0 \\ \frac{1}{f_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{L}{f_1} & L \\ -\frac{1}{f^*} & 1 + \frac{L}{f_2} \end{pmatrix}$$

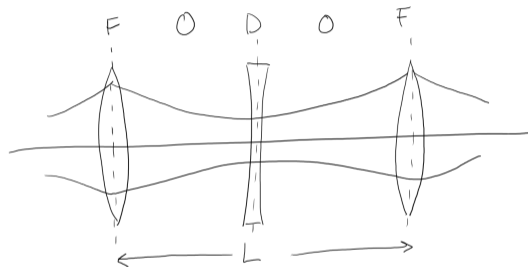
where

$$\frac{1}{f^*} = \frac{1}{f_2} - \frac{1}{f_2} + \frac{1}{f_1 f_2}$$

For example when $f_1 = f_2$ we get focusing in both planes through the quadrupole doublet.

FODO lattice example

We can construct a lattice with a series of horizontally focusing (F) and defocusing quadrupoles (D) spaced by drifts (O) to form a so called FODO cell



$$\mathbf{M}_{\text{FODO}} = \begin{pmatrix} 1 & \frac{L}{2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{L}{2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$
$$\mathbf{M}_{\text{FODO}} = \begin{pmatrix} 1 - \frac{L}{2f} - \frac{L^2}{4f^2} & L + \frac{L^2}{4f} \\ -\frac{L}{2f^2} & 1 + \frac{L}{2f} \end{pmatrix}$$

Stability criteria

For a particle to follow a stable trajectory through N periods and k orbits we need the following product of matrices to not diverge

$$\{M(s)\}^{Nk}.$$

To check this we write out the eigenvalue problem for coordinates (y, y') as \mathbf{Y} and Twiss matrix $\mathbf{M}(s)$ as

$$\mathbf{M}\mathbf{Y} = \lambda\mathbf{Y}.$$

The solutions are then

$$\det(\mathbf{M} - \lambda I) = 0,$$

so we get

$$\lambda^2 - \lambda(a + d) + 1 = 0.$$

Stability criteria cont.

We can find that the Twiss matrix has $\det \mathbf{M} = 1$ and

$$\cos \mu = \frac{1}{2} \text{Tr} \mathbf{M} = \frac{1}{2}(a + d),$$

then we get

$$\lambda = \cos \mu \pm i \sin \mu = e^{\pm i\mu}.$$

So for a non-exponential growth or decay μ must be real and so

$$\left| \frac{1}{2} \text{Tr} \mathbf{M} \right| \leq 1,$$

and

$$|\lambda| = 1,$$

where λ is complex, $|a + ib| = \sqrt{a^2 + b^2}$. So, to test stability of a Twiss matrix \mathbf{M} we can compute these values numerically.

Equation for off-momentum particles

So far we have dealt with particles of the same momentum, now let's add an off-momentum particle with momentum deviation $\Delta p/p$. For simplicity we have only bends and no gradients, then equation of motion is

$$x'' + \frac{1}{\rho^2}x = \frac{1}{\rho} \frac{\Delta p}{p}.$$

We define a special trajectory $D(s)$ for which $\Delta p/p = 1$ which we call the dispersion function and we get

$$D''(s) + \frac{1}{\rho^2}D(s) = \frac{1}{\rho} \frac{\Delta p}{p}.$$

Then our new particle trajectory considering the possibility of off-momentum particles is

$$x(s) = x_0(s) + x_D = x_0(s) + D(s) \frac{\Delta p}{p}$$

Matrix for off-momentum particles

To account for this momentum deviation we add an extra term to our matrix to transform the particle motion in both planes and in momentum

$$\begin{pmatrix} x(s) \\ x'(s) \\ z(s) \\ z'(s) \\ \Delta p/p \end{pmatrix} = M \cdot \begin{pmatrix} x_0 \\ x'_0 \\ z_0 \\ z'_0 \\ \Delta p/p \end{pmatrix}$$

Since off-momentum particles will not have same the path length L of the reference orbit, they will have a path difference ΔL . We can define this relative path length change as a ratio of the relative momentum change as the **momentum compaction factor**

$$\alpha_c = \frac{\Delta L/L}{\Delta p/p}$$

Real accelerators do not behave like Hill's equation, so we need better physics and computer models!

While it was possible in the 1950s and even later, to design an accelerator with pencil and paper, it is impossible today to design a beast like the LHC without the help of a tracking/design code.

– Etienne Forest, From Tracking Code to Analysis (2016).

Historical side note: design of first synchrotron



Mark Oliphant, Birmingham, 1950s.



Mark Oliphant notebook extract, 1943.
University of Adelaide Oliphant archives.

The New Method

THE ACCELERATION OF PARTICLES TO VERY HIGH ENERGIES.

Introduction.

The properties of elementary particles with energies up to 10 or 20 million electron-volts (10-20 Mev.) have been investigated extensively during recent years. In the war period this work has been intensified in some directions, although it has lapsed in others. A great deal of work remains to be done, but it is probable that the main outlines of nuclear physics in this region are now clear. In any event there is ample equipment in existence in USA and elsewhere to fill in most of the gaps in our knowledge, while much will be accomplished in the government laboratories which will be set up in various countries to exploit the possibilities of nuclear fission.

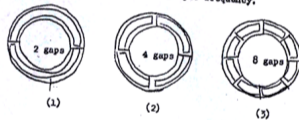
The new method.

The essential feature of the proposals is that the particle should be constrained to move in a circle of constant radius, thus enabling the use of an annular ring of magnetic field of the correct form but over a total volume which is small enough to require only moderate power for its excitation. The magnetic field would be varied in such a way that the radius of curvature remains constant as the particles gain energy through successive accelerations by an alternating electric field applied between coaxial hollow electrodes, as in the cyclotron. The varying magnetic field performs the function of the guiding field in the betatron, but the acceleration is provided by an applied potential rather than by a changing flux. In this way it is possible to apply much higher accelerations per revolution. The changing magnetic field can be produced by an application of modern pulse technique as developed for radar purposes, while the accelerating potential can be provided by the same general method. Essentially, very large powers are available during the acceleration of a single burst of particles, a relatively long quiescent period between pulses reducing the average power consumed to a reasonable value.

At energies of 1000 MeV., or more, electrons and protons do not differ markedly in effective mass or velocity. Hence the greatest differences in technique required for the two particles will exist at the lower range of energies through which the particles are accelerated. These differences render it necessary to proceed in somewhat different ways in the case of ~~these~~ electrons and protons.

...particles should be constrained to move in a circle of constant radius thus enabling the use of an annular ring of magnetic field... which would be varied in such a way that the radius of curvature remains constant as the particles gain energy through successive accelerations by an alternating electric field applied between coaxial hollow electrodes.

Accelerating Electrodes. The particles may be accelerated by passage across one or more gaps between electrodes, across which an a.c. voltage is applied at the proper frequency.



The A.C. frequency is, for the Larmor frequency, $\nu = 1.2 \times 10^7$.

$$\begin{array}{l} 1) \nu = 1.2 \times 10^7 \text{ cycles/second, } \lambda = 25 \text{ metres.} \\ 2) \nu = 2.4 \times 10^7 \text{ " " " " } \lambda = 12.5 \text{ metres.} \\ 3) \nu = 4.8 \times 10^7 \text{ " " " " } \lambda = 6.25 \text{ metres.} \\ \text{etc.} \end{array}$$

The gap voltages required to add an energy of 0.5 MeV per revolution are

- 1) 250,000 volts,
- 2) 125,000 " "
- 3) 62,500 " "

These frequencies and voltages can be obtained by coupling to an oscillator operating continuously, and their production is especially easy with pulsed operation.

The Synchrotron—A Proposed High Energy Particle Accelerator

EDWIN M. McMILLAN

University of California, Berkeley, California

September, 5, 1945

ONE of the most successful methods for accelerating charged particles to very high energies involves the repeated application of an oscillating electric field, as in the cyclotron. If a very large number of individual accelerations is required, there may be difficulty in keeping the particles in step with the electric field. In the case of the cyclotron this difficulty appears when the relativistic mass change causes an appreciable variation in the angular velocity of the particles.

The device proposed here makes use of a "phase stability" possessed by certain orbits in a cyclotron. Consider, for example, a particle whose energy is such that its angular velocity is just right to match the frequency of the electric field. This will be called the equilibrium energy. Suppose

McMillan, Phys Rev 68 p143 (1945)

Concerning Some New Methods of Acceleration of Relativistic Particles

V. VEKSLER

Lebedev Physical Institute of the Academy of Sciences, Moscow, U.S.S.R.

February 16, 1946

TWO papers^{1,2} appearing in 1944 under the above title the author of the present letter pointed out two new principles of acceleration of relativistic particles which generalize the resonance method.

New possibilities for the resonance acceleration of particles in a constant magnetic field are described in the first of these papers, and the possibility of resonance acceleration in magnetic fields which increase with time is also noted.

This latter case is specially examined in the second paper. It is shown that phase stability automatically sets in if the time variation of the field is sufficiently small; relation between the amplitude of the variable electric fields and the rate of variation of the magnetic field is established.

It is also pointed out that the radiation losses in such acceleration do not violate phasing mechanism. Finally in a detailed paper³ an accelerator of heavy particles based on a variation in frequency is analyzed.

Thus the foregoing papers cover completely the contents of the note by McMillan⁴ in which no reference is made to my investigations.

Construction of a 30-Mev accelerator with varying magnetic field is now nearing completion at the Physical Institute of the Academy of Sciences, U.S.S.R.

¹ V. Veksler, *Comptes Rendus (Doklady)*, Acad. Sci. U.S.S.R. **43**, No. 8, 444 IX (1944) (communicated April 25, 1944).

² V. Veksler, *Comptes Rendus (Doklady)*, Acad. Sci. U.S.S.R. **44**, No. 9, 393 (1944) (communicated July 19, 1944).

³ V. Veksler, *J. Phys. (U.S.S.R.)* **9**, No. 3, 153 (1945) (received March 1, 1945).

⁴ E. McMillan, *Phys. Rev.* **68**, 143 (1945).

Veksler, Phys Rev 69 p244 (1946)

What about AUS?

Marcus Laurence Elwin Oliphant (1901–2000)

Australian physicist

Inventor of the synchrotron

Pioneered centimeter radar and the atomic bomb



Mark Oliphant was an impressive individual, tall, with thick white hair. He spoke convincingly and energetically and in later life described himself as “a belligerent pacifist.” Educated at Adelaide University, he joined Ernest Rutherford’s group at the Cavendish Laboratory, Cambridge in 1927. He and Rutherford were the first to identify tritium and helium-3 produced by bombarding light nuclei with protons and deuterons.

By 1937 he was a Professor at Birmingham and, charged with overseeing Britain’s coastal defenses, saw the clear need to use a radar wavelength much shorter than the 170 cm then current. His department set about developing a suitable power source and before long John Randall and Harry Boot from his department invented the magnetron. This was a device that made it possible for aircraft to carry their own radar and upon which the fate of the war hinged. He took this precious gift to the USA in 1941 where there were the resources to develop and mass produce it.

Andrew Sessler and Edmund Wilson, *Engines of Discovery*
– *A Century of Particle Accelerators*, World Scientific, Singapore (2007).

The Best Tool

The best tool for optics design is your own brain.

- Trust but verify the results of a calculation.
- *A trades person never blames their tools.*
- Remember a useful design will need to be engineered someday, so take practical considerations into account with your optics design quite early on in the process. E.g. magnets probably can't be touching...
- Try to get to know the applicability and limits of the tool you use, example disclaimer:

Limitations of the beam TRANSPORT code:

For TRANSPORT the equation of motion is solved by applying a Taylor series up to second (or third) order and using the matrix formalism to compute the propagation of the beam through a beam line.

Therefore TRANSPORT calculations are usually describing a beam accurate enough in the paraxial approximation {sines and tangents of the angles can be replaced with the angles (valid up to about 100 mr or 5 degrees)}.

As long as the 5 first order sine- and cosine-functions (c_x , s_x , d_x , c_y and s_y) and their derivatives are small as feasible through the used magnetic or electrostatic elements, then aberrations (2nd and higher order terms) remain reasonable small.

If above conditions are not fulfilled, then a ray-tracing program, which solves the equation of motion through 4th order Runge-Kutta integration should be used for checking the validity of the results computed with the beam TRANSPORT code.

MADX example description

From Todd Satogata:

There are many other codes available for beamline design, particle tracking, electromagnetic field calculations, plasma calculations, and other diverse areas of research in accelerator physics. MADX does some things extremely well (particularly optics design and matching), and some things reasonably well (tracking or pushing particles through magnets like in large accelerators). There are some things it does not do very well though:

- Acceleration and tracking simultaneously. (esme and BLoND are better longitudinal dynamics engines.)
- Not very good approximations for large excursions, particularly large as found in FFAGs.
- Tracking through spatial descriptions of measured magnetic fields (“field maps”: elegant and zgoubi are better at this).
- Very complicated magnet geometries (where field maps are usually better descriptions than multipole coefficients anyway).

It is likely that you have been exposed to some already and will be exposed to many more over the course of your career.

Transport Codes

A convenient list is available on the Wikipedia Accelerator Code Page

https://en.wikipedia.org/wiki/Accelerator_physics_codes.

Here is a summary list of some of the maintained codes:

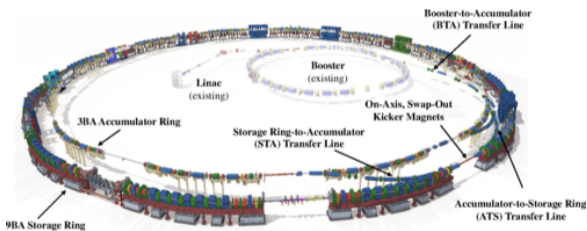
- MADX – CERN defacto standard code <https://mad.web.cern.ch/mad/>
- MATLAB Accelerator Toolbox – from SLAC, now maintained at atcollab <http://atcollab.sourceforge.net/>
- Elegant <https://www.aps.anl.gov/Accelerator-Operations-Physics/Software#elegant>
- OPA <https://ados.web.psi.ch/opa/>
- OPAL <https://gitlab.psi.ch/OPAL/src/wikis/home>
- Zgoubi – developed at Brookhaven <https://zgoubi.sourceforge.io/>, active user community <https://zgoubi-workshop.com/>
- G4beamline – hybrid accelerator and particle/nuclear physics code <http://www.muonsinternal.com/muons3/G4beamline>
- SIMION (commercial code) – <https://simion.com/> (Initially developed by Don McGilvery, my colleague at the Australian Synchrotron, for his PhD in 1978.)

Choose the right tool for the right job!

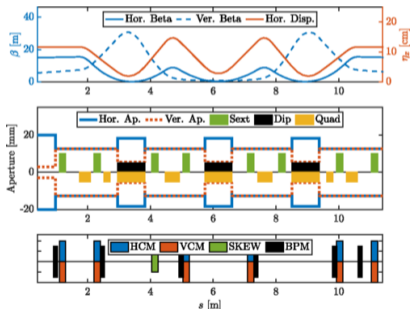
From Code to Commissioning

More realistic codes have enabled machines such as the LHC to be commissioned rapidly with high accuracy. There are also tools that are being developed to simulate commissioning which allow for better understanding machines and helping to foresee issues during the design phase.

- ALS-U accumulator ring commissioning tools, published October 2019
<https://journals.aps.org/prab/abstract/10.1103/PhysRevAccelBeams.22.100702>.

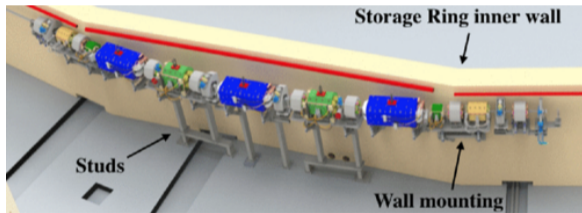


ALS-U new accumulator and storage rings.

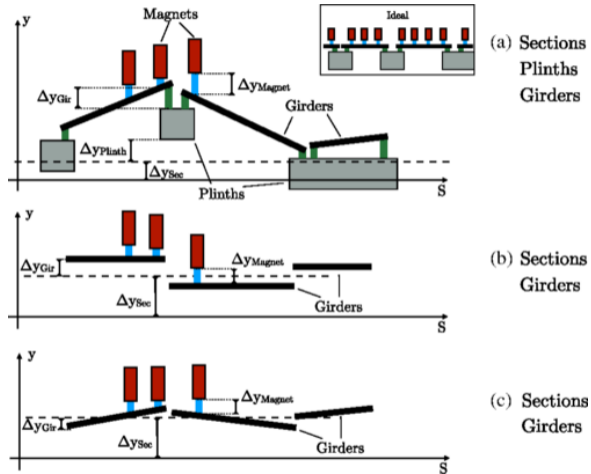


ALS-U AR optics for one sector.

ALS-U AR Example

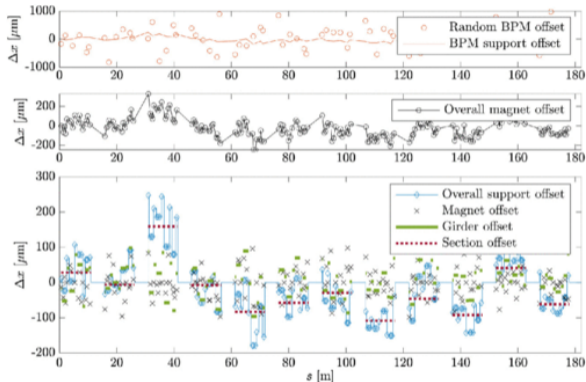


ALS-U AR mechanical design.

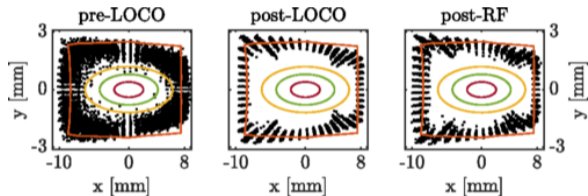


ALS-U AR misalignment model.

ALS-U AR Example



ALS-U AR possible offsets based on mechanical design.

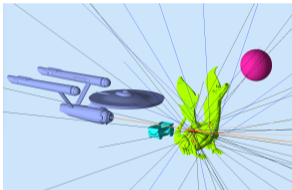


ALS-U AR dynamic aperture improvements using the commissioning model.

Supplementary Codes

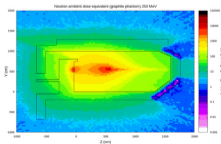
- GEANT4 – possibility to add many physics processes, not just EM fields, e.g. particle decay, say muon beams.

<https://geant4.web.cern.ch/>

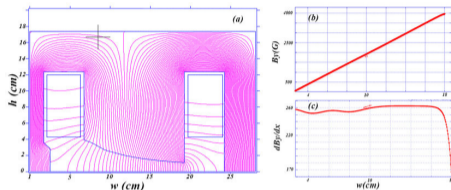


- Fluka – radiation shielding considerations.

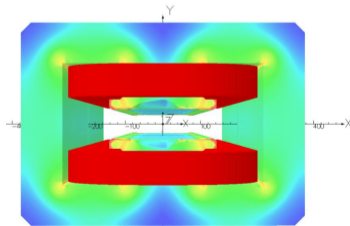
<http://www.fluka.org/>



- Poisson – 2D magnet calculations.



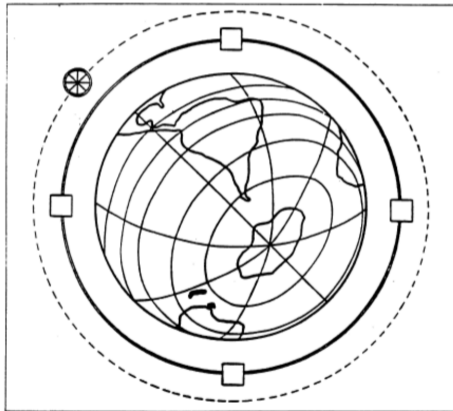
- Opera – 3D magnet calculations.



- Book to get started – Volker Ziemann, Hands-On Accelerator Physics Using MATLAB, CRC Press (2019). <https://www.crcpress.com/Hands-On-Accelerator-Physics-Using-MATLAB/Ziemann/p/book/9781138589940>
- Book with advanced details – Etienne Forest, From Tracking Code to Analysis, Springer (2016). <https://www.springer.com/gp/book/9784431558026>
- USPAS course material <https://uspas.fnal.gov/materials/materials-table.shtml>
- Example class Satogata and Peggs in 2019 <http://toddsatogata.net/2019-USPAS/>
- CAS courses <http://cas.web.cern.ch/> and CDS proceedings, search for CAS <https://cds.cern.ch/>
- Example class from Ziemann tutorial from CAS 2018 <http://ziemann.web.cern.ch/ziemann/teaching/cas/2018/>

Finally

Have fun designing your future collider!



From a 1954 Slide by Enrico Fermi, University of Chicago Special Collections.