

Linear Imperfections Measurements and Correction in Storage Rings

Part I

Contents:

Part I

- Closed Orbit Perturbations:

- Sources
- Corrections

Part II

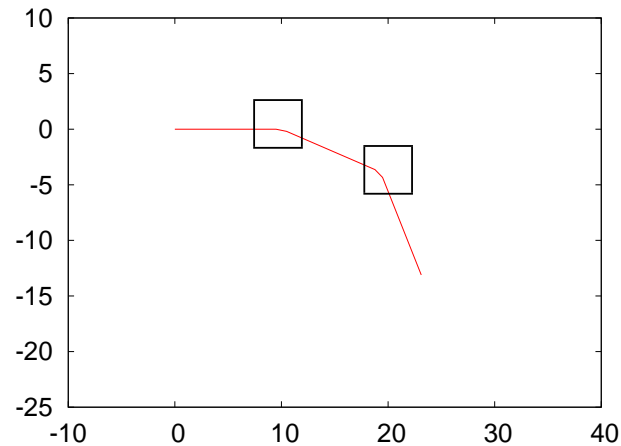
- Optics Perturbations:

- Gradient errors, detection and correction
- Linear coupling: sources, detection and correction

presented by Eliana GIANFELICE

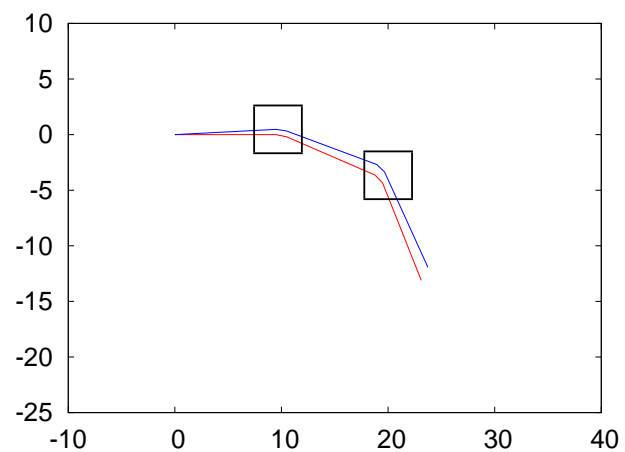
Closed Orbit

The “minimalist” (linear or circular) accelerator is build with bending magnets (uniform magnetic field) which guide the particles and define the *nominal trajectory*



The nominal trajectory in a storage ring is of course closed, ie $\{X, Y, Z\}(C) = \{X, Y, Z\}(0)$ (coordinates wrt the fixed laboratory frame).

But a particle entering with a starting offset/angle follows a different trajectory



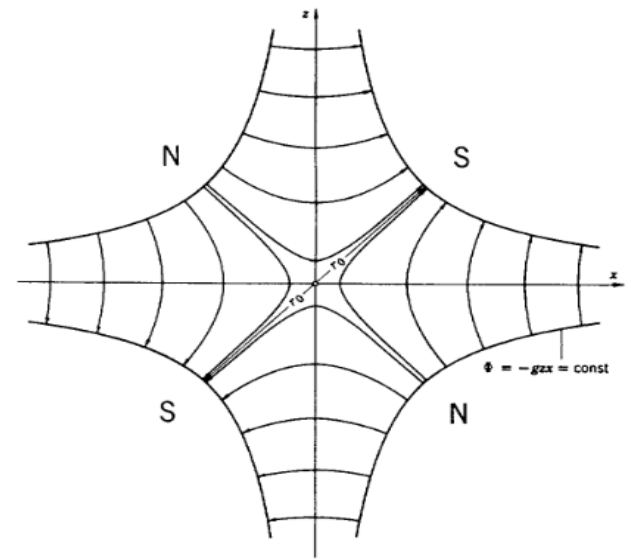
Quadrupole magnets are added for bending back those particles towards the closed orbit (*focusing*).

The field in a quadrupole depends linearly on transverse position. For a normal quad used for focusing:

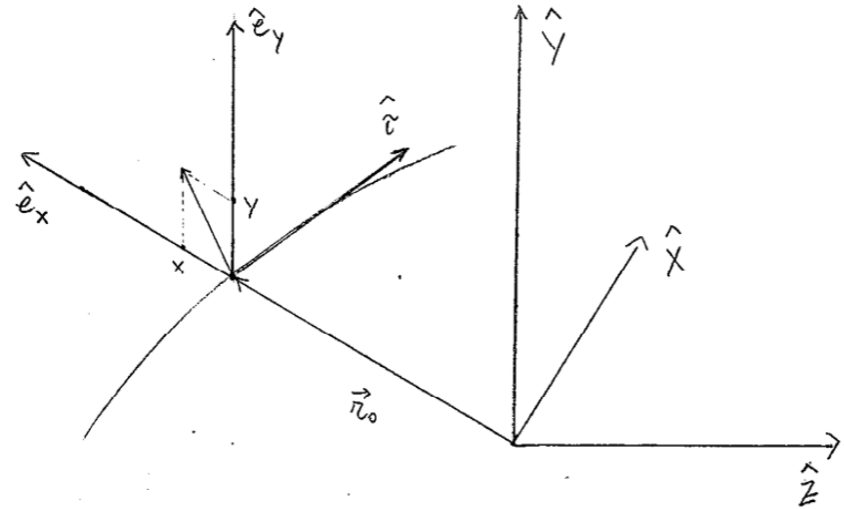
$$B_x = gy \qquad B_y = gx$$

A particle with a horizontal offset, x , wrt the magnet axis is bent horizontally, vertically if the offset is vertical.

The quadrupoles must be aligned to the trajectory defined by the bending dipoles not to affect the nominal trajectory.



It is convenient to describe the particle motion in a right-handed rectangular system $(\hat{\tau}, \hat{e}_x, \hat{e}_y)$ which moves along the nominal trajectory. In such a frame and using the path length s rather than the time t the motion is described by Hill's equations.



Logical steps to get there:

- Use of the *local* coordinate system attached to the reference orbit (assumed here for simplicity lying in the horizontal plane):

$$\left\{ \begin{array}{l} \hat{\tau} \text{ tangent to the orbit pointing in the velocity direction} \\ \hat{e}_y \text{ perpendicular to the plane of the orbit (constant)} \\ \hat{e}_x \equiv \hat{e}_y \times \hat{\tau} \end{array} \right.$$

- Start with the equations of motion under Lorentz force in the laboratory coordinates system

$$\frac{d}{dt} \left[m \frac{d\vec{r}}{dt} \right] = e \frac{d\vec{r}}{dt} \times \vec{B}$$

- Express position of particle in terms of the reference particle position vector \vec{r}_0

$$\vec{r}(s, x, y) = \vec{r}_0(s) + x(s)\hat{e}_x + y(s)\hat{e}_y$$

- Use Frenet formulas

$$\begin{cases} \frac{d}{ds} \hat{e}_x = \hat{\tau} \frac{1}{\rho} \\ \frac{d}{ds} \hat{\tau} = -\hat{e}_x \frac{1}{\rho} \end{cases}$$

- Use s (the distance along the reference trajectory) as parameter instead of time

$$\frac{d}{dt} = \frac{d\ell}{dt} \frac{d}{d\ell} = v \frac{ds}{d\ell} \frac{d}{ds}$$

$$\vec{r}(s + ds) = \vec{r}(s) + d\vec{\ell}$$

$$\frac{d\ell}{ds} \simeq 1 + x/\rho$$

- Expand fields in the local frame $\{\hat{\tau}, \hat{e}_x, \hat{e}_y\}$ and keep the linear components.

Using above ingredients the equations of motion for the generic particle in the local frame attached to the reference trajectory write

$$x'' + \left(\frac{1}{\rho^2} + K \right) x + Ny + 2Hy' = -\frac{e}{p} \Delta B_y \equiv F_x$$

$$y'' - Ky + Nx - 2Hx' = \frac{e}{p} \Delta B_x \equiv F_y$$

with

$$K(s) \equiv \frac{e}{p} \left(\frac{\partial B_y}{\partial x} \right)_{x=y=0} \quad \text{and} \quad N \equiv \frac{1}{2} \frac{e}{p} \left(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y} \right)_{x=y=0}$$



normal quad



skew quad

$$H \equiv \frac{1}{2} \frac{e}{p} B_\tau \quad \leftarrow \quad \text{solenoid}$$

$$\Delta B_z \quad \leftarrow \quad \text{dipolar errors}$$

In general skew quadrupoles are introduced for correction purposes and treated in *perturbation* theory.

Solenoids are usually present in the experiment detectors and their effect is often compensated by anti-solenoids. For machines with polarized beams, they may be also machine components for rotating the particle spin direction.

Setting $\mathbf{N}=\mathbf{H}=0$ the two equations are uncoupled and may be re-written in a symmetric form

$$z'' + K_z z = F_z(s)$$

with

$$K_x \equiv \left(\frac{1}{\rho^2} + K \right) \quad \text{and} \quad K_y \equiv -K$$

and, with our choice of the reference frame, $F_x(s) \equiv -\frac{e}{p} \Delta B_y$ in horizontal plane and $F_y(s) \equiv \frac{e}{p} \Delta B_x$ in the vertical one. In the following the subscript will be omitted when not needed.

By using the periodic solution^a, $\beta(s)$, of

$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + \beta^2 K = 1$$

we may define the new variables η and ϕ

$$\eta \equiv \frac{z}{\sqrt{\beta}} \quad \text{and} \quad \phi(s) \equiv \frac{1}{Q} \int^s \frac{d\bar{s}}{\beta}$$

The parameter Q is chosen so that

$$\phi(C) = \frac{1}{Q} \oint \frac{ds}{\beta} = 2\pi$$

In the new variables the Hill's equation transforms in the equation of a [harmonic oscillator](#)

$$\frac{d^2\eta}{d\phi^2} + Q^2\eta = 0$$

^afor an open line the solution is determined by the starting conditions

To get there one must write z as $\sqrt{\beta}\eta$ and express the derivatives in terms of the new parameter ϕ :

$$\frac{d}{ds} = \frac{1}{Q\beta_0} \frac{d}{d\phi}$$
$$\frac{d^2}{ds^2} = \frac{1}{(Q\beta_0)^2} \frac{d^2}{d\phi^2} + \frac{d}{ds} \left(\frac{1}{Q\beta_0} \right) \frac{d}{d\phi} = \frac{1}{(Q\beta_0)^2} \frac{d^2}{d\phi^2} - \frac{1}{Q^2\beta_0^3} \frac{d\beta_0}{d\phi} \frac{d}{d\phi}$$

The general solution may be written as the sum of a cosinus and sinus

$$\eta(\phi) = A \cos Q\phi + B \sin Q\phi$$

The *tune* Q is then the number of free oscillations per turn and Q/T_{rev} is the *betatron frequency*.

As Q can't be integer (see later), in general the trajectories do not close after one turn. It is

$$\eta(\phi + n2\pi) = A \cos Q(\phi + n2\pi) + B \sin Q(\phi + n2\pi)$$

In presence of dipolar error fields it is

$$\frac{d^2\eta}{d\phi^2} + Q^2\eta = Q^2\beta^{3/2}F(s(\phi))$$

which is a forced oscillator.

The **general solution** is the sum of the **general solution of the homogeneous** part and a **particular solution of the inhomogeneous equation**.

Using the method of Lagrange we find the solution, v , of the in-homogeneous equation with starting conditions $v(0)=v'(0)=0$ as

$$v(\phi) = \int_0^\phi d\sigma f(\sigma) \left[\cos Q\phi \sin Q\sigma - \sin Q\phi \cos Q\sigma \right]$$

with $f(\phi) \equiv \beta^{3/2} F(s(\phi))$. For a closed ring we would like to describe the motion wrt a new periodic trajectory (**closed orbit**).

To do so we use the general solution and impose periodicity. The result is

$$\eta(\phi)_{co} = \frac{Q}{2 \sin \pi Q} \int_\phi^{\phi+2\pi} d\bar{\phi} f(\bar{\phi}) \cos [Q(\pi + \phi - \bar{\phi})] =$$

$$\frac{Q}{2 \sin \pi Q} \int_0^{2\pi} d\bar{\phi} f(\bar{\phi}) \cos [Q(\pi - |\phi - \bar{\phi}|)]$$

which becomes a sum for a finite number of localized errors

$$\eta(\phi)_{co} \simeq \frac{Q}{2 \sin \pi Q} \sum_j f(\bar{\phi}_j) \Delta\bar{\phi}_j \cos [Q(\pi - |\phi - \bar{\phi}_j|)]$$

The actual motion of any particle is then described by

$$z = z_{\beta} + z_{co} = \sqrt{\beta}\eta + z_{co} = \sqrt{\beta} \left[A \cos Q\phi + B \sin Q\phi \right] + z_{co}$$

with

$$z_{co}(s) = \frac{1}{2 \sin \pi Q} \sqrt{\beta(s)} \sum_j \sqrt{\beta_j} \Theta_j \cos [Q(\pi - |\phi(s) - \phi(\bar{s}_j)|)]$$

$$\Theta_j \equiv F \Delta \bar{s}_j = F [Q \beta \Delta \bar{\phi}] \quad \leftarrow \text{from the definition of } \phi \text{ it is: } d\phi = \frac{ds}{Q\beta}$$

- We see why the optics must be designed so that Q is not an integer number!
- In the linear approximation a closed orbit always exists, it may be out of the vacuum chamber though...
- The effect of errors at large β -values location is particularly strong.

nb: For correcting the orbit small dipole correctors are inserted in the lattice. Their effect on the closed orbit is of course described by the same expression.

The magnet positioning precision is finite, although today sophisticated methods allow to align the machine components with a precision in the order of some hundred microns.

The expectation value of the rms closed orbit can be estimated under the assumption that the number of magnets is large enough

$$\langle z_{rms} \rangle = \frac{1}{2\sqrt{2}|\sin \pi Q|} \sqrt{\langle \beta \rangle} \sqrt{\sum_i \beta_i \Psi_i^2}$$

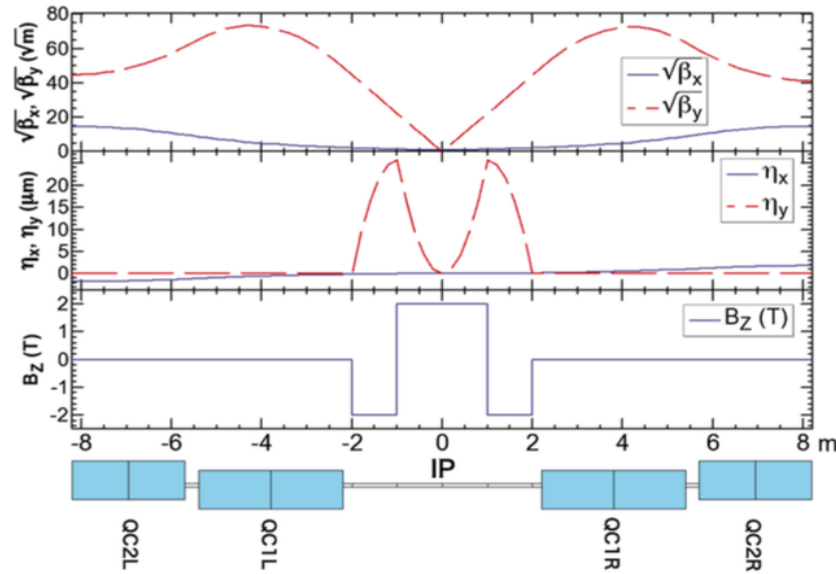
where Ψ_i depends on the kind of error considered. For quadrupole transverse misalignments it is $\Psi_i = (k\ell)_i \delta z_{rms}^Q$ and

$$\langle z_{rms} \rangle = \frac{1}{2\sqrt{2}|\sin \pi Q|} \sqrt{\langle \beta \rangle} \sqrt{\sum_{i=1}^{NQ} \beta_i (k\ell)_i^2} \delta z_{rms}^Q$$

↑
amplification factor

Example: Future Circular Collider e^+e^- ring

FCC-ee IP ($\beta_y^*=2$ mm)



K. Oide et al,
PRAB 19, 111005 (2016)

Optics		amplification factor
45 GeV	all quads	665
(Jan.2018)	w/o IRs quads	124

- The few IRs quadrupoles have an huge effect on the closed orbit.
- It is crucial to have near-by BPMs and correctors to compensate their effects *locally*.

There are some more things to learn from having changed the variables from (z, s) to (η, ϕ) .

For a closed ring, owing to the periodicity, we can expand $z_{co}/\sqrt{\beta} \equiv \eta_{co}$ and $f(\phi) \equiv \beta^{3/2} F(s(\phi))$ in Fourier series

$$\eta(\phi)_{co} = u_0 + \sum_{p=1}^{+\infty} u_p \cos p\phi + v_p \sin p\phi$$

$$f(\phi) = a_0 + \sum_{p=1}^{+\infty} a_p \cos p\phi + b_p \sin p\phi$$

and inserting back into the equation for η we find the relationship between the coefficients of errors and orbit expansions

$$u_0 = a_0$$

$$u_p = \frac{Q^2}{Q^2 - p^2} a_p$$

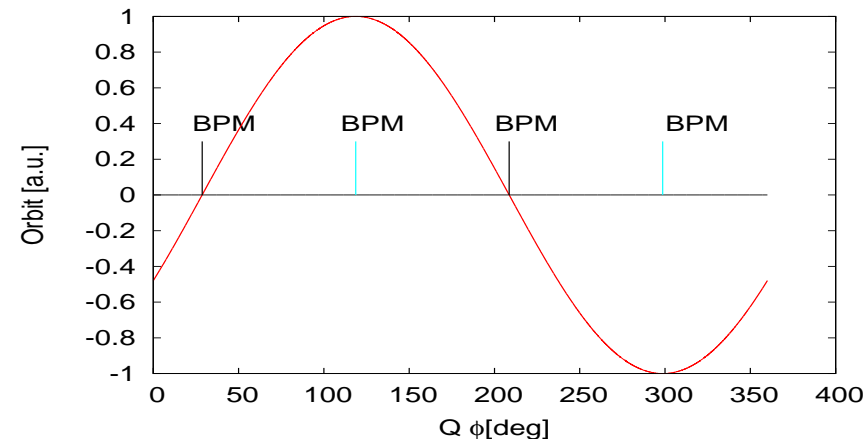
$$v_p = \frac{Q^2}{Q^2 - p^2} b_p$$

Under the reasonable assumption that the errors have a white spectrum

- The orbit is most sensitive to the harmonics close to the betatron frequency Q .
- The Fourier expansion of an un-corrected closed orbit will have large values of the harmonics close to Q .

Beam Position Monitors are used for measuring the beam transverse position. The average over many turns gives the closed orbit. Having used ϕ we see that what matters is the *phase advance*, $Q\phi$ ($\equiv \mu$), rather than the position s :

- The BPMs (and correctors) should be distributed uniformly around the ring wrt to ϕ
- In order to compute all harmonic components we would need an infinite number of sampling points. At least 4 sampling point per wavelength ie one BPM each $Q\Delta\phi=90^\circ$ is recommended.



As $z=\sqrt{\beta\eta}$, locations with large β are preferred.

The “rule” of the 4 BPMs per wavelength concerns the *regular arc* of a storage ring (very often FODO cells).

Additional BPMs must be inserted in special locations as

- Injection/extraction region.
- for colliders, in the collision region in order to
 - monitor the beam position
 - measure the machine optics
 - perform Beam Based Alignment

In writing the analytical expression of the closed orbit we have considered only dipolar errors. They come from:

- Bending magnets
 - field error
 - roll^a, pitch^b, yaw^c
 - longitudinal mis-alignment (very important when the bending angle is large!).
- Quadrupoles
 - quads transverse offset
 - pitch, yaw

Longitudinal displacement of quads affects instead the *optics*.

Quadrupole rolls introduce *betatron coupling*, but if x_{co} and y_{co} are non zero at the quad location they introduce also dipolar errors and couple the closed orbit equations.

^arotation around $\hat{\tau}$

^brotation around \hat{e}_x

^crotation around \hat{e}_y

In practice, how is the closed orbit computed? There are computer codes which may handle many kind of mis-alignments. Many of them use the following approach.

Consider for instance a short corrector. Using a 2×2 matrix formalism it is

$$\vec{z} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \vec{z}_0 + \begin{pmatrix} 0 \\ \Theta_z \end{pmatrix}$$

That is only z' is changed. We can modify the transport matrix for including the kick (the inhomogeneity)

$$\vec{z} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \Theta_z \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z_0 \\ z'_0 \\ 1 \end{pmatrix}$$

The machine element transport matrices are modified for taking into account the errors. Applying the 7×7 (for the most general case) one turn transport matrix to the starting vector $\vec{h}=\underline{0}$ we construct the particular solution found by the Lagrange method.

$$\vec{z}(L) = \underbrace{M(L)\vec{z}(0)}_{\text{gen.sol.hom.}} + \underbrace{\vec{h}(L)}_{\text{part.sol.inhom.}}$$

and imposing that \vec{z} is the periodic solution

$$\vec{z}(L) = M(L)\vec{z}(0) + \vec{h}(L) = \vec{z}(0) \quad \rightarrow \quad (I - M)^{-1}\vec{h}(L) = \vec{z}(0)$$

with M unperturbed transport matrix and $\vec{h}(L)$ transported by the enlarged matrix.

For taking into account the effect of non linear elements the procedure must be iterated. For instance, normal sextupole (used for chromaticity correction):

$$B_x = Sxy = S(x_0 + x_\beta)(y_0 + y_\beta) = Sx_0y_0 + Sx_0y_\beta + Sy_0x_\beta + Sx_\beta y_\beta$$

$$B_y = \frac{1}{2}S(x^2 - y^2) = \dots = \frac{1}{2}S(x_0^2 - y_0^2) + \frac{1}{2}Sx_0x_\beta - Sy_0y_\beta + \frac{1}{2}S(x_\beta^2 - y_\beta^2)$$

- First one finds the closed orbit without sextupoles.
- Knowing x_0 and y_0 at the location of the sextupoles their extra kicks are evaluated and the closed orbit recomputed. In general x_0 and y_0 , and therefore the sextupole contribution, will be different.
- Repeat as long as it (hopefully) converges.

The particle motion in the storage ring is described by

$$z = z_{co} + \underbrace{z_{\beta}}_{\text{free oscillation}}$$

It is important to keep z_{co} as small as possible for

- making good use of the available aperture
- staying clear of unwanted non-linearities

This is done by using small dipole correctors. We recall that change of the closed orbit measured at the i th BPM due to an unitary kick of the i th corrector is

$$\Delta x_i = T_{ij} = \frac{\sqrt{\beta_i^m \beta_j^c}}{2 \sin \pi Q} \cos [Q(\pi - |\phi_i^m - \phi_j^c|)]$$

which *in principle* can be computed from the nominal optics functions and do not depend from the particular orbit. For a very distorted machine the response matrix can be measured and used for correcting the orbit.

For the computation of the needed strength for a global orbit correction there are many approaches:

- **Harmonic method**: it consists in extracting the closed orbit harmonics from the BPM measurement and introducing equal and opposite harmonics by exciting dipole correctors.
- **MICADO** method (implemented for instance in MADX): it consists in finding the best corrector, namely the corrector which minimizes $\sum_{i=1}^M (z_i^m + z_i^c)^2$. Within the remaining correctors a second one is added which together with the previously found one minimizes $\sum_{i=1}^M (z_i^m + z_i^{c1} + z_i^{c2})^2$ and so on.

- Least Square Fit

The response of the orbit at the i th BPM i due to the j th corrector is given by

$$z_i = T_{i,j} \Theta_j^c = \frac{Q}{2 \sin \pi Q} \sqrt{\beta_i^m \beta_j^c} \cos(\pi Q - |\mu_i^m - \mu_j^c|) \Theta_j^c$$

Unless the number of monitors, M , and the number of correctors, N , are equal, the system of eqs.

$$z_i^m + \sum_{j=1}^N T_{ij} \Theta_j^c = 0$$

can't be inverted.

For $N > M$ there are infinite solutions: one can use the *method of Lagrange multipliers* for selecting the solution which minimizes $\sum_{j=1}^N (\Theta_j^c)^2$.

For $M > N$ there are too many constraints, we can find a least square solution by minimizing

$$S = \sum_{i=1}^M (z_i^m + z_i^c)^2 \quad \text{with} \quad z_i^c = \sum_{j=1}^N T_{ij} \Theta_j^c \quad \begin{array}{l} M \equiv \# \text{of BPMs} \\ N \equiv \# \text{of correctors} \end{array}$$

If $S(\Theta_1^c, \dots, \Theta_N^c)$ is a minimum it must be $\partial S / \partial \Theta_k^c = 0 \quad \forall k = 1, \dots, N$ ie

$$\sum_{i=1}^M (z_i^m + \sum_{j=1}^N \Theta_j^c) T_{ik} = 0 \quad \forall k = 1, \dots, N$$

or in matrix notation

$$T^T \vec{z}^m + T^T T \vec{\Theta}^c = 0$$

$T^T T$ is a $N \times N$ matrix which can be, if not singular, inverted. The solution is than

$$\vec{\Theta}^c = - \left(T^T T \right)^{-1} T^T \vec{z}^m$$

One may add a term $w \sum_{j=1}^N (\Theta_j^c)^2$ to S for limiting the values of Θ_j^c (the length of $\vec{\Theta}^c$). w would be a factor to be determined empirically.

There is a way for short cutting the problem of assessing whether a matrix has an inverse, that is the Singular Value Decomposition (SVD).

Any $M \times N$ matrix A may be written as

$$A = U\Sigma V^T$$

with

U $M \times M$ orthogonal matrix ie $(U^T U)_{ij} = \delta_{ij}$ $i, j \leq M$

V $N \times N$ orthogonal matrix

Σ $M \times N$ diagonal matrix

For instance

$$\begin{array}{l} M = 2 \\ N = 3 \end{array} \quad \Sigma = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \end{pmatrix} \quad \begin{array}{l} M = 3 \\ N = 2 \end{array} \quad \Sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{pmatrix}$$

The diagonal elements of Σ are called *singular values*.

- The M columns of U are orthonormal eigenvectors of AA^T .
- The N columns of V are orthonormal eigenvectors of $A^T A$.
- The singular values are the square root of the eigenvalues of $A^T A$ (or AA^T , they share the same non-zero eigenvalues).

If $M = N$ then we can write

$$A^{-1} = (V^T)^{-1} \Sigma^{-1} U^{-1} = V \Sigma^{-1} U^T$$

Therefore if A is ill defined some of the elements of Σ are vanishing (or small). In general when $N \neq M$, $V \Sigma^{-1} U^T$ is the *pseudo-inverse* (or generalized inverse) which

- for $M > N$ gives the least square solution,
- for $M < N$ gives the minimum length solution, ie the minimum of $\sum_{j=1}^N (\Theta_j^c)^2$.

The ratio $\check{\sigma} / \hat{\sigma}$ (*condition number*) determines the accuracy of the inversion and it should be as much as possible close to one: the small singular values must be zeroed until a reasonable solution is found.

There are routines for performing a SVD. The decomposition is not unique though! *MATLAB* and LAPACK dgesvd subroutine use the above definitions.

Since $\mathbf{T}^T\mathbf{T}$ is a squared matrix one can also solve the eigenvalues problem. Find the eigenvalues for the matrix $\mathbf{A} \equiv \mathbf{T}^T\mathbf{T}$:

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^{-1}$$

with $\mathbf{\Sigma}$ diagonal matrix of the eigenvalues of \mathbf{A} and and the N column of \mathbf{U} are the eigenvectors of \mathbf{A} . As \mathbf{A} is real and symmetric \mathbf{U} is orthogonal and $\mathbf{U}^{-1}=\mathbf{U}^T$

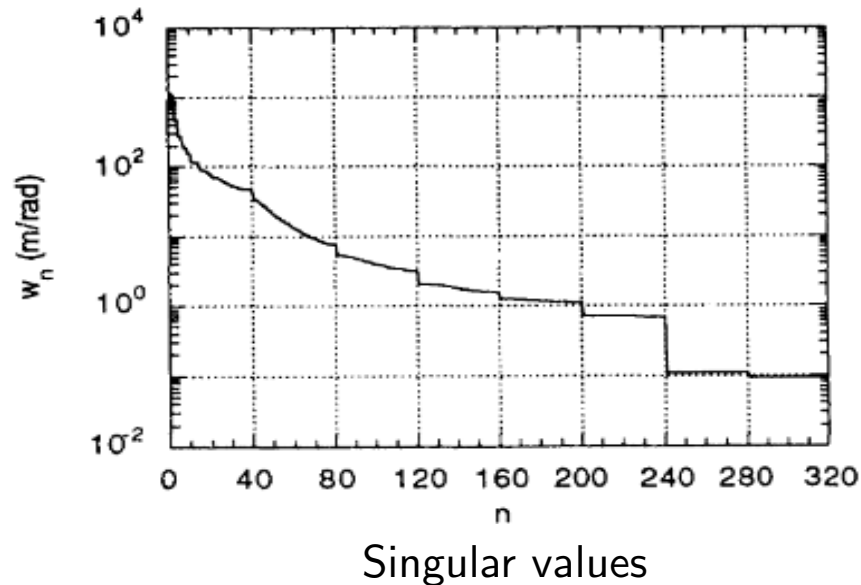
$$\mathbf{A}^{-1} = \mathbf{U}\mathbf{\Sigma}^{-1}\mathbf{U}^T$$

As for the SVD, the singularities are confined in the diagonal matrix $\mathbf{\Sigma}$. One should replace by zero the smallest eigenvalues to get a reasonable solution.

The SVD uses all correctors and BPMs. Ignoring the small singular values doesn't mean that we are eliminating correctors and/or monitors responsible for the bad condition number.

Is there a way for eliminating redundant BPMs and correctors *a priori*?

The case of Advanced Photon Source (Argonne National Lab): 360 BPMs and 320 correctors.



from Y. Chung et al.,
PAC 1993

There is a step at 240.

The condition number is $8e-5$. Eliminating from the analysis the 80 correctors with the smaller value of

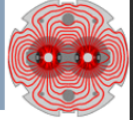
$$E(j) \equiv \sum_n \sigma_n V_{jn}$$

the condition number increases to $7e-4$.

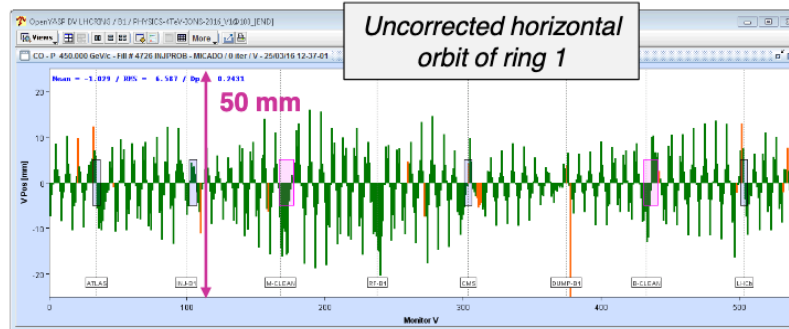
An example from the LHC



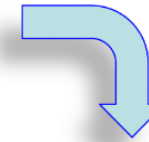
LHC orbit correction example



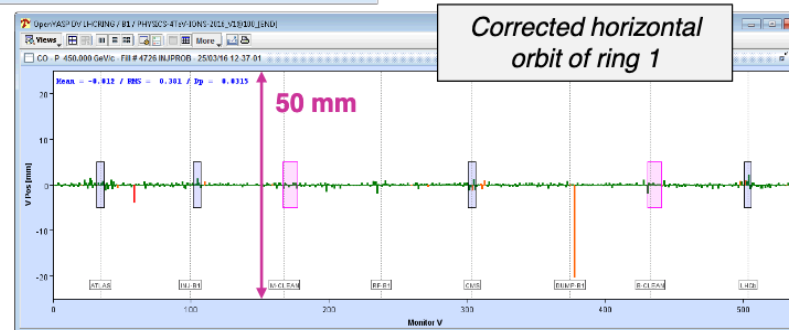
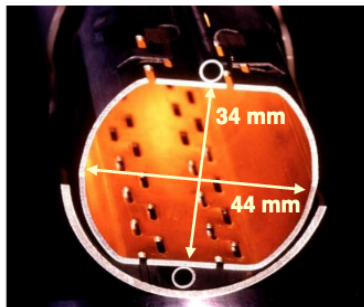
- The raw orbit at the LHC can have huge errors, but the correction (based partly on MICADO) brings the deviations down by more than a factor 20.



MICADO & Co



LHC vacuum chamber



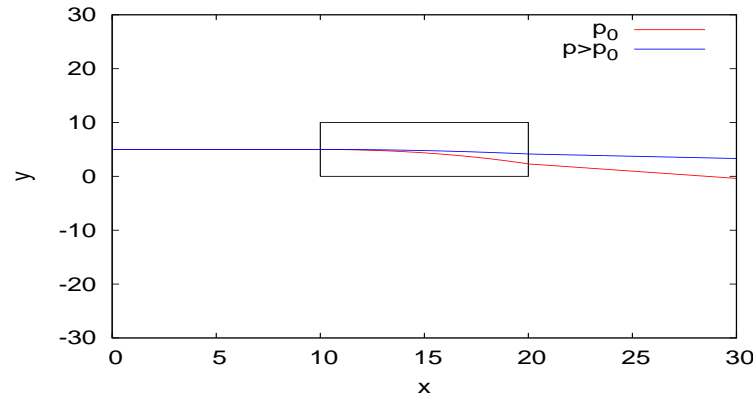
At the LHC a good orbit correction is vital !

30

J. Wenninger, JUAS 2019.

Dispersion correction

The dispersion, $D_z(s)$, describes the dependence of the particle trajectory upon its momentum. It originates from the bending magnets.



In a closed ring, the periodic dispersion is the closed orbit of a particle with a relative momentum offset $\Delta p/p_0=1$ wrt the nominal one: the equation of the periodic dispersion has the same form as the equation for the closed orbit, the in-homogeneity being $1/\rho$

$$z'' + K_z z = \frac{1}{\rho}$$

- Quadrupoles do not generate dispersion, they modified it.
- If the closed orbit is not vanishing at their locations, quadrupoles introduces extra dipolar fields and become source of extra (*spurious*) dispersion, ΔD_z :

$$\frac{d^2 \Delta D_z}{ds^2} + K_z(s) \Delta D_z = F_z(s)$$

with $F_z(s) = K_z z_{co}^q$

The solution has the same form of the closed orbit:

$$\Delta D_z(s) = \frac{\sqrt{\beta_z(s)}}{2 \sin \pi Q_z} \sqrt{\beta_z^q(K\ell)} y_{co}^q \cos(\pi Q_z - |\mu_z(s) - \mu_z^q|)$$

closed orbit at the quad

Contributions from many quads trivially add up.

Because ΔD depends on the closed orbit which larger harmonics are around Q , the spectrum of the disturbance is not white. The harmonics of $D/\sqrt{\beta}$ relates to those of the dipolar errors originating the non-vanishing closed orbit as $Q^4/(Q^2 - p^2)^2$.

In general if the closed orbit is well corrected the spurious dispersion should result well corrected too. However, especially for e^\pm machines where a very low vertical emittance is required, it may be necessary to implement a dispersion correction or a “dispersion free correction” which consists in trying to minimize closed orbit and dispersion simultaneously.

By its definition, the dispersion can be measured by measuring the closed orbit corresponding to different energies. The beam energy may be varied by changing the RF frequency at fixed dipole field:

$$\frac{dp}{p} = \frac{\gamma^2 \gamma_T^2}{\gamma_T^2 - \gamma^2} \frac{df}{f}$$

The measurement of a *difference orbit* doesn't suffer from possible BPMs offsets and therefore correction of the dispersion instead of the closed orbit can be a good way for correcting both at once^a. This is done again using the small dipole correctors.

Effect of the j th corrector on dispersion at the i th BPM is (neglecting the sextupoles)

↙ dipole contribution (small)

$$N_{ij} = T_{ij} \Theta_j^c +$$

↙ quadrupole contribution

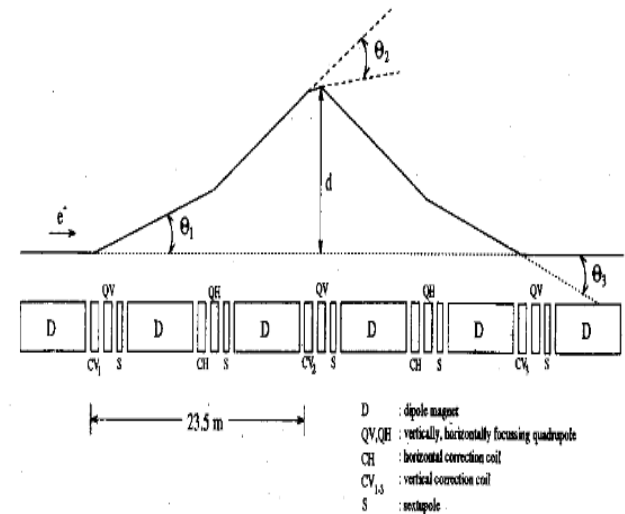
$$\frac{\sqrt{\beta_i^m \beta_j^c}}{2 \sin \pi Q} \sum_{n=1}^{NQ} \frac{\beta_n^q (K \ell)_n}{2 \sin \pi Q} \cos(\pi Q - |\mu_i^m - \mu_n^q|) \cos(\pi Q - |\mu_n^q - \mu_j^c|) \Theta_j^c$$

^adispersion distortions may also originate from the roll angle of dipoles and quadrupoles around the longitudinal axis. In the second case the correction of dispersion is *not* equivalent to the correction of the closed orbit.

Local steering: orbit bumps

Orbit bumps may be used for correcting local orbit distortions or more in general for locally changing the closed orbit:

- beam steering at injection/extraction;
- avoiding obstacles;
- diagnostic purposes.



The simplest one is a 3-correctors bump:

- first corrector kicks the beam away from the closed orbit;
- second one steers it back towards the closed orbit at the location of the third corrector;
- third one closes the orbit bump with a kick $\Theta_3 = -z'(s_3)$.

The analytical expressions for the needed Θ_2 and Θ_3 can be found by imposing

$$T(\bar{s})_1\Theta_1 + T(\bar{s})_2\Theta_2 + T(\bar{s})_3\Theta_3 = 0 \quad \text{with } \bar{s} < s_1 \text{ and } \bar{s} > s_3$$

or imposing

$$\sum_{i=1}^3 T_i(s_3)\Theta_i = \sum_{i=1}^3 T'_i(s_3)\Theta_i = 0$$

A fourth corrector is needed for controlling both transverse position and slope at a given point.

Closed bumps are a simple way for detecting optics errors: if a quadrupole involved in the bump is faulty the bump will be not closed!

Beam Based Alignment

- Accelerator components are aligned with respect to an *ideal orbit*. Magnet misalignments are specified and field are expanded with respect to this design orbit.
- In general orbit correction algorithms try to minimize the beam offset with respect to the design orbit.

It may be convenient to steer the beam through the *center of the quadrupoles*. If close to each quadrupole there is a BPM perfectly centered with the quadrupole axis this is accomplished by the usual orbit correction.

However BPMs may have offsets, mechanical and electronic: the aim of BBA is determining such offset.

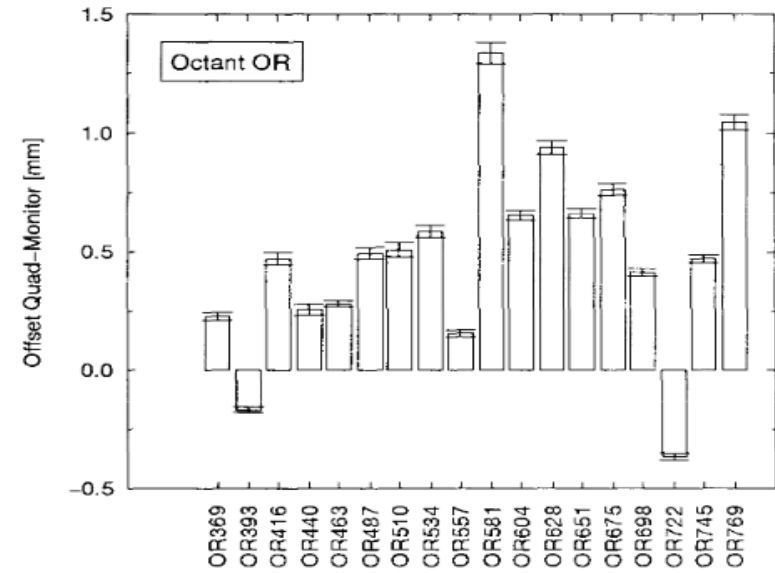
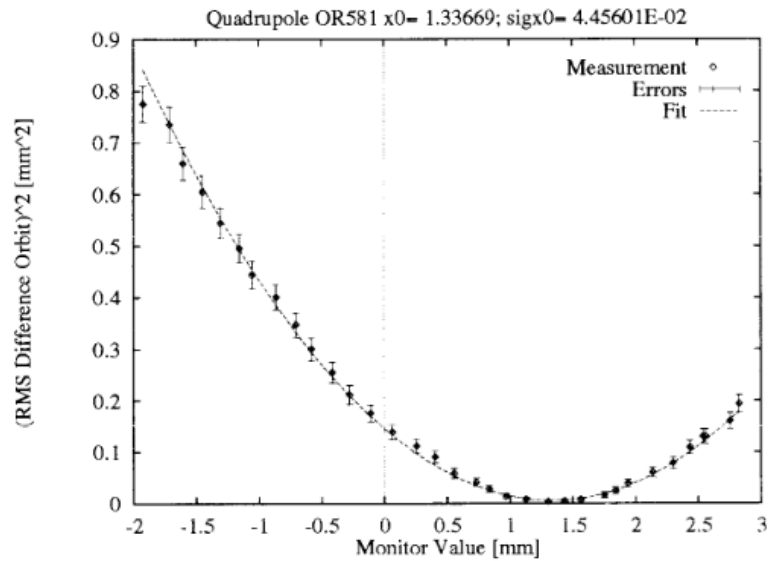
The quantity to be minimized by the orbit correction application is $x_{bpm} + \delta_{bpm,q}$ where x_{bpm} is the BPM reading and $\delta_{bpm,q}$ is the position of the BPM center wrt quadrupole.

For measuring $\delta_{bpm,q}$ we use the fact that if the beam is centered in the quadrupole a change of the field will not change the closed orbit. For finding the center we move the beam across the quadrupole.

The procedure for measuring $\delta_{bpm,q}$ would look like

- Skew quadrupoles and sextupoles inside the bump are switched-off.
- A reference orbit is acquired.
- A orbit bump with maximum at the BPM is excited; the difference orbit outside the bump region wrt the reference orbit (w/o bump) is minimized by empirically tweaking the last bump corrector. This gives an empirically corrected coefficient for that corrector.
- The quadrupole strength is changed by a “reasonable” amount.
- The bump size is varied and the rms orbit difference *outside* the bump and the BPM reading recorded.
- These rms values are fitted for finding the minimum ie $\delta_{bpm,q}$.

This procedure has been tested for instance at DESY HERA-e.



M. Böge et al., IWAA95.

Summary

- We have introduced the concept of closed orbit in a storage ring using the co-moving frame
 - Sources, equation of motion and solution.
 - Correction algorithms.
 - Spurious dispersion.
- Some orbit manipulations (orbit bumps) have been described.
 - The use of orbit bumps for BBA has been described