
JINR MEMBER STATES
Armenia
Azerbaijan
Belarus
Bulgaria
Cuba
Czech Republic
DPRK
Georgia
Kazakhstan
Moldova
Mongolia
Poland
Romania
Russia
Slovakia
Ukraine
Uzbekistan
Vietnam
Agreements are signed on
the governmental level with (associated members)
Egypt
Germany
Hungary
Italy
RSA
Serbia
105
rutherfordium ← dubnium → seaborgium
Ta
↑
Db
↓
(Upp)
Db
105P
157N
Periodic Table - Extended Periodic Table

Физические явления в фи-ноль переходе. Маятник Капицы и переворот магнитного момента

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Outline

1. Introduction
2. Numerical methods
3. Phi-0 junctions
4. Kapitza pendulum
5. Magnetization reversal
6. Ferromagnetic resonance
7. Dynamics of magnetization along IVC
8. Devil’s staircase in SFS structure
9. Kapitsa pendulum features in other models

Mechanism for the φ_0 - Josephson junction realization.

Very special situation is possible when the weak link in Josephson junction is a non-centrosymmetric magnetic metal with broken inversion symmetry !
Suitable candidates : MnSi, FeGe
ot thin ferromagnetic films on the substrate.

Josephson junctions with time reversal symmetry: $j(-\varphi) = -j(\varphi)$;
i.e. higher harmonics can be observed $\sim j_n \sin(n\varphi)$ –the case of the π junctions.

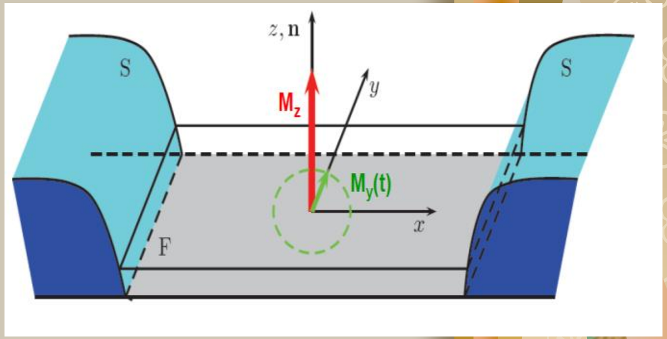
Without this restriction a more general dependence is possible
$j(\varphi) = j_0 \sin(\varphi - \varphi_0)$.

Rashba-type spin-orbit coupling $\alpha(\vec{\sigma} \times \vec{p}) \cdot \vec{n}$

$\vec{n}$ is the unit vector along the asymmetric potential gradient.

A. Buzdin, PRL, 101,107005 (2008)

Energy of the system:



$$E_{tot} = -\frac{\phi_0}{2\pi}\varphi I + E_s(\varphi,\varphi_0) + E_M(\varphi_0)$$

$$E_S(\varphi,\varphi_0) = E_J[1-\cos(\varphi-\varphi_0)]$$

$$E_M = \frac{-KV}{2}\left(\frac{M_Z}{M_0}\right)^2 \qquad E_J = \frac{\phi_0 I_c}{2\pi} \qquad \varphi_0 = l\frac{v_0}{v_F}\frac{M_y}{M_0},$$

A. Buzdin.
Phys. Rev. Lett .
101, 107005 (2008) .
F . Konschelle, A. Buzdin.
Phys. Rev. Lett.
102, 017001 (2009) .

Landau-Lifshitz-Gilbert equation:

$$\frac{dM}{dt} = -\gamma[M\times H_{eff}] + \frac{\alpha}{M_0}[M\times\frac{dM}{dt}]$$

$$H_{eff} = \frac{K}{M_0}\left[\Gamma\sin\left(\varphi - r\frac{M_y}{M_0}\right)\vec{y} + \frac{M_z}{M_0}\vec{z}\right]$$

$$\Gamma = Gr = \frac{E_J}{KV}l\frac{v_0}{v_F}, \qquad G = \frac{E_J}{KV},\ \ r = l\frac{v_0}{v_F},\ \ l = 4\frac{hL}{\hbar v_F}$$

System of equations for magnetization dynamics

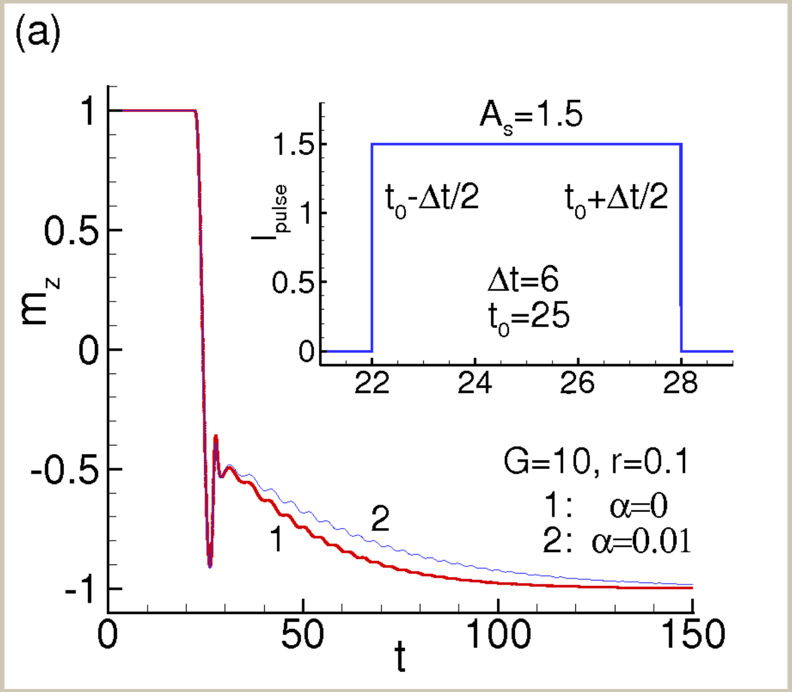
$$
\begin{cases}
\dot{m}_x = \dfrac{1}{1+\alpha^2}\{-m_y m_z + Grm_z \sin(\omega t - rm_y) - \alpha[m_x m_z^2 + Grm_x m_y \sin(\omega t - rm_y)]\} \\
\dot{m}_y = \dfrac{1}{1+\alpha^2}\{m_x m_z - \alpha[m_y m_z^2 - Gr(m_z^2 + m_x^2)\sin(\omega t - rm_y)]\} \\
\dot{m}_z = \dfrac{1}{1+\alpha^2}\{-Grm_x \sin(\omega t - rm_y) - \alpha[Grm_y m_z \sin(\omega t - rm_y) - m_z(m_x^2 + m_y^2)]\}
\end{cases}
$$

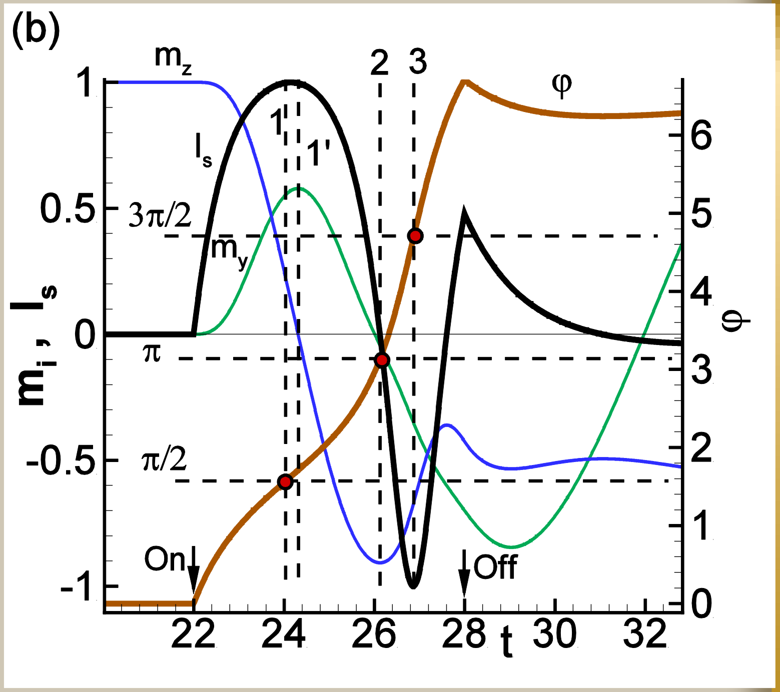
Naturally, we may expect that the most interesting situation corresponds to the case when the magnetic anisotropy energy does not exceed too much the Josephson energy. From the measurements [18] on permalloy with very weak anisotropy, we may estimate $K \sim 4 \times 10^{-5}$ K · Å^{-3}. On the other hand, typical value of L in S/F/S junction is $L \sim$ 10 nm and $\sin\ell/\ell \sim 1$. Then, the ratio of the Josephson over magnetic energy would be $E_J/E_M \sim 100$ for $T_c \sim$ 10 K. Naturally, in the more realistic case of stronger anisotropy, this ratio would be smaller, but it is plausible to expect a great variety of regimes from $E_J/E_M \ll 1$ to $E_J/E_M \gg 1$.

F . Konschelle, A. Buzdin. Phys. Rev. Lett . 102, 017001 (2009) .

Transition dynamics in the system under rectangular current pulse

$$I_{pulse} = w\frac{d\varphi}{dt} + \sin(\varphi - rm_y)$$

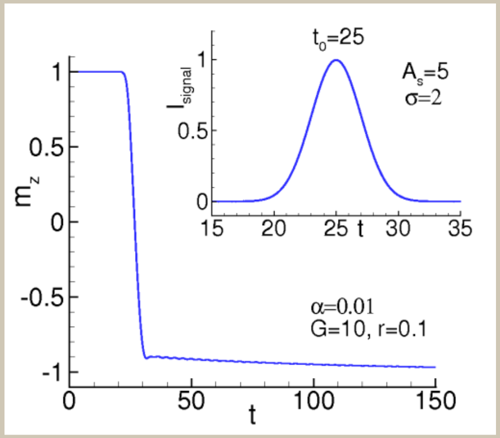


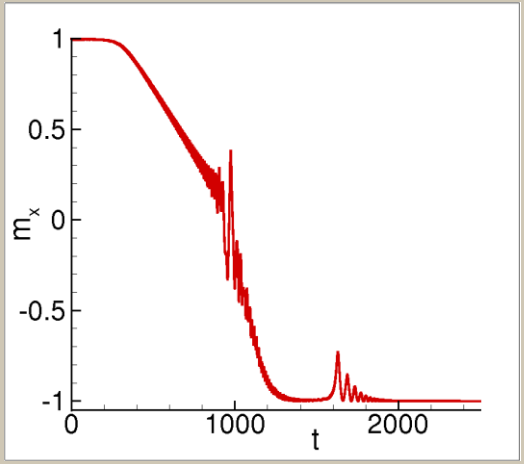


$$\sin\frac{1}{2}\left[\int_{t_{min}}^{t_{max}} V(t')dt' - r\left[m_y(t_{max}) - m_y\,(t_{min})\right]\right] = 1$$

Transition dynamics under electric current pulse of the Gaussian form

$$I_{pulse} = A\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(t-t_0)^2}{2\sigma^2}\right)$$



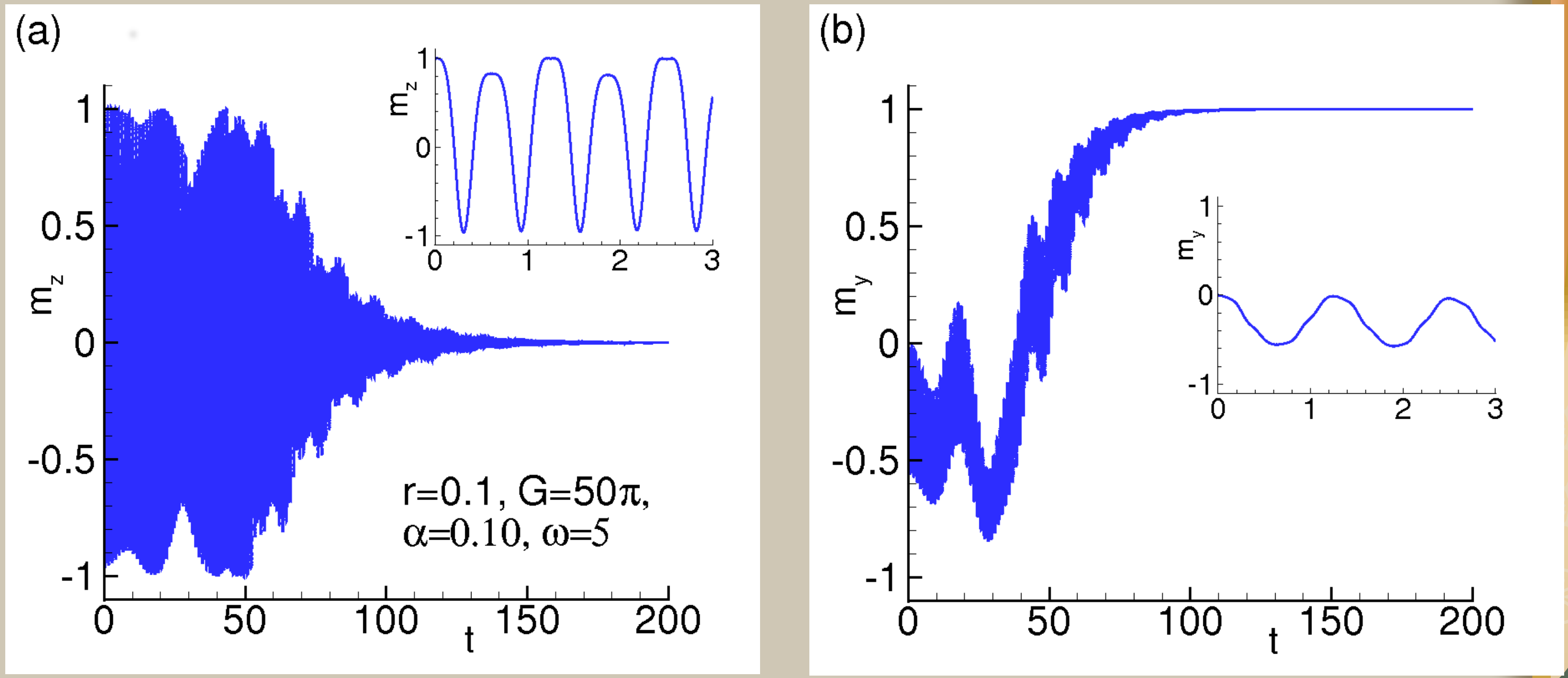


$$\bar{V} = 1.5 - 0.00075\bar{t},$$

$$\varepsilon = \bar{k} = 0.05,\ \eta = 0.01$$

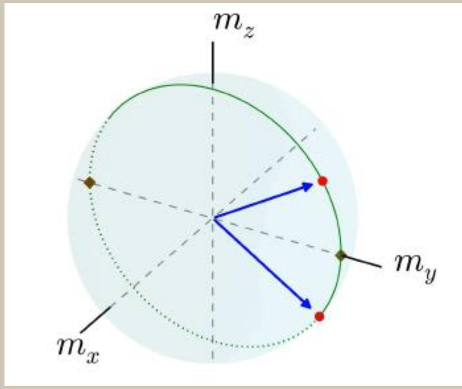
L.Kai, E.M.Chudnovsky,
PRB, 82,104429 (2010)

Dynamics of magnetization components without signal

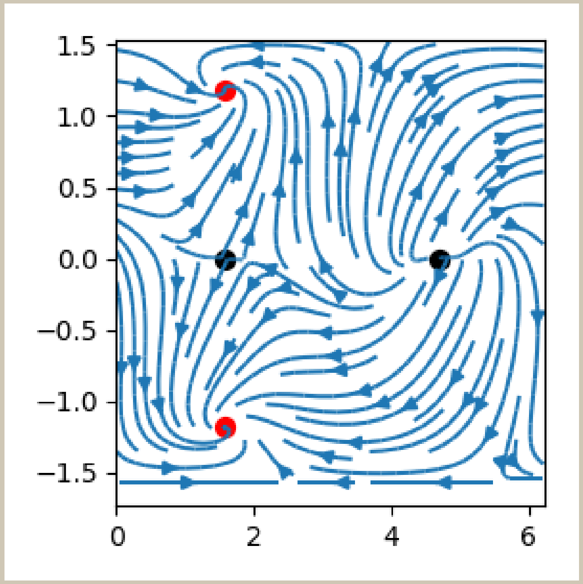


Yu.M. Shukrinov, I.R. Rahmonov, K. Sengupta, A.Buzdin,
APL, 110, 182407, 2017

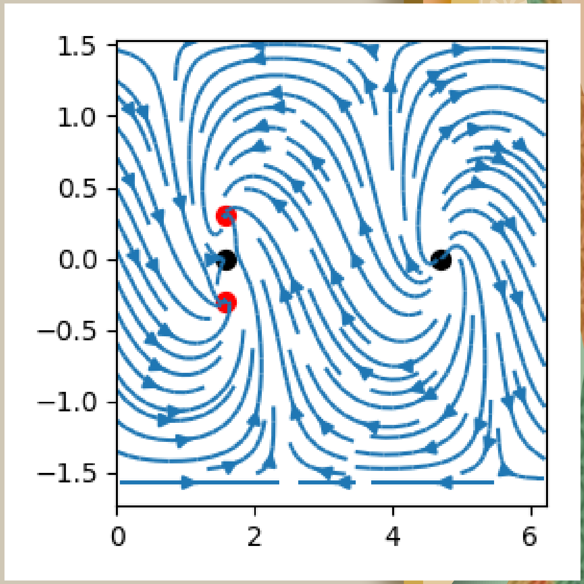
Demonstration of equilibrium points



Arrows show stable points, other two are unstable.



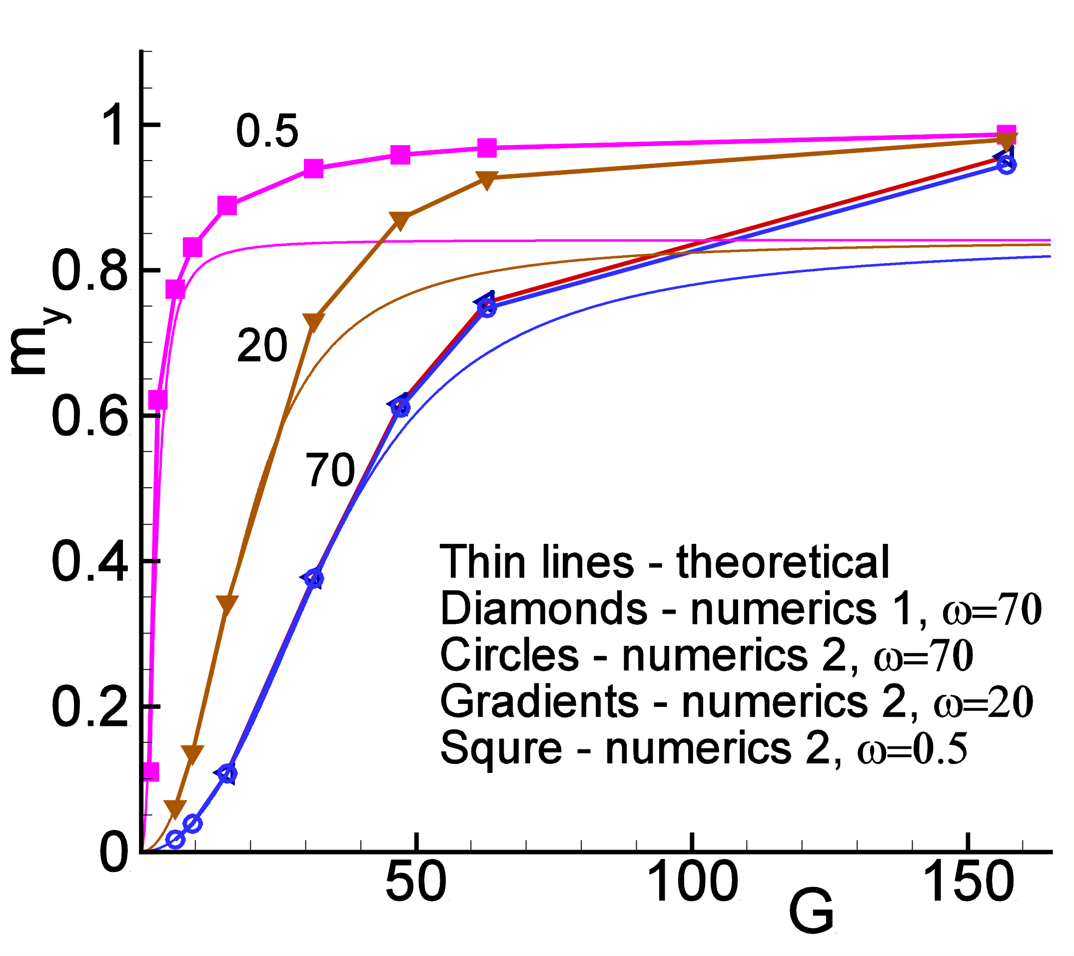
G=31.4



G=157.0

Yu. M. Shukrinov, A. Mazanik, I. R. Rahmonov, A. E. Botha and A. Buzdin. Re-orientation of easy axis in Phi-0–junction, Europhysics Letters, EPL, 122 (2018) 37001

Comparison of theoretical and numerical calculations



Yu. M. Shukrinov, A. Mazanik, I. R. Rahmonov, A. E. Botha and A. Buzdin. Re-orientation of easy axis in Phi-0–junction, Europhysics Letters, EPL, 122 (2018) 37001

- Dynamics of magnetization along IVC and ferromagnetic resonance

Magnetization dynamics along IV-characteristics of JJ.

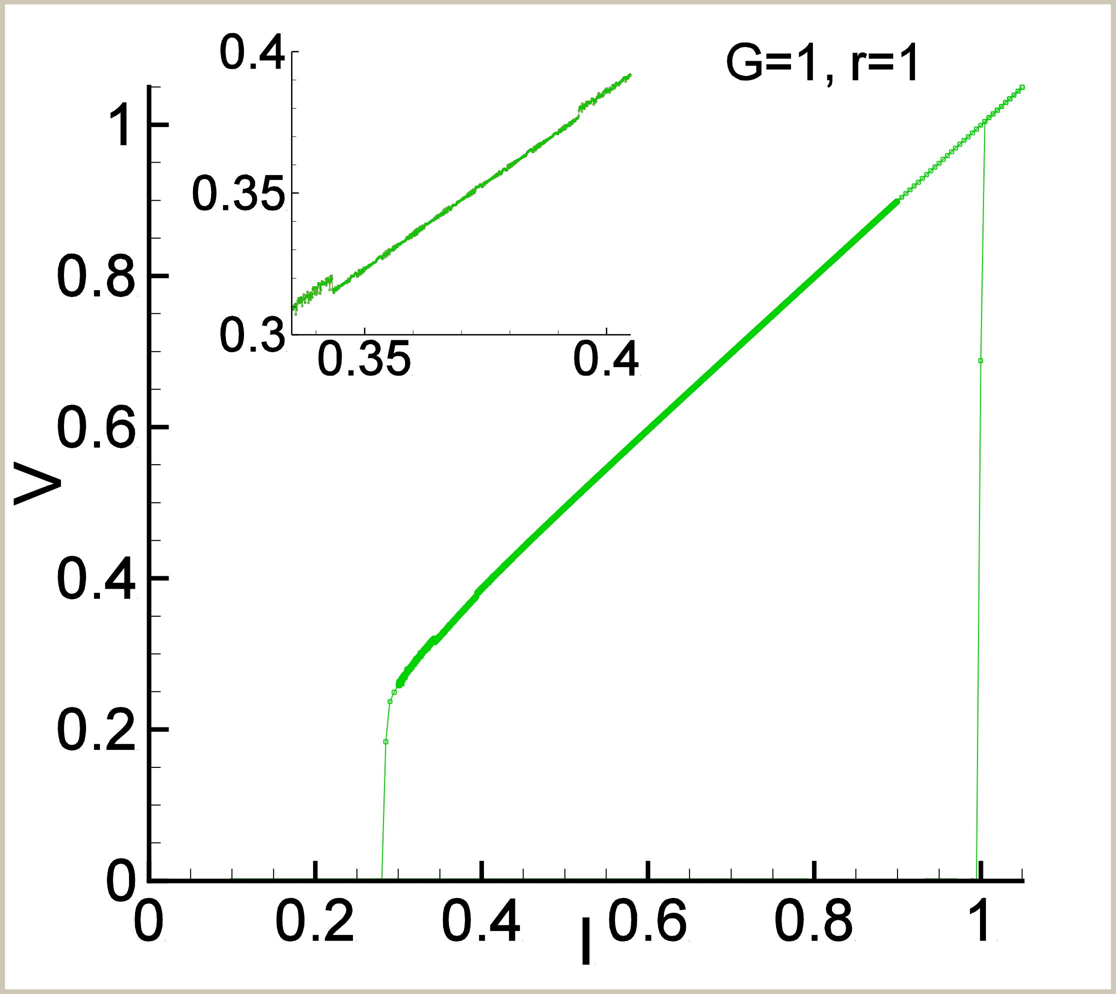
System of equations

$$\begin{cases} \dot{m}_x = \dfrac{1}{1+\alpha^2}\{-m_y m_z + Grm_z \sin(\omega t - rm_y) - \alpha[m_x m_z^2 + Grm_x m_y \sin(\omega t - rm_y)]\} \\ \dot{m}_y = \dfrac{1}{1+\alpha^2}\{m_x m_z - \alpha[m_y m_z^2 - Gr(m_z^2 + m_x^2)\sin(\omega t - rm_y)]\} \\ \dot{m}_z = \dfrac{1}{1+\alpha^2}\{-Grm_x \sin(\omega t - rm_y) - \alpha[Grm_y m_z \sin(\omega t - rm_y) - m_z(m_x^2 + m_y^2)]\} \end{cases}$$

$$\frac{dV}{dt} = \frac{1}{\beta_c}\left[I - V - \sin\left(\varphi - rm_y\right) + A\sin(\omega t)\right],$$

$$\frac{d\varphi}{dt} = V$$

Result of simulation of IV-characteristic, beta=0.2, alpha=0



Ferromagnetic resonance, w=1

At G<<1, $(m_x, m_y) \ll 1$

- Effect of damping: a damped resonance at $\boldsymbol{\omega}$ --> 1

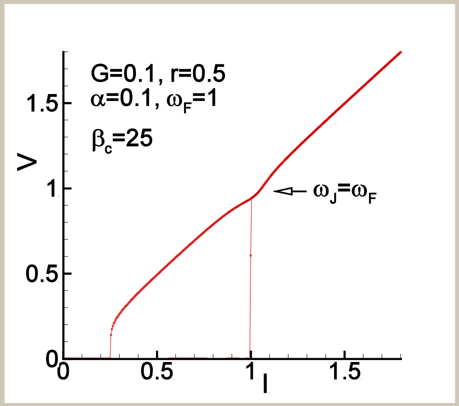
$$\begin{cases} \dot{m}_x = \dfrac{1}{1+\alpha^2}\{-m_y + Gr\sin\omega t - \alpha m_x\} \\ \dot{m}_y = \dfrac{1}{1+\alpha^2}\{m_x - \alpha[m_y - Gr\sin\omega t]\} \end{cases}$$

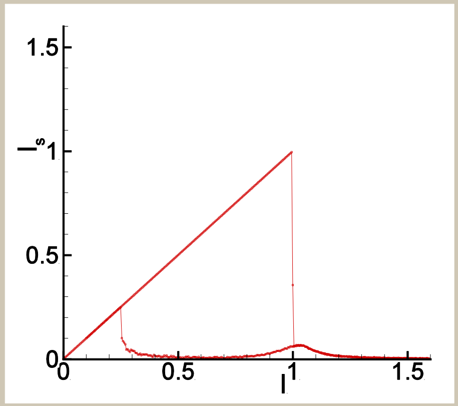
$$m_y(t) = \frac{\omega_+ - \omega_-}{r}\sin\omega t - \frac{\alpha_+ + \alpha_-}{r}\cos\omega t$$

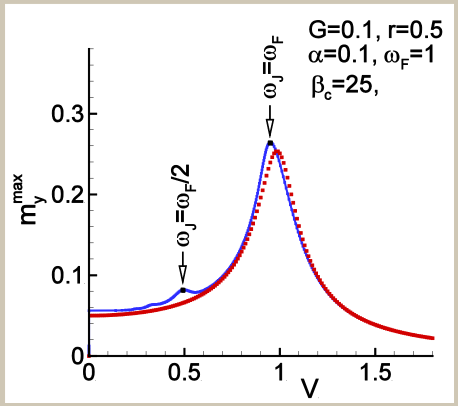
$$\Omega_\pm = (\omega \pm 1)^2 + \alpha^2\omega^2$$

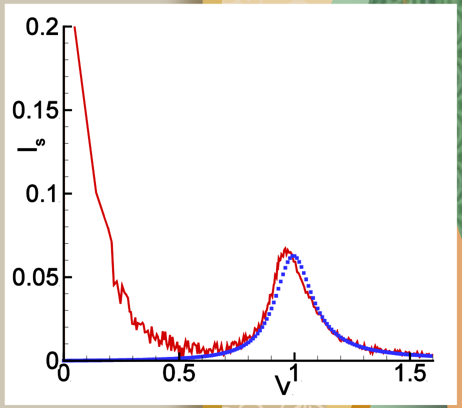
$$\omega_\pm = \frac{Gr^2}{2}\frac{\omega \pm 1}{\Omega_\pm} \quad \textit{and} \quad \alpha_\pm = \frac{Gr^2}{2}\frac{\alpha\omega}{\Omega_\pm}$$

$$I_0(\alpha) = \frac{\alpha G r^2 \omega}{4}\left(\frac{1}{\Omega_-} + \frac{1}{\Omega_+}\right)$$





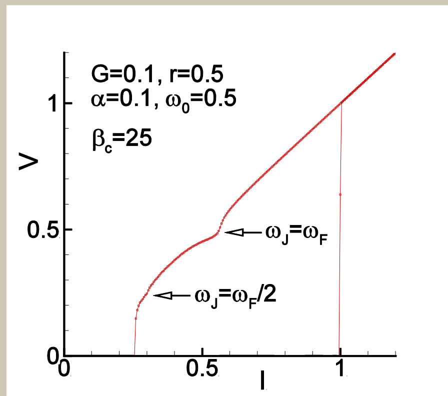


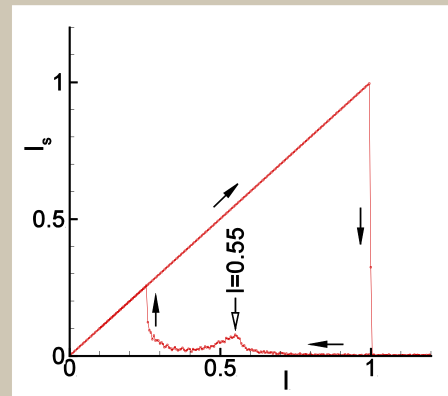


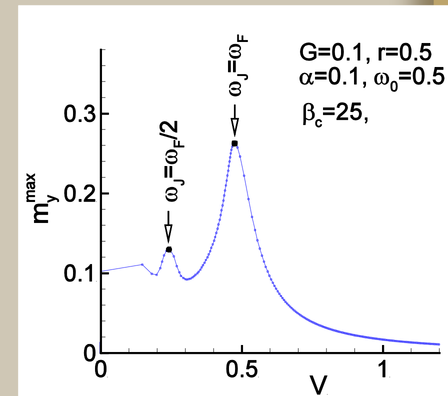
I. Rahmonov, A. Mazanik, Yu. M. Shukrinov, K. Sengupta

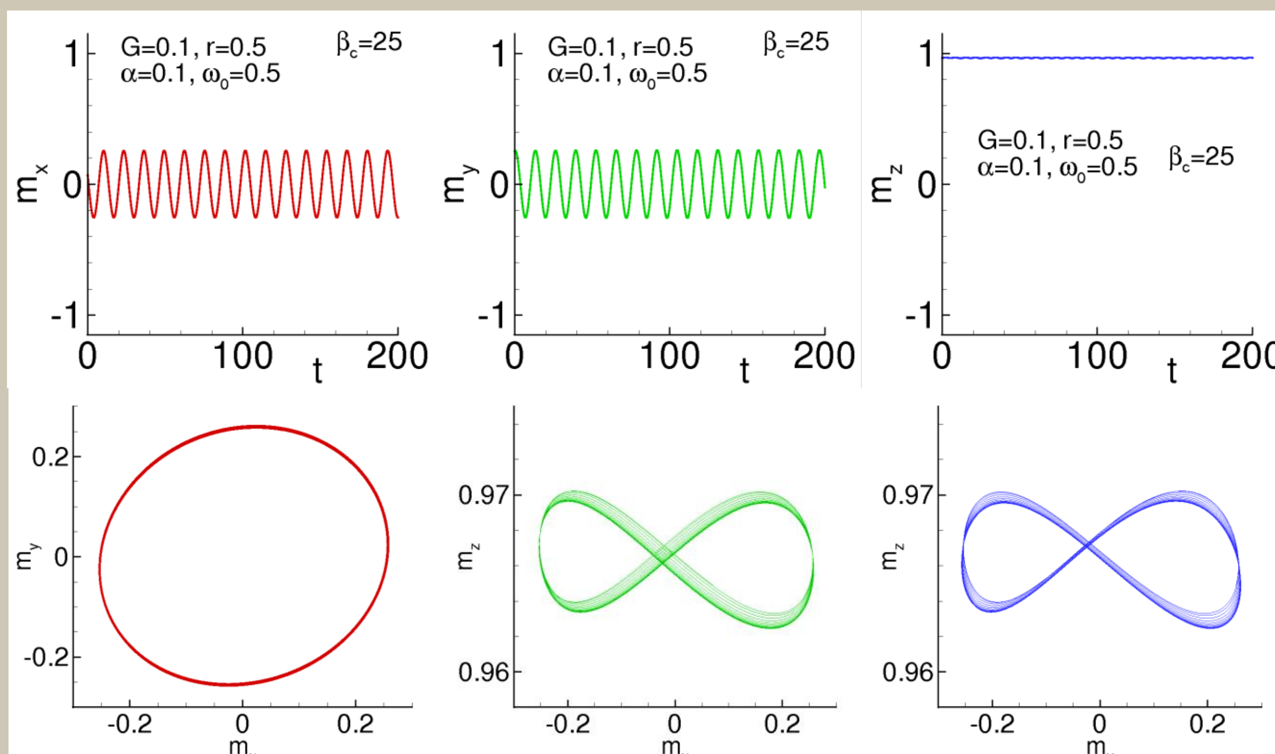
In preparation, 2019

Ferromagnetic resonance, w=0.5

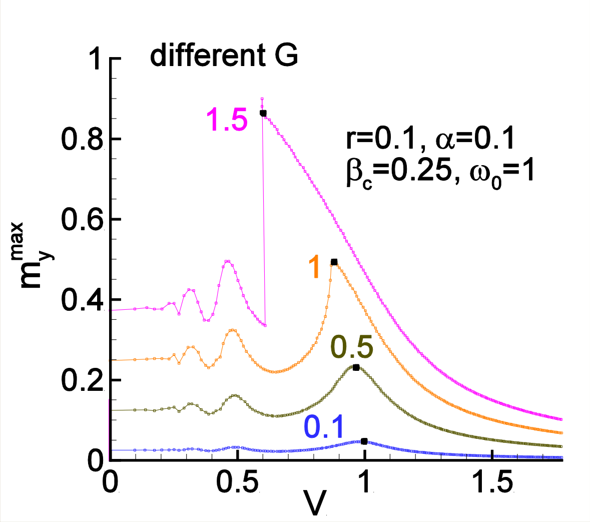


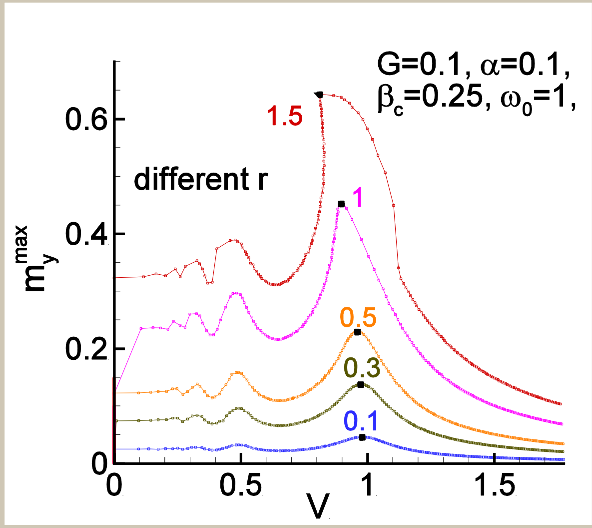


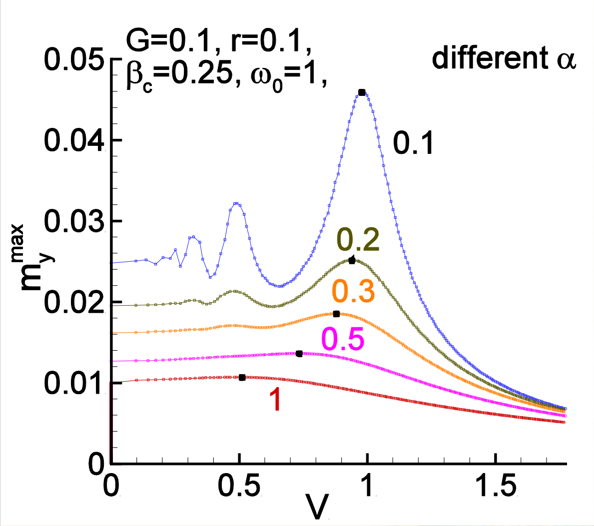




Ferromagnetic resonance, w=1
Effect of G, r, alpha







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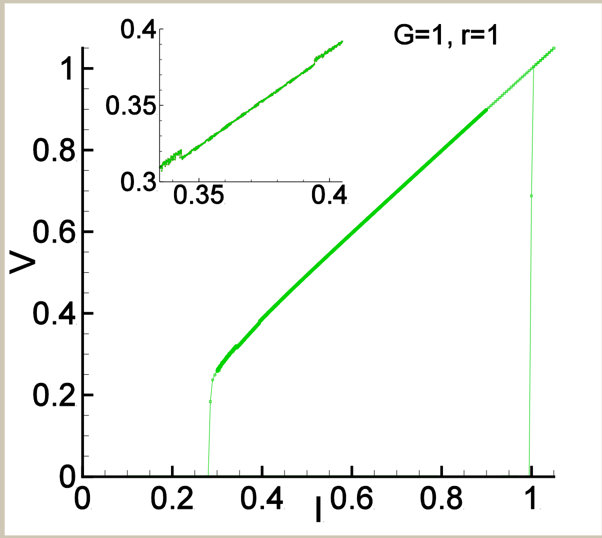
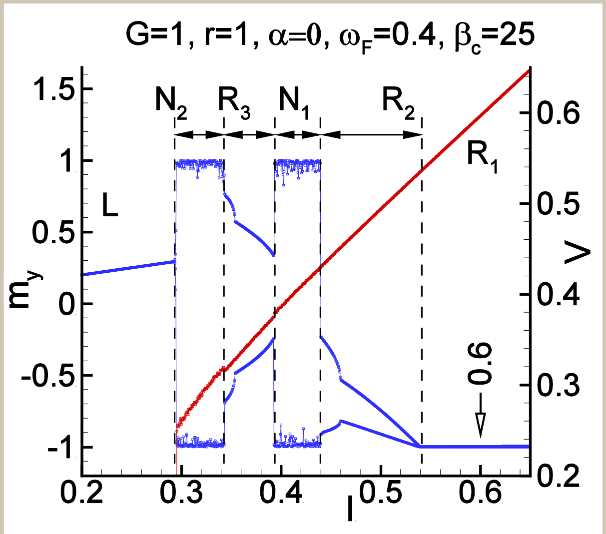
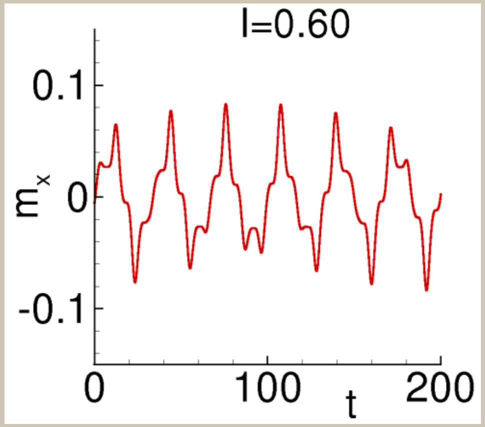
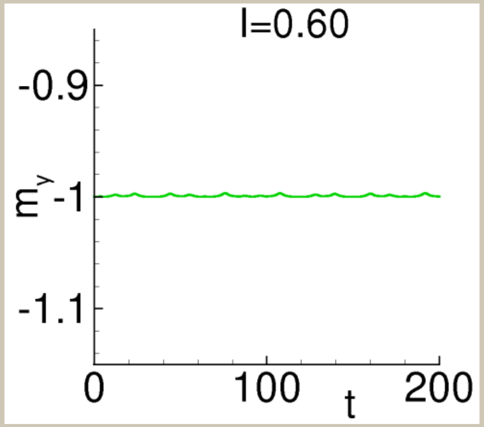
In preparation, 2019

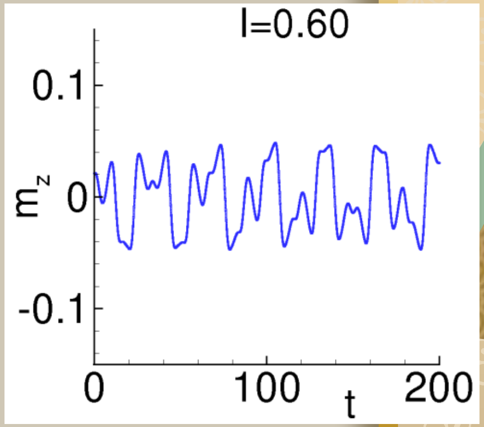
Maximal and minimal m_y along IVC











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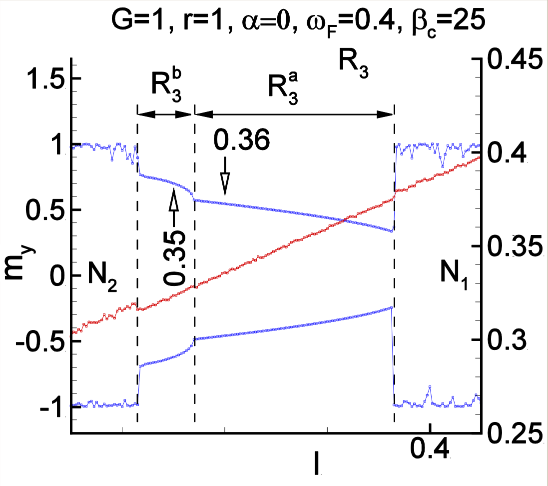
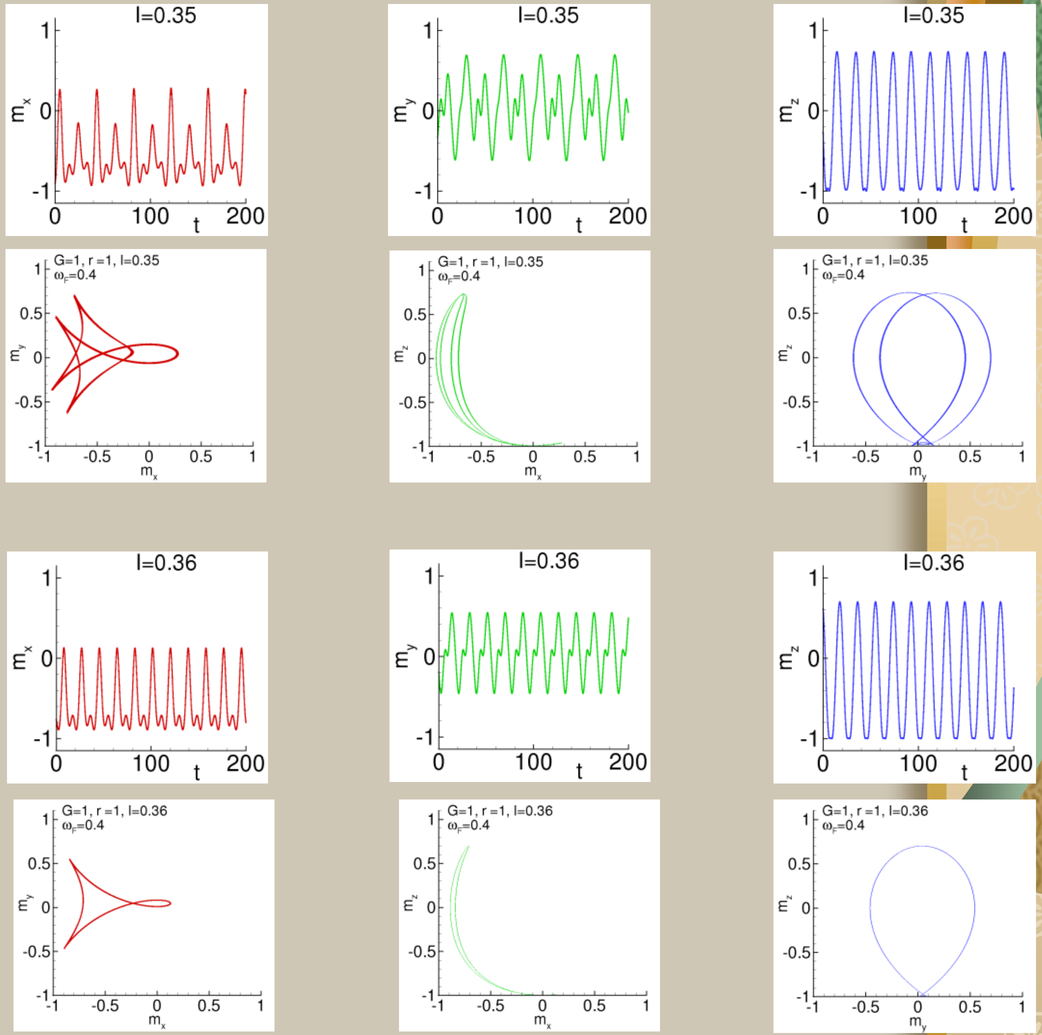
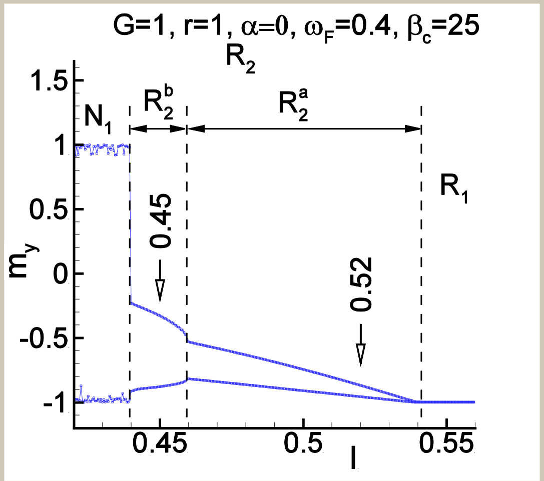
In preparation, 2019

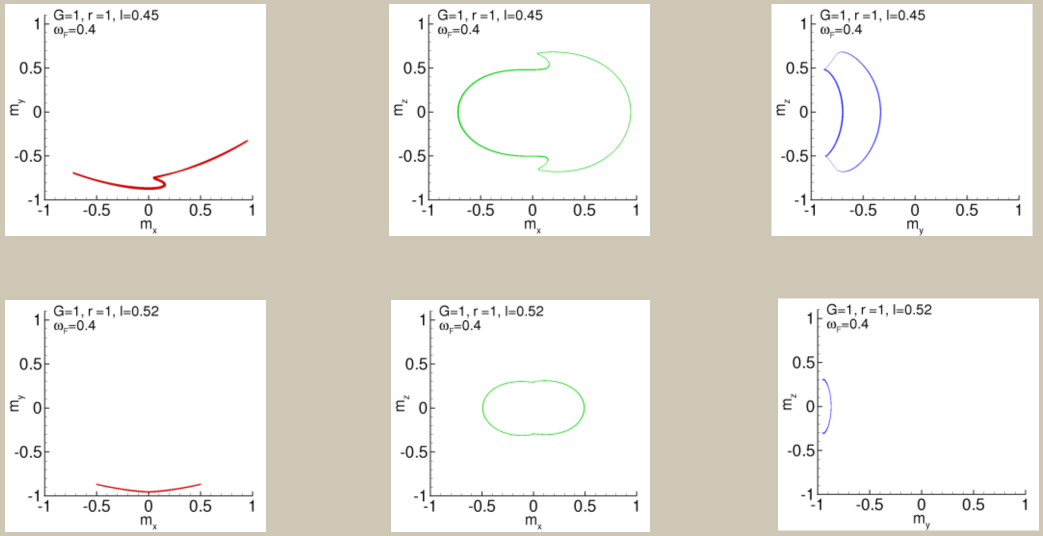
Maximal and minimal m_y along IVC



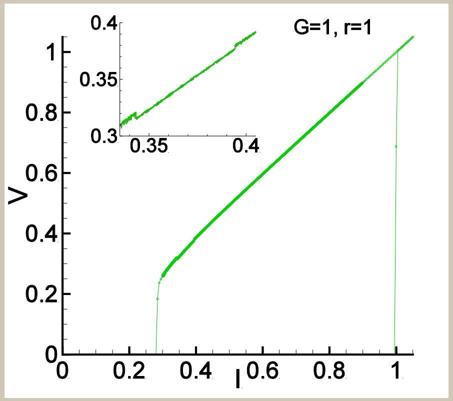


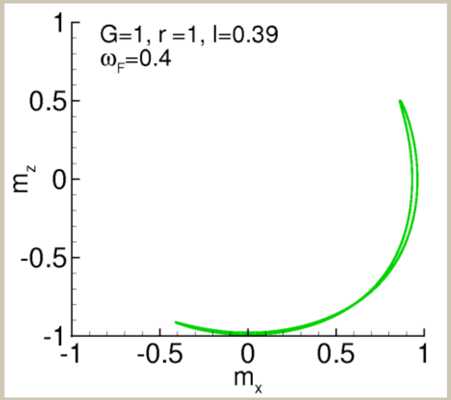
Maximal and minimal m_y along IVC

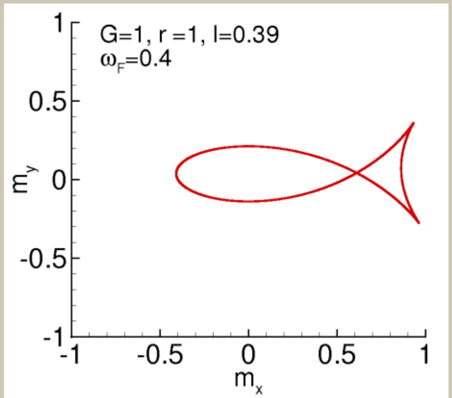




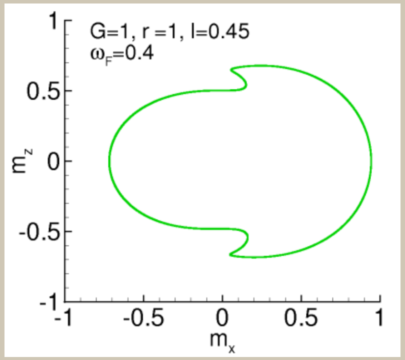
I = 0.39

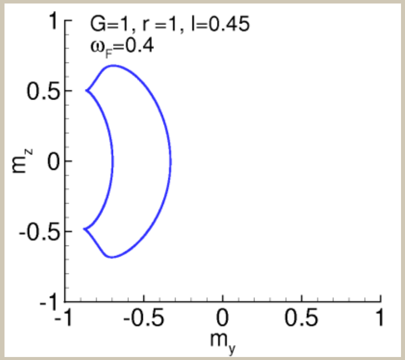


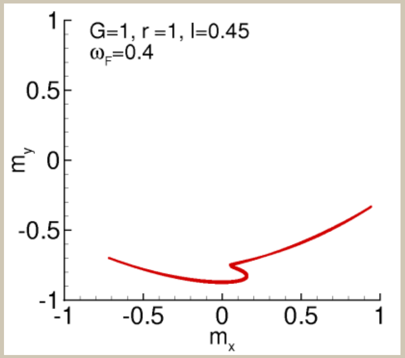




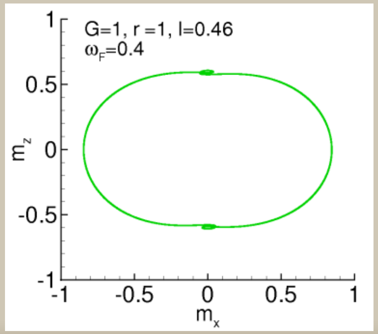
I = 0.45

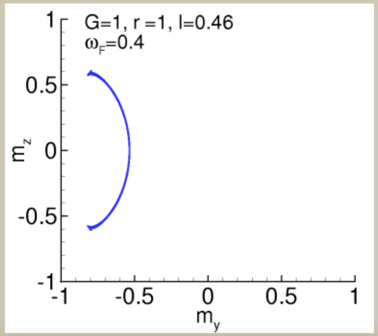


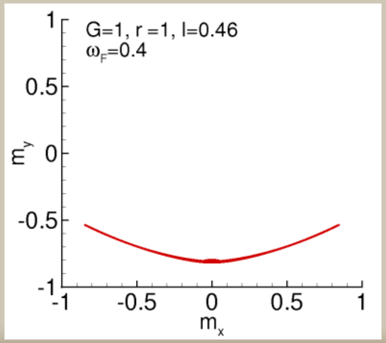




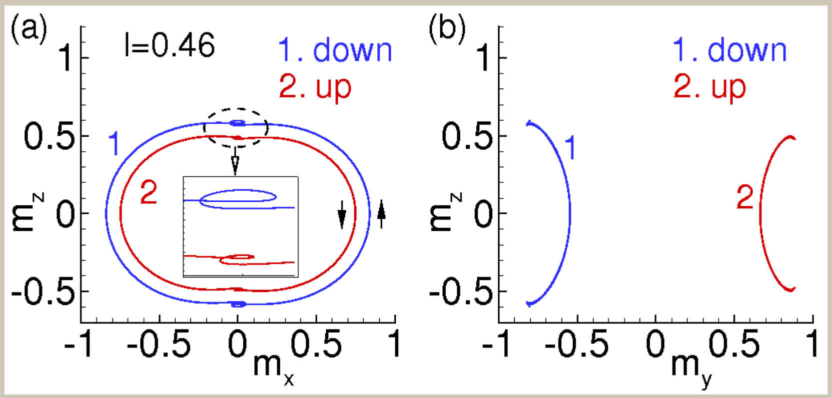
I = 0.46

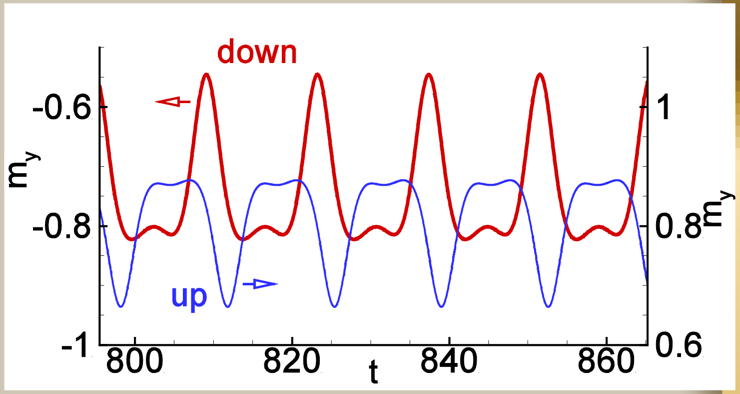




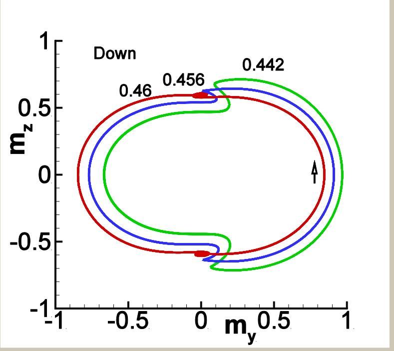


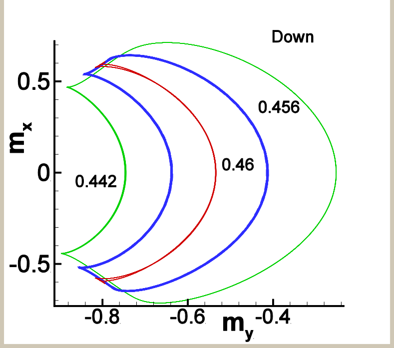
Topological features: Changing direction of precession by changing of current direction along the IV-characteristics



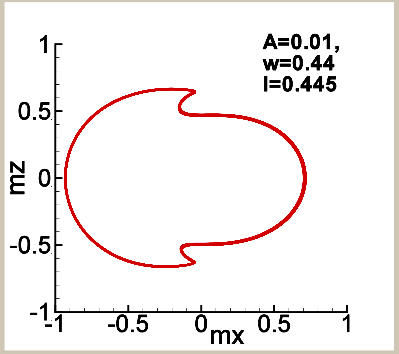


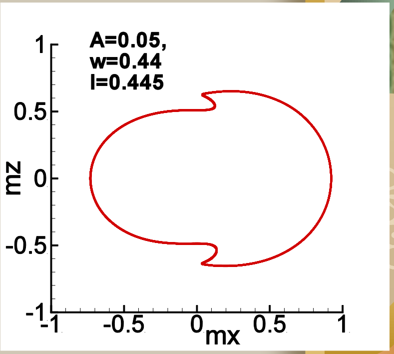
Transformations along IV-characteristics



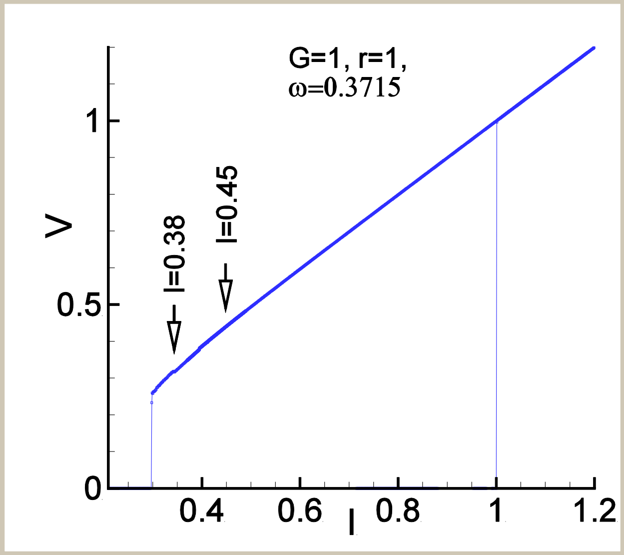


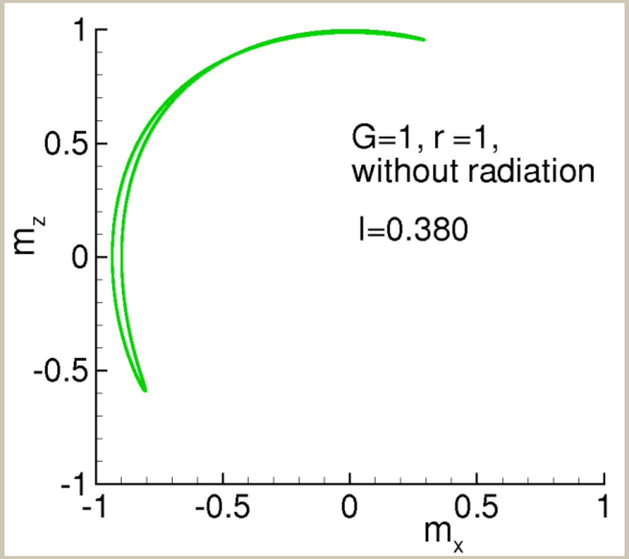
Left-right transformations

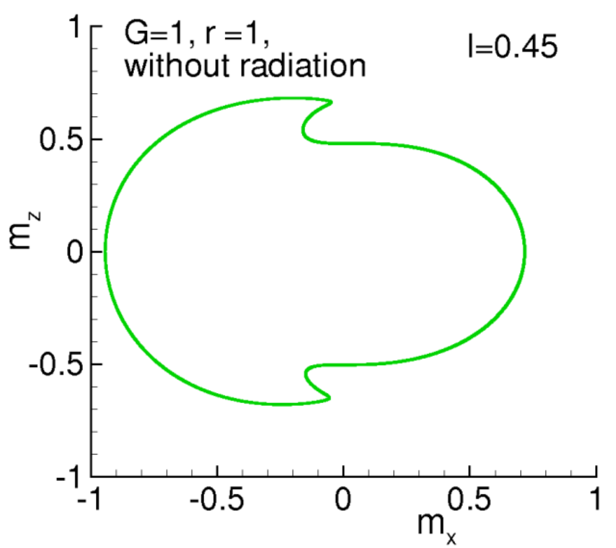


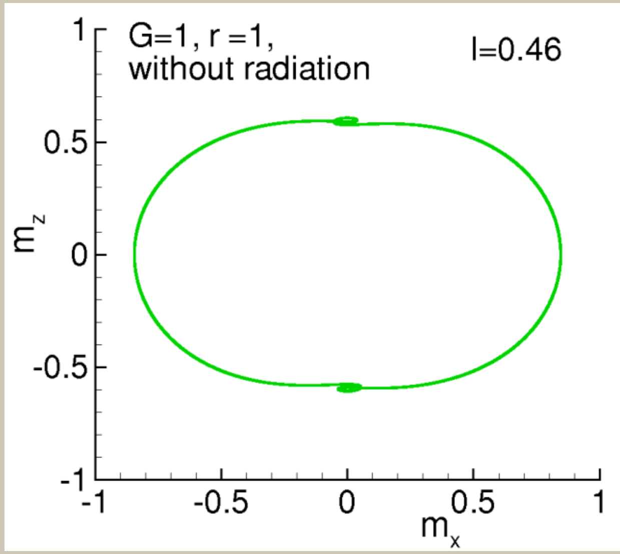


Without radiation

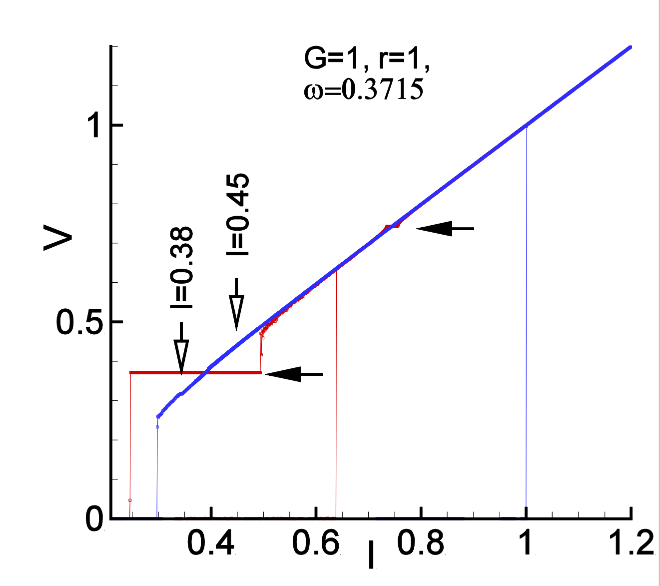


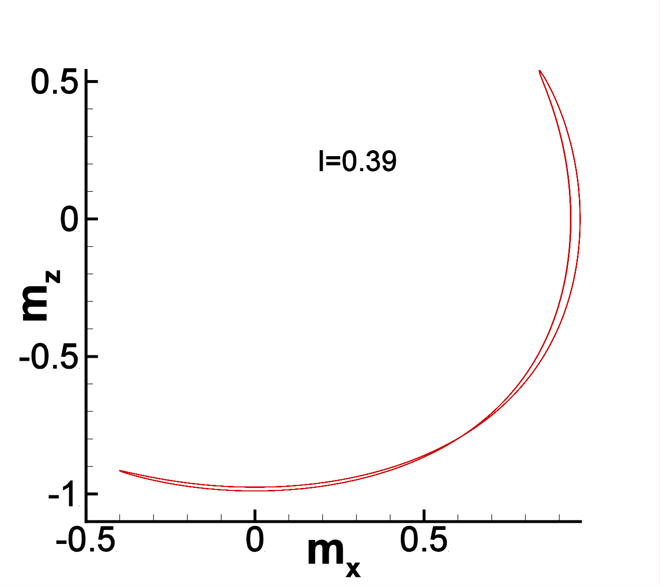


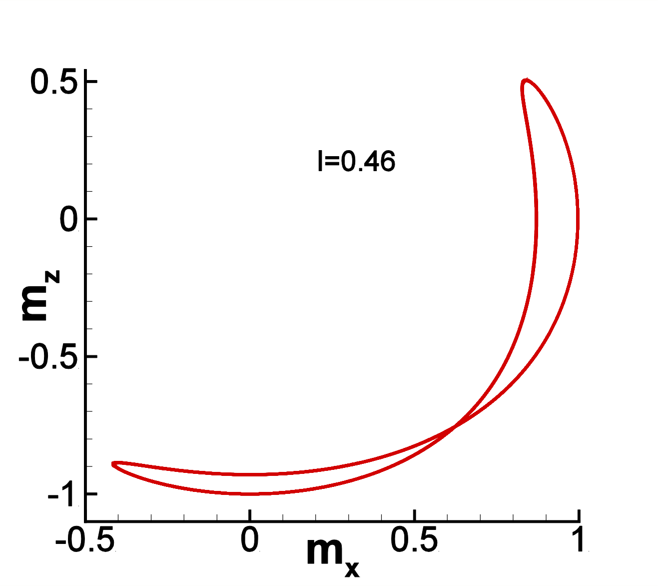




Under radiation, W=0.3715, A=1





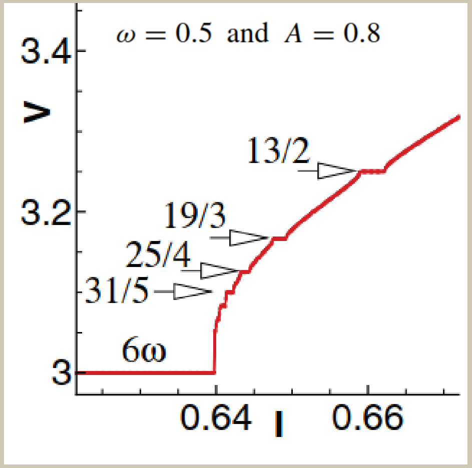


Devil's staircase in Superconductor/Ferromagnetic/Superconductor Josephson Junction

Devil's staircase and its importance

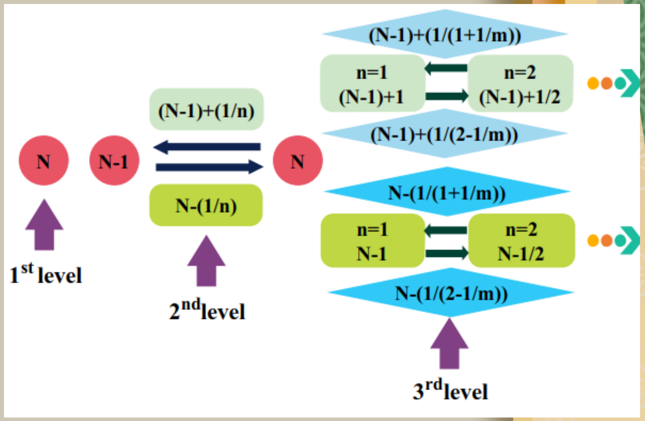
$$\dot{V} + \sin(\varphi) + \beta\dot{\varphi} = I + A\sin(\omega t),$$

$$\dot{\varphi} = V.$$



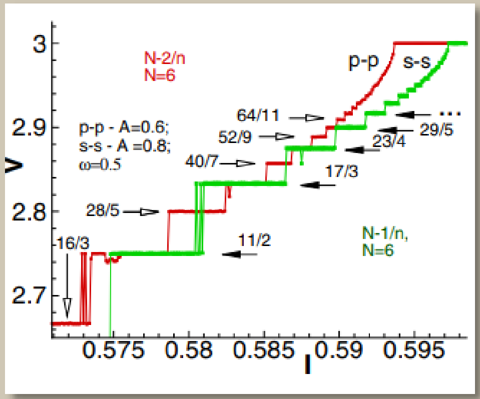
The position of the steps follow:

$$V = \left(N \pm \frac{1}{n \pm \frac{1}{m \pm \frac{1}{p \pm ..}}} \right) \Omega$$



where N,n,m,p,... are positive integers.

Yu. M. Shukrinov, et al,
PRB 88, 214515 (2013)

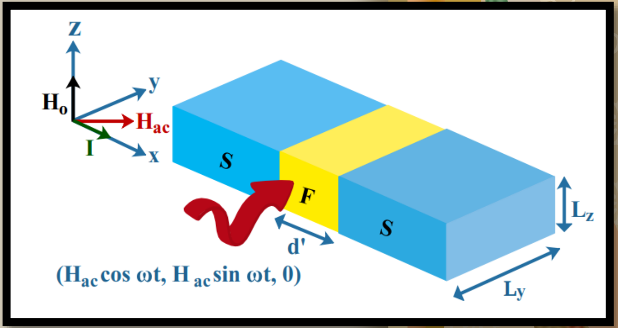


M. Maiti et al.,PRB 92, 224501(2015)

Model and Formalism

RCSJ

$$I/I_c^0 = \frac{\Phi_o^2 \sin\theta \sin\left(\frac{4\pi^2 dM_z(t)L_y}{\Phi_o}\right)\sin\left(\frac{4\pi^2 dM_y(t)L_z}{\Phi_o}\right)}{16\pi^4 d^2 L_z L_y M_z(t) M_y(t)} + \frac{\Phi_0}{2\pi R I_c^0}\frac{d\theta(y,z,t)}{dt} + C\frac{\Phi_0}{2\pi I_c^0}\frac{d^2\theta(y,z,t)}{dt^2}$$



LLG

$$(1+\alpha^2)\frac{d\mathbf{M}}{dt} = -(\gamma \mathbf{M}\times\mathbf{H}_e + \frac{\gamma\alpha}{|\mathbf{M}|}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_e)])$$

S. Hikino, et al., **SUST, 24(2), 024008, 2011**

The effective field is calculated from

$$H_e = -\frac{1}{v}\frac{\partial E}{\partial M} \qquad E = E_s + E_M + E_{ac}$$

$$\begin{aligned} E_s &= -\frac{\Phi_0}{2\pi}\theta(y,z,t)I + E_J\left[1-\cos(y,z,t)\right], \\ E_M &= -vH_0M_z(t), \\ E_{ac} &= -vM_x(t)H_{ac}\cos(\omega t) - vM_y(t)H_{ac}\sin(\omega t) \end{aligned}$$

M. Nashaat, A. E. Botha, Yu. M. Shukrinov., **PRB Vol. 97, No. 22 (2018).**

- **Energies**

$$H_o = \Omega_o/\gamma\,,\quad h_{ac} = \frac{H_{ac}}{H_o},\quad h_e = \frac{H_e}{H_o}\,,\quad \epsilon_J = \frac{E_J}{\mathrm{v}\,|M|\,H_o}$$

- **Magnetization**

$$m_{x,y,z} = M_{x,y,z}/M_0 \qquad \sum_{\alpha=x,y,z} m_\alpha^2(t) = 1$$

- **Time and Frequencies**

$$t = \tau\omega_c\,,\ \omega_c = 2\pi I_c^0 R/\Phi_o\,,\quad \Omega = \omega/\omega_c,\quad \Omega_o = \omega_o/\omega_c,$$

- **Magnetic fluxes**

$$\phi_{sy} = \frac{4\pi^2 L_y d|M|}{\Phi_o},\quad \phi_{sz} = \frac{4\pi^2 l_z d|M|}{\Phi_o}$$

➢ Non-linearized LLG

$$\phi_s = \frac{4\pi^2 L_z d h_{ac} M_z}{\Phi_0}$$

➢ linearized LLG

❑Numerical Scheme

$$\frac{d\mathbf{m}}{dt} = -\frac{\Omega_0}{(1+\alpha^2)}\Big(\mathbf{m}\times\mathbf{h}_e + \alpha\left[\mathbf{m}\times(\mathbf{m}\times\mathbf{h}_e)\right]\Big),$$

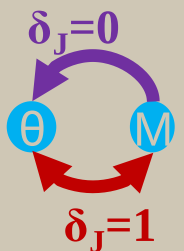
$$\mathbf{h}_e = h_{ac}\cos(\Omega t)\quad \hat{\mathbf{e}}_x + (h_{ac}\ sin(\Omega t) + \Gamma_{ij}\epsilon_J\cos\theta)\quad \hat{\mathbf{e}}_y + (1+\Gamma_{ji}\epsilon_J\cos\theta)\quad \hat{\mathbf{e}}_z,$$

and

$$\Gamma_{ij} = \frac{\sin(\phi_{si}m_j)}{m_i(\phi_{si}m_j)}\left[\cos(\phi_{sj}m_i) - \frac{\sin(\phi_{sj}m_i)}{(\phi_{sj}m_i)}\right]\delta_J,$$

where i=y, j=z and

$$\delta_J = \begin{cases} 0 : if \;\; \mathbf{h}_e = \mathbf{h}_0 + \mathbf{h}_{ac} \\ 1 : if \;\; \mathbf{h}_e = \mathbf{h}_0 + \mathbf{h}_{ac} + \mathbf{h}_s \end{cases}$$



$$I/I_c^0 = \frac{\sin(\phi_{sy}m_z)\sin(\phi_{sz}m_y)}{(\phi_{sy}m_z)(\phi_{sz}m_y)}\sin\theta + \frac{d\theta}{dt}$$

❑linearized Scheme

$$m_y = \frac{-2\alpha\frac{\Omega^2}{\Omega_0^2}\cos(\Omega t) + \left(1-\eta_1\frac{\Omega^2}{\Omega_0^2}\right)\sin(\Omega t)}{\left(1-\eta_2\frac{\Omega^2}{\Omega_0^2}\right)^2 + \Delta_J\left(1-\eta_1\frac{\Omega^2}{\Omega_0^2}\right) + 4\alpha^2\frac{\Omega^2}{\Omega_0^2}},$$

$\Delta_J = \epsilon_J\phi_{sz}^2\cos\theta(t)/3$, $\eta_1 = 1-\alpha^2$ and $\eta_2 = 1+\alpha^2$

Resonance frequency:

$$\alpha = 0\begin{cases} \Omega_r = \pm\Omega_0, & \Delta_J = 0, \\ \Omega_r = \pm\frac{\sqrt{2+\Delta_J}}{\sqrt{2}}\Omega_0, & \Delta_J \neq 0. \end{cases}$$

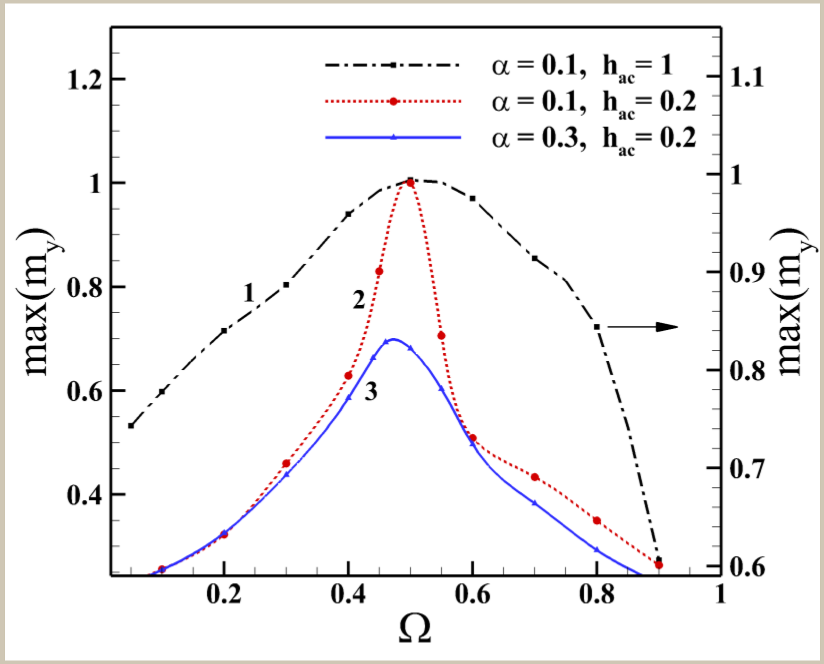
$$\alpha \neq 0\begin{cases} \Omega_r = \pm\frac{\sqrt{1-\alpha^2}}{1+\alpha^2}\Omega_0, & \Delta_J = 0, \\ \Omega_r = \pm\frac{\sqrt{(1-\alpha^2)(2+\Delta_J)}}{\sqrt{2}(1+\alpha^2)}\Omega_0, & \Delta_J \neq 0. \end{cases}$$

$$I/I_c = \frac{\sin(\phi_s m_y)}{(\phi_s m_y)}\sin\theta + \frac{d\theta}{dt}$$

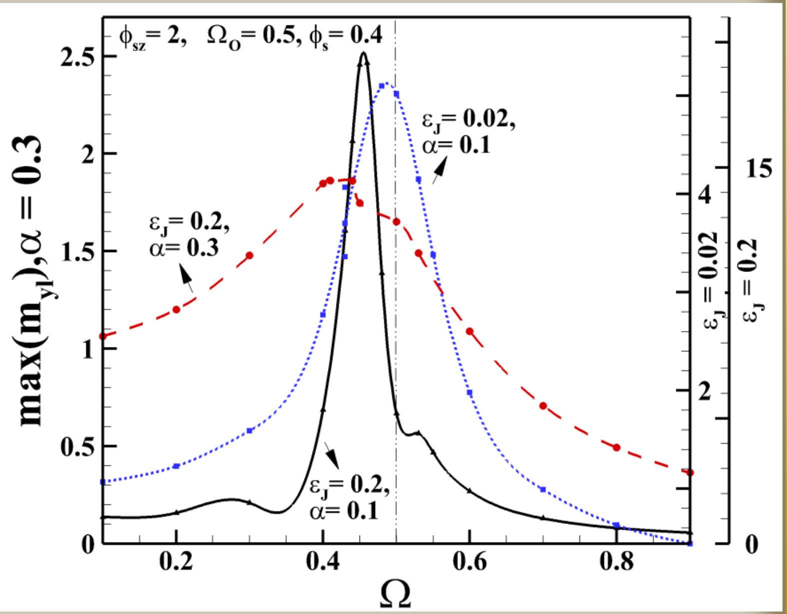
M. Nashaat, A. E. Botha, Yu. M. Shukrinov., PRB Vol. 97, No. 22 (2018).

Manifestation Of FMR

❑Numerical Scheme

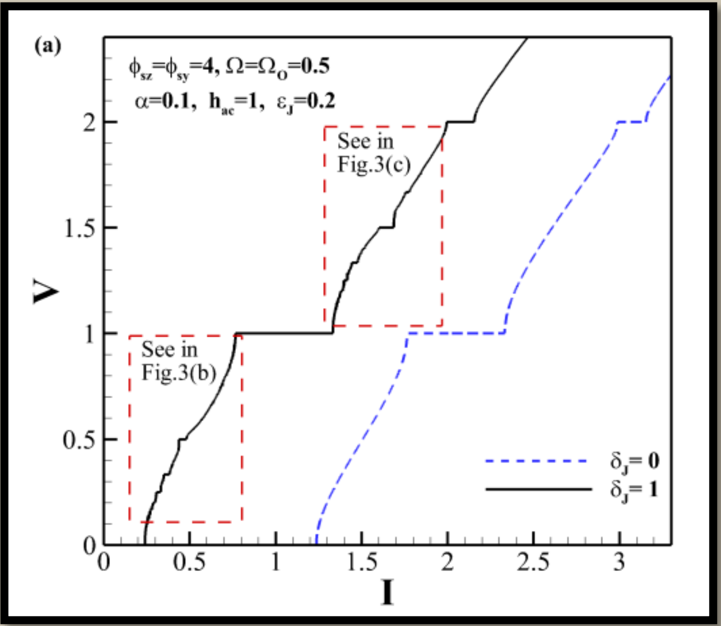


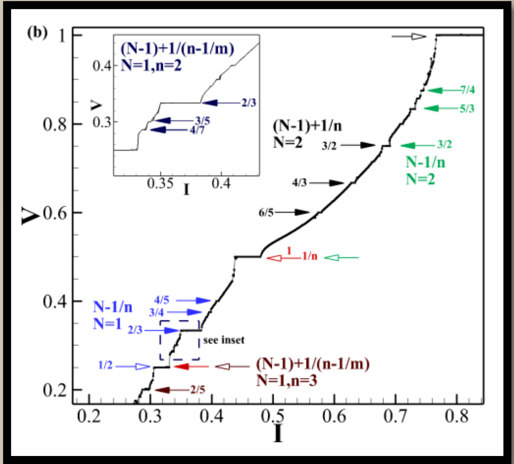
❑Linearized Scheme

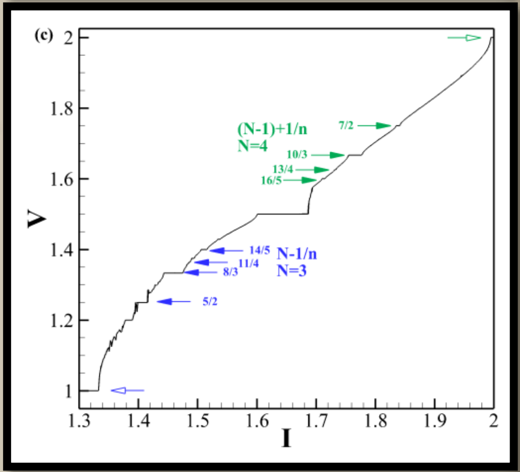


M. Nashaat, A. E. Botha, and Yu. M. Shukrinov, PRB Vol. 97, No. 22 (2018)

Manifestation of Devil's staircase structure in IV-characteristics of SFS JJ.







The position of the subharmonic steps follow continued fraction formula:

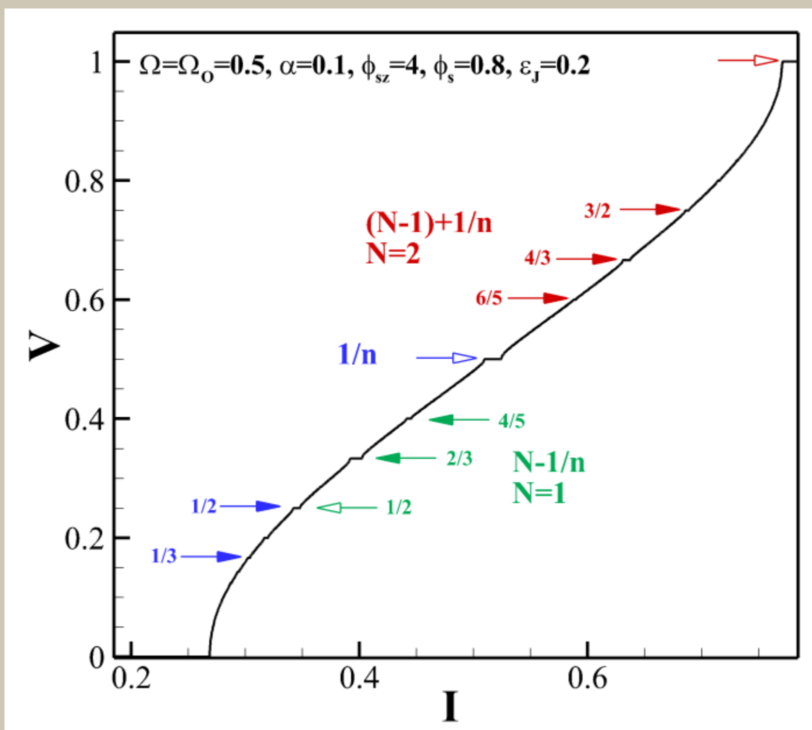
$$V = \left(N \pm \frac{1}{n \pm \frac{1}{m \pm \frac{1}{p \pm ..}}} \right) \Omega$$

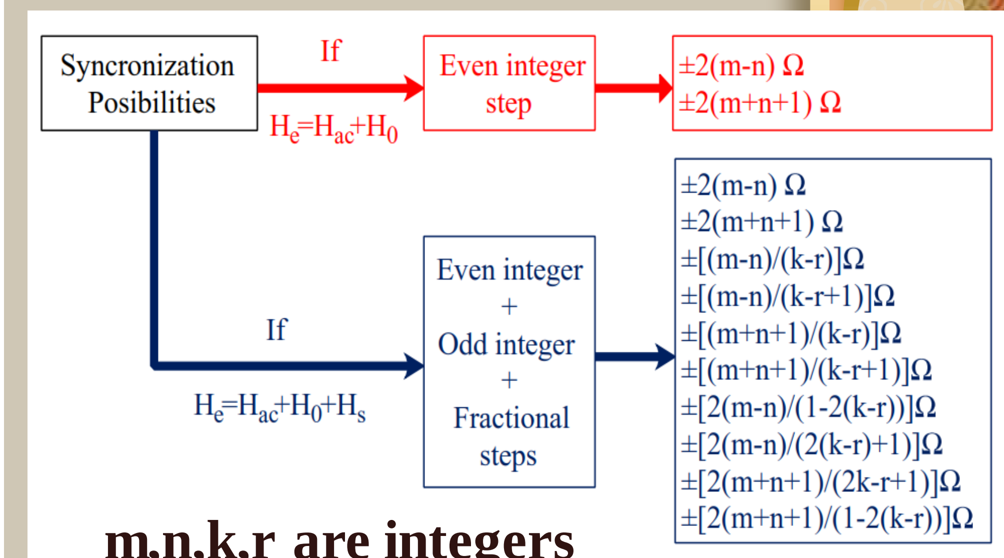
M. Nashaat, A. E. Botha, and Yu. M. Shukrinov, **PRB Vol. 97, No. 22 (2018)**

Locking conditions

$$I/I_c^0 = \frac{\sin(\phi_s m_y)}{\phi_s m_y}\sin\theta + \frac{d\theta(\tau)}{d\tau}$$

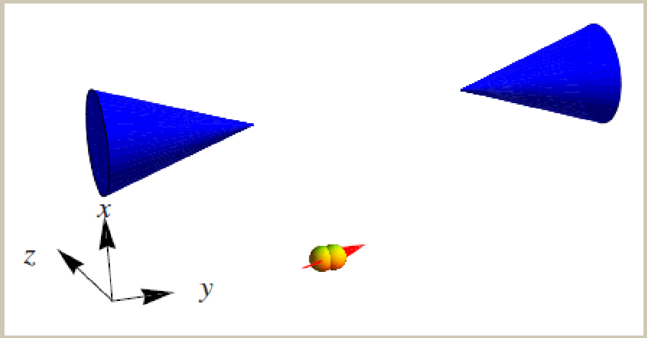
$$m_y = \frac{-2\alpha\frac{\Omega^2}{\Omega_0^2}\cos(\Omega t) + \left(1-\eta_1\frac{\Omega^2}{\Omega_0^2}\right)\sin(\Omega t)}{\left(1-\eta_2\frac{\Omega^2}{\Omega_0^2}\right)^2 + \Delta_J\left(1-\eta_1\frac{\Omega^2}{\Omega_0^2}\right) + 4\alpha^2\frac{\Omega^2}{\Omega_0^2}}$$





M. Nashaat, A. E. Botha, and Yu. M. Shukrinov, PRB Vol. 97, No. 22 (2018)

Interaction of superconducting current with nanomagnet

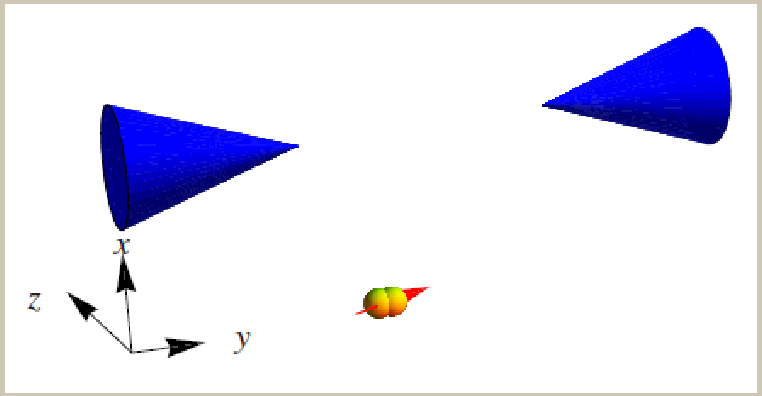


L.Kai, E.M.Chudnovsky, PRB, 82,104429 (2010)

- **Energy of nanomagnet**

$$\varepsilon_M = K(M) - M \cdot B_0$$

K(M) is the magnetic anisotropic
B_0 is the external magnetic field



- **Energy of weak link**

$$\varepsilon_J = -E_J \cos\gamma$$

where γ is the gauge invariant phase

$$\frac{d\gamma}{dt} = \frac{2eV(t)}{\hbar}$$

V(t) is the total voltage.

- **In the presence of the external voltage V_0**

$$\gamma = \gamma_0 + \gamma_A$$

$$\frac{d\gamma_0}{dt} = \frac{2eV_0(t)}{\hbar}$$

$$\gamma_A = -\frac{2\pi}{\Phi_0}\int_1^2 dr\, A(r,t)$$

$$A_M = \frac{\mu_0}{4\pi}\frac{M \times r}{r^3}$$

vector potential created at point **r** from the nanomagnet

$$A_B = \frac{1}{2}(B \times r)$$

vector potential created by the external field

L.Kai, E.M.Chudnovsky, PRB, 82,104429 (2010)

❑ Dynamical equations

LLG

$$\frac{dm_x}{d\tau} = \frac{\Omega_F}{(1+\alpha^2)}\left[h_y\left(m_z - \alpha m_x m_y\right) - h_z\left(\alpha m_x m_z + m_y\right) + \alpha h_x\left(m_y^2 + m_z^2\right)\right]$$

$$\frac{dm_y}{d\tau} = \frac{\Omega_F}{(1+\alpha^2)}\left[-h_x\left(\alpha m_x m_y + m_z\right) + h_z\left(m_x - \alpha m_y m_z\right) + \alpha h_y\left(m_x^2 + m_z^2\right)\right]$$

$$\frac{dm_z}{d\tau} = \frac{\Omega_F}{(1+\alpha^2)}\left[\alpha \tilde{h}_z\left(m_x^2 + m_y^2\right) - h_y\left(m_x + \alpha m_y m_z\right) + h_x\left(m_y - \alpha m_x m_z\right)\right]/D$$

$$D = 1 + \frac{\Omega_0 \alpha \epsilon k}{1 + M^2\alpha^2}\left(m_x^2 + m_y^2\right) \qquad k = \frac{2\pi}{\Phi_0}\frac{L}{\sqrt{L^2 + a^2}}$$

where ε = G k, the components of the effective fields read

$$h_x = 0,$$

$$h_y = m_y,$$

$$\boxed{h_z = \tilde{h}_z - \epsilon k \dot{m}_z, \quad \tilde{h}_z = \epsilon\left[\sin(\omega_j t - k m_z) + \omega_j\right]}$$

Effective field due to tunneling current through the weak link

Phase dynamics

- **Voltage biased (VB) junction**

$$I(t) = \sin[Vt - km_z] + V - k\frac{dm_z}{dt},$$

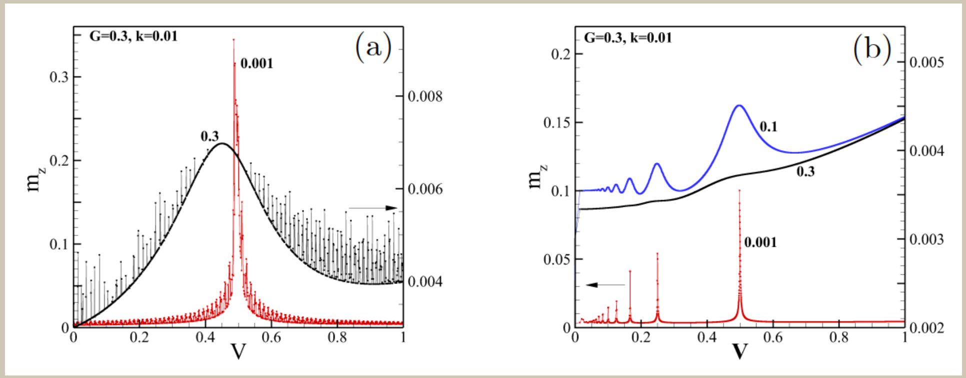
$$I_{av} = \frac{1}{T_{max}} \int_0^{T_{max}} I(t)dt.$$

- **Current biased (CB) junction**

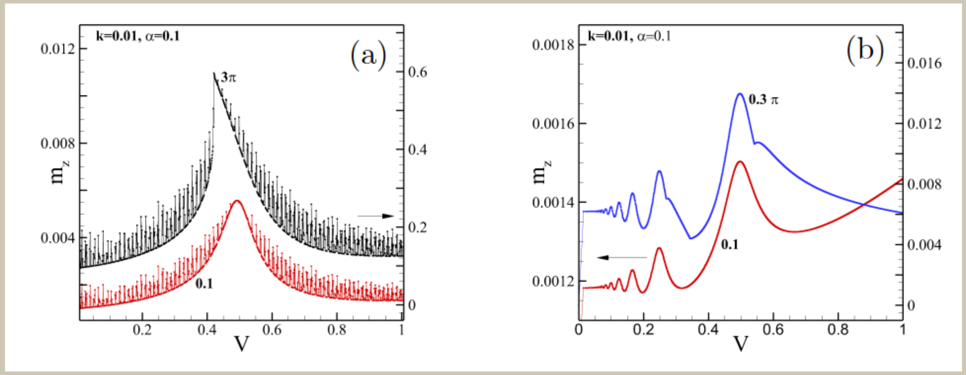
$$I = \sin[Vt - km_z] + V - k\frac{dm_z}{dt} + \beta_c\frac{dV}{dt},$$

$$V_{av} = \frac{1}{T_{max}} \int_0^{T_{max}} V(t)dt.$$

FMR



Effect of α on the FMR for mz (mx is qualitatively same),
(a) VB- junction, (b) CB-junction. The numbers indicate the value of α.



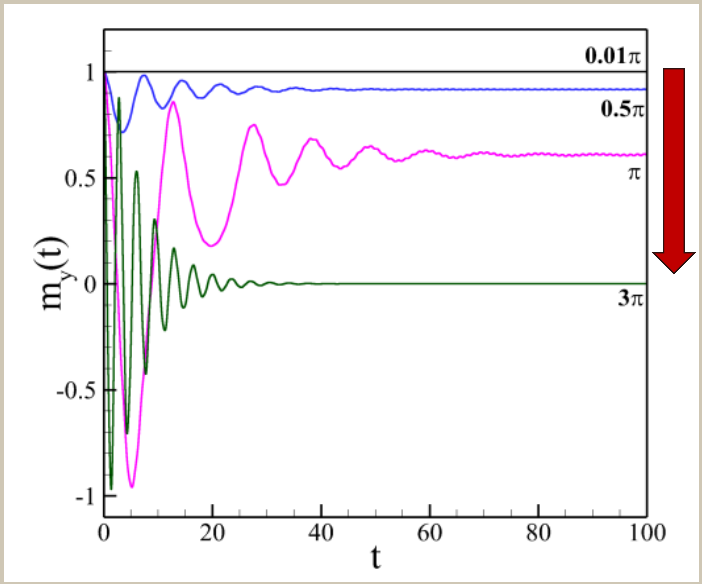
Effect of G on the FMR for ;
(a) VB- junction, (b) CB-junction. The numbers indicate the value of G

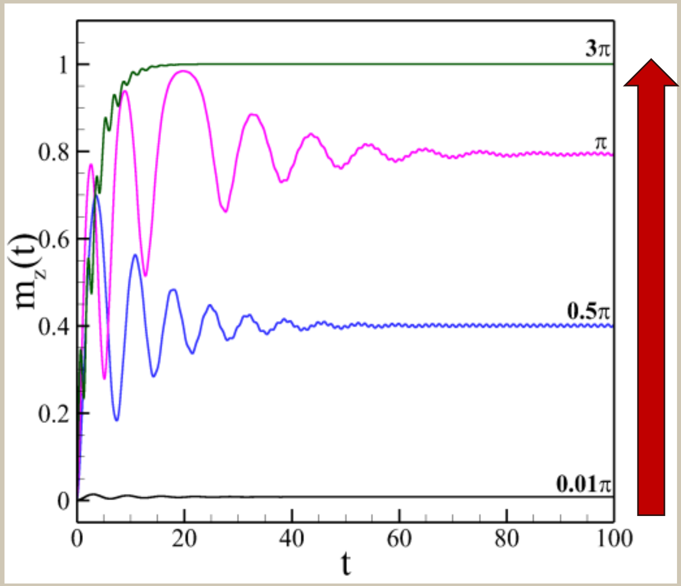
M. Nashaat, I. Rahmonov, K. Kulikov, M. Sameh, Yu. M. Shukrinov.,

In preparation, 2019

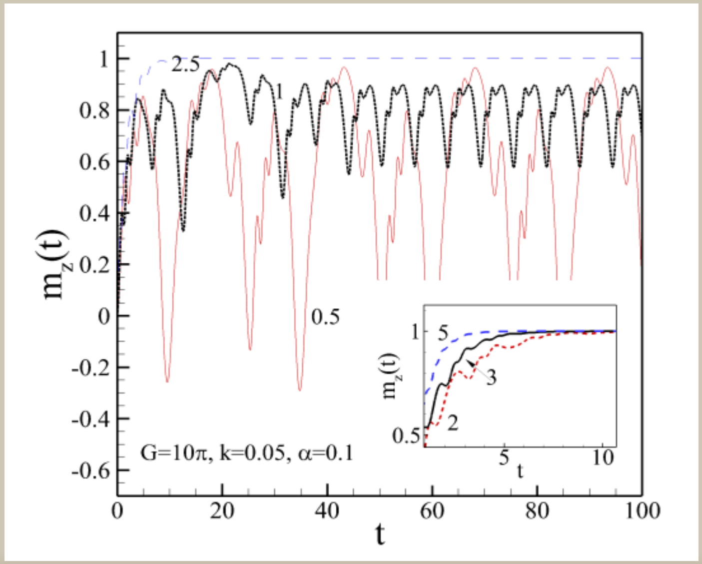
Reorientation of easy axis (capture Kapitza pendulum features)

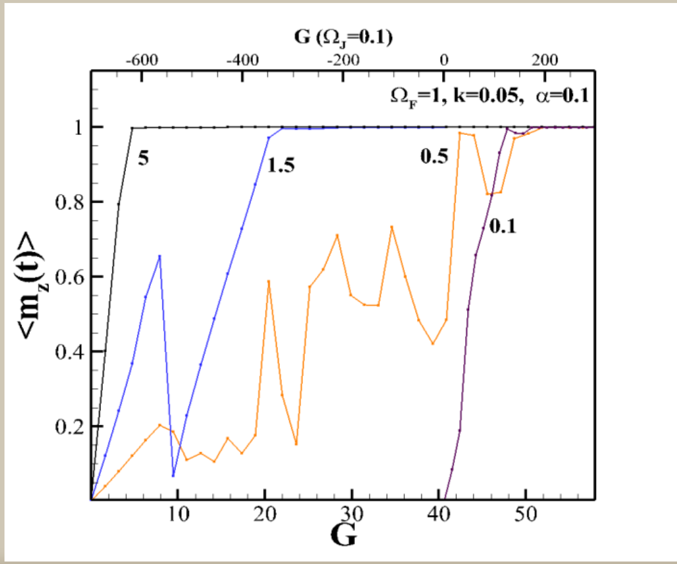
- Increasing the value of $G = \varepsilon_J/\varepsilon_M$, k = 0.05, α = 0.1





- Increasing the value of Josephson frequency, k = 0.05, α = 0.1





Conclusions

- Reversal magnetization at different parameters of electric current pulse and JJ is shown.
- We have demonstrated variation of magnetic precession along IV-characteristic of RCS junction.
- External radiation fix a character of precession along Shapiro step.
- Kapitza pendulum features in JJ+nanomagnet system is demonstrated.
- Coupling between spin wave and Josephson phase in SFS junction through the Josephson energy and gauge invariant phase difference between the S-layers leads to Devil’s staircase structure.
- The position of the subharmonic steps follow continued fraction formula similar to SIS structure.
- The structure of the subharmonic steps can be tuned by changing the frequency of the constant magnetic field, junction dimension and Gilbert damping.

Thank you
for your attention!