New Bayesian analysis of hybrid EoS constraints with mass-radius data for CS

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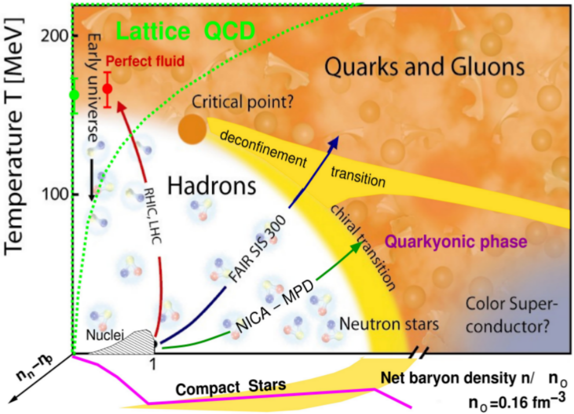
What if we have twins

Important questions

- Does hybrid neutron star exist?
- Does CEP exist on QCD phase diagram?

Existence of CEP at the QCD Phase Diagram

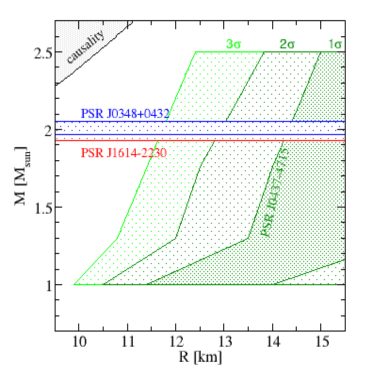


Topic for discussion!

Observational Constraints

Mass and Radius Constraints

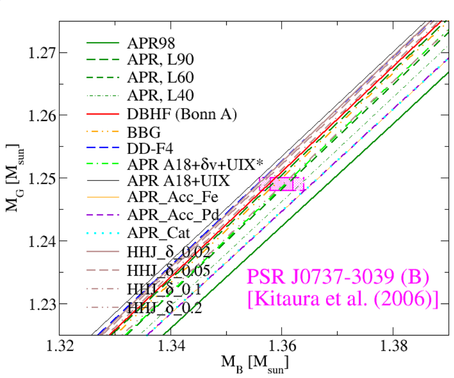
Radius and maximum mass constraints are given from PSR J0437-4715 (Bogdanov. Ast. J. **762**, 96) and PSR J0348+0432 (Antoniadis *et al.* Sci. **340**, 6131) correspondingly.



Observational Constraints

Gravitational Binding Energy Constraint

A constraint on the gravitational binding energy is taken from the neutron star B in the binary system J0737-3039 (B).



Observational Constraints

Three Statistically Independent Constraints

- A radius constraint from the nearest millisecond pulsar PSR J0437-4715 [S. Bogdanov. Astrophys. J. 762, 96 (2013)].
- A maximum mass constraint from PSR J0348+0432 [J. Antoniadis et al. Science 340, 6131 (2013)].
- A constraint on the gravitational binding energy from the neutron star *B* in the binary system PSR J0737-3039 (B) [F. Kitaura et al. A. & A. 450, 345 (2006)].

EoS Parametrization

AHP scheme of hybrid EoS

$$p(\epsilon) = p^I(\epsilon)\,\Theta\left(\epsilon_c^I - \epsilon\right) + p^I\left(\epsilon_c^I\right)\Theta\left(\epsilon - \epsilon_c^I\right)\Theta\left(\epsilon_c^I - \epsilon + \Delta\epsilon\right) + p^{II}(\epsilon)\,\Theta\left(\epsilon - \epsilon_c^I - \Delta\epsilon\right),$$

where $p^I(\epsilon)$ is given by a pure hadronic EoS, and $p^{II}(\epsilon)$ represents the high density matter introduced here as quark matter given in the bag-like form.

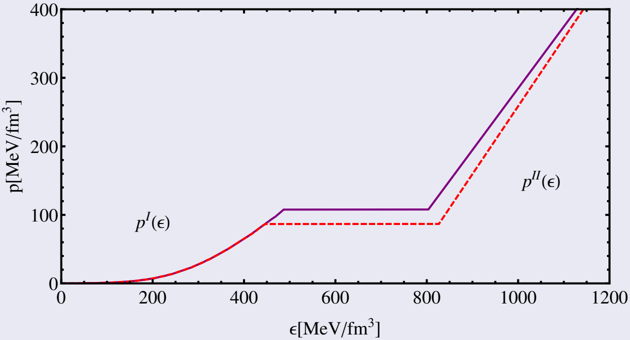
Bag-Like Form of QM EoS

$$p^{II}(\epsilon) = c_{QM}^2\epsilon - B,$$

where c_{QM}^2 is the squared speed of sound in quark matter and B is the bag constant.

EoS Parametrization

AHP scheme of hybrid EoS



EoS Parametrization

Hybrid EoS Pareameters

$$400 \leq \epsilon_c \left[MeVfm^{-3}\right] \leq 1000 \quad : \quad \epsilon_c(k) \qquad k = 1 \ldots N_1 = 10$$

$$0 \leq \gamma = \frac{\Delta\epsilon}{\epsilon_c} \leq 1 \quad : \quad \gamma(l) \qquad l = 1 \ldots N_2 = 10$$

$$0.3 \leq c^2_{QM} \leq 1 \quad : \quad c^2_{QM}(m) \qquad m = 1 \ldots N_3 = 10$$

Vector of Parameters

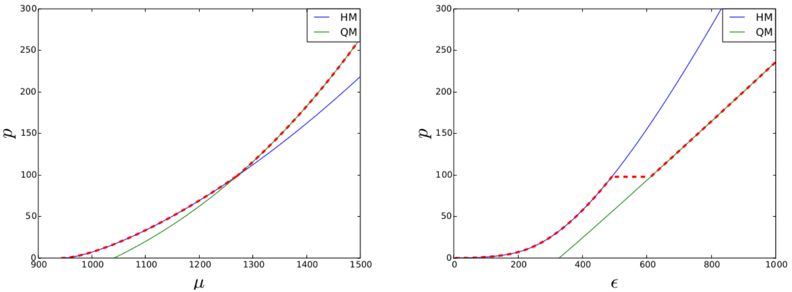
For the BA, we have to sample the above defined parameter space and to that end we introduce a vector of the parameter values:

$$\pi_i = \overrightarrow{\pi}\left(\epsilon_c(k), \gamma(l), c^2_{\rm QM}(m)\right),$$

$i = 1 \ldots N$ (here $N = \prod_{q=1}^{3} N_q$) and $i = N_1 \times N_2 \times k + N_2 \times l + m$

EoS Parametrization

Maxwell construction of hybrid EoS



(A) $p(\mu)$ functions.

(B) $p(\epsilon)$ functions.

EoS Parametrization

Maxwell Construction of hybrid EoS

$\mu_H = \mu_Q = \mu$ and $p_H(\mu) = p_Q(\mu)$,
where $p_H(\mu)$ is DD2 and $p_Q(\mu)$ is NJL8 quark EoS
(here $\eta_2 = 0.03$).

Hybrid EoS Pareameters

$0 \leq \eta_4 \leq 20 \quad : \quad \eta_4(i) \quad i = 0 \ldots N = 21$

Vector of Parameters

For the BA, vector of parameters is defined as following:
$\pi_i = \vec{\pi}\left(\eta_4(i)\right), i = 1 \ldots N.$

Qualification of the EoS models from Observation

Goal of the BA

To find posterior probabilities of the set of π_i taking into account the observational constraints.

Unification of priori probabilities

$P(\pi_i) = 1$ for $\forall i$.

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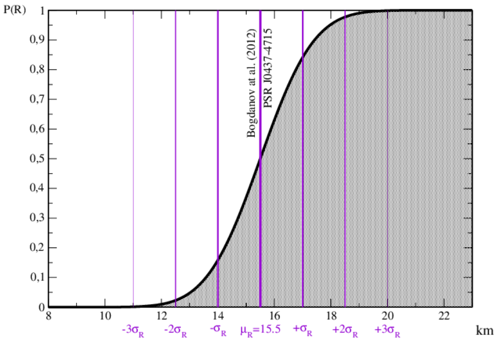
Calculation of Probabilities

Probability of Corresponding to Radius Constraint for π_i

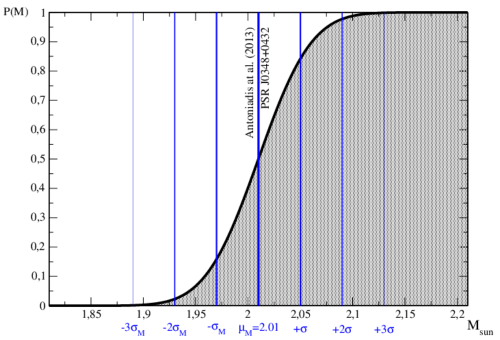
$P(E_B|\pi_i) = \Phi(R_i, \mu_B, \sigma_B)$, here R_i is max radius given by π_i.
$\mu_B = 15.5$ km and $\sigma_B = 1.5$ km [?].



Calculation of Probabilities

Probability of Corresponding to Mass Constraint for π_i

$P\left(E_A \middle| \pi_i\right) = \Phi(M_i, \mu_A, \sigma_A)$, here M_i is max mass given by π_i.
$\mu_A = 2.01\ \mathrm{M}_\odot$ and $\sigma_A = 0.04\ \mathrm{M}_\odot$ [?].



Calculation of Probabilities

Probability of Corresponding to $M - M_B$ Constraint for π_i

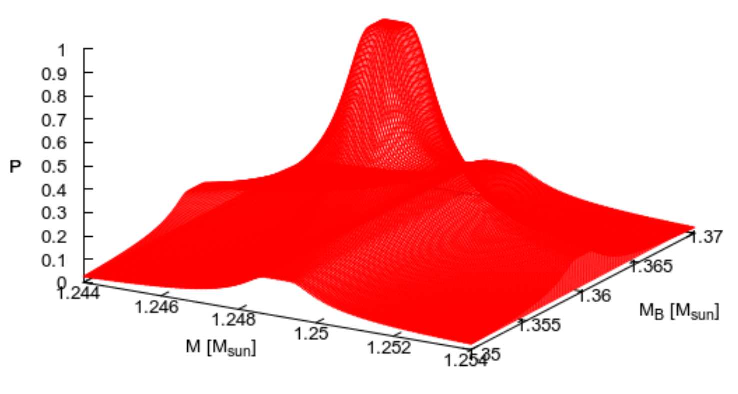
We need to estimate the probability for the closeness of a theoretical point $M_i = (M_i, M_{Bi})$ to the observed point $\mu_K = (\mu_G, \mu_B)$. The required probability can be calculated using the following formula

$$P\left(E_K \left|\pi_i\right.\right) = \left[\Phi\left(\xi_G\right) - \Phi\left(-\xi_G\right)\right] \cdot \left[\Phi\left(\xi_B\right) - \Phi\left(-\xi_B\right)\right],$$

where $\Phi\left(x\right) = \Phi\left(x, 0, 1\right)$, $\xi_G = \sigma_{M_G}/d_{M_G}$ and $\xi_B = \sigma_{M_B}/d_{M_B}$, with d_{M_G} and d_{M_B} being the absolute values of components of the vector $\mathbf{d}_{\mathrm{i}} = \mu - \mathbf{M}_i$, where $\mu_{\mathbf{B}} = \left(\mu_G, \mu_B\right)^T$ is given in

Calculation of Probabilities

Probability of $M - M_B$ for π_i



Calculation of Probabilities

Probability of All Constraints for π_i

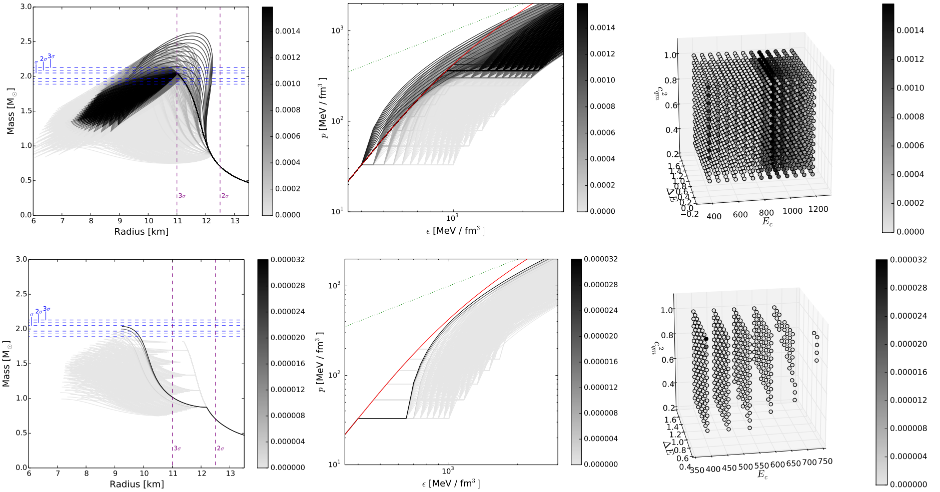
Taking to the account assumption that these measurements are independent on each other we can calculate complete conditional probability:

$$P\left(E\left|\pi_i\right.\right) = P\left(E_A\left|\pi_i\right.\right) \times P\left(E_B\left|\pi_i\right.\right) \times P\left(E_K\left|\pi_i\right.\right)$$

Calculation of *a posteriori* Probabilities of π_i

Now, we can calculate posterior probability of π_i:

$$P\left(\pi_i\left|E\right.\right) = \frac{P\left(E\left|\pi_i\right.\right) P\left(\pi_i\right)}{\sum\limits_{j=0}^{N-1} P\left(E\left|\pi_j\right.\right) P\left(\pi_j\right)}$$



Bayesian analysis of the AHP HEoS models based on pure APR.

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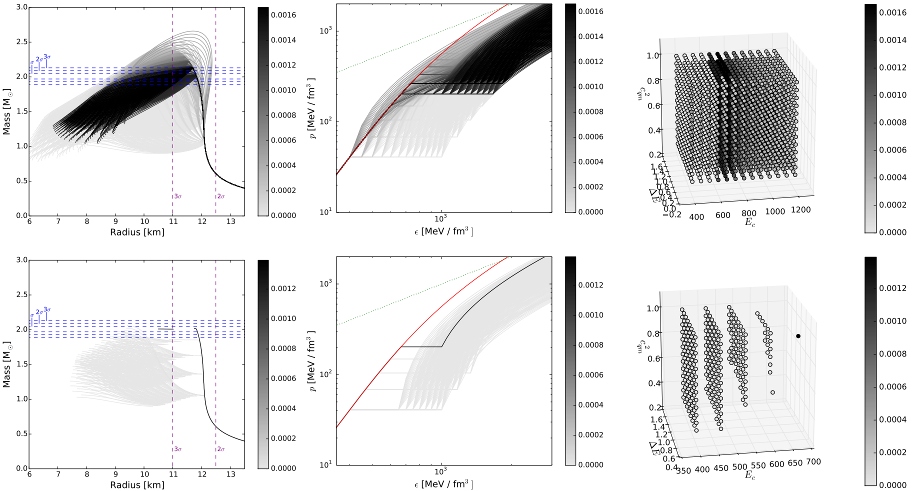

BA of AHP HEoS models with excluded volume APR.

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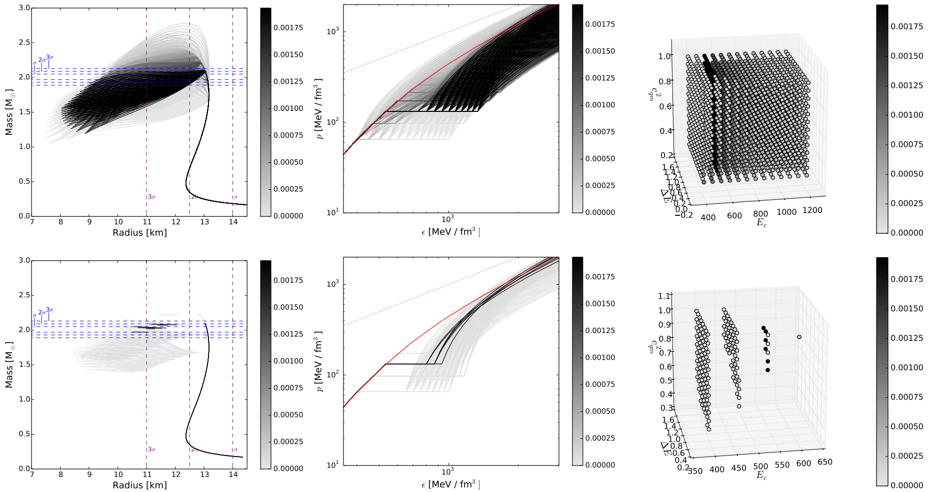

BA of AHP HEoS models based on pure DD2.

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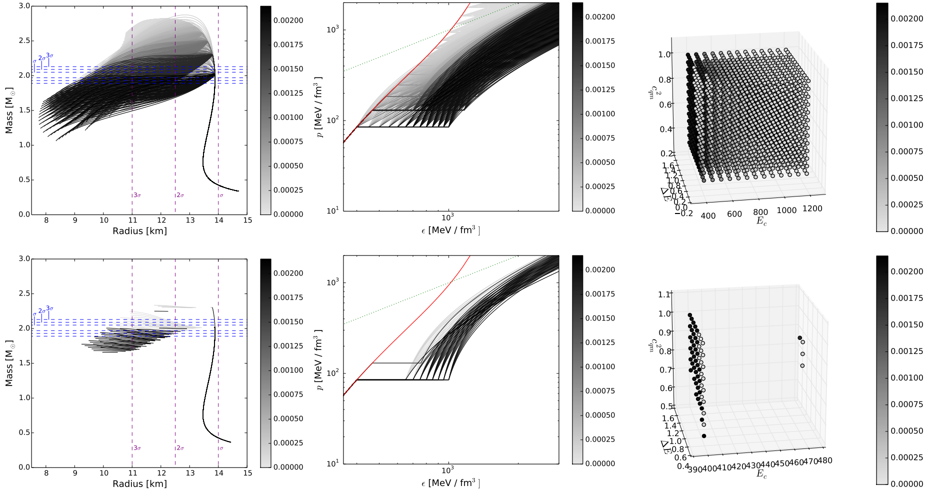

BA of AHP HEoS models with excluded volume DD2, $n_{cp} = 0.9\ \mathrm{fm}^{-3}$.

"Now let us travel into future. It is year **2017**, some new, reliable NS radius measurement methods are discovered and were used to find the size of two most massive pulsars, which still are PSR J0348+0432 and PSR J1614-2230. **The community was shocked** when received the results of observations: one radius is 13 $\pm$ 0.5 km, while the other is 11 $\pm$ 0.5 km!"

– *Michał Sokołowski*, Master Thesis, 2014

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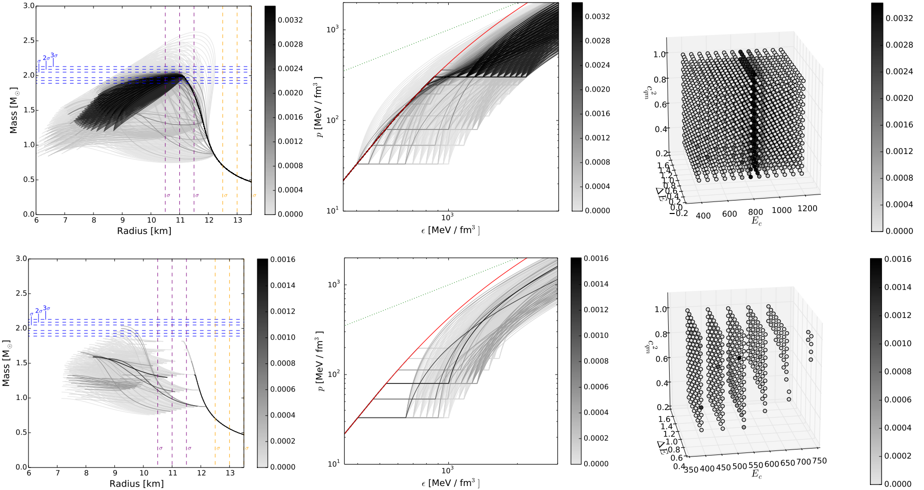
Fictitious radius measurements

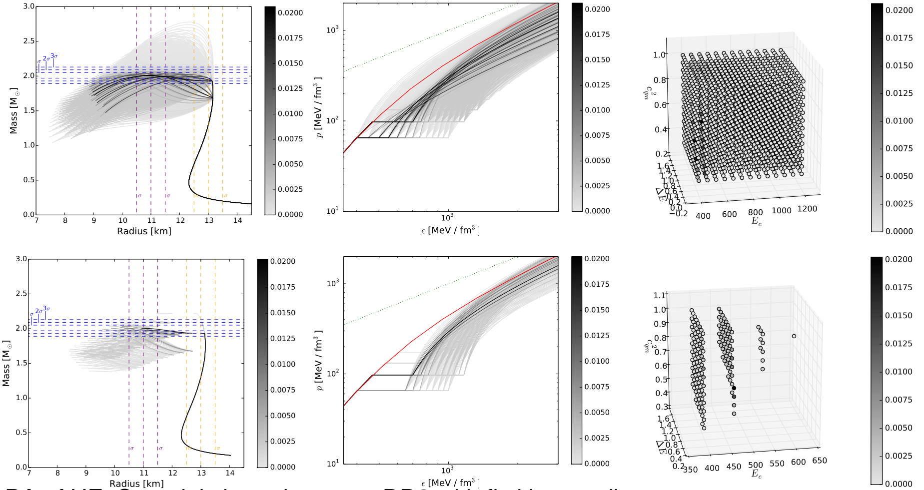
For masses $M_1 = 2.01 \pm 0.04\ \mathrm{M}_\odot$ and $M_2 = 1.93 \pm 0.04\ \mathrm{M}_\odot$ we suggested folowing radius mesurements:

- $R_1 = 11\ \mathrm{km}$,
- $R_2 = 13\ \mathrm{km}$

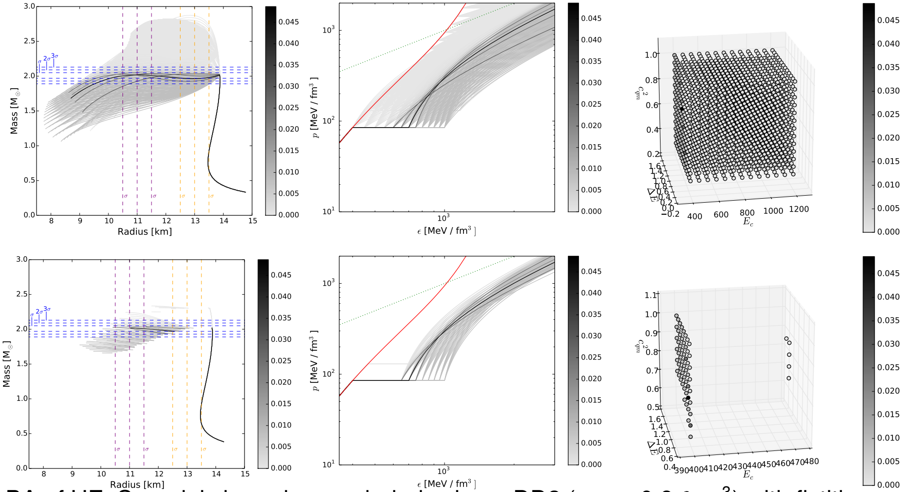
with $\sigma_{1,2} = 0.5\ \mathrm{km}$.



BA of HEoS models based on pure APR with fictitious radius measurements.



BA of HEoS models based on pure DD2 with fictitious radius measurements.



BA of HEoS models based on excluded volume DD2 ($n_{cp} = 0.9\ \mathrm{fm}^{-3}$) with fictitious radius measurements.

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Conclusions

- BA of HEoS (AHP construction) is focused on possibility of high mass neutron stars.
- Stiff hadronic EoS is necessary to achieve high mass twins.
- Radius measurements can be used to detect twin stars, therefore, to select HEoS.