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Different Approaches for Elastic Imaging using Multiprocessor Computing Systems

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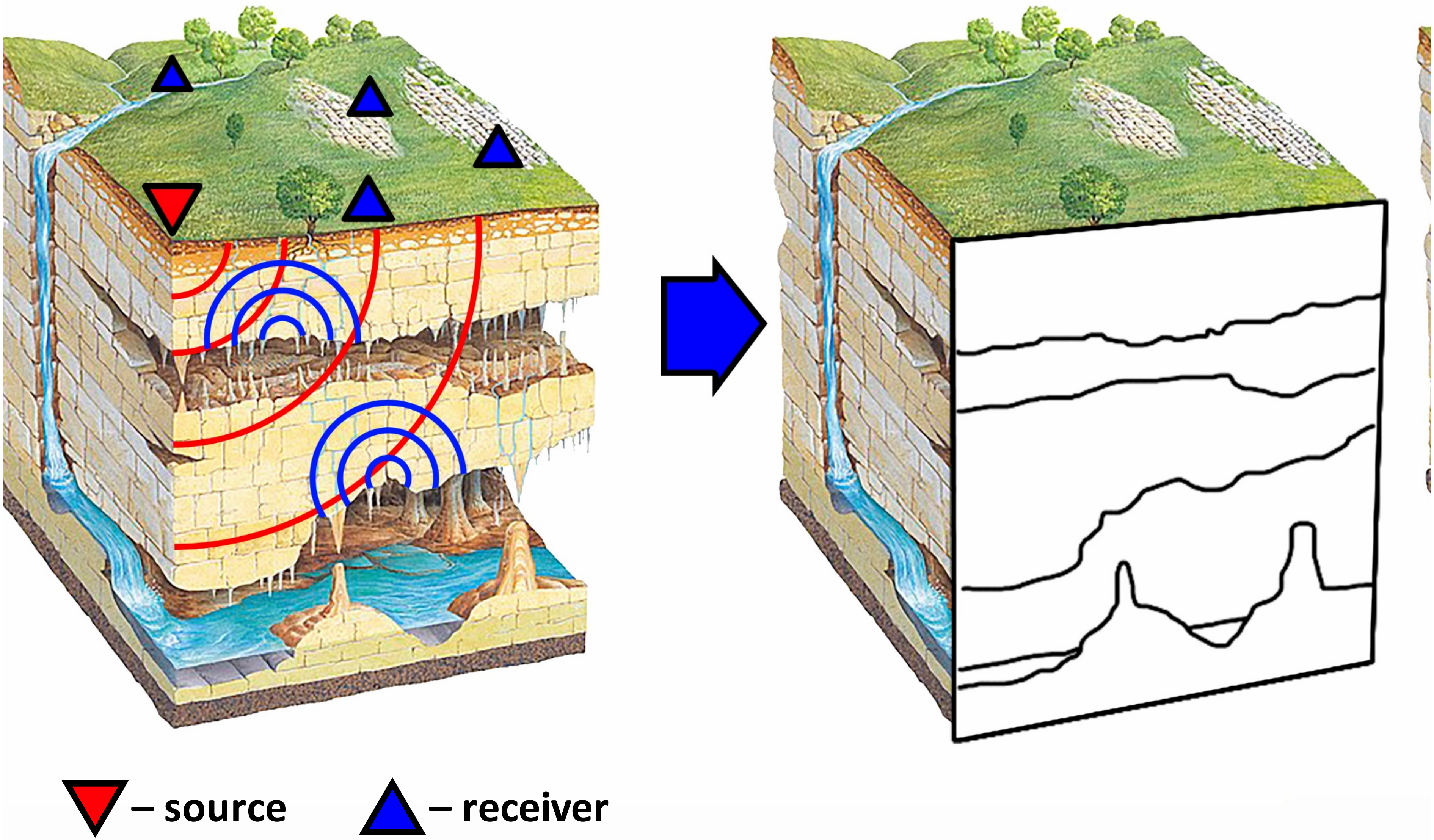
Seismic migration imaging



Different Approaches

- Kirchhoff integral / Born approximation
- Full-wave simulation

Can be formulated for acoustic and **elastic** media

Born Approximation.
Elastic Case

Lame equation:

$$\hat{\Lambda}\vec{u} - \frac{\partial^2 \vec{u}}{\partial t^2} = -\frac{1}{\rho}\vec{F}, \qquad \hat{\Lambda} = c_p^2 \nabla\nabla \cdot - c_s^2 \nabla\times\nabla\times$$

Background and anomalous parts:

$$c_\alpha^2 = c_{\alpha,b}^2 + \Delta c_\alpha^2, \qquad \Delta c_\alpha^2\Big|_{r\notin V} = 0, \qquad \alpha \in \{p, s\},$$

$$\hat{\Lambda} = \hat{\Lambda}_b + \Delta\hat{\Lambda}, \qquad \vec{u} = \vec{u}^i + \vec{u}^s$$

Equations for incident and scattered fields:

Born approximation

$$\hat{\Lambda}_b \vec{u}^i - \frac{\partial^2 \vec{u}^i}{\partial t^2} = -\frac{1}{\rho}\vec{F}, \qquad \hat{\Lambda}_b \vec{u}^s - \frac{\partial^2 \vec{u}^s}{\partial t^2} = -\Delta\hat{\Lambda}\left(\vec{u}^i + \vec{u}^s\right)$$

Homogeneous space: $c_{\alpha,b} = const,\ V = \mathbb{R}^3$

$$s = c^{-1},$$

$$\widehat{D}_p^i = \mathrm{grad}^i \mathrm{div}^i, \qquad \widehat{D}_s^i = -\mathrm{rot}^i \mathrm{rot}^i,$$

$$\nabla^i = \left(\partial_{x^i}\, \partial_{y^i}\, \partial_{z^i}\right)^{\mathrm{T}}$$

Green's tensor:

$$\widehat{G}_\alpha^L = \widehat{D}_\alpha \hat{g}_\alpha = \widehat{D}'_\alpha \hat{g}_\alpha,$$

$$\hat{g}_\alpha = \left\{\chi\left(t' - t - s_{\alpha,b}|\vec{r}' - \vec{r}|\right) - \chi(t' - t)\right\} \frac{\hat{I}}{4\pi|\vec{r}' - \vec{r}|},$$

$$\chi(t) = \max(0, t)$$

Permanently polarized point source :

$$\vec{F}(\vec{r},t) = \delta(\vec{r}-\vec{r}_0) f''(t) \vec{f},$$

Forward modeling (whole space):

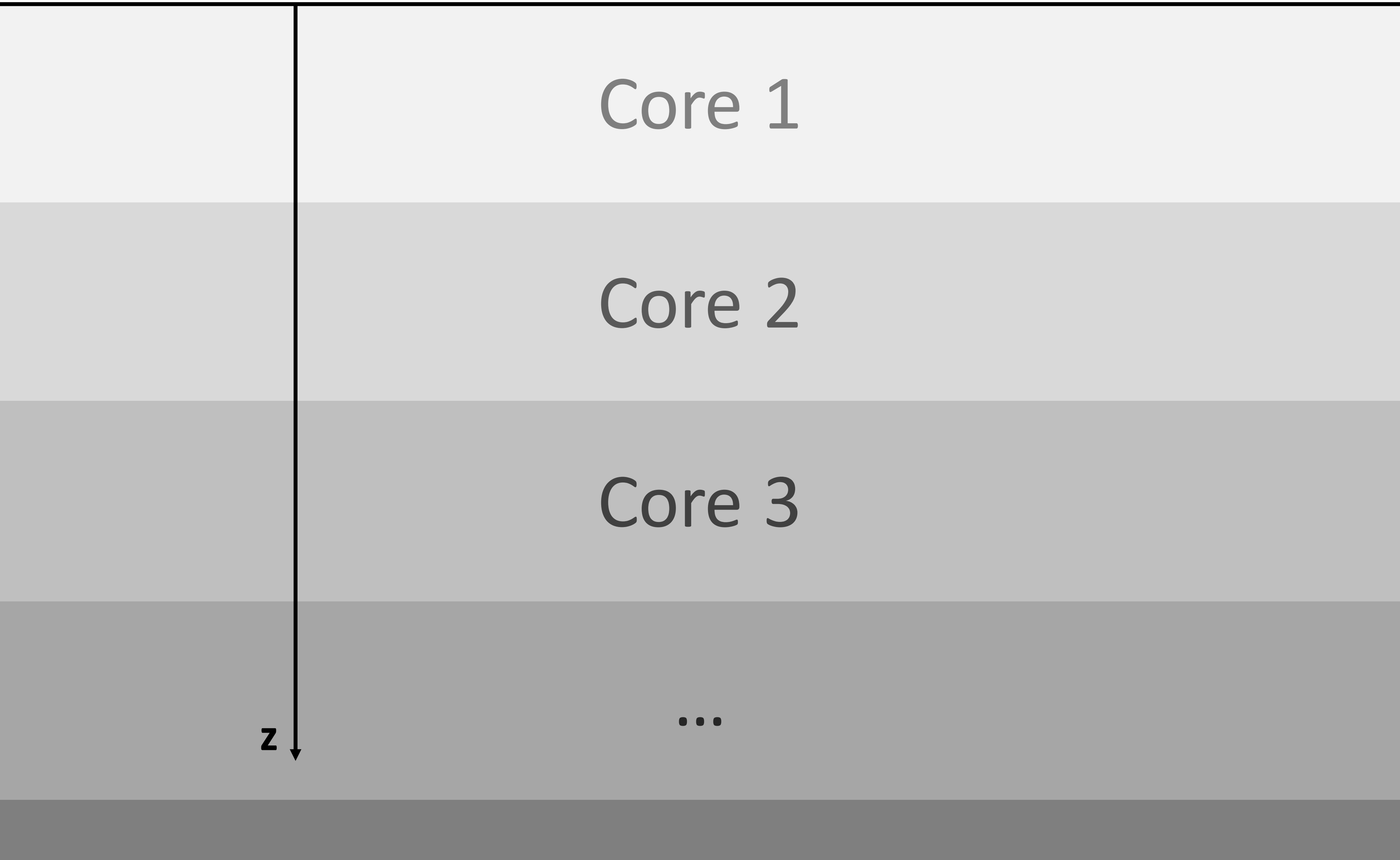
$$\vec{u}_{\alpha}^{s,B}(\vec{r}',t') = \sum_{\beta} \frac{1}{\rho(\vec{r}_0)} \widehat{D}'_{\alpha} \widehat{D}^{0}_{\beta} \int_V \Delta c_{\beta}^2(\vec{r}) \frac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}|\vec{r}' - \vec{r}|\right)}{16\pi^2 c_{\beta,b}^2 |\vec{r}_0 - \vec{r}||\vec{r}' - \vec{r}|} \vec{f}\, dV$$

Migration (whole space):

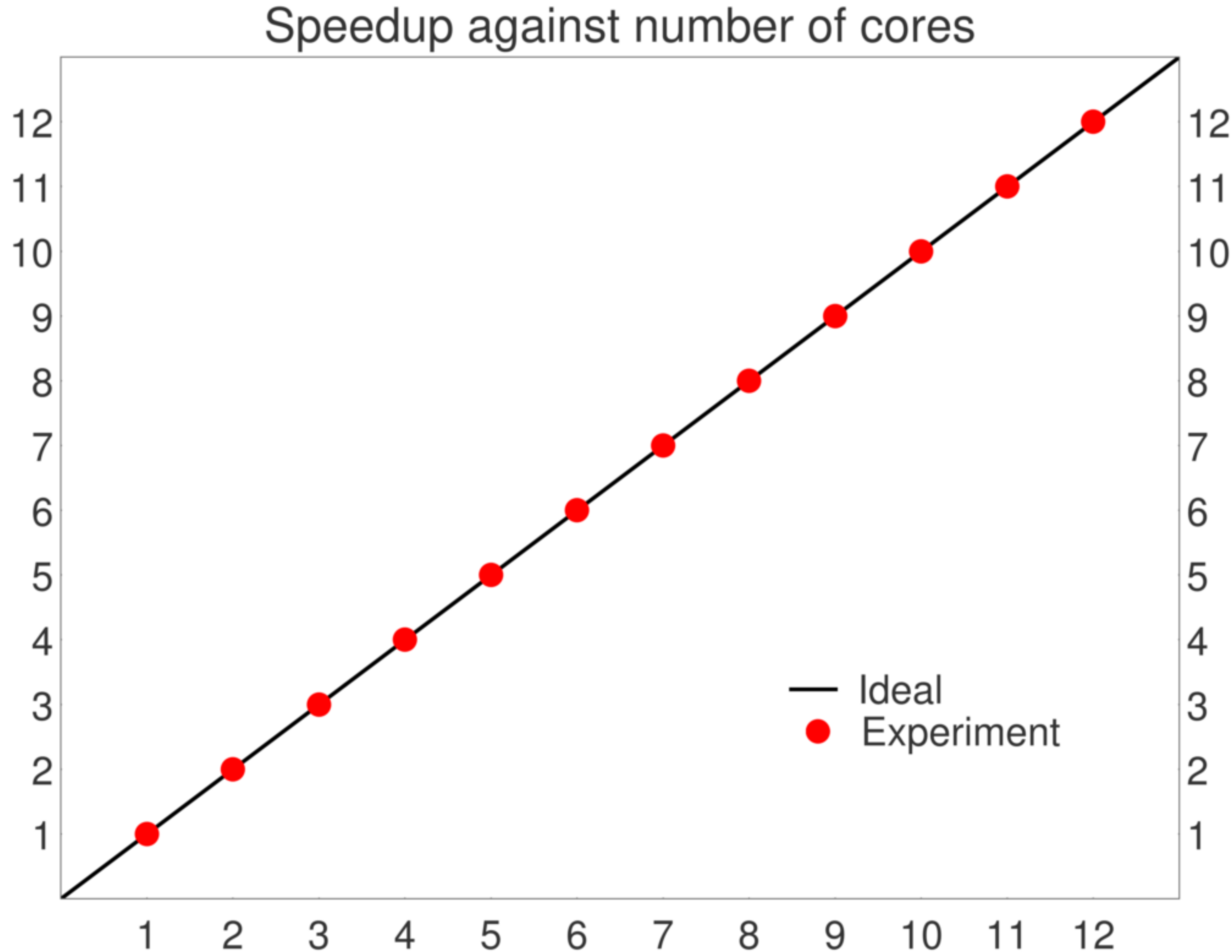
$$\Delta c_{\beta,\mathrm{migr}}^2(\vec{r}) = \sum_{\alpha} \int_S \int_T \frac{\vec{d}(\vec{r}',t')}{\rho(\vec{r}_0)} \cdot \widehat{D}'_{\alpha} \widehat{D}^{0}_{\beta} \frac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}|\vec{r}' - \vec{r}|\right)}{16\pi^2 c_{\beta,b}^2 |\vec{r}_0 - \vec{r}||\vec{r}' - \vec{r}|} \vec{f}\, dt'\, dS$$

Complexity $\sim \mathrm{O}\left(N_x N_y N_z N_t \log(N_x) \log(N_y)\right)$

Simplest Parallelization



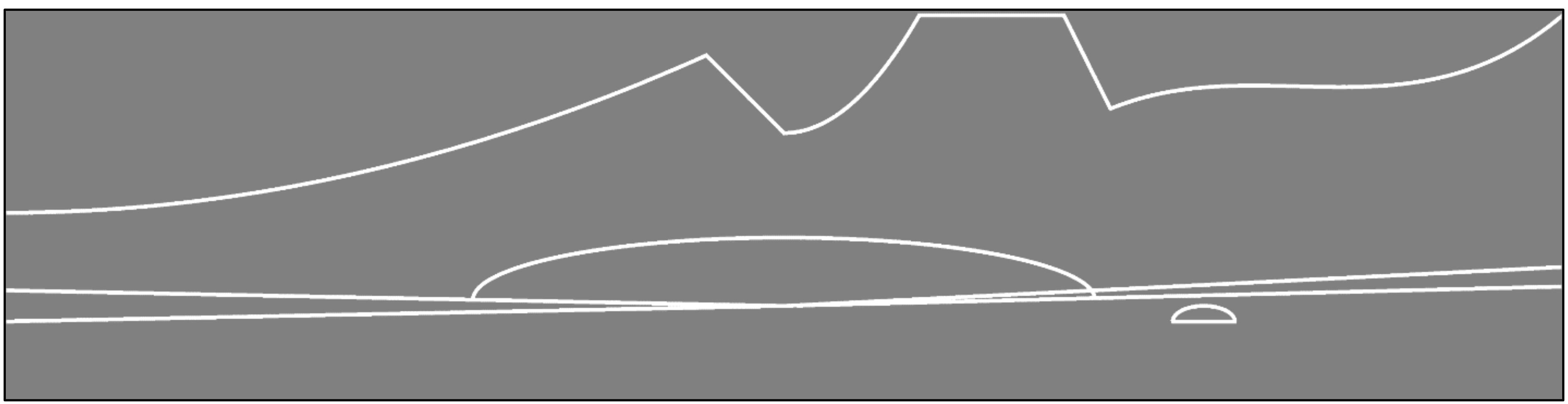
Number of cores	Time of calculation, secs	Memory used, GB
1	17437	0.52
2	8717	0.83
3	5831	1.07
4	4362	1.36
5	3526	1.62
6	2920	1.96
7	2498	2.17
8	2215	2.39
9	1950	2.72
10	1793	3.05
11	1609	3.34
12	1494	3.59

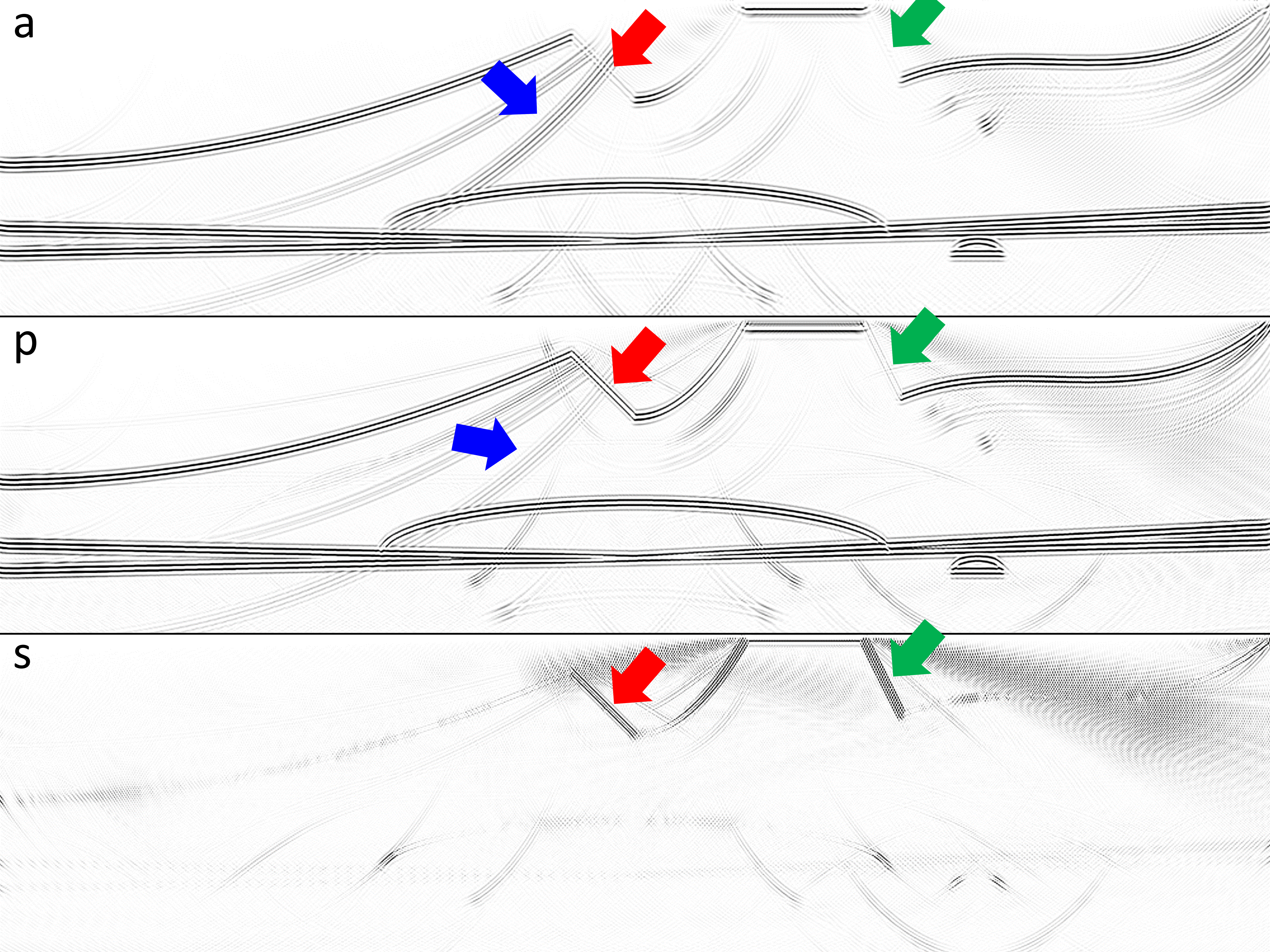

Speedup against number of cores
Ideal
Experiment
1
2
3
4
5
6
7
8
9
10
11
12

Test Model

Media
$x \times z = 10 \times 2.5$ km,
$y = const,$
$c_{p,b} = 2.5$ km/s,
$c_{s,b} = 1.25$ km/s,
$\Delta c_\alpha^2 / c_{\alpha,b}^2 = 0.01,$
$\rho = 2.5$ t/m³

Data
(only z-component
of scattered field)
$z = 15$ m,
$\Delta x = 10$ m,
$\Delta t = 2$ ms,
$t \in [0,4]$ s,
$F(t) = (1 - 2\pi^2 f_M^2 t^2) \cdot e^{-\pi^2 f_M^2 t^2},$
$f_M = 25$ Hz,
$\vec{f} = (0\ 0\ 1)^T$





a
p
s

Full-wave Simulation.
Elastic Case

Mathematical Model and Numerical Method

Elastic Parameters:

- ρ – density
- λ, μ – Lame parameters
- V – velocity
- T – stress tensor

We use grid-characteristic method on structured meshes to solve direct seismic problem

$$\begin{cases}
\rho \dfrac{\partial V_x}{\partial t} = \dfrac{\partial T_{xx}}{\partial x} + \dfrac{\partial T_{xy}}{\partial y} + \dfrac{\partial T_{xz}}{\partial z}, \\
\rho \dfrac{\partial V_y}{\partial t} = \dfrac{\partial T_{yx}}{\partial x} + \dfrac{\partial T_{yy}}{\partial y} + \dfrac{\partial T_{yz}}{\partial z}, \\
\rho \dfrac{\partial V_z}{\partial t} = \dfrac{\partial T_{zx}}{\partial x} + \dfrac{\partial T_{zy}}{\partial y} + \dfrac{\partial T_{zz}}{\partial z}, \\
\dfrac{\partial T_{xx}}{\partial t} = (\lambda + 2\mu)\dfrac{\partial V_x}{\partial x} + \lambda(\dfrac{\partial V_y}{\partial y} + \dfrac{\partial V_z}{\partial z}), \\
\dfrac{\partial T_{yy}}{\partial t} = (\lambda + 2\mu)\dfrac{\partial V_y}{\partial y} + \lambda(\dfrac{\partial V_x}{\partial x} + \dfrac{\partial V_z}{\partial z}), \\
\dfrac{\partial T_{zz}}{\partial t} = (\lambda + 2\mu)\dfrac{\partial V_z}{\partial z} + \lambda(\dfrac{\partial V_x}{\partial x} + \dfrac{\partial V_y}{\partial y}), \\
\dfrac{\partial T_{xy}}{\partial t} = \mu(\dfrac{\partial V_x}{\partial y} + \dfrac{\partial V_y}{\partial x}), \\
\dfrac{\partial T_{xz}}{\partial t} = \mu(\dfrac{\partial V_x}{\partial z} + \dfrac{\partial V_z}{\partial x}), \\
\dfrac{\partial T_{yz}}{\partial t} = \mu(\dfrac{\partial V_y}{\partial z} + \dfrac{\partial V_z}{\partial y}).
\end{cases}$$

Research Software*

- Seismic waves simulation in elastic media
- Taking into account heterogeneities (cavities, layers, fractures)
- C++, micro-optimisations (SIMD, SSE, AVX)
- Parallelization with OpenMP and MPI

(~ 80 % up to 16 000 cores)

* Khokhlov N.I., MIPT

Migration Image

Z-kernel $K_Z = K_\rho + K_\kappa + K_\mu$

Density-Kernel $K_\rho(\boldsymbol{x}) = \rho(\boldsymbol{x}) \int \partial_t \boldsymbol{s}^\dagger(\boldsymbol{x}, -t) \partial_t \boldsymbol{s}(\boldsymbol{x}, t)\, dt$

K-kernel $K_\kappa(\boldsymbol{x}) = -\kappa(\boldsymbol{x}) \int [\nabla \cdot \boldsymbol{s}^\dagger(\boldsymbol{x}, -t)][\nabla \cdot \boldsymbol{s}(\boldsymbol{x}, t)]\, dt$

M-Kernel $K_\mu(\boldsymbol{x}) = -2\mu(\boldsymbol{x}) \int \sum_{i,j} D_{ij}^\dagger(\boldsymbol{x}, -t) D_{ij}(\boldsymbol{x}, t)\, dt$

$$\mathbf{D} = \frac{\nabla \boldsymbol{s} + (\nabla \boldsymbol{s})^T}{2} - \frac{\nabla \cdot \boldsymbol{s}}{3} \mathbf{I}$$

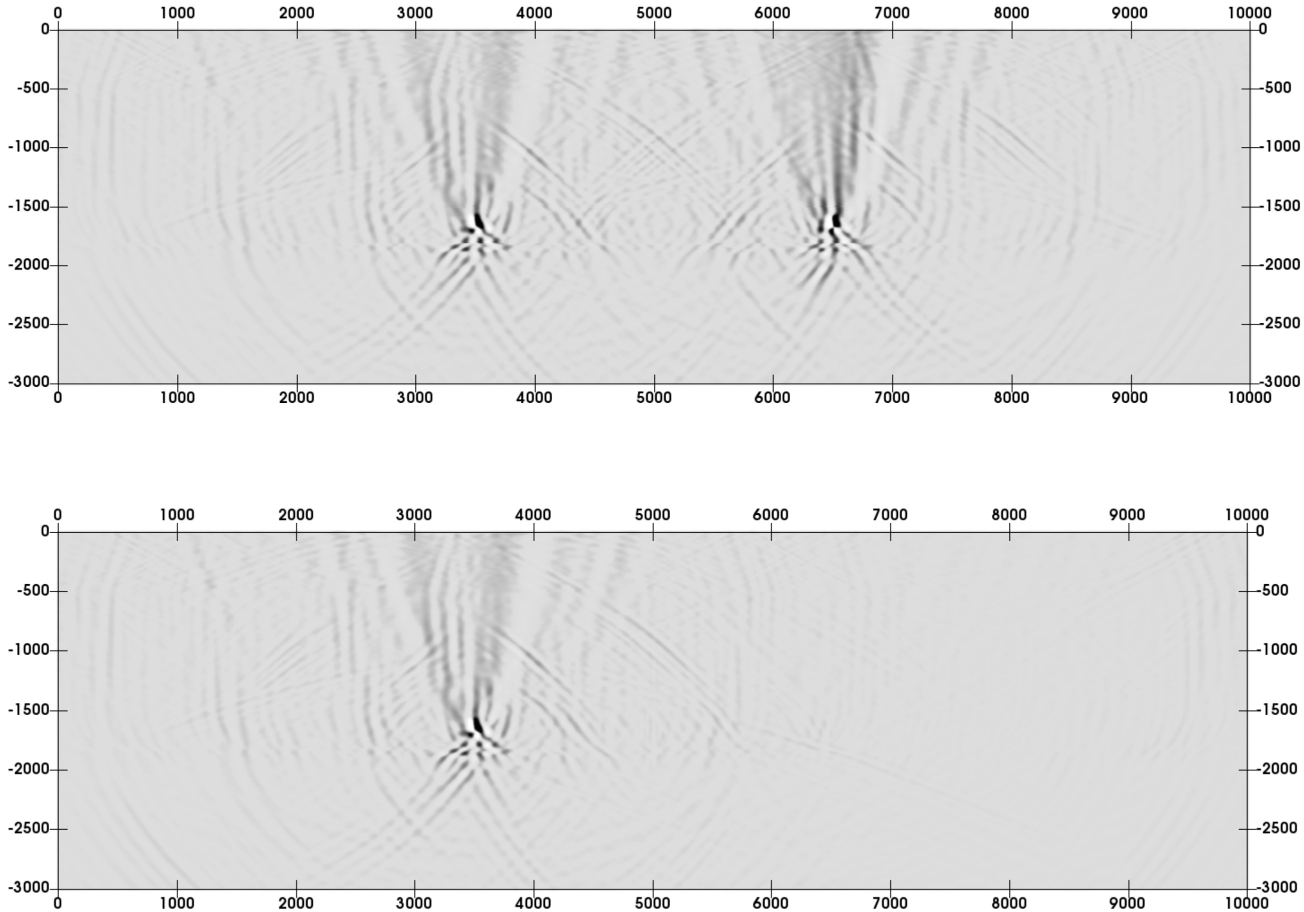
Direct and adjoint fields $\boldsymbol{s}(\boldsymbol{x}, t)$ и $\boldsymbol{s}^\dagger(\boldsymbol{x}, t)$

Adjoint source $\boldsymbol{f}^\dagger = \sum_r [\boldsymbol{s}(\boldsymbol{x}_r, -t) - \boldsymbol{d}(\boldsymbol{x}, -t)] \delta(\boldsymbol{x} - \boldsymbol{x}_r)$

Experimental and simulated data $\boldsymbol{d}(\boldsymbol{x}_r, t)$ и $\boldsymbol{s}(\boldsymbol{x}_r, t)$

Luo et al., 3D coupled acoustic-elastic migration with topography and bathymetry based on spectral-element and adjoint methods *// Geophysics – 2013. – Vol. 78(4).*

Examples of Simulation



Migration images of geological medium with 2 subvertical fluid-filled cracks. Background model without cracks (up) and with 1 crack (down).

Results

- Algorithm for elastic migration based on Born approximation was proposed. Research software was developed and tested on multilayered media.
  - On multi-core shared memory system the efficiency of parallelization is close to 100 %
  - On elastic images steep interfaces are visualized better than on acoustic images
- Algorithm for elastic migration based on full-wave simulation with grid-characteristic method was proposed. Research software was developed and tested on fractured media.
  - With the simplest background model geological heterogeneities were successfully recovered
  - New filed data can be taken into account with the usage of proposed algorithm

Thank you for your attention

Extra Slides

Homogeneous half-space with free surface:

$$c_{\alpha,b} = const, \qquad V = \{(x,y,z)\colon z \geq 0\},$$

$$2\mu \frac{\partial u_z}{\partial z} + \lambda \operatorname{div} \vec{\mathrm{u}} = \mu\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) = \mu\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) = 0$$

Green's tensor:

$$\hat{G}_\alpha^{L,H} = \hat{D}'_\alpha\left[\hat{g}_\alpha - \underline{\hat{g}_\alpha}\right],$$

$$\underline{\hat{g}_\alpha} = \left\{\chi\left(t' - t - s_{\alpha,b}\left|\vec{r}' - \underline{\vec{r}}\right|\right) - \chi(t' - t)\right\}\frac{\hat{I}}{4\pi\left|\vec{r}' - \underline{\vec{r}}\right|},$$

$$\underline{\vec{r}} = (x, y, -z)^{\mathrm{T}}$$

Forward modeling:

$$\vec{u}_{\alpha}^{s,B}(\vec{r}\,',t') = \sum_{\beta} \frac{1}{\rho(\vec{r}_0)} \widehat{D}'_{\alpha} \int_V \Delta c_{\beta}^{2}(\vec{r}) \left\{ \begin{array}{l} \widehat{D}_{\beta}^{0} \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}|\vec{r}\,' - \vec{r}|\right)}{16\pi^2 c_{\beta,b}^{2}|\vec{r}_0 - \vec{r}||\vec{r}\,' - \vec{r}|} \\ -\widehat{D}_{\beta}^{\underline{0}} \dfrac{f\left(t' - s_{\beta,b}\left|\vec{r}_0 - \underline{\vec{r}}\right| - s_{\alpha,b}|\vec{r}\,' - \vec{r}|\right)}{16\pi^2 c_{\beta,b}^{2}\left|\vec{r}_0 - \underline{\vec{r}}\right||\vec{r}\,' - \vec{r}|} \\ +\widehat{D}_{\beta}^{\underline{0}} \dfrac{f\left(t' - s_{\beta,b}\left|\vec{r}_0 - \underline{\vec{r}}\right| - s_{\alpha,b}\left|\vec{r}\,' - \underline{\vec{r}}\right|\right)}{16\pi^2 c_{\beta,b}^{2}\left|\vec{r}_0 - \underline{\vec{r}}\right|\left|\vec{r}\,' - \underline{\vec{r}}\right|} \\ -\widehat{D}_{\beta}^{0} \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}\left|\vec{r}\,' - \underline{\vec{r}}\right|\right)}{16\pi^2 c_{\beta,b}^{2}|\vec{r}_0 - \vec{r}|\left|\vec{r}\,' - \underline{\vec{r}}\right|} \end{array} \right\} \vec{f}\, dV,$$

$$\widehat{D}_{\beta}^{0} \rightarrow \widehat{D}_{\beta}^{\underline{0}} \sim \partial_{z_0} \rightarrow -\partial_{z_0}$$

Migration:

$$\Delta c^2_{\beta,\mathrm{migr}}(\vec{r}) = \sum_{\alpha} \int_S \int_T \frac{\vec{d}(\vec{r}\,',t')}{\rho(\vec{r}_0)} \cdot \hat{D}'_{\alpha} \left\{ \begin{array}{l} \hat{D}^{0}_{\beta} \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}|\vec{r}\,' - \vec{r}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \vec{r}||\vec{r}\,' - \vec{r}|} \\ -\hat{D}^{\underline{0}}_{\beta} \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}| - s_{\alpha,b}|\vec{r}\,' - \vec{r}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}||\vec{r}\,' - \vec{r}|} \\ +\hat{D}^{\underline{0}}_{\beta} \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}| - s_{\alpha,b}|\vec{r}\,' - \underline{\vec{r}}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}||\vec{r}\,' - \underline{\vec{r}}|} \\ -\hat{D}^{\underline{0}}_{\beta} \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}|\vec{r}\,' - \underline{\vec{r}}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \vec{r}||\vec{r}\,' - \underline{\vec{r}}|} \end{array} \right\} \vec{f}\, dt'\, dS\,,$$

$$\hat{D}^{0}_{\beta} \to \hat{D}^{\underline{0}}_{\beta} \sim \partial_{z_0} \to -\partial_{z_0}$$