

# Sorting of events according to their shapes

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Results shown here are worked out by  
**Renata Kopečná, Jakub Cimerman**

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# Motivation

- Each collision event evolves differently and has different final state
- Measured quantities are usually averaged over a large number of events
- Calculated quantities should be averaged over large number of events
- potential problems:
  - not enough events in the theoretical sample
  - different averaging procedure in theory and in experiment
  - in comparison of averages interesting information may get lost (e.g. critical fluctuations)
- Can we have a more exclusive approach to data analysis?

# Anisotropic expansion

(only nuclear collisions and assume non-flow effects under control)

- generic effect: blue-shift  
⇒ more particles and higher  $p_t$  in direction of stronger transverse flow
- link between the observable spectrum and the expansion of the fireball
- expansion results from the pressure gradients
- anisotropic expansion  $\Leftarrow$  anisotropic pressure gradients in initial conditions
- initial conditions evolved into final distribution—nothing added
- some contribution from hard partons depositing momentum during expansion

# Mapping of $\varepsilon_n$ 's and $v_n$ 'n

- spatial anisotropy

$$\varepsilon_{m,n} e^{in\Psi_{m,n}} = \int r dr d\phi r^m e^{in\phi} \rho(r, \phi)$$

use  $\varepsilon_n = \varepsilon_{n,n}$

- to a very good extent  $\langle v_n \rangle = k \langle \varepsilon_n \rangle$   
[F.G. Gardim *et al.*, Phys. Rev. C **85** (2012) 024908]
- also mapping between the values in individual events and between probability distributions  
valid for various initial conditions and ideal as well as viscous hydro  
[H. Niemi *et al.*, Phys. Rev. C **87** (2013) 54901]

# Each event is different: selection of events

Each event undergoes different evolution

There is always averaging over others but the selected order event plane.

Various event shapes are averaged out!

Can we see non-averaged events?

An example:

- $R_o^2(\phi)$  for a fireball with both 2nd and 3rd order flow anisotropy
- varying angles between 2nd and 3rd event plane

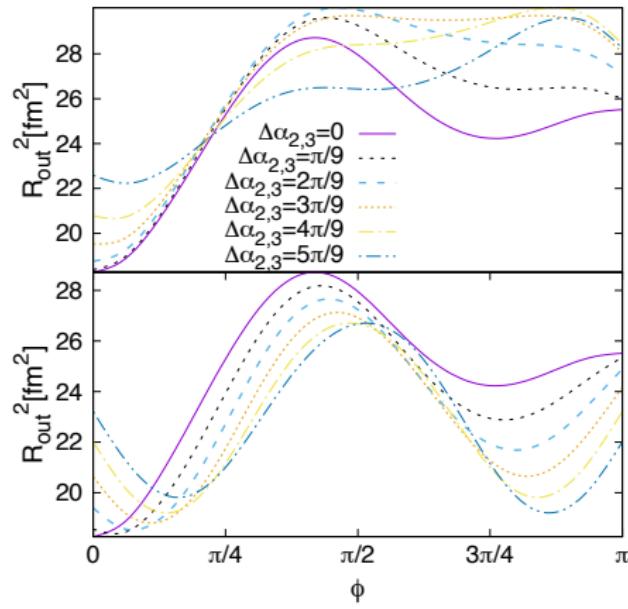


Figure: Sándor Lököš

# Event Shape Engineering

- Two subevents
  - Subevent *a*: event selection
  - Subevent *b*: physical analysis
- Helps avoiding nonphysical biases (nonflow effects)
- Information loss
- Event selection according to the magnitude of the **reduced flow vector**  $q_n$

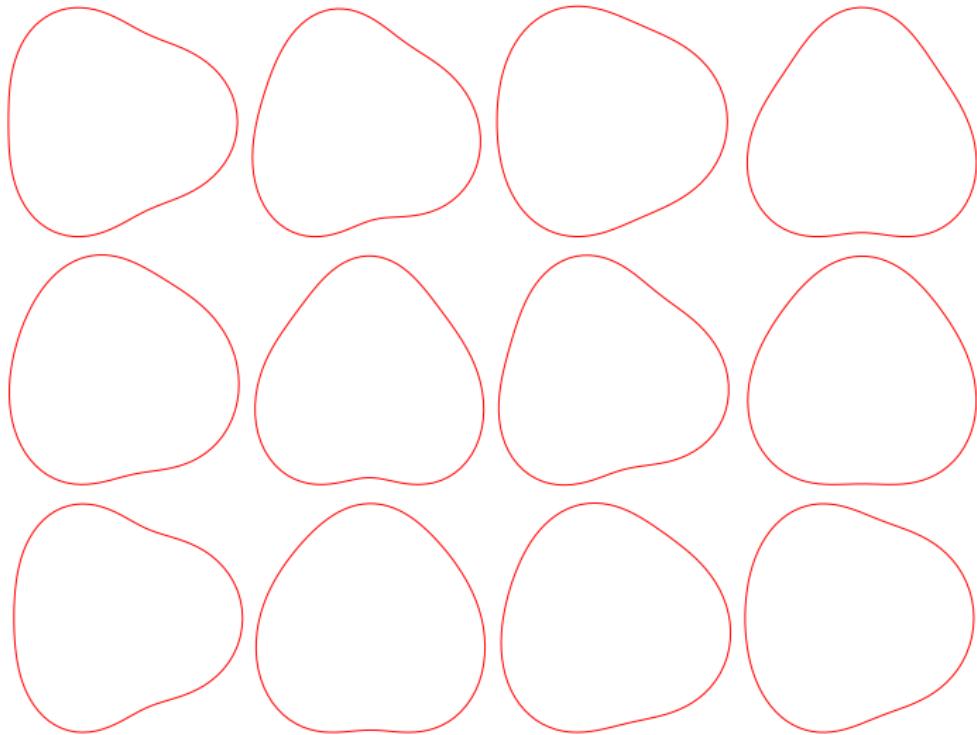
$$\vec{Q}_n = \left( \sum_{i=1}^M \cos(n\phi_i), \sum_{i=1}^M \sin(n\phi_i) \right),$$

$$q_n = |\vec{Q}_n|/\sqrt{M}.$$

[J. Schukraft, A. Timmins, S. A. Voloshin, Phys. Lett. B 719 (2013) 394-398]

# Event shapes

How to do Event Shape Engineering among these shapes...?



... ordered

$$v_2 = 0.04$$

$$v_3 = 0.04$$

$$v_2 = 0.06$$

$$v_3 = 0.04$$

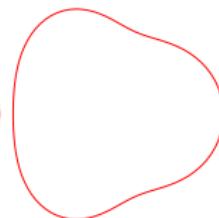
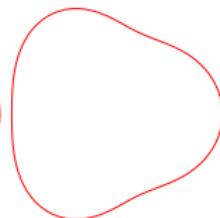
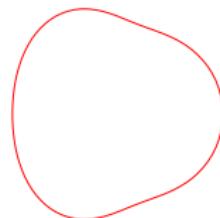
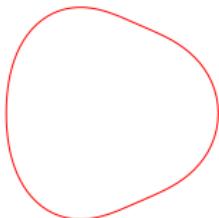
$$v_2 = 0.04$$

$$v_3 = 0.06$$

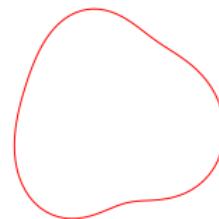
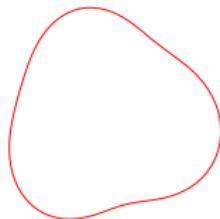
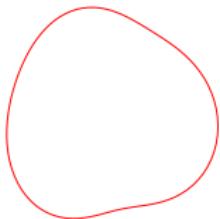
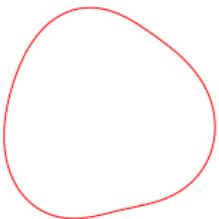
$$v_2 = 0.06$$

$$v_3 = 0.06$$

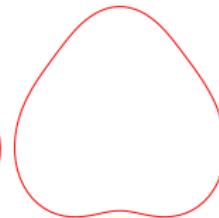
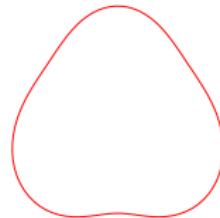
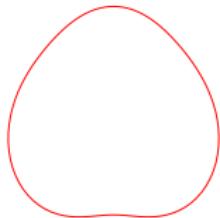
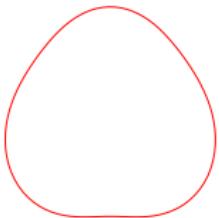
$$\Psi_{23} = 0$$



$$\Psi_{23} = 0.7$$



$$\Psi_{23} = 1.57$$



# Event Shape Sorting

- sorts events in such a way, that events with similar histograms (in azimuthal angle, e.g.) end up close to each other
- divides the totality of events into (customarily 10) event bins
- no need to specify a sorting variable, unlike Event Shape Engineering [J. Schukraft, A. Timmins, S. A. Voloshin, Phys. Lett. B 719 (2013) 394-398]

Algorithm based on:

S. Lehmann, A.D. Jackson, B. Lautrup, arXiv:physics/0512238

S. Lehmann, A. D. Jackson and B. E. Lautrup, Scientometrics **76** (2008) 369  
[physics/0701311 [physics.soc-ph]]

Published in

R. Kopečná, B. Tomášik: Eur. Phys. J. A **52** (2016) 115.

# Event Shape Sorting: the algorithm

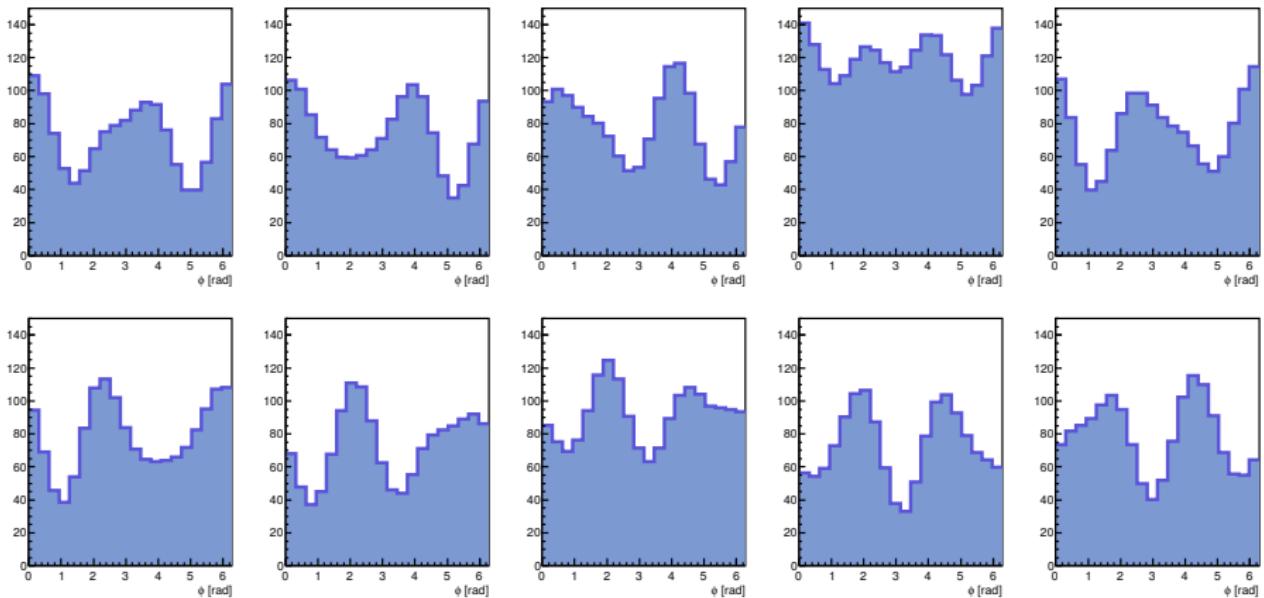
We will sort events according to their histograms in azimuthal angle.

- ① (Rotate the events appropriately)
- ② Sort your events as you wish
- ③ Divide sorted events into quantiles (we'll do deciles)
- ④ Determine average histograms in each quantiles
- ⑤ For each event  $i$  calculate Bayesian probability  $P(i|\mu)$  that it belongs to quantile  $\mu$
- ⑥ For each event calculate average  $\bar{\mu} = \sum_{\mu} \mu P(i|\mu)$
- ⑦ Sort events according to their values of  $\bar{\mu}$
- ⑧ If order of events changed, return to 3. Otherwise sorting converged.

S. Lehmann, A.D. Jackson, B. Lautrup, arXiv:physics/0512238

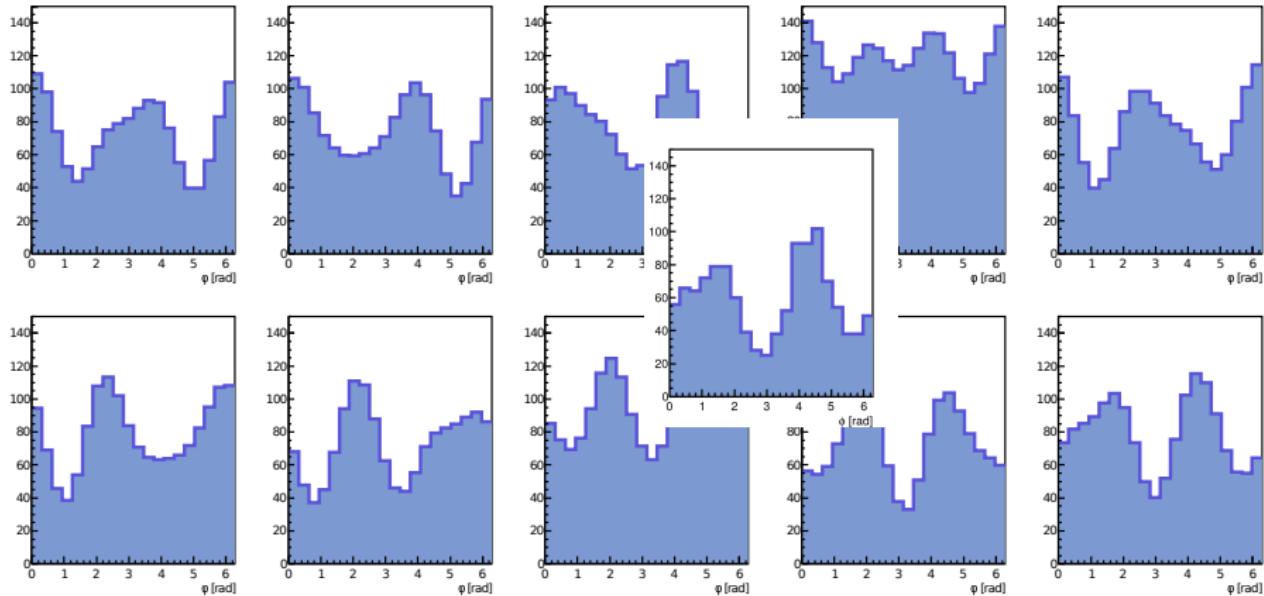
S. Lehmann, A. D. Jackson and B. E. Lautrup, *Scientometrics* **76** (2008) 369  
[physics/0701311 [physics.soc-ph]]

# Assigning event to event bin



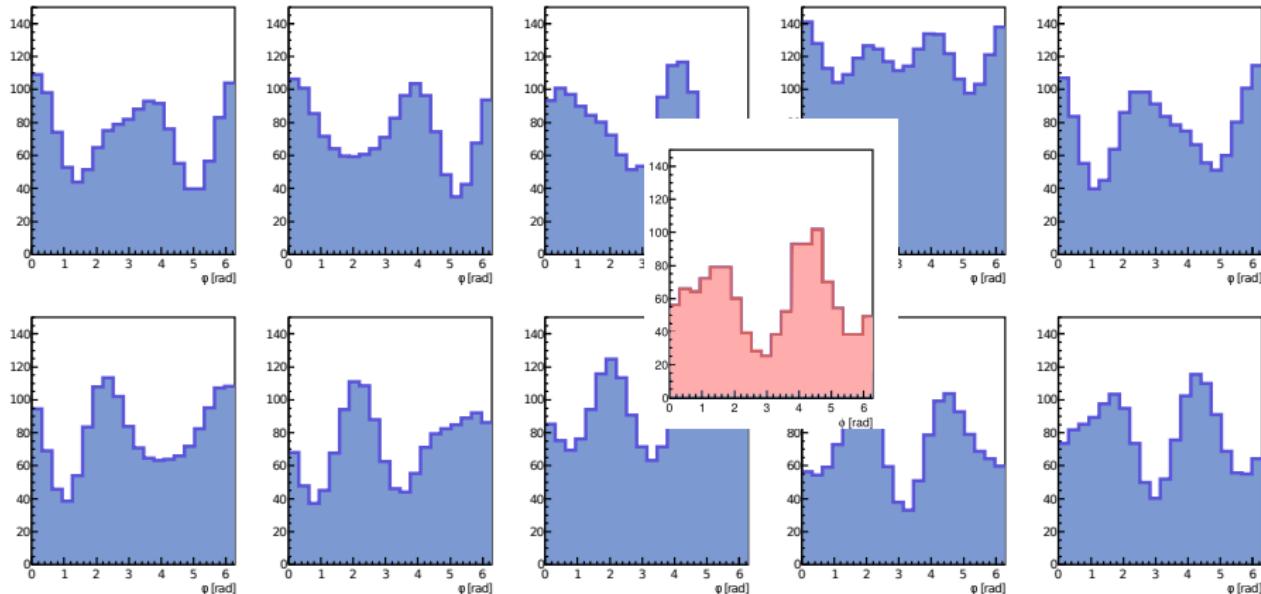
# Assigning event to event bin

To which event bin is this event similar?



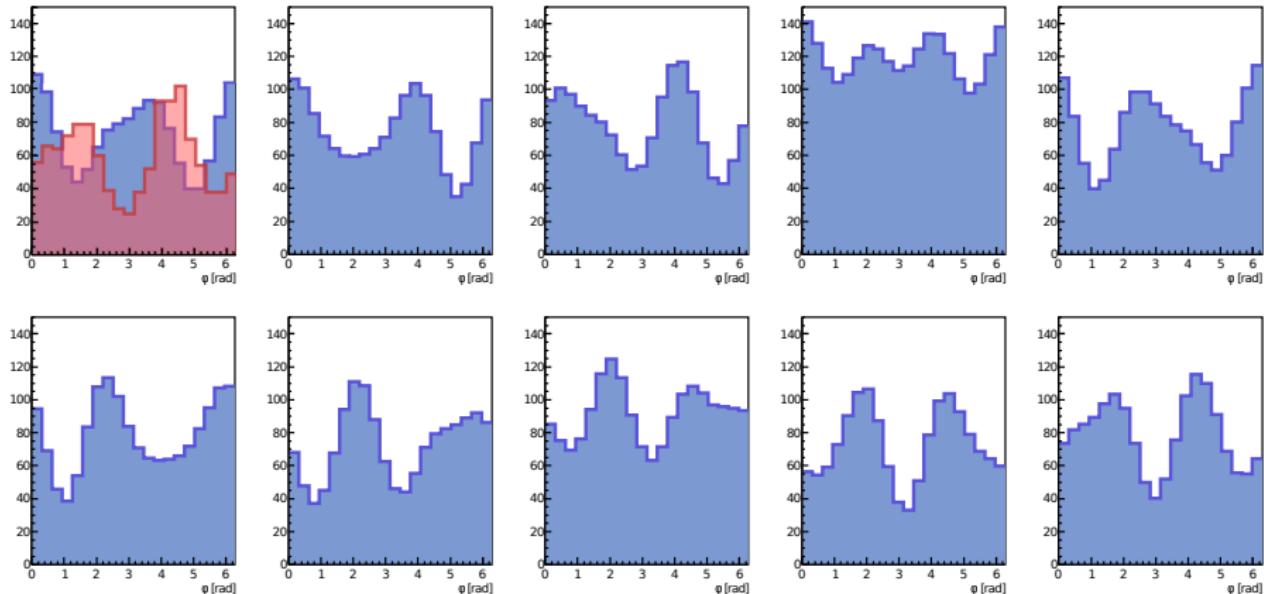
# Assigning event to event bin

Calculate Bayesian probability that the event belong to each event bin



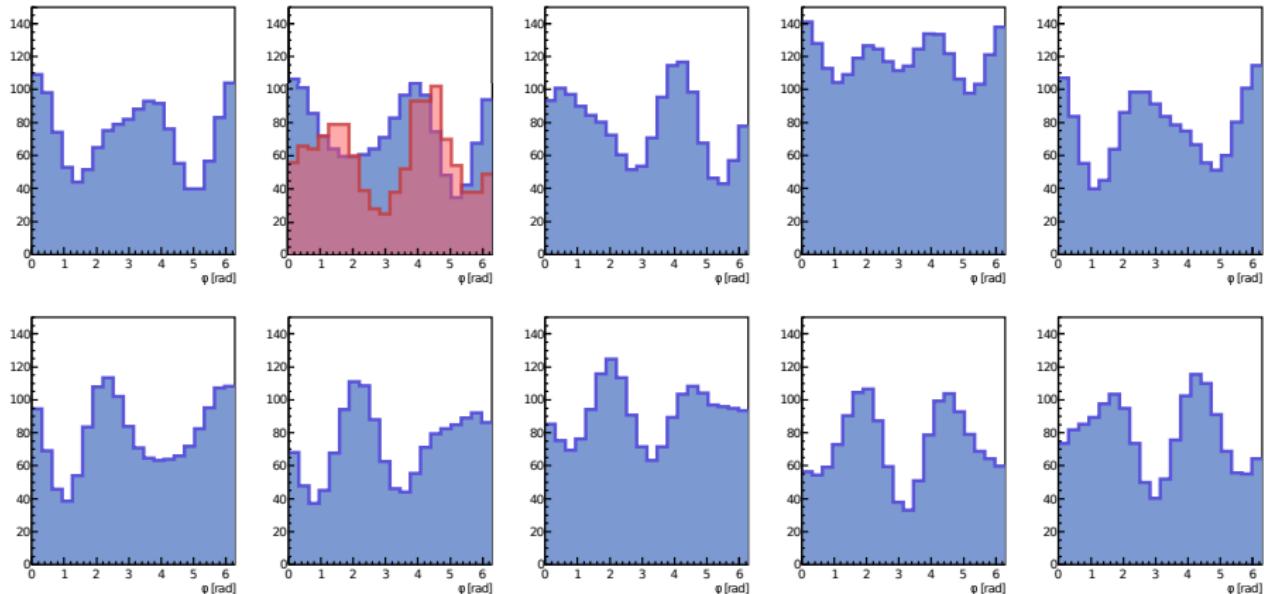
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 1



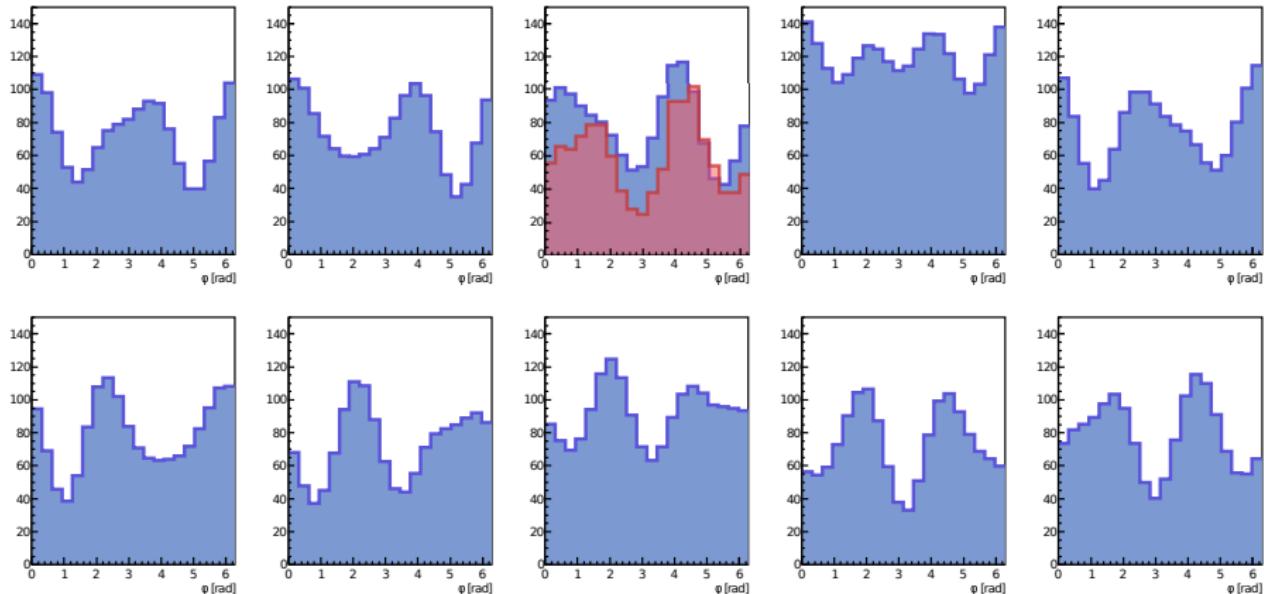
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 2



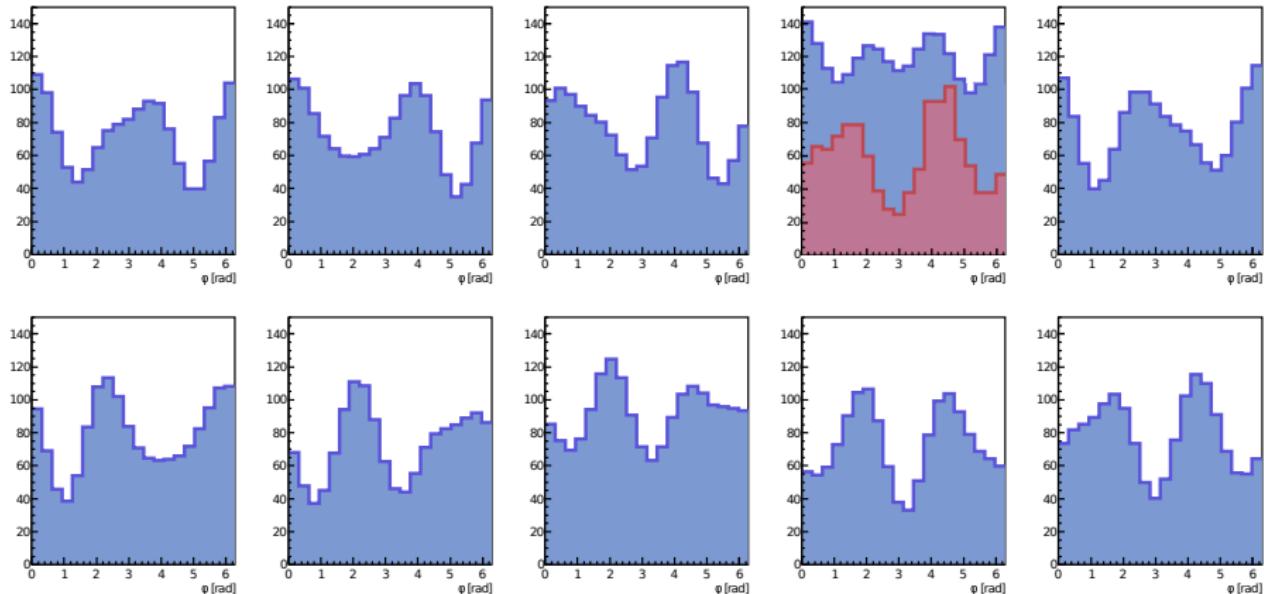
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 3



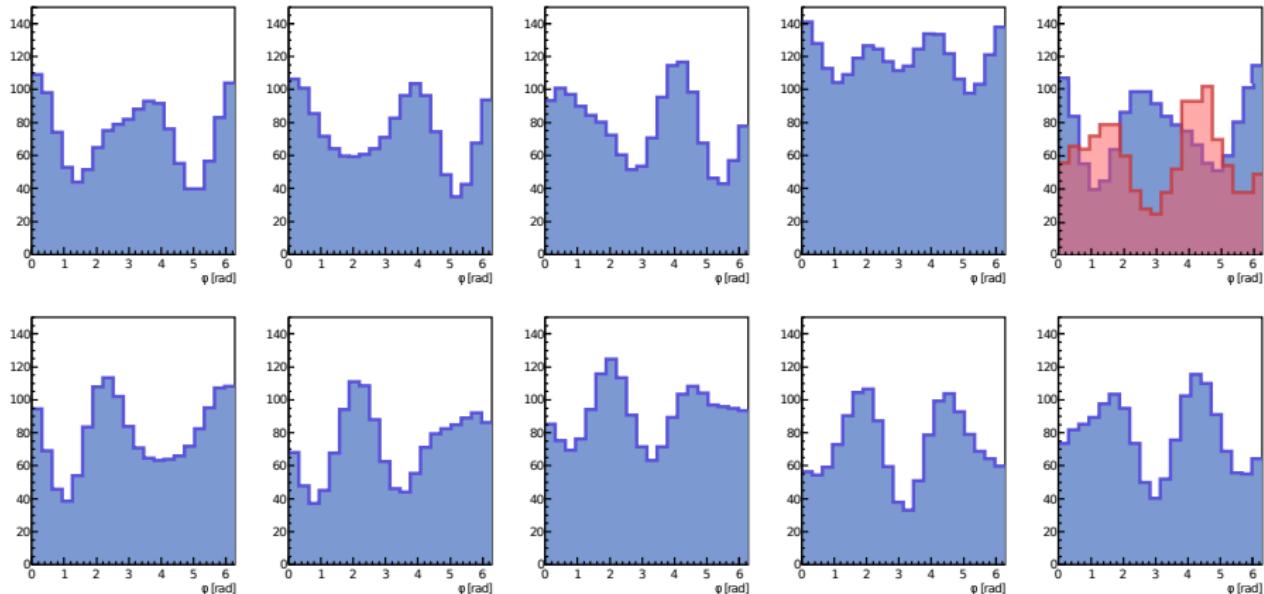
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 4



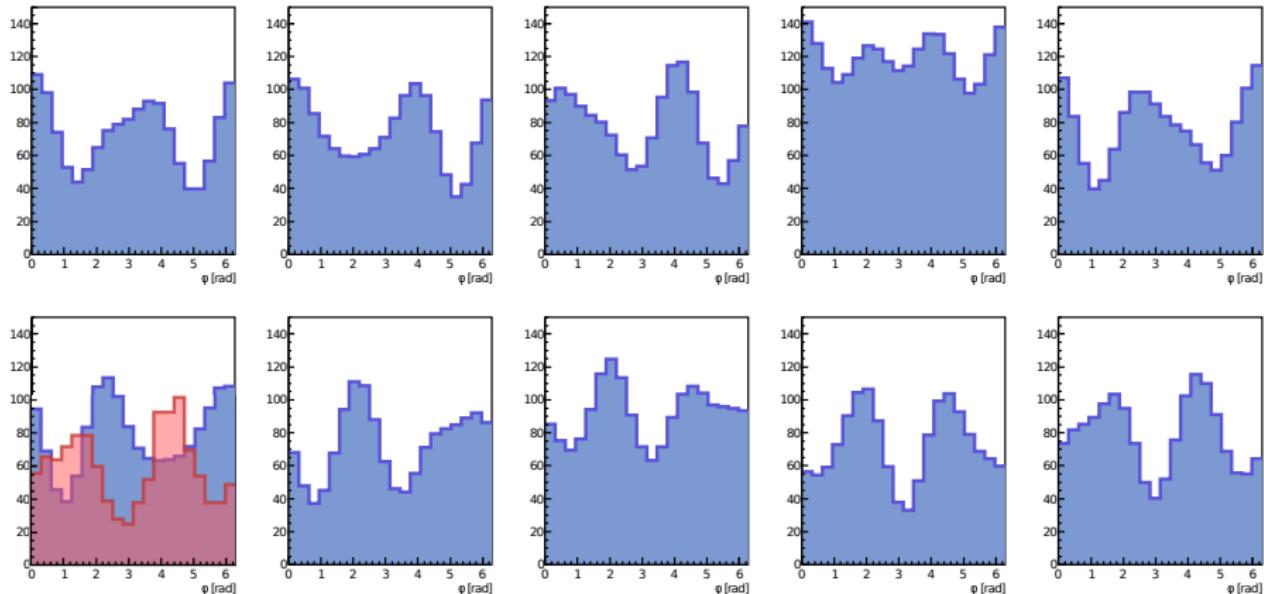
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 5



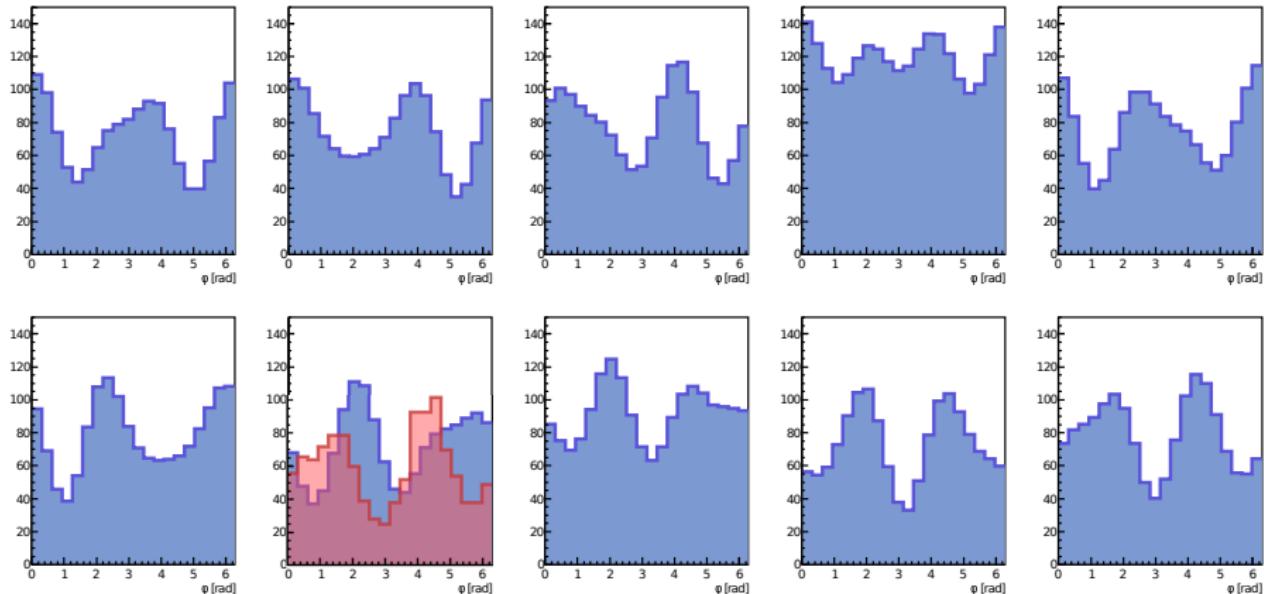
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 6



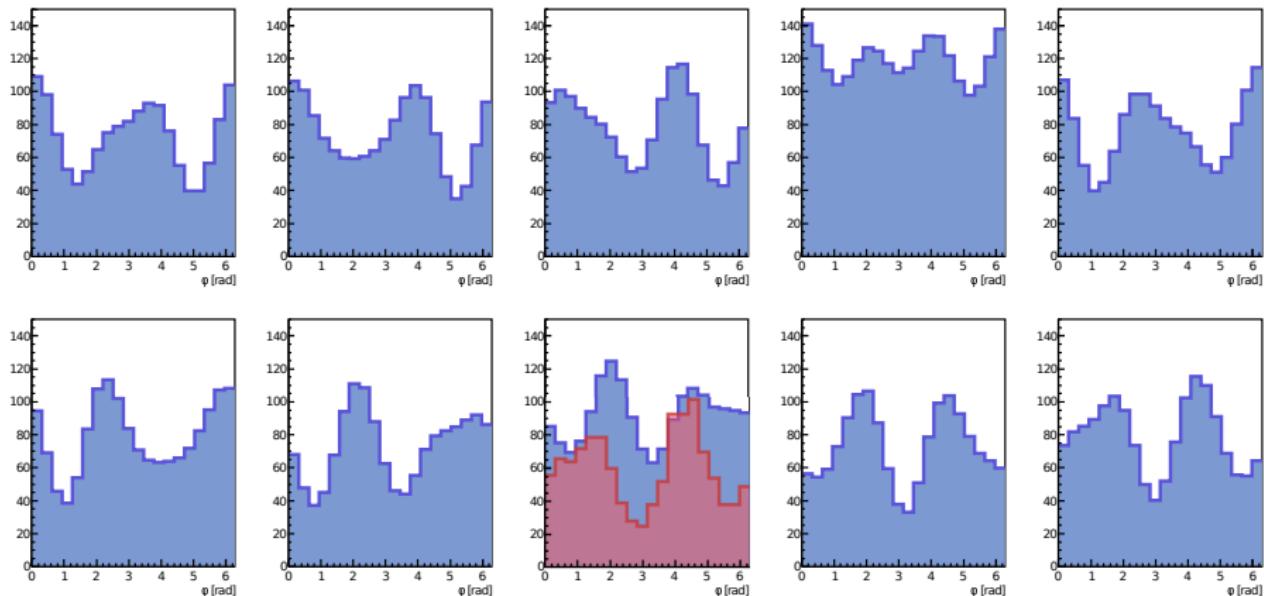
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 7



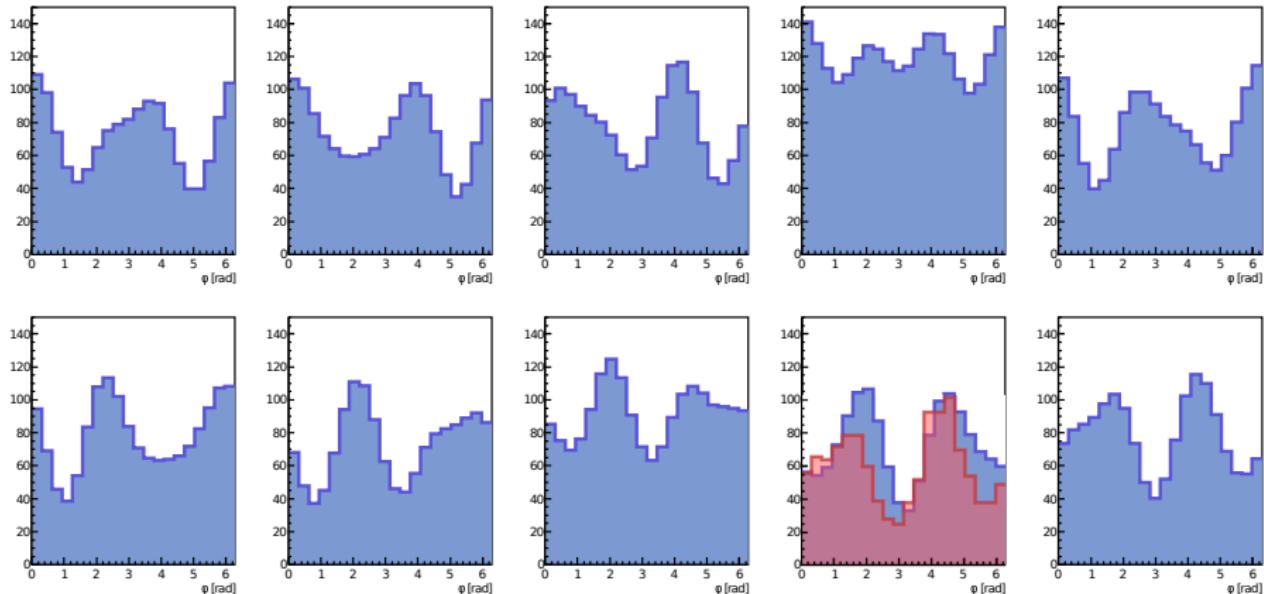
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 8



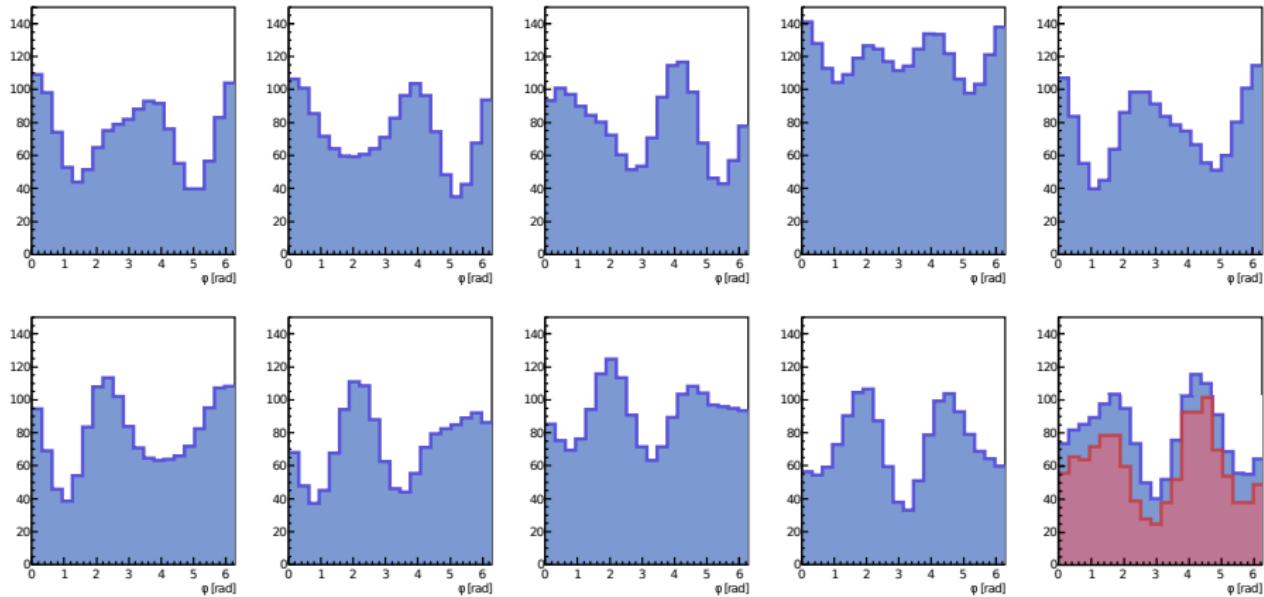
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 9



# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 10



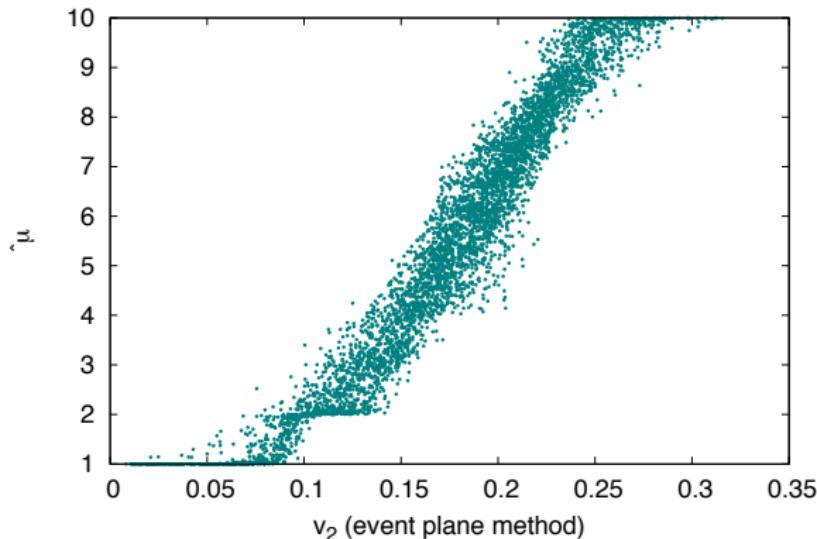
# Iterations of event bins

## Sorted events: Gradual change of event shape

- 2000 events, AMPT centrality 0–20%,  $\sqrt{s_{NN}} = 2.76$  TeV
- each frame averaged over 50 events and shifted by 10 events wrt previous frame
- change of colour = change of event bin

## Toy model: only elliptic flow

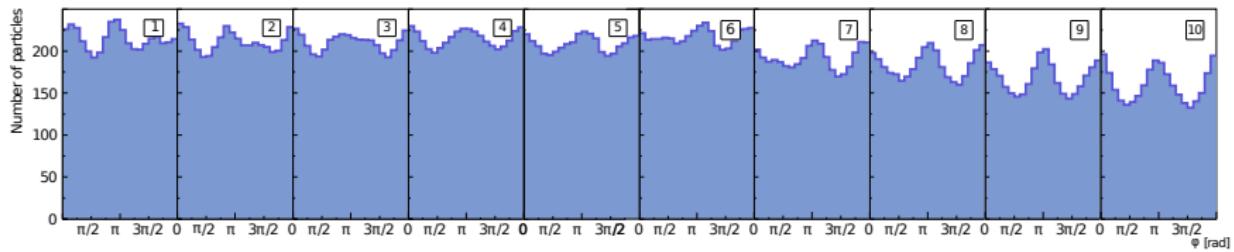
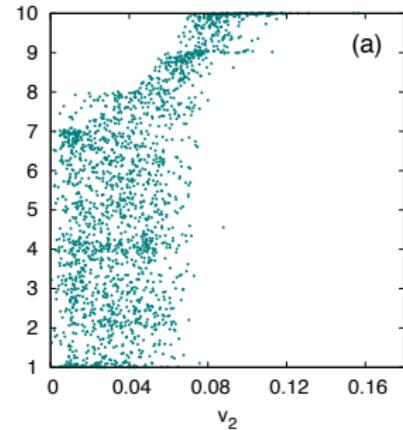
- in the toy model, azimuthal distribution of pions with only elliptic flow is generated
- correlation between  $v_2$  and  $\mu$ : 0.959
- $v_2$  is good sorting variable



## Sorted AMPT central events (LHC energy)

Event shape sorting goes beyond characterisation of events according to single variable (e.g.  $v_2$  or  $q_2$ )

- simulated 2000 central 0–20% events from AMPT for  $\sqrt{s_{NN}} = 2.76$  TeV
  - correlation between sorting variable  $\mu$  and elliptic flow  $v_2$



# Extension of the blast-wave model to higher orders

The emission function:

$$S(x, p) d^4x = \frac{m_t \cosh(Y - \eta)}{(2\pi)^3} d\eta \, r dr \, d\varphi \frac{\tau \, d\tau}{\sqrt{2\pi \Delta \tau^2}} \\ \times \exp\left(-\frac{(\tau - \tau_0)^2}{2\Delta \tau^2}\right) \exp\left(-\frac{p^\mu u_\mu}{T}\right) \Theta(1 - \bar{r})$$

Transverse size

$$\bar{r} = \frac{r}{R(\varphi)} \quad R(\varphi) = R_0 \left( 1 - \sum_{n=2}^{\infty} a_n \cos(n(\varphi - \varphi_n)) \right)$$

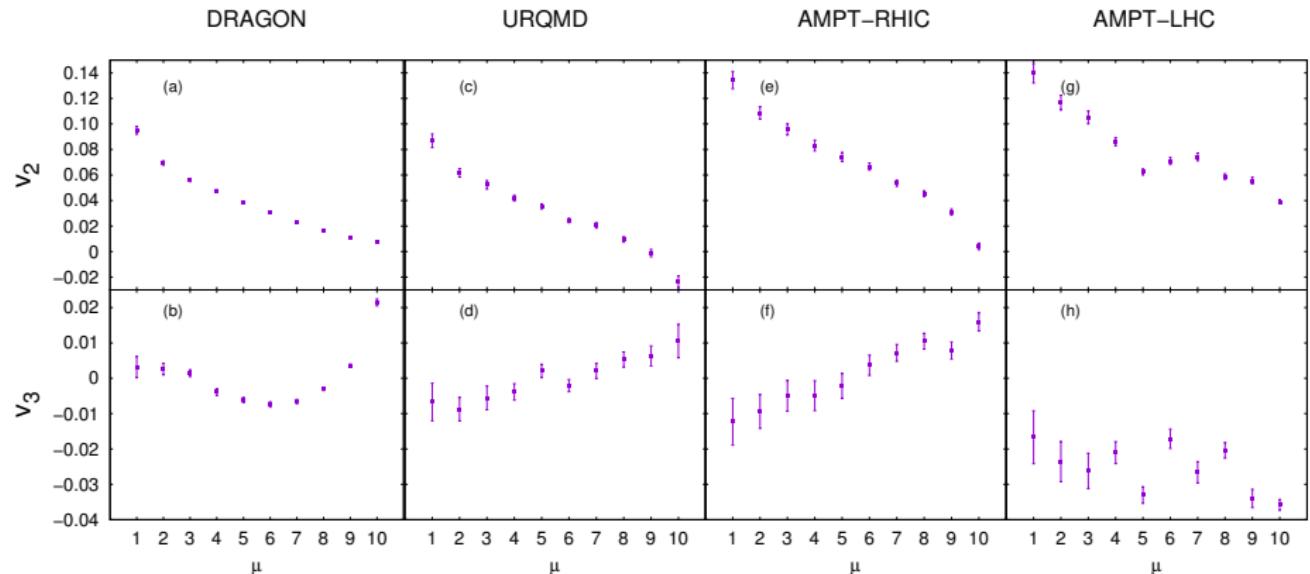
Transverse expansion goes into  $u_\mu$ , parametrized by transverse rapidity

$$\rho(\bar{r}, \varphi) = \bar{r} \rho_0 \left( 1 + \sum_{n=2}^{\infty} 2\rho_n \cos(n(\varphi - \varphi_n)) \right)$$

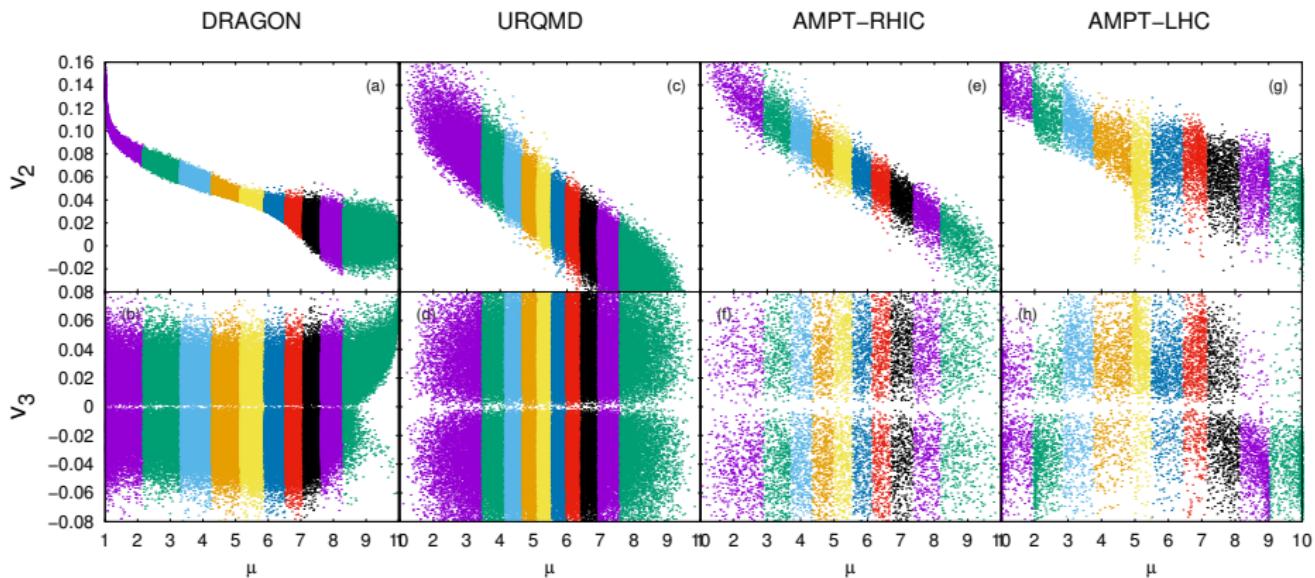
## Models which will be compared

- DRAGON** 150 000 events by DRAGON with anisotropies  
 $a_2, \rho_2 \in (-0.1; 0.1)$ ,  $a_3, \rho_3 \in (-0.03; 0.03)$   
(DRAGON is MC final state generator based on blast-wave model with included resonances)
- UrQMD** 100 000 events by UrQMD in AuAu collisions with energy  
 $\sqrt{s_{NN}} = 200$  GeV, impact parameter 7 – 10 fm
- AMPT-RHIC** 10 000 events by AMPT in AuAu collisions with energy  
 $\sqrt{s_{NN}} = 200$  GeV, impact parameter 7 – 10 fm
- AMPT-LHC** 10 000 events by AMPT in PbPb collisions with energy  
 $\sqrt{s_{NN}} = 2760$  GeV, impact parameter 7 – 10 fm

# Anisotropic flow in similar events



# Anisotropic flow in similar events



## Femtoscopy: trivia

Correlation radii are parameters if the measured correlation function:

$$C(q, K) - 1 = \exp(-R_o^2(K)q_o^2 - R_s^2(K)q_s^2 - R_l^2(K)q_l^2)$$

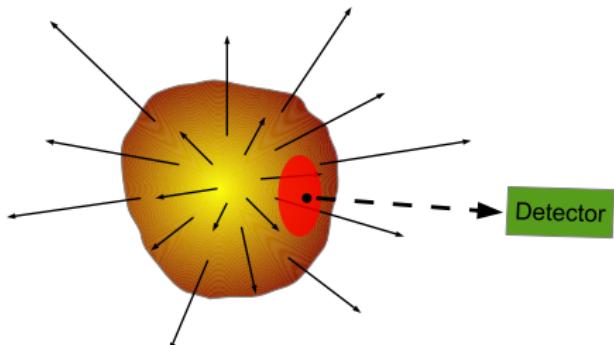
- $K$ : average pair momentum,  
 $q$ : relative pair momentum
- no cross terms at  
midrapidity at high energies
- out-side-long coordinate  
frame

out perpendicular to  
beam, along  $K_t$

long beam direction

side perpendicular to  
out and long

The correlation radii measure the sizes of the homogeneity regions



## Azimuthal dependence of correlation radii

Correlation radii are customarily decomposed into Fourier series

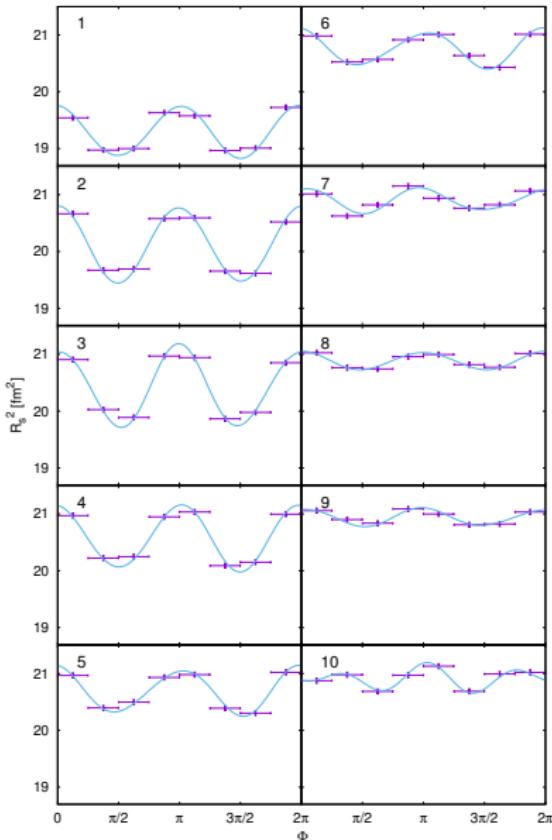
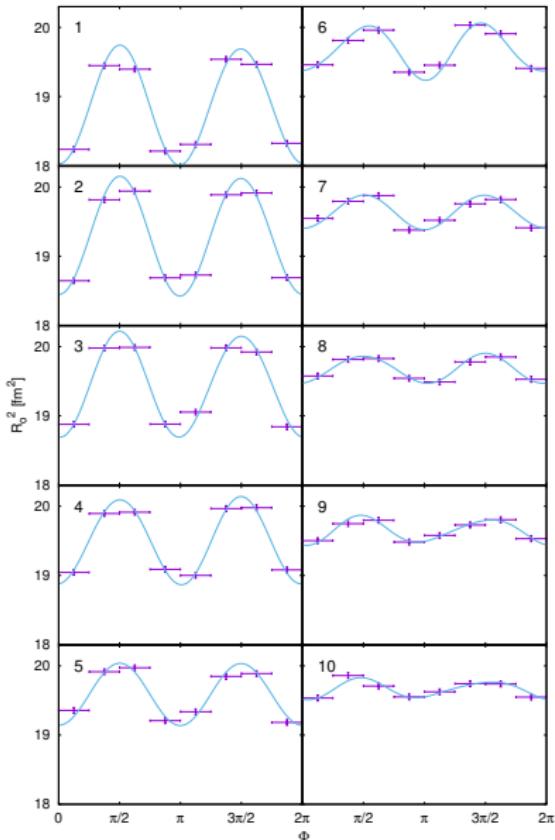
$$R_i^2(\phi) = R_{i,0}^2 + \sum_{n=1}^{\infty} R_{i,n}^2 \cos(n(\phi - \phi_n))$$

where  $i = o, s, l$ .

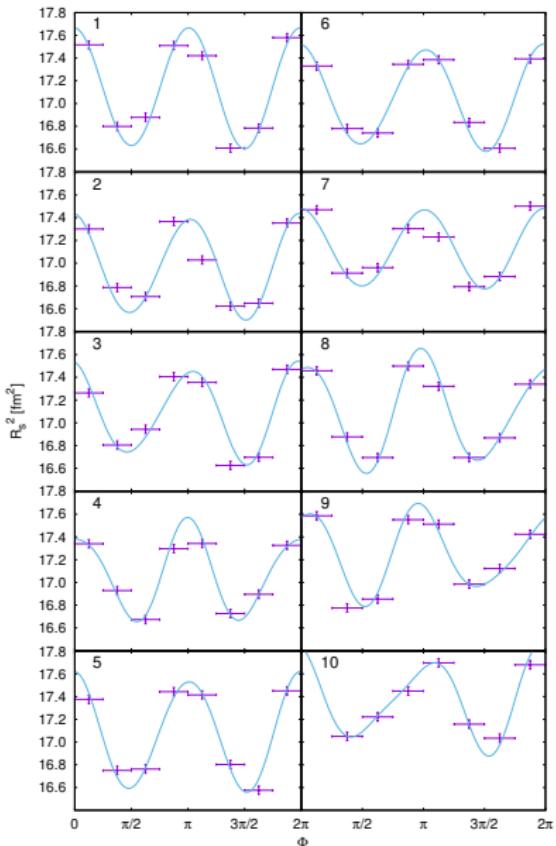
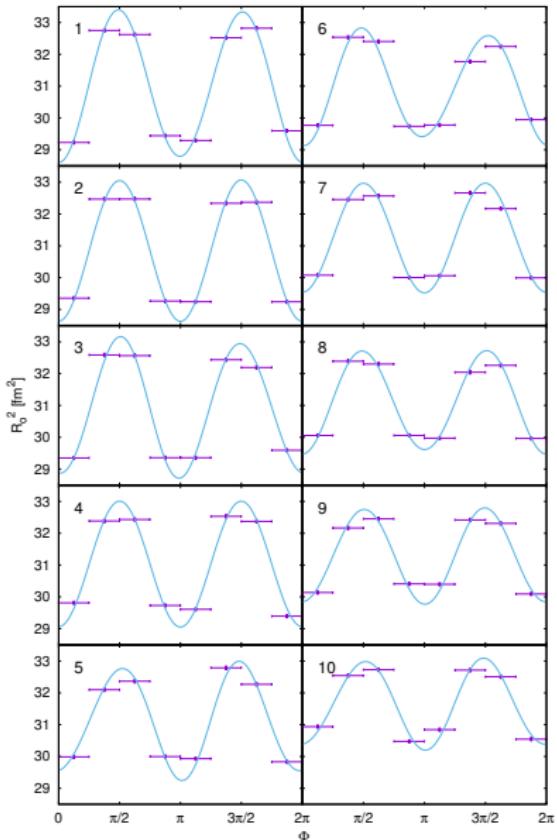
Real effect of oscillations: amplitudes divided by 0th term allow to scale out the absolute sizes

$$R_i^2(\phi) = R_{i,0}^2 \left( 1 + \sum_{n=1}^{\infty} \frac{R_{i,n}^2}{R_{i,0}^2} \cos(n(\phi - \phi_n)) \right)$$

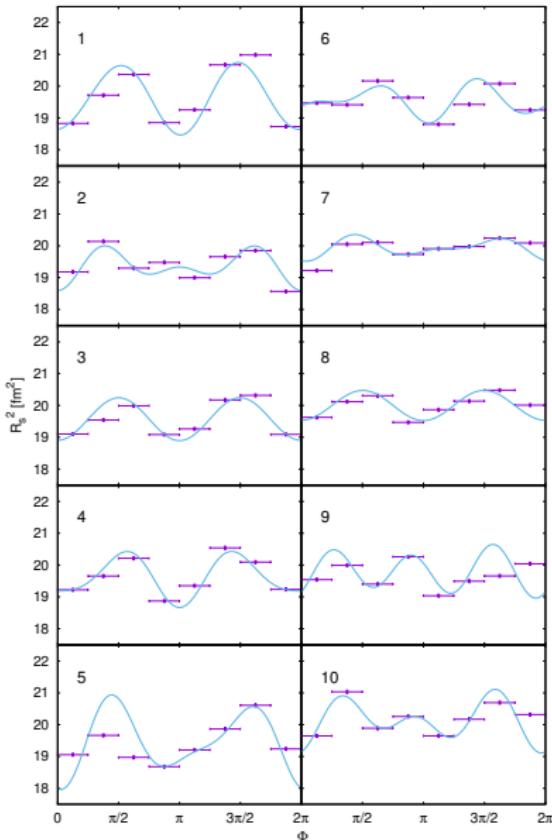
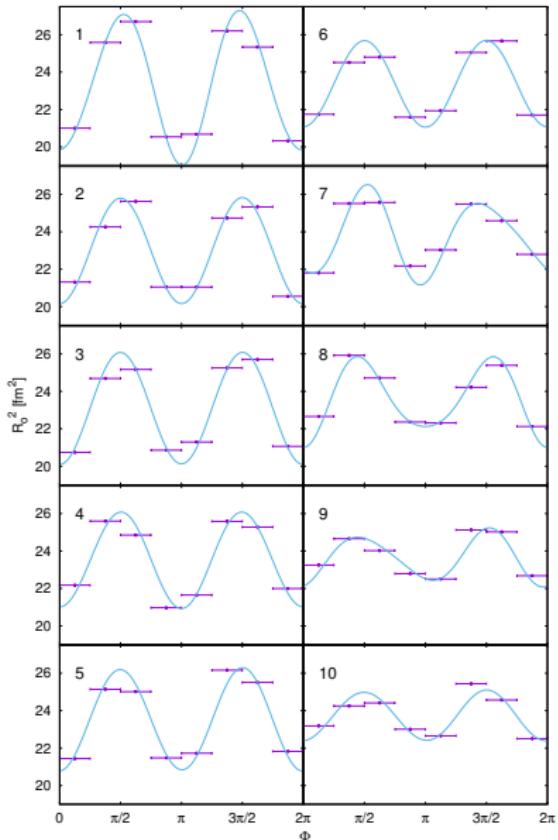
# Correlation radii from DRAGON in different event bins



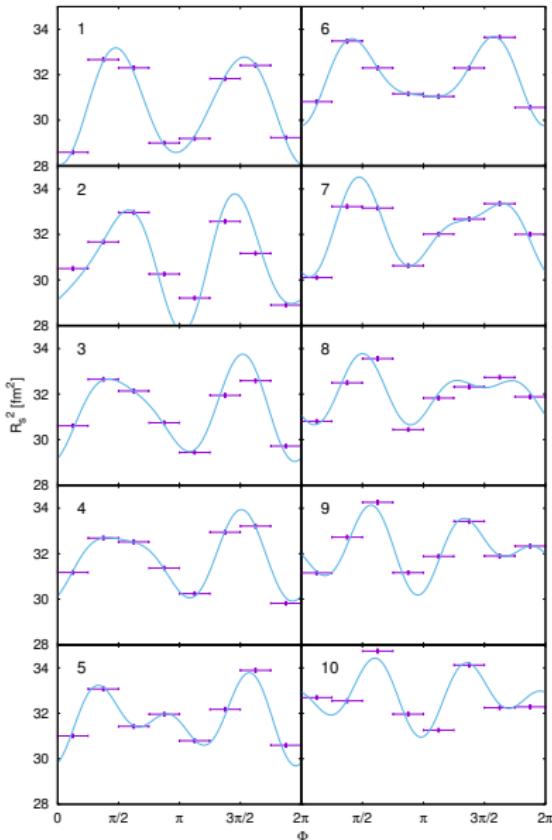
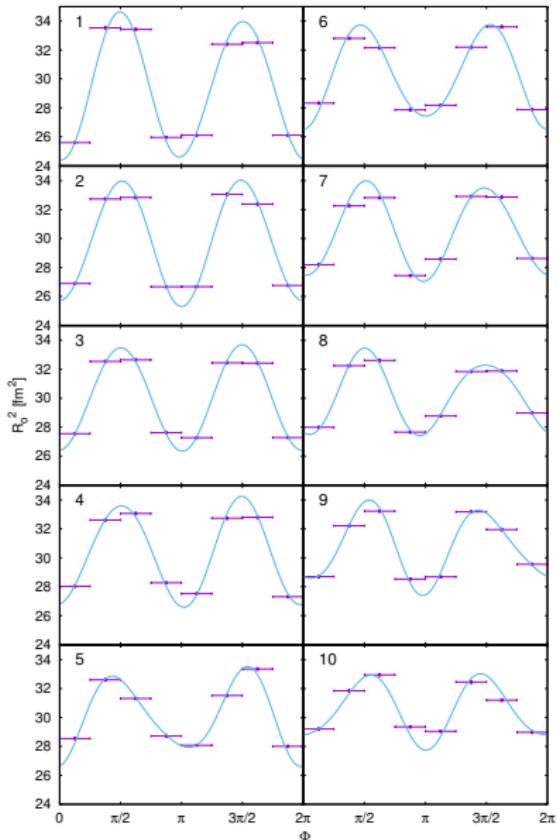
# Correlation radii from UrQMD in different event bins



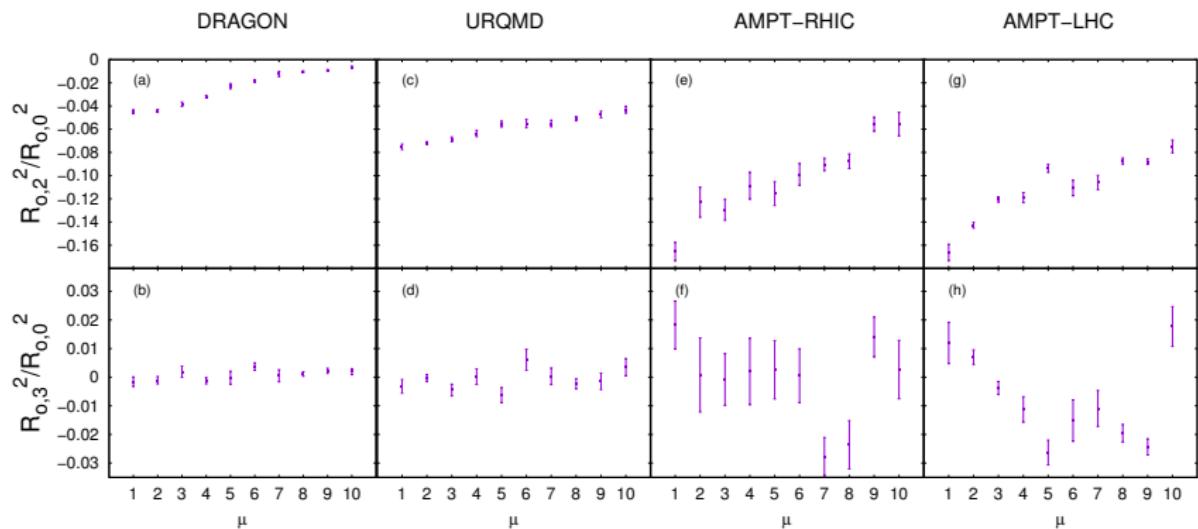
# Correlation radii from AMPT-RHIC in different event bins



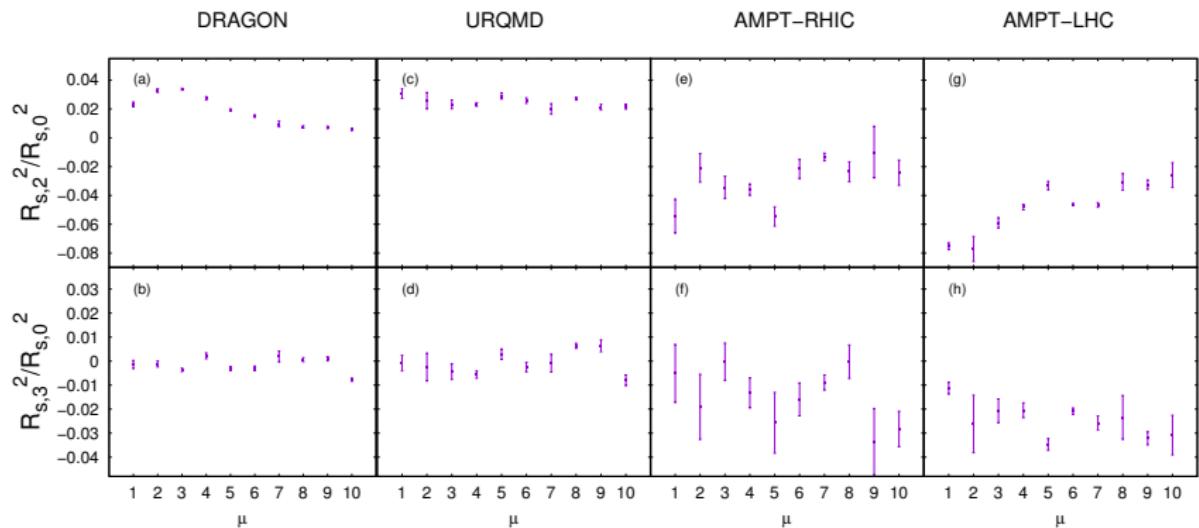
# Correlation radii from AMPT-LHC in different event bins



# Oscillation amplitudes for $R_o^2$



# Oscillation amplitudes for $R_s^2$



**INDIAN-SUMMER  
SCHOOL OF PHYSICS**  
Prague, Czech Republic

# Phenomenology of Hot and Dense Matter for Future Accelerators

September 3–7, 2018

Registration deadline:  
**June 1, 2018**

[iss2018.fjfi.cvut.cz](http://iss2018.fjfi.cvut.cz)

Paul Romatschke:  
**Hydrodynamic evolution, flow, bulk properties**

Nestor Armesto:  
**Hadron structure phenomenology**

Konrad Tywoniuk:  
**Jet suppression and energy loss**

Tuomas Lappi:  
**Initial stages of the collision, low-x physics**

Mateusz Płoskon:  
**Overview of experimental results, EIC perspective**

Chihiro Sasaki:  
**Theory of the deconfinement transition**

Selected students from COST member states will be given a grant to cover expenses financed by the THOR COST action CA15213. Attendance is limited to 45 participants.

**Venue:**

Faculty of Nuclear Sciences  
and Physical Engineering  
Czech Technical University in Prague  
Brehova 7, 11519 Prague 1

**Organizers**

Jan Cepila (chair), Boris Tomášik,  
Jesus Guillermo Contreras, Jaroslav Bielčík,  
Miroslav Myska, Jana Bielčíková,  
Jiří Mareš, Jiří Adam, Petr Bydžovský



# What is it good for?

- More selective comparison of data to theory.
- Looking for and selecting rare events.
- Construction of mixed-events background for correlation functions.
- Allows single-event femtoscopy?
- ...

Published in

R. Kopečná, B. Tomášik: Eur. Phys. J. A **52** (2016) 115.  
J. Cimerman, B. Tomášik: in preparation.