Effective $\gamma\gamma G$ vertex and scattering of photons in quark-gluon plasma

M. Bordag[1], V. Skalozub[2]

[1]Leipzig University, Germany

[2]Dnipro National University, Ukraine

1. effective vertex, composed from the one gluon and two photon lines,
2. non-zero quark triangle diagram
3. background: A_0-condensate and effective charge Q^3_μ

Motivation

At present, it is assumed that after deconfinement phase transition, i.e., at high temperature, a quark-gluon plasma (QGP) is formed. It consists of quarks, gluons (or corresponding quasi particles). Numerous calculations (analytic, numeric, lattice, SD-equations) suggest that the QGP is not a free gas. Instead, in QGP background is expected to have a rather complicated structure, including, for instance, an A_0-condensate (Polyakov loop),

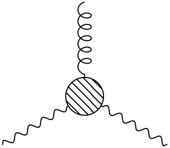
$$L = \frac{1}{N}\mathrm{Tr}\left[\mathbf{P}\exp\left(ig\int_0^\beta d\tau A_0(\tau,\mathbf{x})\right)\right].$$

$A_0 \neq 0$ can be considered also as order parameter of the deconfinement phase transition (DPT). The condensation of the A_0 was demonstrated in lattice simulations and also in analytic calculations. $A_0 \neq 0$ violates the $Z(3)$ and gauge symmetries.

The effective potential of QCD with A_0-background was investigated in a large number of papers, for example [**?**], [**?**], and more recently in [**?**]. It was found that A_0 is important at temperatures $T \sim (1 \ldots 2.5)\, T_d$ (see, e.g., [**?**]). Implications of an A_0-condensate were recently investigated with respect to CP-violating processes in [**?**].

Present talk

In the present talk we discuss the scattering of photons off the A_0-condensate and the gluon background field, resulting in an effective $\gamma\gamma G$-vertex,



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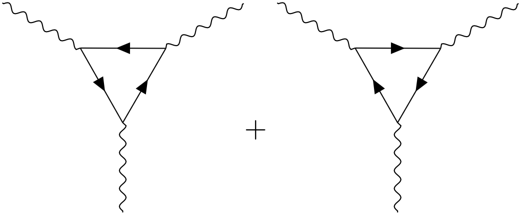
This is in contrast to the three photon vertex,



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which is forbidden by Furry's theorem. Since quarks carry both, electric and color charges, a quark loop could connect white and colored objects

QCD with A_0-background

Below, we consider the Feynmann rules and the C-symmetry, which is the tool to derive Furry's theorem.
the quark Lagrangian reads

$$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu \not{\boldsymbol{D}}_\mu - m\right)\psi,$$

where, for SU(2) we have two spinor components, $\psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$, and the covariant derivative is $\boldsymbol{D}_\mu = (\partial_\mu - ieA_\mu)\,\mathbb{1} - ig\boldsymbol{Q}_\mu$.
Here, A_μ is the electromagnetic field, $\boldsymbol{Q}_\mu = Q^a_\mu T^a$ are the gluon fields

for SU(2) we have $T^a = \frac{1}{2}\sigma^a$ (Pauli matrices)
and

$$\boldsymbol{D}_\mu = (\partial_\mu - ieA_\mu)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - i\frac{g}{2}\begin{pmatrix}Q^3_\mu & Q^1_\mu - iQ^2_\mu\\ Q^1_\mu + iQ^2_\mu & -Q^3_\mu\end{pmatrix}$$

we introduce the charged gluon fields

$$Q^\pm_\mu = \frac{1}{\sqrt{2}}\left(Q^1_\mu \pm iQ^2_\mu\right)$$

Inserting into the Lagrangian we divide into free and interaction parts,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm int}$$

where

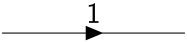
$$\mathcal{L}_0 = \overline{\psi}\left((i\gamma^\mu \partial_\mu - m) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - i\frac{g}{2} A_0 \delta_{\mu,0} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right) \psi$$
$$= \overline{\psi}_1 \left(\tilde{\not{p}}_1 - m \right) \psi_1 + \overline{\psi}_2 \left(\tilde{\not{p}}_2 - m \right) \psi_2$$

(in momentum representation) with $(\tilde{p}_{1,2})_\mu = p_\mu \pm A_0 \delta_{\mu,0}$ and

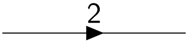
$$\mathcal{L}_{\rm int} = (\overline{\psi}_1, \overline{\psi}_2) \left(\gamma^\mu \left[-ieA_\mu \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - i\frac{g}{2} Q^3_\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right.\right.$$
$$\left.\left. - i\frac{g}{\sqrt{2}} \begin{pmatrix} 0 & Q^-_\mu \\ Q^+_\mu & 0 \end{pmatrix} \right] \right) \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Feynmann rules

from this Lagrangian we get the following Feynmann rules,
lines

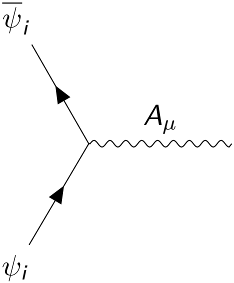


$= \frac{-i}{\not{p}-A-m+i0}$,

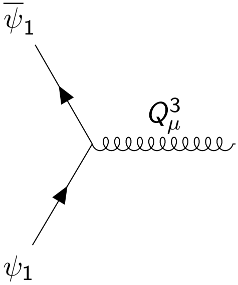


$= \frac{-i}{\not{p}+A-m+i0}$,

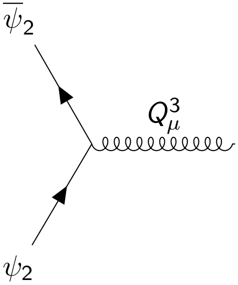
vertices



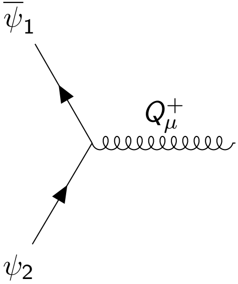
$$= \quad -ie\gamma_\mu, \qquad (i = 1, 2),$$



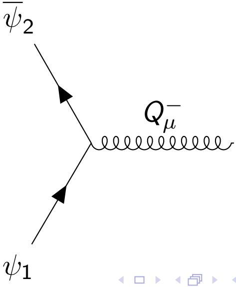
$= -i\frac{g}{2}\gamma_\mu,$



$= i\frac{g}{2}\gamma_\mu,$



$= -i\frac{g}{\sqrt{2}}\gamma_\mu,$



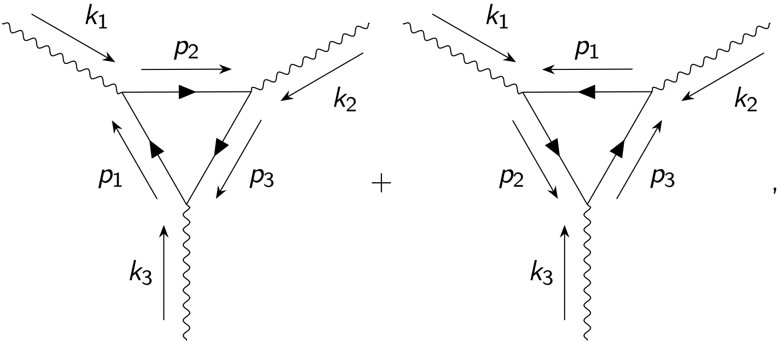
$= -i\frac{g}{\sqrt{2}}\gamma_\mu,$

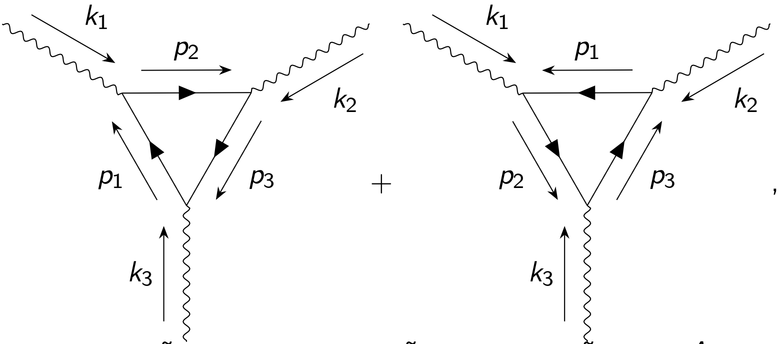
$\gamma\gamma G$-scattering

In QGP we expect non zero gluon fields Q_μ^a and scattering of photons on those fields

simplest process of such kind is given by Fermion triangle diagram, which is usually zero due to Furry's theorem

remember, from Feynmann rules, together with a Fermion loop we get also the same loop with lines in opposite direction





$$p_1 = \tilde{p}, \qquad p_1 = \tilde{p}, \qquad \tilde{p} = p - A_0,$$
$$p_2 = \tilde{p} + k_1, \qquad p_2 = \tilde{p} + k_3,$$
$$p_3 = \tilde{p} - k_3, \qquad p_3 = \tilde{p} - k_1,$$

$$\Gamma_1(A) = \int \frac{dp}{(2\pi)^4} \mathrm{Tr} \gamma_{\mu_1} S(p_2 - A_0) \gamma_{\mu_2} S(p_3 - A_0) \gamma_{\mu_3} S(p_1 - A_0),$$
$$\Gamma_2(A) = \int \frac{dp}{(2\pi)^4} \mathrm{Tr} \gamma_{\mu_1} S(p_1 - A_0) \gamma_{\mu_2} S(p_2 - A_0) \gamma_{\mu_3} S(p_3 - A_0),$$

and have $\Gamma = \Gamma_1(A) + \Gamma_2(A)$

Transformation

use $\mathrm{Tr}(\dots) = \mathrm{Tr}(\dots)^\top$ and

$$C = i\gamma^2\gamma^0, \quad (\gamma^\mu)^\top = -C^{-1}\gamma^\mu C, \quad S(p-A)^\top = -C^{-1}S(-p+A)C$$

as well as do $p \to -p$ in integration,
get this way,

$$\Gamma_2(A) = \int \frac{dp}{(2\pi)^4} \mathrm{Tr}(-1)^3 \gamma_{\mu_1} S(p_2 + A_0)\gamma_{\mu_2} S(p_3 + A_0)\gamma_{\mu_3} S(p_1 + A_0),$$

momenta from Γ_2, in this order, are just the momenta in Γ_1 and we get

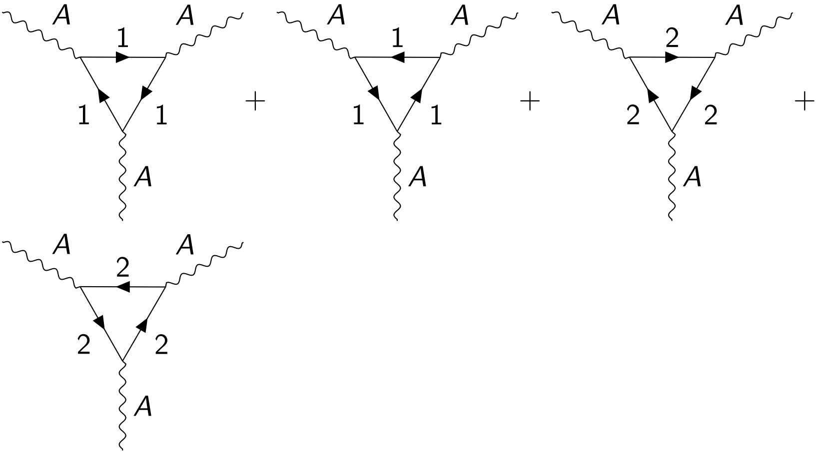
$$\Gamma_2(A) = -\Gamma_1(-A)$$

This way, we get

$$\Gamma = \Gamma_1(A) - \Gamma_1(-A)$$

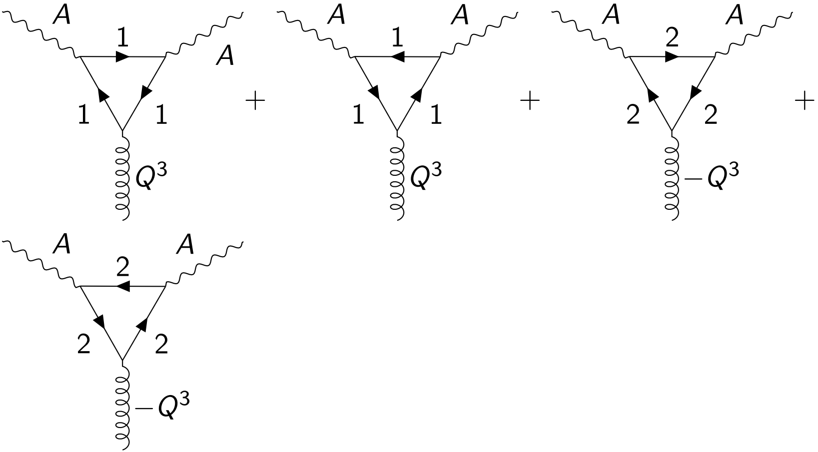
In QED we have $\Gamma = \Gamma_1(0) - \Gamma_1(0) = 0$, which is just Furry's theorem.

In QCD we have



$$
\begin{aligned}
\Gamma &= \Gamma_1(A) + \Gamma_2(A) + \Gamma_1(-A) + \Gamma_2(-A) \\
&= \Gamma_1(A) - \Gamma_1(-A) + \Gamma_1(-A) - \Gamma_1(A) \ = 0
\end{aligned}
$$

2 photons and the neutral gluon



$$\begin{aligned}\Gamma &= \Gamma_1(A) + \Gamma_2(A) - \Gamma_1(-A) - \Gamma_2(-A)\\ &= \Gamma_1(A) - \Gamma_1(A) - \Gamma_1(-A) + \Gamma_1(A) \qquad = 2\left(\Gamma_1(A) - \Gamma_1(-A)\right)\end{aligned}$$

this contribution, uneven in A_0, gives the effect we are interested in

For finite temperature, turn to Euclidean version
note the $4^{\rm th}$ components are $p_4 = 2\pi T l - A_0$ and $k_4 = 2\pi T n$, where A_0 is the background potential. The convention for the momenta are $\underline{p}_1 = \underline{p}$, $\underline{p}_2 = \underline{p} + \underline{k}_1$ and $\underline{p}_3 = \underline{p} - \underline{k}_3$ with $\sum_{i=1}^{3} \underline{k}_i = 0$.
The temperature dependent part of the vertex is

$$\Delta_T \Gamma(k_1, k_2, k_3) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2\Gamma} \sum_{i=1}^{3} \left(n_{A_0} M_i + \text{c.c.} \right),$$

where

$$M_1 = \frac{Z(i\Gamma, \mathbf{p})}{[\underline{k}_1^2 + 2ik_{14}\Gamma + 2\mathbf{p}\mathbf{k}_1]\ [\underline{k}_3^2 - 2ik_{34}\Gamma - 2\mathbf{p}\mathbf{k}_3]},$$

$$M_2 = \frac{Z(-k_{14} + i\Gamma, \mathbf{p} - \mathbf{k}_1)}{[\underline{k}_1^2 - 2ik_{14}\Gamma - 2\mathbf{p}\mathbf{k}_1 + 2\mathbf{k}_1^2]\ [\underline{k}_2^2 + 2ik_{24}\Gamma + 2\mathbf{p}\mathbf{k}_2]},$$

$$M_3 = \frac{Z(k_{34} + i\Gamma, \mathbf{p} + \mathbf{k}_3)}{[\underline{k}_3^2 + 2ik_{34}\Gamma + 2\mathbf{p}\mathbf{k}_3 + 2\mathbf{k}_3^2]\ [\underline{k}_2^2 - 2ik_{24}\Gamma - 2\mathbf{p}\mathbf{k}_2]},$$

($\pm$c.c. means to add/subtract the complex conjugated, $\Gamma = \sqrt{p^2 + m^2}$)

Boltzmann factor

$$n_{A_0} = \frac{\epsilon}{\exp(\beta(\Gamma + iA_0)) - \epsilon}.$$

($\epsilon = -1$ for fermions) this is the only place where A_0 enters.
The factors in the numerator are

$$\begin{aligned} Z(p_4, \not p) &= \mathrm{Tr}\ \gamma_{\mu_1}(\not p_{\mu_2} + m)\gamma_{\mu_2}(\not p_3 + m)\gamma_{\mu_3}(\not p_1 + m) \\ &= 4m^2\left(\delta_{\mu_1\mu_2}(p + k_3) + \delta_{\mu_1\mu_3}(p + k_1 - k_3) + \right. \\ &\qquad \left. \delta_{\mu_2\mu_3}(p - k_2)\right) + O(m^0), \end{aligned}$$

the m^2-contribution is leading given a large quark mass
Further evaluation will include taking the high-T limit, carrying out the angular integration (one can be carried out analytically), and taking the contribution linear in A_0.

Conclusions

We have demonstrated that in the QGP processes of scattering of photons on the plasma background are possible.
The basic contribution comes from the triangle quark loop which couples 'white' (photon) and 'color' (gluon) states.
Basic ingredients are the A_0-condensate (Polyakov loop) and a color neutral background field Q_0^3 (effective charge), which can be found from the minimum of the effective action or from lattice calculations.
Next steps will be to insert known values (estimates) for the condensate, the effective charge (Q_0^3), and the quark mass in order to get numbers.

Thank you for attention