Nuclear liquid-gas phase transition in realistic models of neutron star matter

Konstantin A. Maslov
in collaboration with E. E. Kolomeitsev and D. N. Voskresensky

National Research Nuclear University (MEPhI), Moscow, Russia
Joint Institute for Nuclear Research (JINR), Dubna, Russia
Matej Bel University, Banska Bystrica, Slovakia

JINR, 2018

Supported by RSF project 17-12-01427
& BASIS foundation (K.M.)

Introduction

- The equation of state (EoS) of strongly interacting hadronic matter in various regimes of density n, temperature T and isospin asymmetry $\beta = \frac{n_n - n_p}{n}$ is necessary for description of:
  - Neutron stars (NSs) : $T = 0$, $n \gg n_0$, asymmetric $\beta \sim 1$
  - Heavy-ion collisions (HICs): $T \sim m_\pi$, $n \gg n_0$, nearly symmetric $\beta \sim 0$
  - Supernova explosions: $T \sim (20 - 50)$ MeV, $n \gg n_0$, asymmetric $0 < \beta \lesssim 1$
- Constraints from the NS observations can be used to select a model parametrization to be used for generalization to finite temperatures for being used in HIC/supernovae simulations
  This requires a unified hadronic EoS with many degrees of freedom included
- Any EoS is characterized by a maximum NS mass it can support from a gravitational collapse
  A viable EoS model should pass the observed maximum NS mass constraint $M > 2.01 \pm 0.04\, M_\odot$ and many other $T = 0$ constraints.

Hyperon/Δ puzzle

For realistic hyperon interaction with an increase of the density already at $n \gtrsim 2 \div 3\, n_0$ the conversion nucleons convert to more massive baryon species:

- Hyperons [N.K. Glendenning ApJ 293 (1985)], recent review [I. Vidana arXiv:1803.00504]
- Δ-isobars [A. Drago et al. Phys.Rev. C90 (2014), B.-J. Cai et al. Phys.Rev. C92 (2015)]

In standard realistic models the maximum NS mass decreases below the observed values.

Problem can be resolved in relativistic mean-field (RMF) models by taking into account a hadron mass and couplings in-medium modifications + inclusion of ϕ-meson

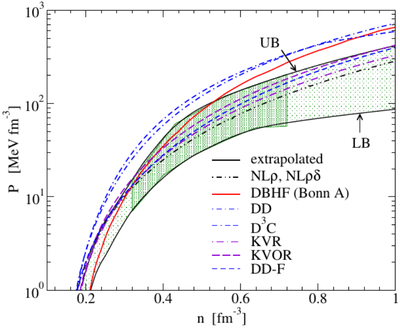
- Hyperons: [K. A. Maslov, E. E. Kolomeitsev and D. N. Voskresensky, Phys. Lett. B **748**, 369 (2015)]
- Δ-puzzle: [E. E. K., K. A. M. and D. N. V., NPA 961 (2017)]

High-density EoS: contradicting constraints

Constraint for the pressure, obtained from analyses of transverse and elliptic flows in heavy-ion collisions

Passed by rather soft EoSs

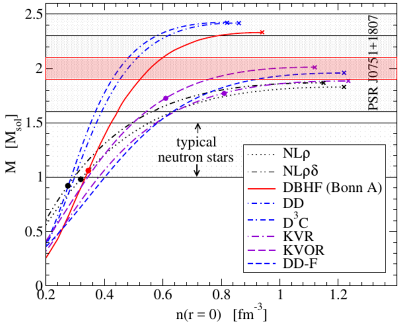
[P. Danielewicz, R. Lacey, W.G. Lynch, Science 298 (2002)]



The maximum NS mass constraint favors stiff EoS

NS cooling data ⇒ direct URCA (DU) is not operative for most stars

⇒ constraint for the proton fraction



figures from [T. Klahn et al. PRC74 (2006)]

Low-density EoS: liquid-gas phase transition

The 1^{st} order PT from the nuclear liquid to the gas of nucleons – well-known phase transition at low temperature and densities below the nuclear saturation density.
In the isospin-symmetric matter the equilibrium conditions read:

$$P^I = P^{II}, \quad \mu_B^I = \mu_B^{II}$$

Not hard to describe within RMF models:

- Low densities $n \leq n_0$ – no baryons except nucleons
- Low temperatures $T \lesssim 20$ MeV – can neglect thermal excitations of mesons

Important for describing low-energy ion collisions and supernovae

Outline

1. RMF framework with scaled hadron masses and couplings at finite temperature
2. Liquid-gas PT in the symmetric matter
3. Isospin-asymmetric case
4. Summary

EoS frameworks

Microscopic

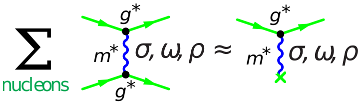
- Based on baryon-baryon potential + a many-body method
- Robust at low densities, large uncertainties at large densities
- Non-relativistic – acausal at large densities

Phenomenological

- Relatively simple models with parameters fitted to describe the experimental data / robust theoretical results
- Causal for all densities - important for NSs and HICs

Relativistic mean-field models

Meson-exchange picture of the interaction with classical meson fields



Additional flexibility needed to describe all the data

- Density-dependent couplings
- Various meson fields and self-interactions
- Field-dependent couplings and meson masses

RMF model with scaled hadron masses and couplings

E. E. Kolomeitsev and D. N. Voskresensky NPA 759 (2005) 373

- Walecka-type model with in-medium change of masses and coupling constants of all hadrons in terms of the scalar field σ:

$$m_i^* = m_i \Phi_i(\sigma),\ g_{mB}^* = g_{mB}\chi_m(\sigma),$$
$$m = \{\text{mesons}\},\ B = \{\text{baryons}\},\ i = B \cup m$$

- Common decrease of hadron masses [Brown, Rho Phys. Rev. Lett. 66 (1991) 2720; Phys. Rept. 363 (2002) 85]:

$$\frac{m_N^*}{m_N} \simeq \frac{m_\sigma^*}{m_\sigma} \simeq \frac{m_\omega^*}{m_\omega} \simeq \frac{m_\rho^*}{m_\rho}$$

- In the infinite matter only $\eta_m(\sigma) = \frac{\Phi_m^2(\sigma)}{\chi_m^2(\sigma)}$ enter the EoS - we define them phenomenologically to pass the constraints

Below we use the dimensionless scalar field $f(n) \equiv \dfrac{g_{\sigma N}\chi_\sigma(\sigma)\sigma}{m_N}$

Generalized relativistic mean-field model

E. E. K., K. A. M. and D. N. V., NPA 961 (2017)

$$
\begin{aligned}
&\mathcal{L} = \mathcal{L}_{\rm bar} + \mathcal{L}_{\rm mes} + \mathcal{L}_l,\\
&\mathcal{L}_{\rm bar} = \sum_{i=b\cup r} (\bar{\Psi}_i \left(iD_\mu^{(i)}\gamma^\mu - m_i \Phi_i(\sigma)\right) \Psi_i,\\
&D_\mu^{(i)} = \partial_\mu + i g_{\omega i}\chi_{\omega i}(\sigma)\omega_\mu + i g_{\rho i}\chi_{\rho i}(\sigma)\vec{t}\vec{\rho}_\mu + i g_{\phi i}\chi_{\phi i}(\sigma)\phi_\mu,\\
&\{b\} = (N, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}, \Delta^-, \Delta^0, \Delta^+, \Delta^{++})\\
&\mathcal{L}_{\rm mes} = \frac{\partial_\mu\sigma\partial^\mu\sigma}{2} - \frac{m_\sigma^2\Phi_\sigma^2(\sigma)\sigma^2}{2} - U(\sigma) +\\
&+\frac{m_\omega^2\Phi_\omega^2(\sigma)\omega_\mu\omega^\mu}{2} - \frac{\omega_{\mu\nu}\omega^{\mu\nu}}{4} + \frac{m_\rho^2\Phi_\rho^2(\sigma)\vec{\rho}_\mu\vec{\rho}^\mu}{2} - \frac{\rho_{\mu\nu}\rho^{\mu\nu}}{4} +\\
&+\frac{m_\phi^2\Phi_\phi^2(\sigma)\phi_\mu\phi^\mu}{2} - \frac{\phi_{\mu\nu}\phi^{\mu\nu}}{4},\\
&\omega_{\mu\nu} = \partial_\nu\omega_\mu - \partial_\mu\omega_\nu, \quad \vec{\rho}_{\mu\nu} = \partial_\nu\vec{\rho}_\mu - \partial_\mu\vec{\rho}_\nu + g_\rho\chi'_\rho[\vec{\rho}_\mu\times\vec{\rho}_\nu],\\
&\phi_{\mu\nu} = \partial_\nu\phi_\mu - \partial_\mu\phi_\nu,\\
&\mathcal{L}_l = \sum_l \bar{\psi}_l(i\partial_\mu\gamma^\mu - m_l)\psi_l, \quad \{l\} = (e, \mu).
\end{aligned}
$$

Finite T: Pressure

$$P[\mu_B,\mu_Q,f,T] = T\sum_b (2S_b+1)\int\limits_0^\infty \frac{dp\,p^2}{2\pi^2}\ln[1+e^{-\beta(\epsilon_b^*(p)-\mu_b^*)}] - \frac{m_N^4 f^2}{2C_\sigma^2}\eta_\sigma(f)$$

$$+\frac{C_\omega^2}{2m_N^2\eta_\omega(f)}n_V^2 + \frac{C_\rho^2}{2m_N^2\eta_\rho(f)}n_I^2, \quad \epsilon_b^*(p) = \sqrt{p^2+m_b^{*2}}, \quad \beta = 1/T$$

$$\mu_b^* = \mu_B - x_{\omega b}\frac{C_\omega^2 n_V}{m_N^2\eta_\omega(f)} - t_{3b}x_{\rho b}\frac{C_\rho^2 n_I}{m_N^2\eta_\rho(f)} + Q_b\mu_Q + S_b\mu_S$$

$$n_V = \sum_b x_{\omega b}n_b, \quad n_I = \sum_b x_{\rho b}t_{3b}n_b, \quad n_b = (2S_b+1)\int\limits_0^\infty \frac{dp\,p^2}{2\pi^2} f_b(p;\mu_b^*,T),$$

$$f_b(p;\mu,T) = \frac{1}{1+e^{\beta(\epsilon_b^*(p)-\mu)}}, \quad Q_b, S_b - \text{charge and strangeness of a baryon } b$$

Scaling functions

In the homogeneous medium $\eta_M = \Phi_M^2(f)/\chi_{Mb}^2(f)$,

$$\Phi_N(f) = \Phi_m(f) = 1-f, \text{ universal scaling of hadron masses}$$

$$\frac{\partial P}{\partial f} = 0 - \text{e.o.m. for the scalar field}$$

Working models

Initial model: KVOR [E.E.K., D.N.V. NPA 759 (2005)] described many constraints, but only without hyperons $\Rightarrow$ need for enhancement
Contributions of ω, ρ mesons to pressure couple to the scalar field.
"Cut" mechanism: rapid decrease of $\eta_m(f)$ quenches the growth of the scalar field $f(n)$ and leads to the stiffening of an EoS
[K.A.M, E.E.K., D.N.V. PRC 92 (2015)].

KVORcut03	MKVOR*
Based on KVOR	More involved parameterization
Sharp decrease in $\eta_\omega(f)$	Sharp decrease in $\eta_\rho(f)$
Stiff in NS matter	Stiff in NS matter.
and nuclear matter	soft in nuclear matter

- Pass the flow constraint
- Pass the maximum NS mass constraint with both **hyperons** (with help of ϕ-meson) and Δ-**isobars** included

Scalar sector version of this method was successfully employed in recent work [H.Pais, C.Providência PRC94 (2016), M.Dutra et al. PRC93 (2016), Y.Zhang et al. PRC97 (2018)]

Low-density: bulk nuclear matter properties

Energy per particle expansion:

$$\mathcal{E} = \mathcal{E}_0 + \frac{K}{18}\epsilon^2 - \frac{K^{'}}{162}\epsilon^3 + ... + \beta^2 \left(\mathcal{E}_{\rm sym} + \frac{L}{3}\epsilon + \frac{K_{\rm sym}}{18}\epsilon^2 ...\right),$$

$$\epsilon = (n - n_0)/n_0, \quad \beta = [(n_n - n_p)/n_0]_{n_0}$$

Coefficients are accessible experimentally and are to be used to determine $C_\sigma, C_\omega, C_\rho$ and parameters of the scaling function $\eta_\sigma(f)$.
We adopt the values consistent with available data within uncertainties

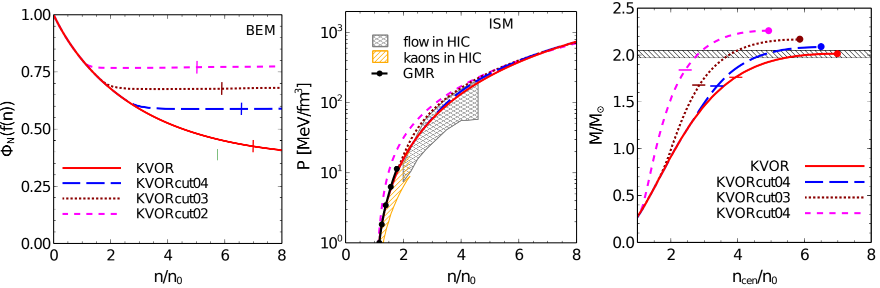
$$n_0 = 0.16 \text{ fm}^{-3}, \quad \mathcal{E}_0 = -16 \text{ MeV}, \quad K = 250 \text{ MeV},$$

KVORcut03: $\mathcal{E}_{\rm sym} = 32$ MeV, $\quad m_N^*(n_0)/m_N = 0.805$
MKVOR : $\mathcal{E}_{\rm sym} = 30$ MeV, $\quad m_N^*(n_0)/m_N = 0.73$

KVORcut models

Example of introducing a sharp decrease into $\eta_\omega(f)$:

$$\eta_\omega^{\mathrm{KVOR}}(f) \to \eta_\omega^{\mathrm{KVOR}}(f) - \frac{a_\omega}{2}[1 + \tanh(b_\omega(f - f_{\mathrm{cut},\omega}))]$$

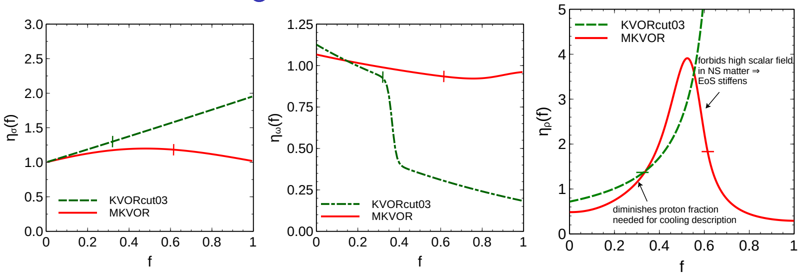


- KVOR model can be stiffened enough to have a high maximum NS mass
- KVORcut03 is the most realistic (flow constraint)

MKVOR model

The procedure can be applied to the isospin-asymmetric part ($\eta_\rho(f)$)
Does not change symmetric matter EoS, but stiffens the asymmetric part

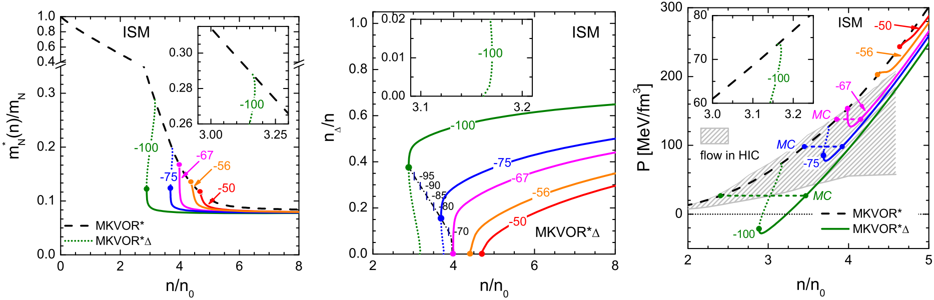
Choice of the scaling functions



$\eta_\sigma(f)$: governs low density ($n \lesssim 2.5\, n_0$) behavior – needed for passing flow constraint
$\eta_\omega(f)$: needed to pass flow constraint at higher n
$\eta_\rho(f)$: sharp increase at low f lowers proton fraction – needed for DU constraint
sharp decrease at $f \gtrsim 0.6$ – "cut"-mechanism for stiffening the EoS of NS matter

MKVOR*: $T = 0$ features with Δ

E. E. K., K. A. M. and D. N. V., NPA 961 (2017)



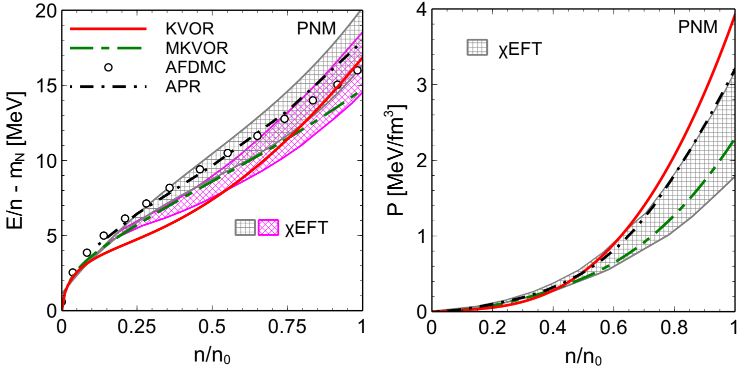
- 1^{st} order phase transition for $U_\Delta < -56\,\mathrm{MeV}$.
- Could manifest itself as an increase of the pion yield at typical energies and momenta corresponding to the $\Delta \to \pi N$ decays
- For $U_\Delta < -65$ MeV the pressure curve lies within the constraint.

MKVOR/MKVOR* are the same in the liquid-gas density area

Low-density behavior of EoSs

Comparison with the results of:

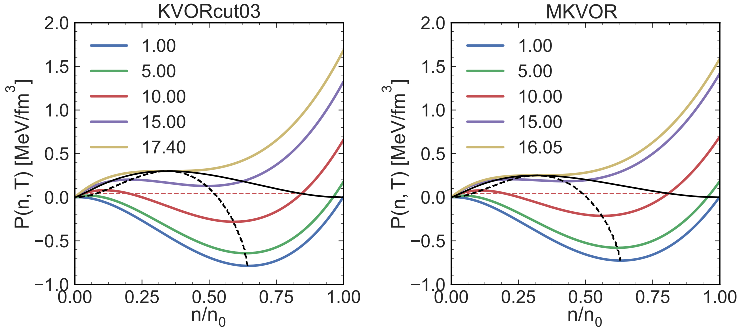
- Chiral effective field theory (χEFT), [K, Hebeler et al. EPJ A50 (2014)]
- Auxiliary field diffusion Monte-Carlo [S. Gandolfi et al. MNRAS 404 (2010)]
- APR EoS



MKVOR is consistent with χEFT at low densities despite the parameterization was chosen basing only on the high-density properties

Liquid-gas PT - results

Pressure for various temperatures T [MeV]

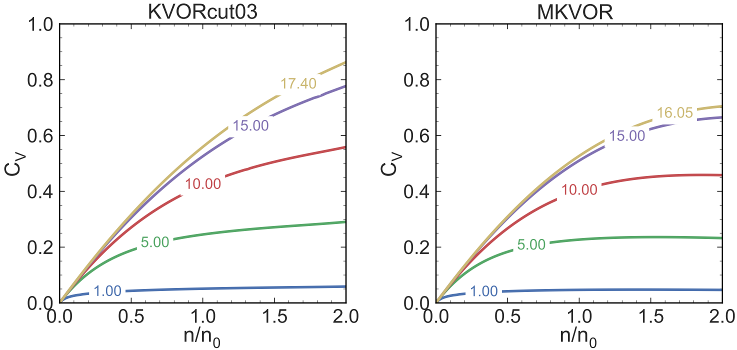


Solid lines - phase coexistence region, dashed lines - spinodal region with $v_s^2 = \frac{dP}{dE} < 0 \Rightarrow$ mechanically unstable

Heat capacity

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V$$

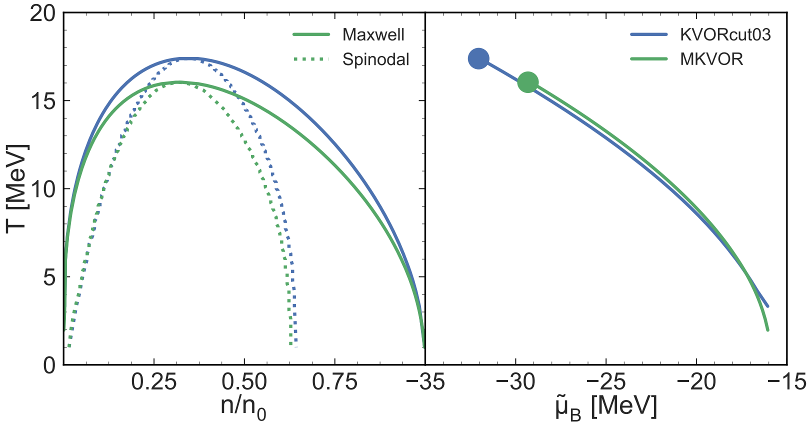
C_V as a function of the density and temperature is continuous:



$C_V = A(\frac{T}{T_c} - 1)^\alpha \Rightarrow$ mean-field universality class: $\alpha = 0$

Critical temperature

$T_c[\text{KVORcut03}] = 17.4$ MeV, $T_c[\text{MKVOR}^*] = 16.04$ MeV

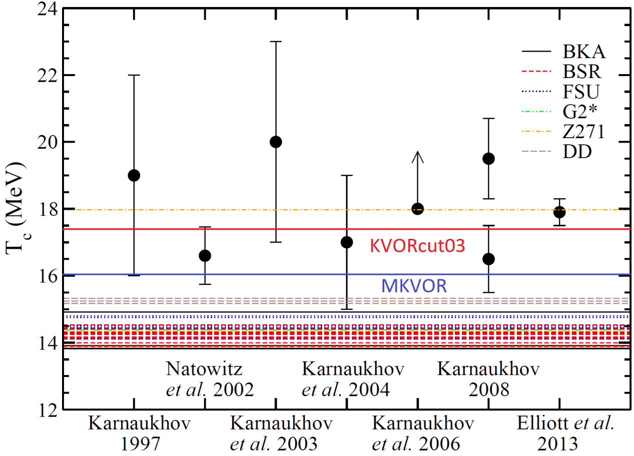


$$\tilde{\mu}_B \equiv \mu_B - m_N$$

T_c for MKVOR is lower than for KVORcut03 because of the lower effective nucleon mass $m_N^*(n_0)$ at saturation. It supports the results of a systematic RMF parameter variation [O. Lourenço et al. PRC 94(4) (2016)]

Experimental T_c

Figure adapted from [O. Lourenço et al. PRC 95 (2017)]

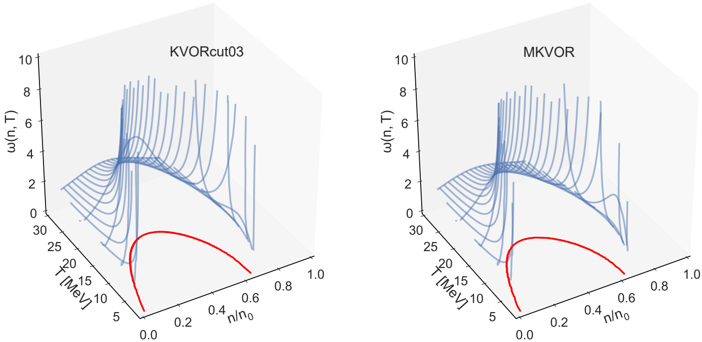


T_c in our models is higher than in most part of traditional RMF models, which is supported by the data

Isospin symmetric case - scaled variance

Quantity characterizing the particle number fluctuations in an event-by-event analysis, $\langle \ldots \rangle$ – event-by-event averaging

$$\omega[n,T] = \frac{\langle (N - \langle N \rangle)^2 \rangle}{\langle N \rangle} = \frac{T}{n}\left(\frac{\partial n}{\partial \mu}\right)_T$$



Red line - border of the spinodal region where the variance diverges

Liquid-gas at finite isospin density

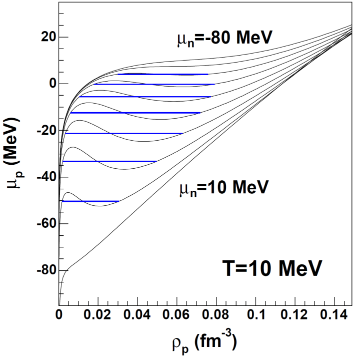
Heavy nuclei are not symmetric (e.g. $Y_p = Z/A \simeq 0.4$ for Au + Au); supernova simulations require EoS of warm asymmetric matter

Construction of the PT:

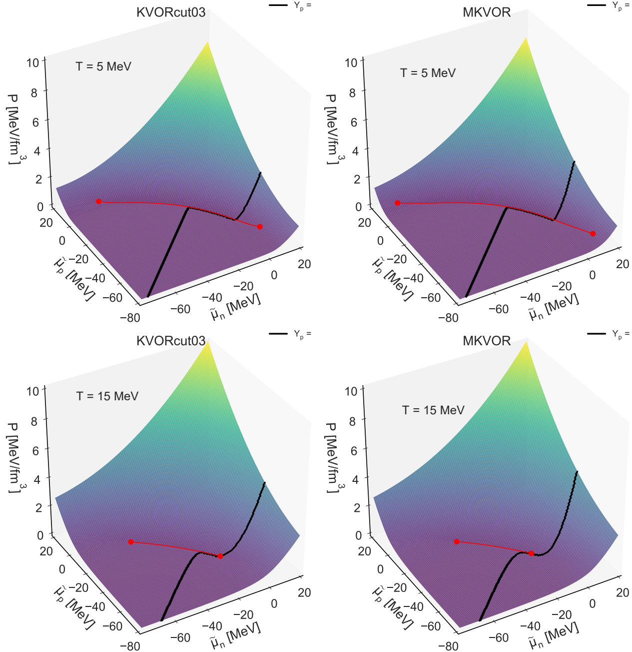
- Continuity of two chemical potentials:

$$\mu_B^I = \mu_B^{II}, \quad \mu_Q^I = \mu_Q^{II}.$$

- Easy way to solve: use the mixed thermodynamic potential $\Omega'[n_p, \mu_B]$ and perform a Maxwell construction in terms of n_p for a given μ_B [Ducoin Chomaz Gulminelli NPA 771 (2006)]



Phase transition with isospin asymmetry

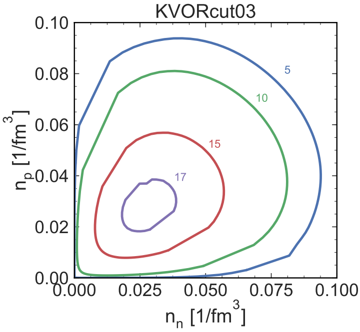


Red line - critical line

Black line - trajectory of constant $Y_p = 0.3$

Critical areas in $n_n - n_p$ plane

Phase coexistence borders for various temperatures



Isospin asymmetry works against the phase transition
Shape of the coexistence borders depends on the L parameter ($L \simeq 40$ MeV for MKVOR and $\simeq 70$ MeV for KVORcut03), not contradicting to findings of [N. Alam et al. PRC 95(5) (2017)] where the variation of L was studied.

Summary

- RMF models with scaled hadron masses and couplings constructed to describe neutron stars give reasonable properties of the nuclear liquid-gas phase transition
- Critical temperature T_c is lower in the MKVOR model due to the lower effective nucleons mass
- Weak model dependence, no anomalies in the MKVOR model

Prospective study

Larger temperatures relevant to HICs at NICA:

- Inclusion of higher hadronic multiplets
- Thermal excitations of mesons

Supernovae:

- Inclusion of nuclear clusters at arbitrary Y_p