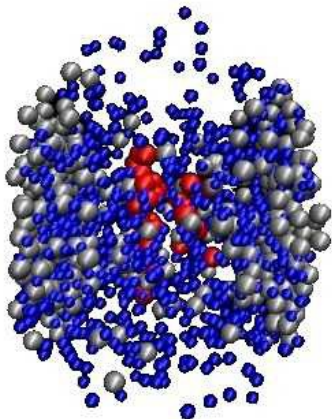


The QCD phase diagram within effective models

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Dubna, 16.04.2018



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H-QM | Helmholtz Research School
Quark Matter Studies



- **Hadronic EoS:**

The Interacting Hadron-Resonance Gas

- **Partonic EoS:**

The Dynamical QuasiParticle Model

- **Hadron-Parton transition in the $T-\mu_B$ -plane**

QCD phase diagram

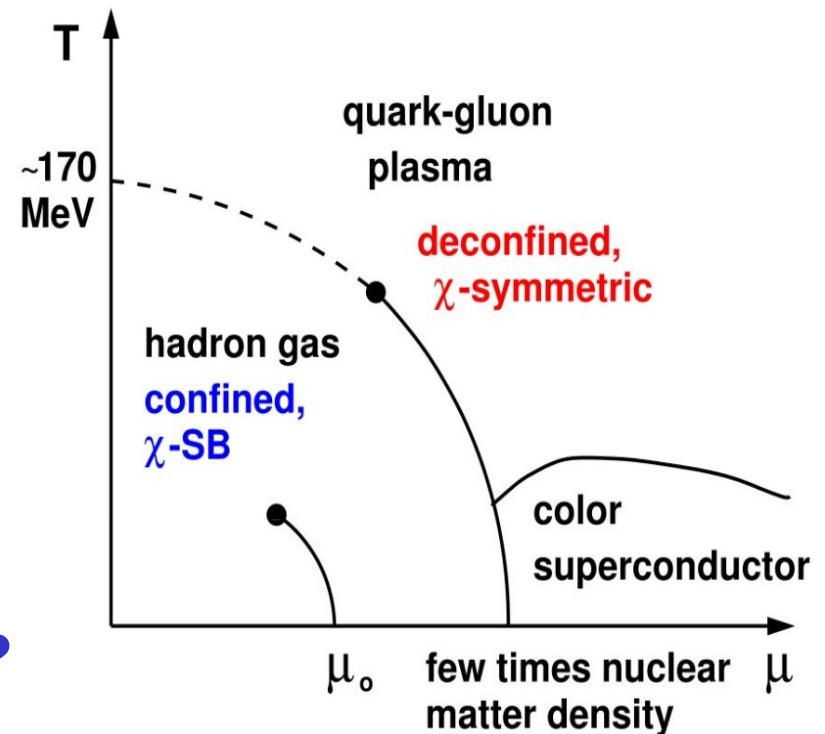
3

The QCD phase diagram consists of a hadronic phase with broken χ -symmetry at low T and μ_B and a partonic phase with restored χ -symmetry at large T and μ_B .

Transition is important for heavy-ion simulations.

FAIR and NICA probe the transition at finite μ_B .

Where is the transition in the T - μ_B plane and of what order?



Degrees of freedom

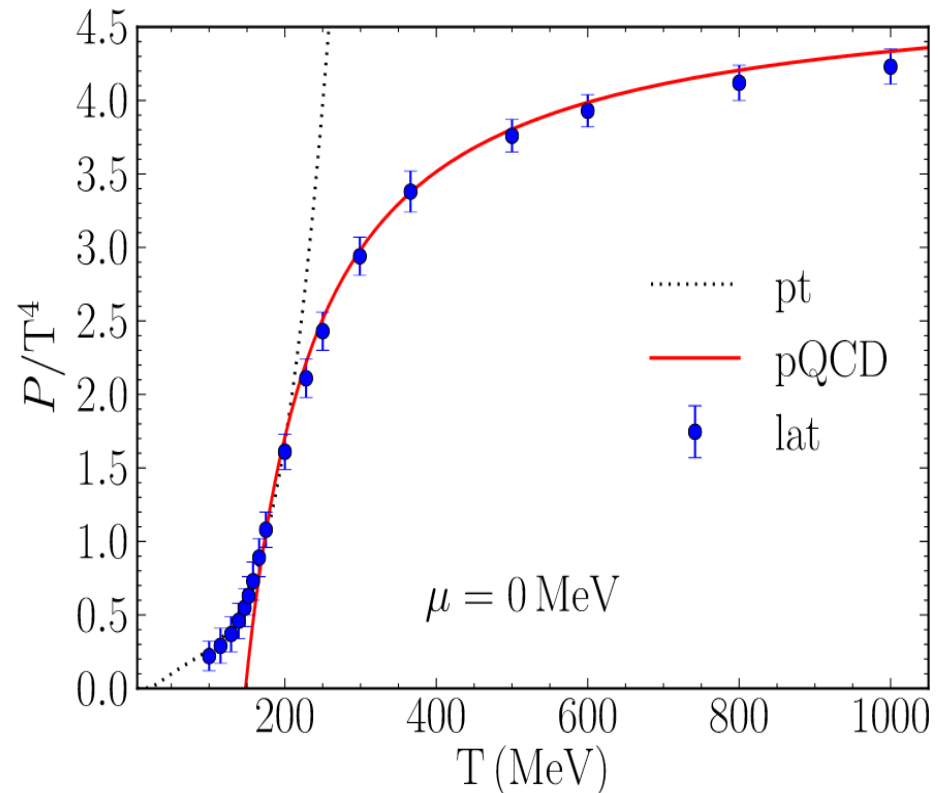
4

LQCD predicts the QCD EoS, but gives no informations about the degrees of freedom.

Hadronic models below T_c

Partonic models above T_c

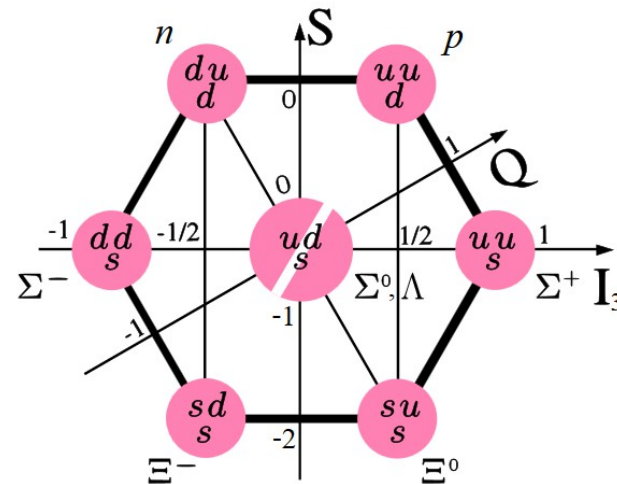
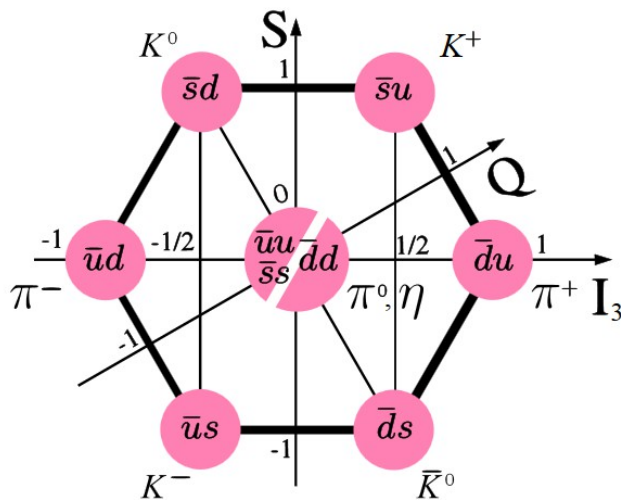
One needs to switch from hadrons to partons to describe the whole EoS.



Hadronic equation of state

Hadronic degrees of freedom 5

- Simplest model is a non-int. hadron resonance gas
- Relevant degrees of freedom at low temperatures are the 0- mesons and the spin 1/2 baryons:



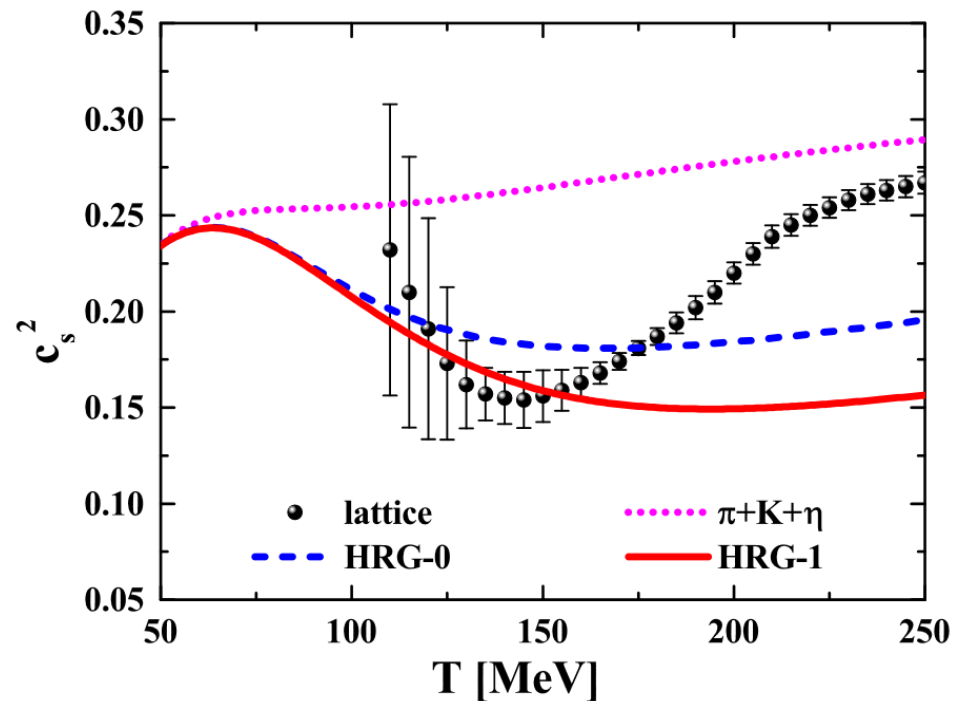
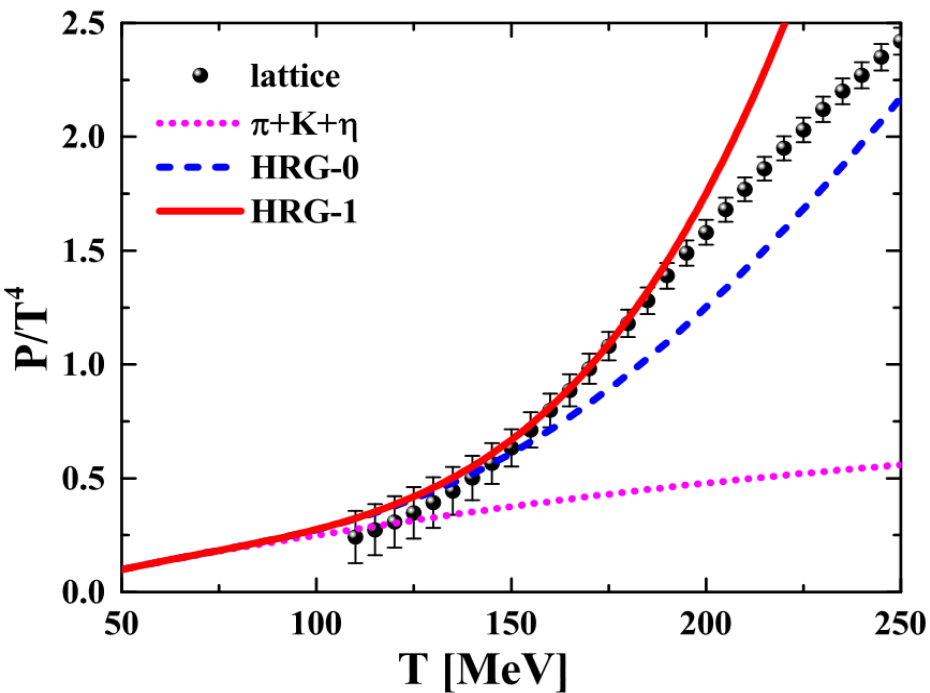
- 1- mesons and 3/2 baryons are important resonances
- Additional hadrons describe attractive interactions

Hadrons in a „standard“ HRG 6

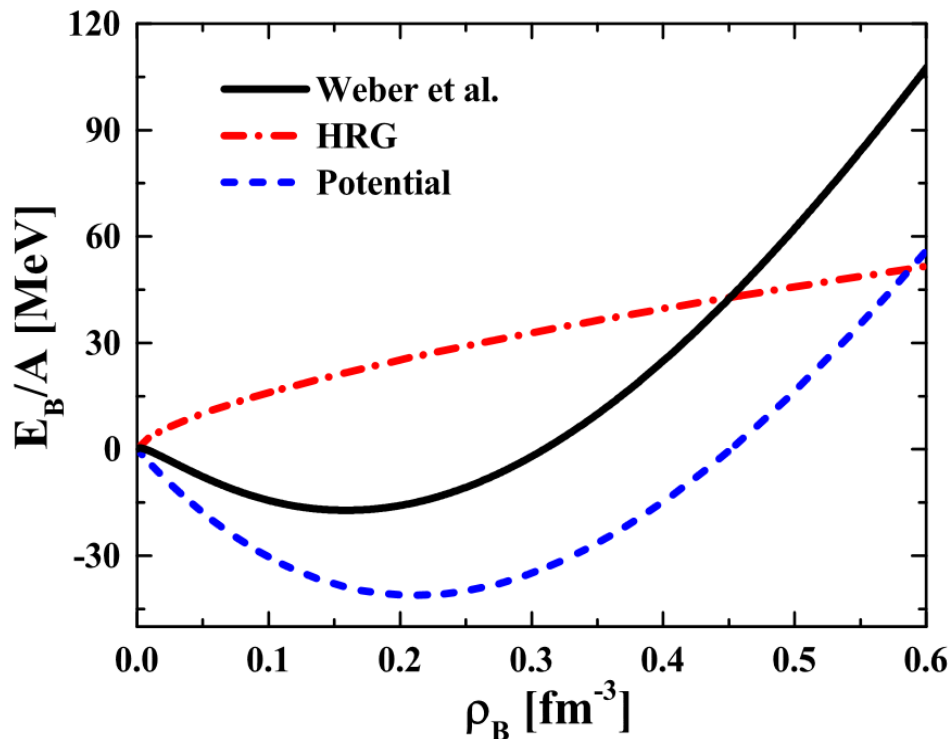
hadron	m_α (GeV)	degen	b_α	hadron	m_α (GeV)	degen	b_α	hadron	m_α (GeV)	degen	b_α
π^0	0.135	1	0	$K^{*0}(1430)$	1.432	10	0	$K_4^*(1780)$	1.776	28	0
π^\pm	0.140	2	0	$N(1440)$	1.440	4	1	$\Lambda(1800)$	1.800	2	1
K^\pm	0.494	2	0	$\rho(1450)$	1.465	9	0	$\Lambda(1810)$	1.810	2	1
K^0	0.498	2	0	$a_0(1450)$	1.474	3	0	$\pi(1800)$	1.812	3	0
η	0.548	1	0	$\eta(1475)$	1.476	1	0	$K_2(1820)$	1.816	20	0
ρ	0.775	9	0	$f_0(1500)$	1.505	1	0	$\Lambda(1820)$	1.820	6	1
ω	0.783	3	0	$\Lambda(1520)$	1.520	4	1	$\Xi(1820)$	1.823	8	1
$K^{*\pm}(892)$	0.892	6	0	$N(1520)$	1.520	8	1	$\Lambda(1830)$	1.830	6	1
$K^{*0}(892)$	0.896	6	0	$f_2'(1525)$	1.525	5	0	$\phi_3(1850)$	1.854	7	0
p	0.938	2	1	$\Xi^0(1530)$	1.532	4	1	$N(1875)$	1.875	8	1
n	0.940	2	1	$N(1535)$	1.535	4	1	$\Delta(1905)$	1.880	24	1
η'	0.958	1	0	$\Xi^-(1530)$	1.535	4	1	$\Delta(1910)$	1.890	8	1
a_0	0.980	3	0	$\Delta(1600)$	1.600	16	1	$\Lambda(1890)$	1.890	4	1
f_0	0.990	1	0	$\Lambda(1600)$	1.600	2	1	$\pi_2(1880)$	1.895	15	0
ϕ	1.019	3	0	$\eta_2(1645)$	1.617	5	0	$N(1900)$	1.900	8	1
Λ	1.116	2	1	$\Delta(1620)$	1.630	8	1	$\Sigma(1915)$	1.915	18	1
h_1	1.170	3	0	$N(1650)$	1.655	4	1	$\Delta(1920)$	1.920	16	1
Σ'	1.189	2	1	$\Sigma(1660)$	1.660	6	1	$\Delta(1950)$	1.930	32	1
Σ^0	1.193	2	1	$\pi_1(1600)$	1.662	9	0	$\Sigma(1940)$	1.940	12	1
Σ^-	1.197	2	1	$\omega_3(1670)$	1.667	7	0	$f_2(1950)$	1.944	5	0
h_2	1.230	9	0	$\omega(1650)$	1.670	3	0	$\Delta(1930)$	1.950	24	1
a_1	1.230	9	0	$\Lambda(1670)$	1.670	2	1	$\Xi(1950)$	1.950	4	1
Δ	1.232	16	1	$\Sigma(1670)$	1.670	12	1	$a_4(2040)$	1.996	27	0
$K_1(1270)$	1.272	12	0	$\pi_2(1670)$	1.672	15	0	$f_2(2010)$	2.011	5	0
f_2	1.275	5	0	Ω^-	1.673	4	1	$f_4(2050)$	2.018	9	0
f_1	1.282	3	0	$N(1675)$	1.675	12	1	$\Xi(2030)$	2.025	12	1
$\eta(1295)$	1.294	1	0	$\phi(1680)$	1.680	3	0	$\Sigma(2030)$	2.030	24	1
$\pi(1300)$	1.300	3	0	$N(1680)$	1.685	12	1	$K_4^*(2045)$	2.045	36	0
Ξ^0	1.315	2	1	$\rho_3(1690)$	1.689	21	0	$\Lambda(2100)$	2.100	8	1
a_2	1.318	15	0	$\Lambda(1690)$	1.690	4	1	$\Lambda(2110)$	2.110	6	1
Ξ^-	1.322	2	1	$\Xi(1690)$	1.690	4	1	$\phi(2170)$	2.175	3	0
$f_0(1370)$	1.350	1	0	$N(1700)$	1.700	8	1	$N(2190)$	2.190	16	1
$\pi_1(1400)$	1.354	9	0	$\Delta(1700)$	1.700	16	1	$N(2200)$	2.250	20	1
$\Sigma(1385)$	1.385	12	1	$N(1710)$	1.710	4	1	$\Sigma(2250)$	2.250	6	1
$K_1(1400)$	1.403	12	0	$K^*(1680)$	1.717	12	0	$\Omega^-(2250)$	2.252	2	1
$\Lambda(1405)$	1.405	2	1	$\rho(1700)$	1.720	9	0	$N(2250)$	2.275	20	1
$\eta(1405)$	1.409	1	0	$f_0(1710)$	1.720	1	0	$f_2(2300)$	2.297	5	0
$K^*(1410)$	1.414	12	0	$N(1720)$	1.720	8	1	$f_2(2340)$	2.339	5	0
$\omega(1420)$	1.425	3	0	$\Sigma(1750)$	1.750	6	1	$\Lambda(2350)$	2.350	10	1
$K_0^*(1430)$	1.425	4	0	$K_2(1770)$	1.773	20	0	$\Delta(2420)$	2.420	48	1
$K_2^{*\pm}(1430)$	1.426	10	0	$\Sigma(1775)$	1.775	18	1	$N(2600)$	2.600	24	1
$f_1(1420)$	1.426	3	0								

Hadronic equation of state 7

- One needs a lot of particles to describe the EoS.
- Speed of sound is wrong above $T=140$ MeV.



- Nuclear matter is a pure hadronic system with well known binding energy: $E_B/A = \epsilon/\rho_N - m_N$
- Non-interacting models fail for the nuclear EoS



Nuclear EoS requires a combination of attractive and repulsive interactions.

A popular model that contains both is the nonlinear Walecka model.

Relativistic mean-field theory 9

- **Nonlinear Walecka interaction for nucleons:**

$$\mathcal{L}_B = \bar{\Psi} (i\gamma_\mu \partial^\mu - M) \Psi$$

$$\mathcal{L}_M = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + O(\omega)$$

$$\mathcal{L}_{int} = \Gamma_\sigma(\rho_0) \bar{\Psi} \sigma \Psi - \Gamma_\omega(\rho_0) \bar{\Psi} \gamma^\mu \omega_\mu \Psi$$

The σ -interaction defines an effective mass:

$$m^* = m - \Sigma^s = m - \Gamma_\sigma(\rho_0) \sigma - \Sigma^{s(r)}$$

The ω -interaction defines an effective μ :

$$\mu^* = \mu - \Sigma^0 = \mu - \Gamma_\omega(\rho_0) \omega - \Sigma^{0(r)}$$

• We solve the model in mean-field approximation:

σ -equation of motion leads to attractive interaction:

$$\frac{\partial U}{\partial \sigma} = \Gamma_{\sigma}(\rho_0)\rho_s = \Gamma_{\sigma}(\rho_0) d \int \frac{d^3p}{(2\pi)^3} \frac{m^*}{E^*} (f(T, \mu_B^*, m^*) + f(T, -\mu_B^*, m^*))$$

ω -equation of motion leads to repulsive interaction:

$$\frac{\partial O}{\partial \omega} = \Gamma_{\omega}(\rho_0)\rho_B = \Gamma_{\omega}(\rho_0) d \int \frac{d^3p}{(2\pi)^3} (f(T, \mu_B^*, m^*) - f(T, -\mu_B^*, m^*))$$

Equation of state:

$$P = P_0(T, \mu^*, m^*) - U(\sigma) + O(\omega) + \Sigma^{0(r)}\rho_B - \Sigma^{s(r)}\rho_s$$

$$E = E_0(T, \mu^*, m^*) + U(\sigma) - O(\omega) + \Gamma_{\omega}(\rho_0)\omega\rho_B + \Sigma^{s(r)}\rho_s$$

RMF describes interactions in the t-channel.

Sufficient at small temperatures.

Resonance formation sets in at higher temperatures.

We describe this interaction as in the HRG and introduce important resonances as non-interacting hadrons:

$$\Omega_{IHRG} = \Omega_{RMF} + \Omega_{HRG} - \Omega_{0,N}$$

$U(\sigma)$ is mass term and selfinteractions of the σ -field:

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}B\sigma^3 + \frac{1}{4}C\sigma^4 + \dots$$

We determine $U(\sigma)$ from the lattice EoS at $\mu_B=0$:

The repulsive interaction vanishes: $\omega = 0, O(0) = 0, \mu_B^* = 0$

Entropy takes a simple form and depends only on m_N^* :

$$S_{IHRG}(T, \mu_B) = S_{HRG} - S_{free}^N + S_{free}^N(T, \mu_B^*, m_n^*)$$

LQCD defines $U(\sigma)$:

$$\left. \begin{array}{l} m_N^*(T) \rightarrow \sigma(T) \\ m_N^*(T) \rightarrow \rho_s(T) \rightarrow \frac{\partial U}{\partial \sigma}(T) \end{array} \right\} \Rightarrow U(\sigma)$$

Attractive interaction

13

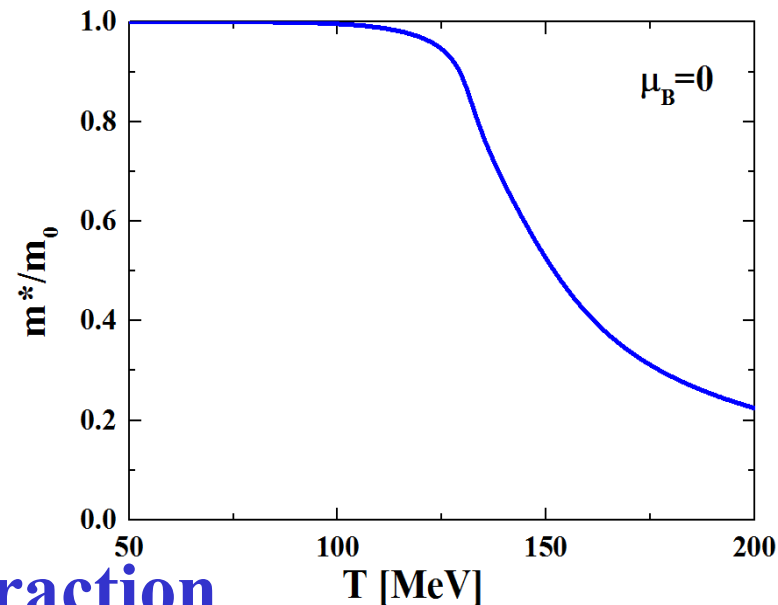
The effective mass contains all the information about the attractive interaction

$$m_N^*(T, \mu_B)$$

g_σ	m_σ [MeV]	B [1/fm]	C
28.64	550	-29.67	3837

σ^4 -term is the dominant contribution to $U(\sigma)$

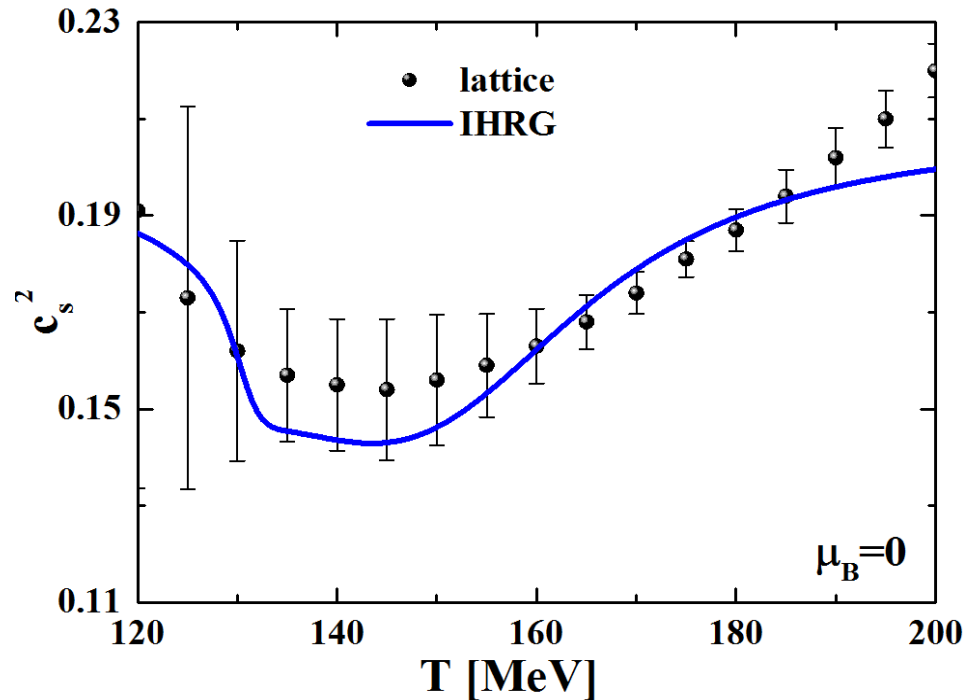
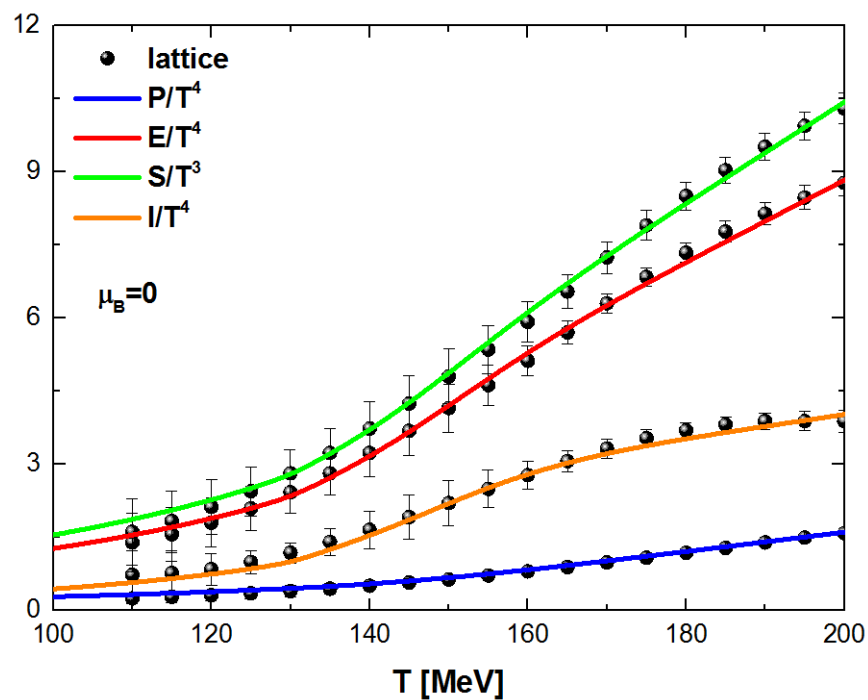
Interaction becomes important close to the phase transition



Attractive σ -interaction
compensates for missing resonances.

- Include important baryons with strong interactions and mesons as noninteracting particles.

Resulting EoS describes hadronic part of the EoS:



So far we include only nucleons.

Generalization possible, see [arXiv: 1803.10546](https://arxiv.org/abs/1803.10546)

Repulsive interaction

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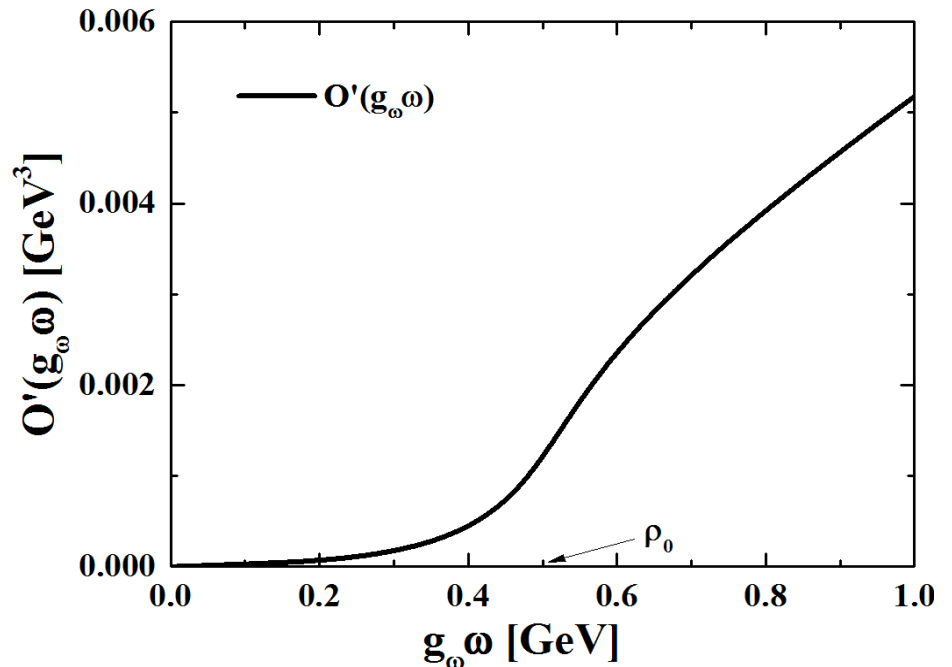
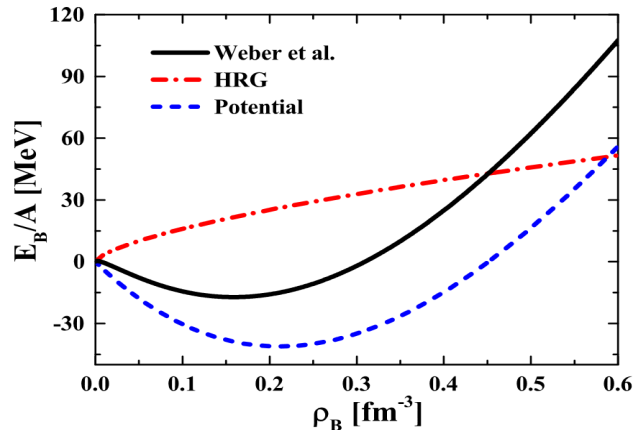
$O(\omega)$ is mass term and selfinteractions of the ω -field:

$$O(\omega) = \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{4}D\omega^4 + \dots$$

Use nuclear EoS as input:

$$O(\omega) = P - P_{g_\omega=0}^{RMF}$$

$$g_\omega\omega = \frac{(E + P) - (E + P)_{g_\omega=0}^{RMF}}{\rho_B}$$

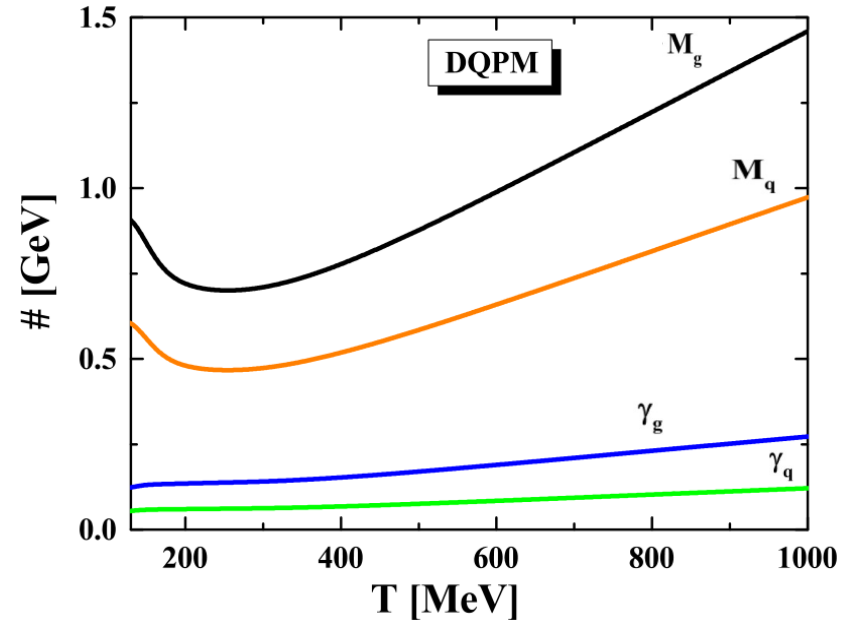
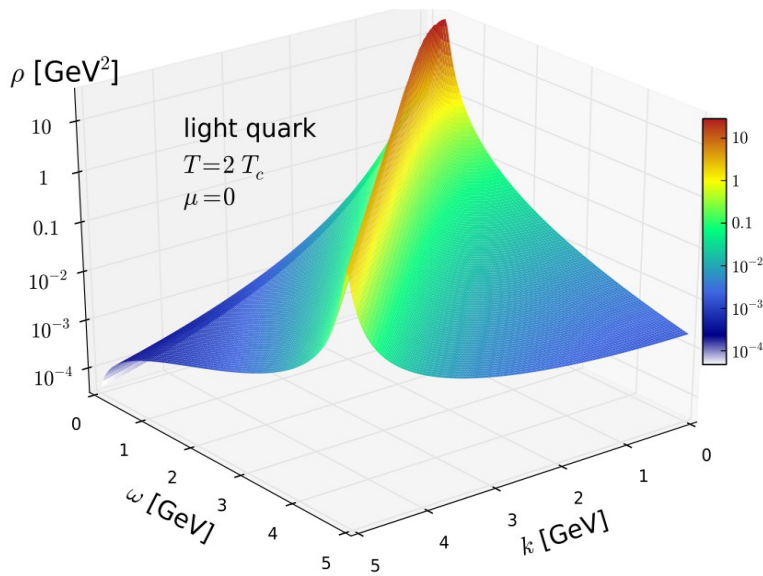


$U(\sigma)$ and $O(\omega)$ define the model in the whole T - μ_B -plane

EoS is consistent with lattice and nuclear EoS

Partonic equation of state

Partons with Breit-Wigner spectral functions:



Mass & width motivated by HTL

$$M \sim g T$$

The width is an additional „parameter“
 to be controlled by „correlators“.

$$\gamma \sim g^2 T$$

Quasiparticle thermodynamics 17

$$A(\omega, \mathbf{p}) = \frac{2\gamma\omega}{(\omega^2 - \mathbf{p}^2 - M^2)^2 + 4\gamma^2\omega^2}$$

$$G(\omega, \mathbf{p}) = \frac{-1}{\omega^2 - \mathbf{p}^2 - M^2 + 2i\gamma\omega} = \frac{-1}{\omega^2 - \mathbf{p}^2 - \Sigma}$$

- **Entropy and density for a given propagator D:**

$$S/V = -d \int \frac{d^3k}{(2\pi)^3} \frac{d\omega}{2\pi} \frac{\partial n_{B/F}}{\partial T} (\text{Im}(\ln G^{-1}) - \text{Re}(G)\text{Im}(\Sigma))$$

$$N/V = -d \int \frac{d^3k}{(2\pi)^3} \frac{d\omega}{2\pi} \frac{\partial n_{B/F}}{\partial \mu} (\text{Im}(\ln G^{-1}) - \text{Re}(G)\text{Im}(\Sigma))$$

In the on-shell limit $\gamma \rightarrow 0$ they reduce to the non-interacting entropy and particle density.

Quasiparticles are very heavy, they can not reproduce the perturbative massless propagators.

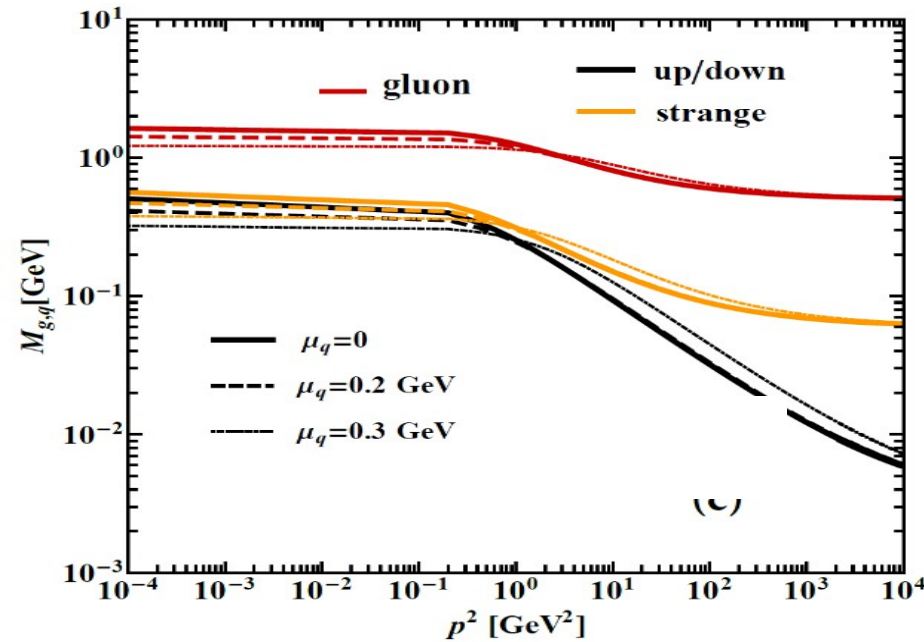
We introduce a mom. dep. correction factor:

$$h(\Lambda, \mathbf{p}) = \frac{1}{\sqrt{1 + \Lambda \cdot \mathbf{p}^2 \cdot (T_c/T)^2}}$$

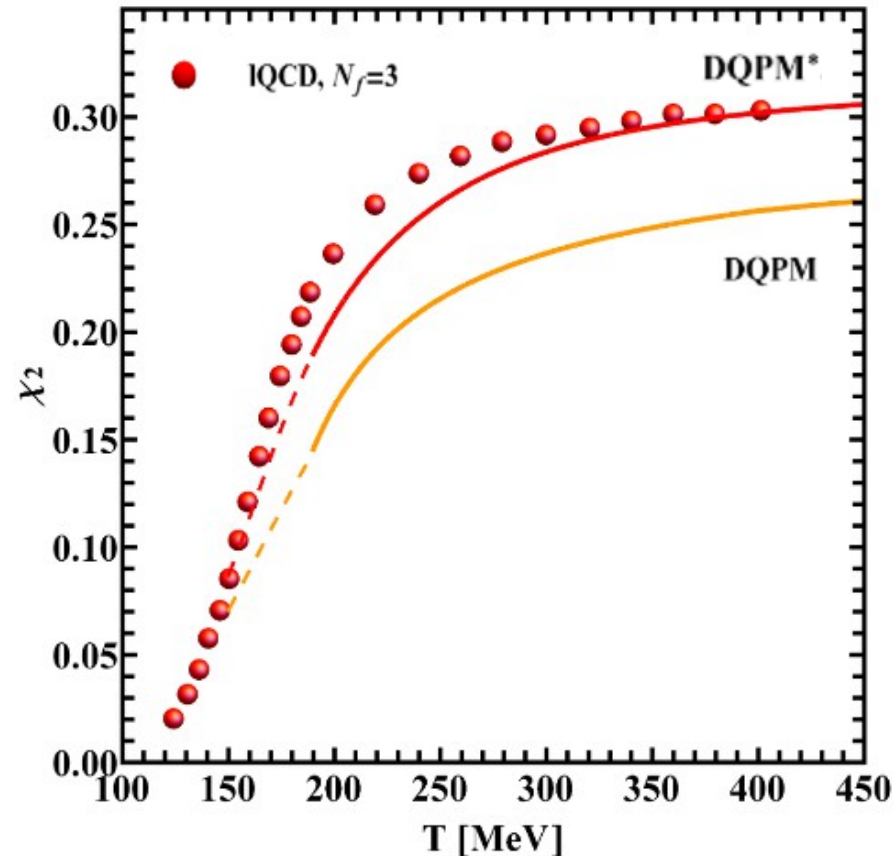
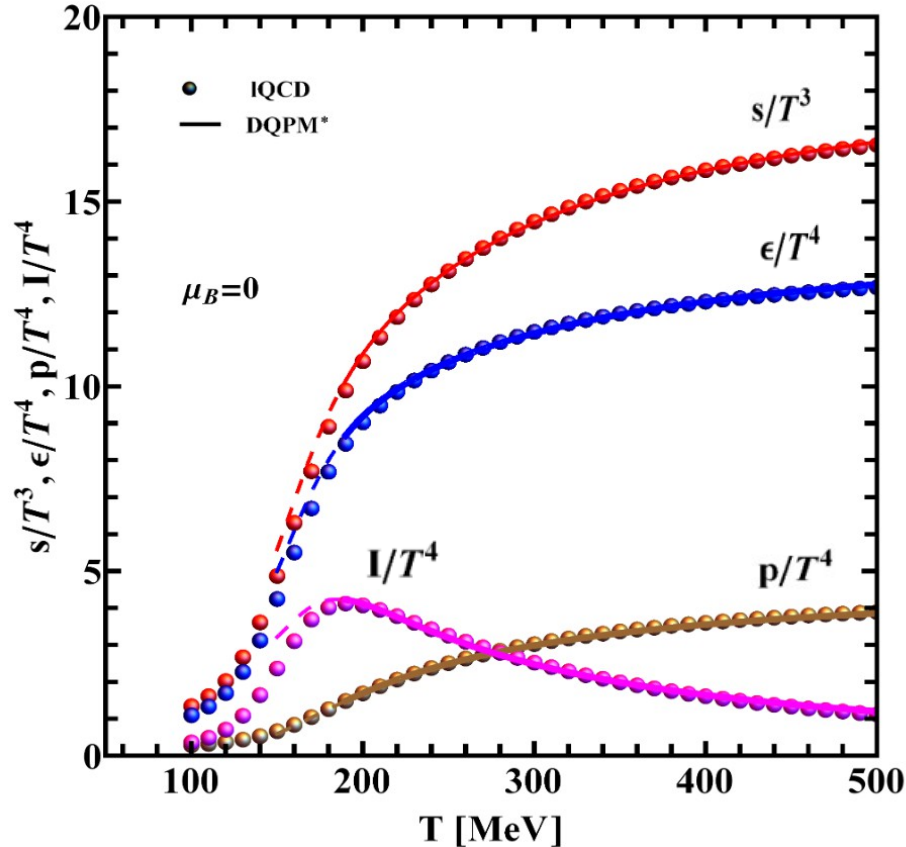
Propagator remains analytic in the upper half plane.

Correct perturbative limit of the effective propagators.

This defines the generalized quasiparticle model DQPM*.



- Mom. dep. DQPM* reproduces the EoS at $T > 170$ MeV.



Momentum dependence improves the susceptibility.

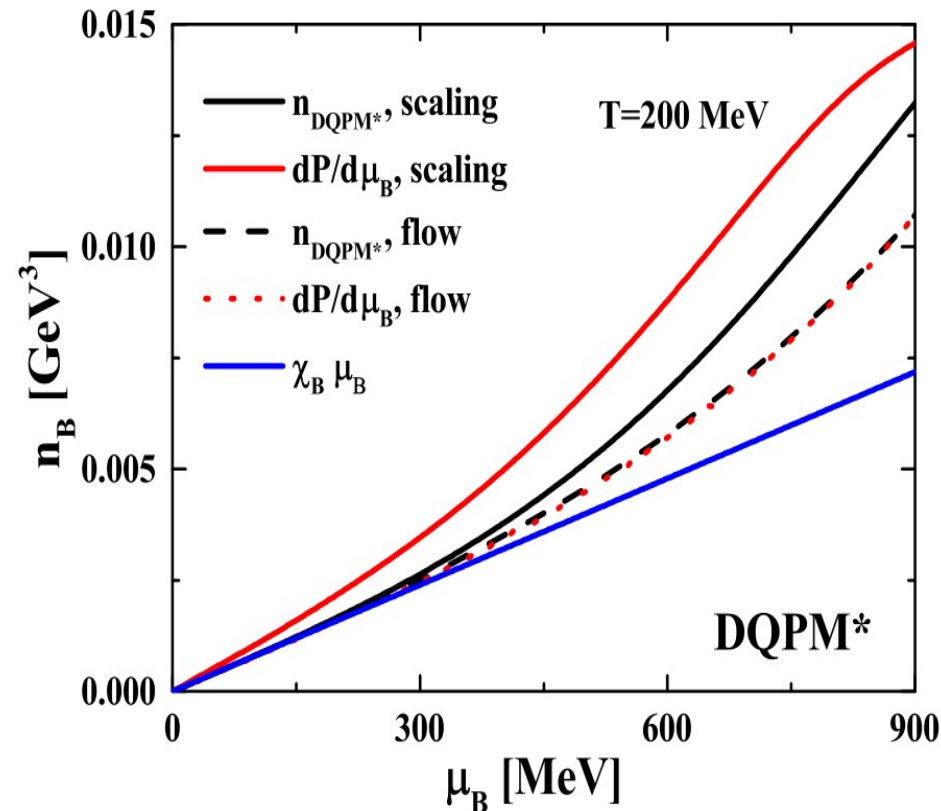
See Phys. Rev. C93 (2016) no. 4, 044914 and Int. J. Mod. Phys. E25 (2016) no. 07, 1642003 for more about the DQPM*.

- Entropy density and particle density are both derived from the same potential.
- They have to fulfill the Maxwell relation:

$$\frac{\partial s}{\partial \mu_B} = \frac{\partial n_B}{\partial T}$$

- This leads to a differential equation for g^2 :

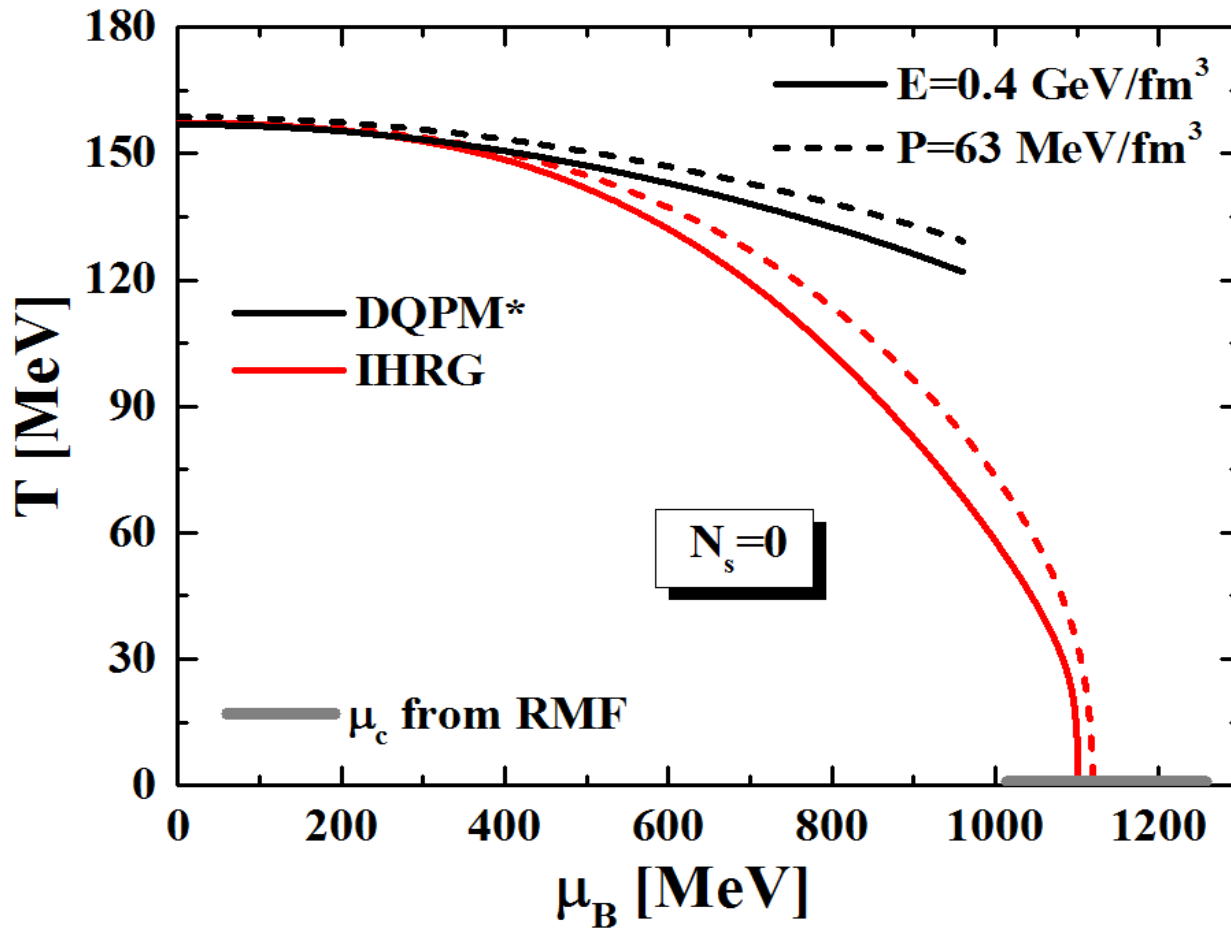
$$a_T \frac{\partial g^2}{\partial T} + a_\mu \frac{\partial g^2}{\partial \mu_B} = a_0$$



Hadron-Parton Transition

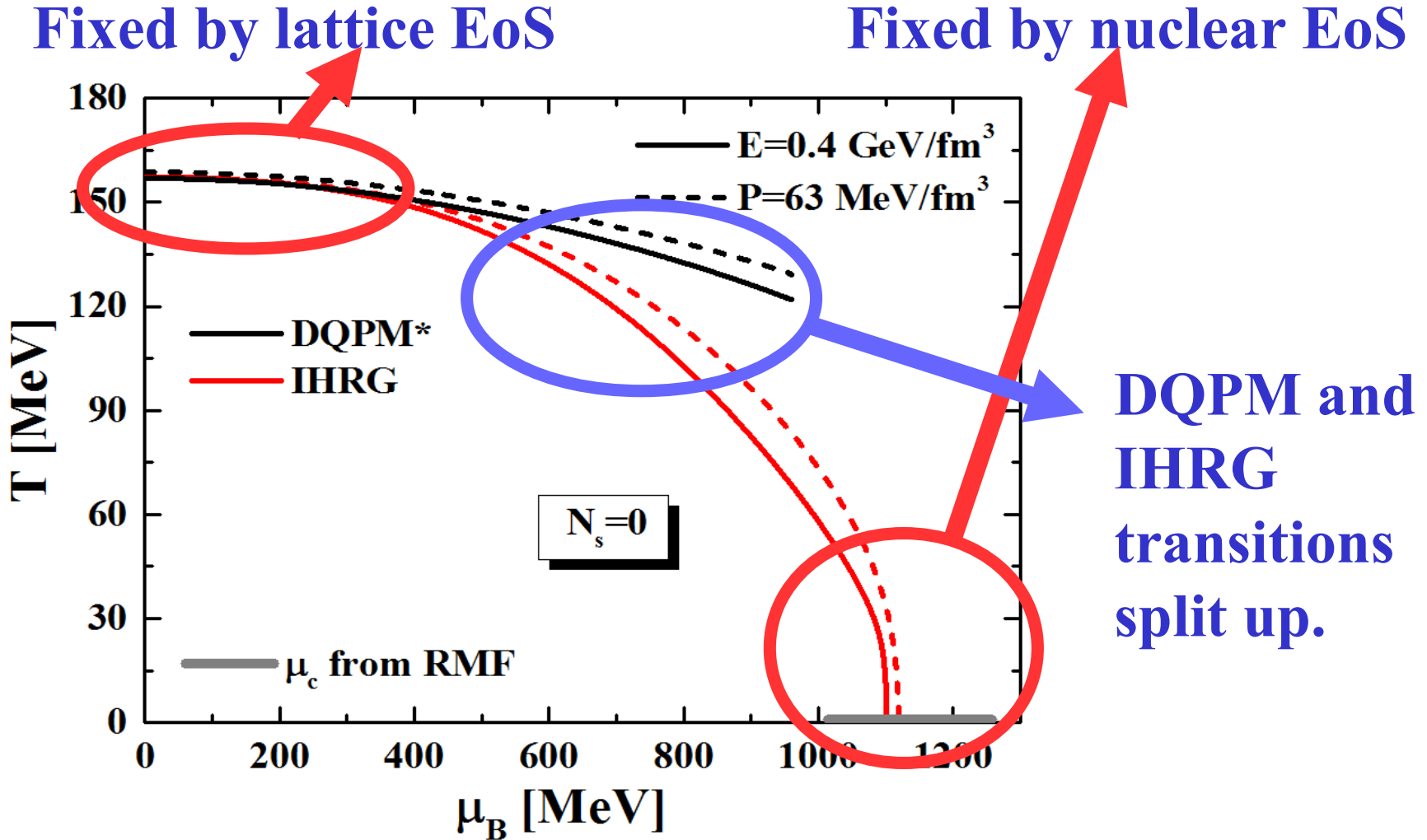
Hadron-Parton transition 21

Transition defined by constant thermodynamics



Conditions
fixed by
IQCD at T_c

Hadron-Parton transition 22



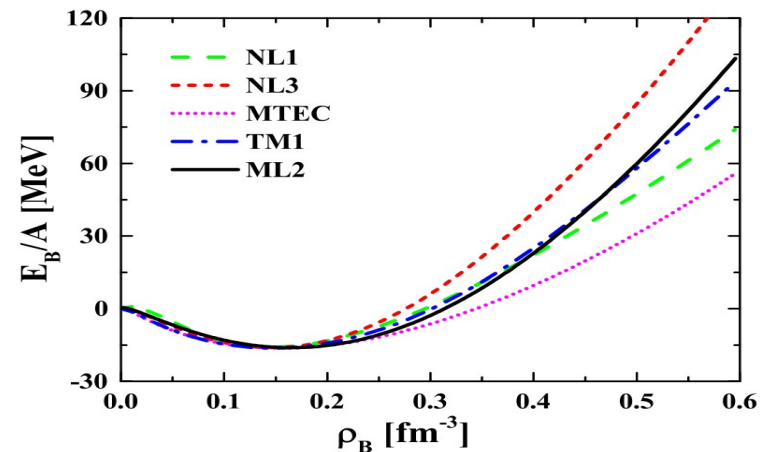
Can we constrain the DQPM at finite density?

Nuclear EoS is only known as a function of density

Repulsive interactions shift chemical potential

$$\mu_B^* = \mu_B - \Sigma_B^0(T, \rho_N)$$

Correct dependence
on μ_B is not known!



- However, IHRG is constrained by the nuclear EoS
- DQPM is only constrained by thermodynamics

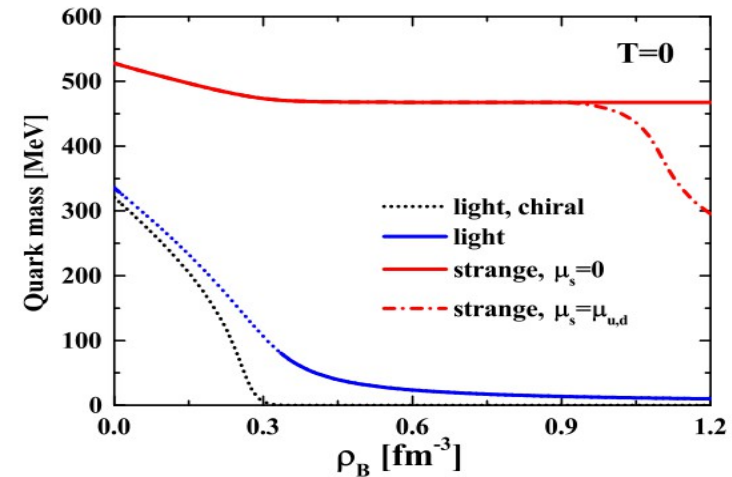
- **DQPM masses need to decrease strongly with μ_B :**

Lower quark masses increase the density, shifts the phase boundary closer to the IHRG

$$M_{q,\bar{q}}^2(T) = \frac{N_c^2 - 1}{8N_c} g^2 \left(T^2 + \frac{\mu_q^2}{\pi^2} \right) \cdot F(\mu_q)$$

Control μ_B dependence with $F(\mu_B)$

$$F(\mu_q) = \exp \left(-B\mu_q^2 - \frac{1}{2}B^2\mu_q^4 \right)$$



Neglect widths: only 10% effect on the EoS

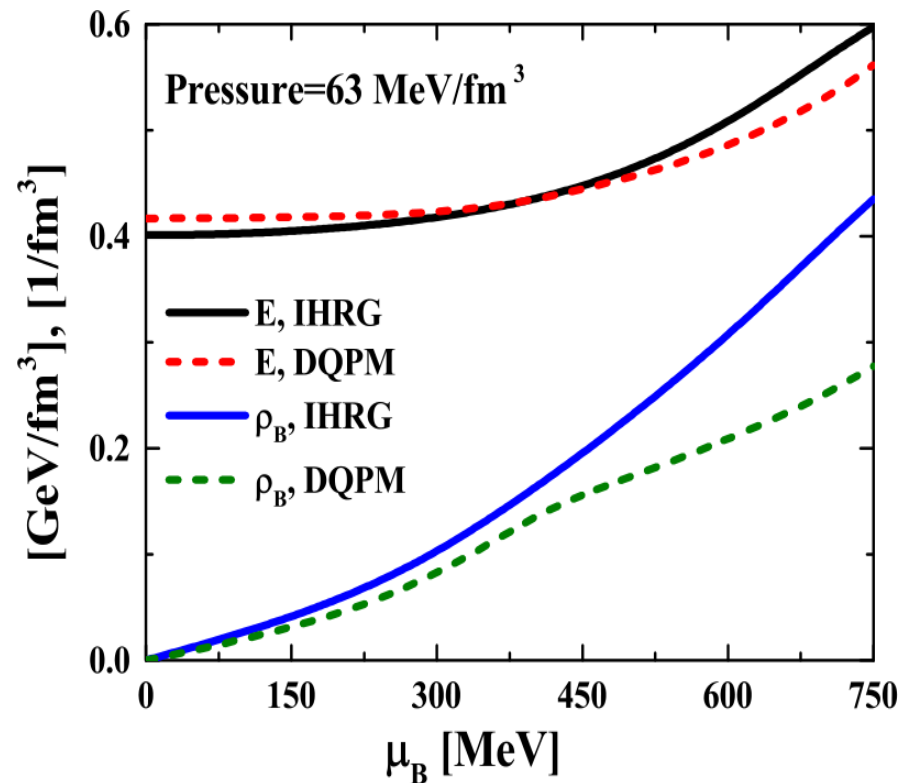
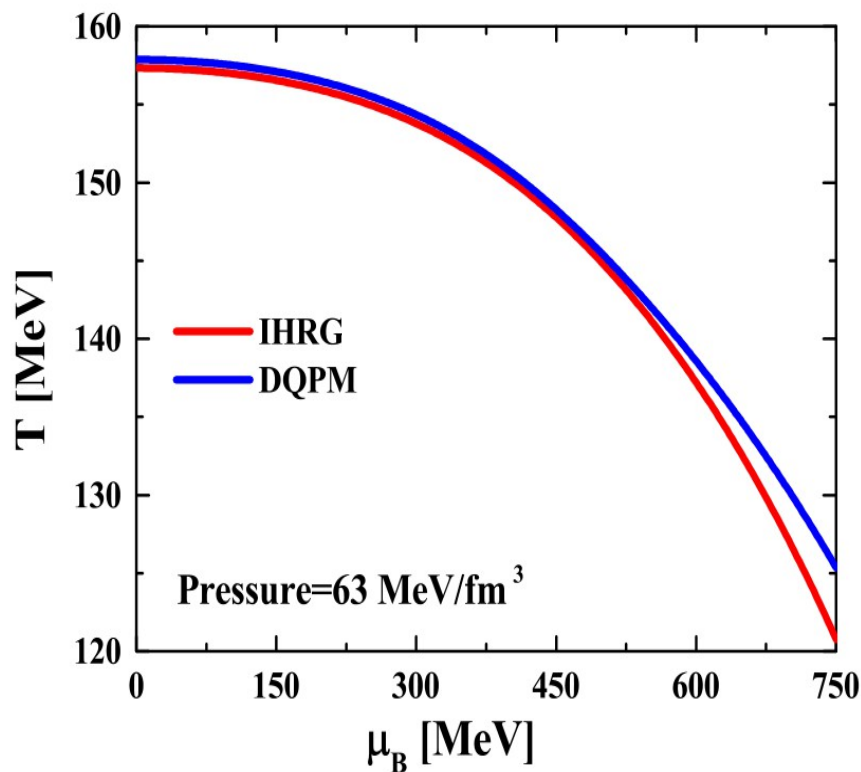
No mom. dep.: only small effect close to T_c

New phase boundary

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$B = 75 \text{ GeV}^{-2}$ gives best result for the phase boundary:

Agreement up to $\mu_B = 450 - 600 \text{ MeV}$:



Further improvement not possible
within quasiparticle models!

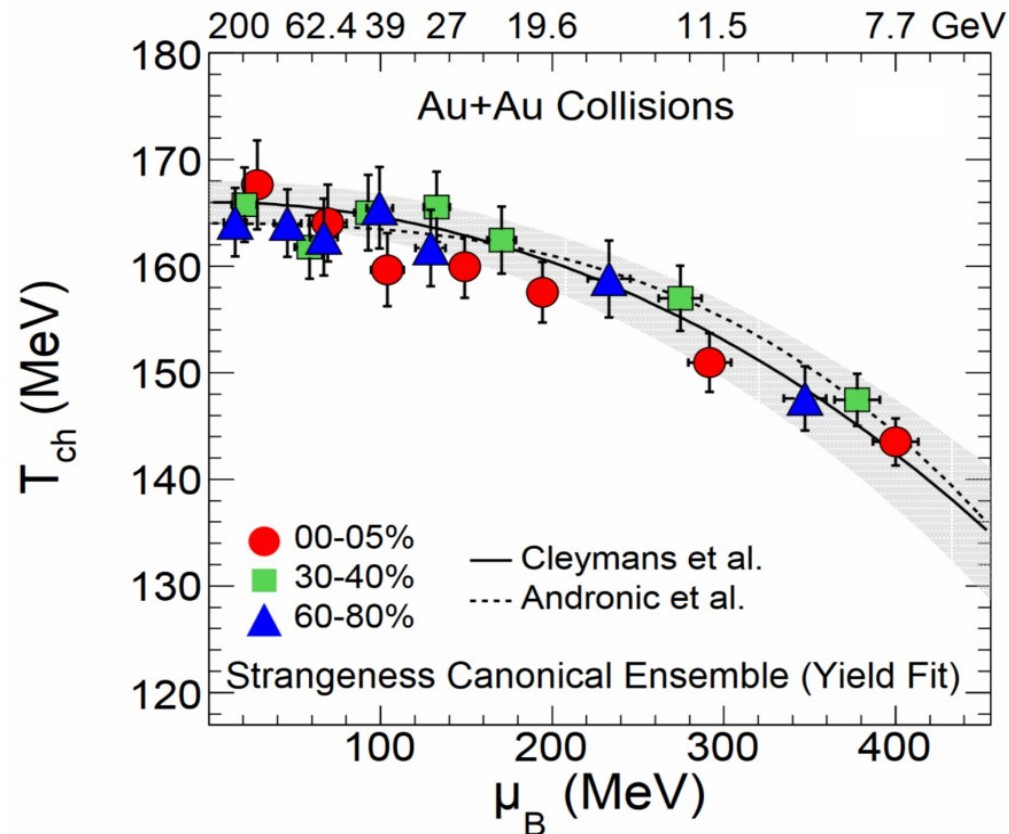
How does this compare to experimental results?

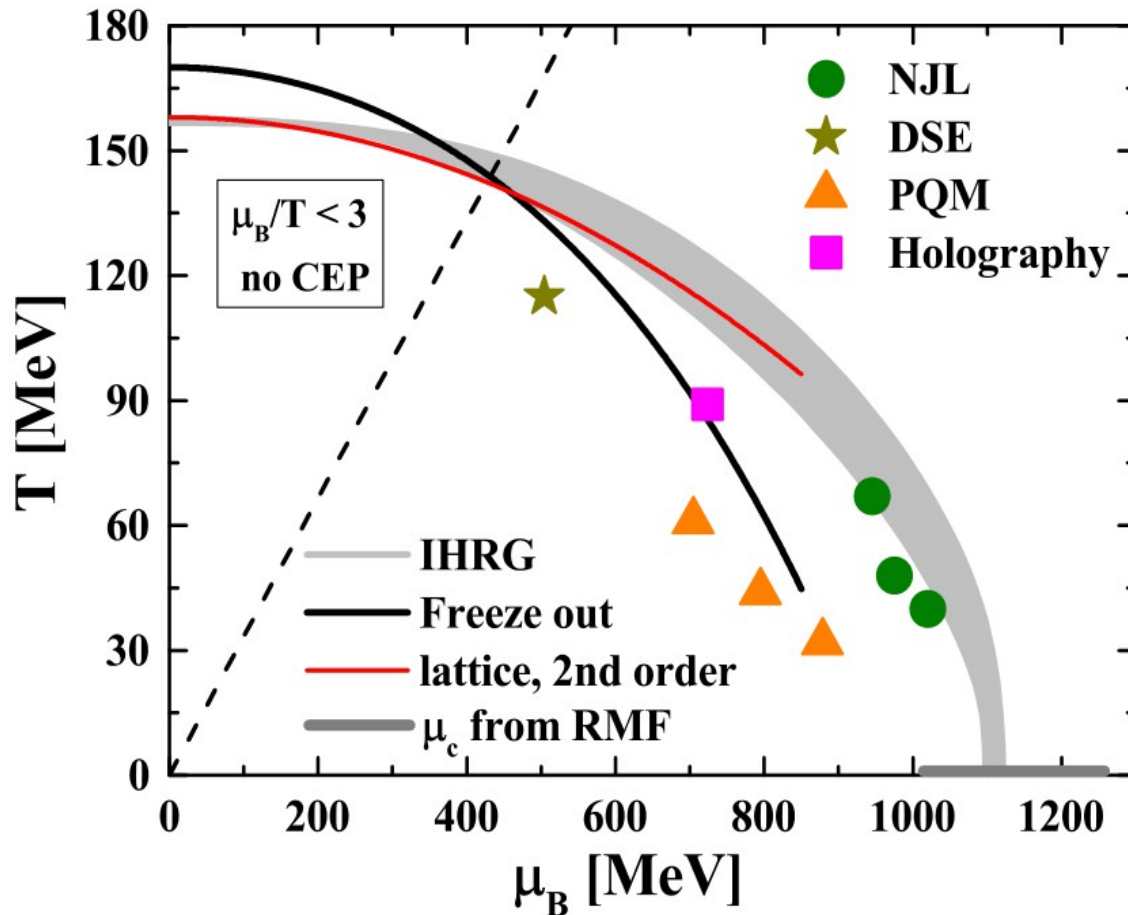
RHIC - BES $\sqrt{s} = 7.7 - 200$ GeV

$\mu_B = 300$ MeV corresponds
to $\sqrt{s} = 12.5$ GeV

$\mu_B = 450$ MeV corresponds
to $\sqrt{s} = 7$ GeV

The DQPM/IHRG
EoS covers the whole
RHIC BES .





RHIC BES can not reach predicted CEP

Most predictions at $\mu_B > 700$ MeV

Corresponds to $\sqrt{s}=2.7$ GeV or $T_{Lab}=3$ GeV

NJL: Nucl. Phys. A 504, 668 (1989); Phys. Rev. C 53, 410 (1996); Phys. Rep. 247, 221 (1994); **DSE:** Phys. Rev. D 90, 034022 (2014); **PQM:** Phys. Lett. B 696, 58 (2011); Phys. Rev. D 96, 016009 (2017); **Holography:** arXiv:1706.00455 [nucl-th]; **Freeze out:** Phys. Rev. C 73, 034905 (2006); **Curvature:** Phys. Rev. D 92, 054503 (2015)

Summary

- **IHRG** is a hadronic model that reproduces the nuclear and the lattice EoS below T_c .
- **DQPM** is a partonic model that reproduces the lattice EoS above T_c .
- Both models share a common phase boundary in the T - μ_B plane up to $\mu_B \approx 600$ MeV.
- Sufficient to cover the physics of the BES program at RHIC.
- Search for the CEP requires even larger μ_B .

Backup

Extension to further Baryons

Generalize the approach to more interacting baryons

Fix ratios of effective masses and μ 's $\frac{m_X}{m_N} = \frac{m_X^*}{m_N^*}, \mu_B^X = \mu_B^N$

Defines the couplings for other baryons:

$$\frac{g_{\sigma X}}{g_{\sigma N}} = \frac{m_X}{m_N} \quad \frac{\partial U}{\partial \sigma} = g_{\sigma} \sum_X \frac{m_X}{m_N} \rho_s^X(T, \mu_B^*, m_X^*)$$

$$g_{\omega X} = g_{\omega N} \quad \frac{\partial O}{\partial \omega} = g_{\omega} \sum_X \rho_B^X(T, \mu_B^*, m_X^*)$$

Include mesons as noninteracting particles:

$$P_{IHRG} = -U(\sigma) + O(\omega) + \sum_X P_{free}^X(T, \mu_B^*, m_X^*) + P_{HRG}^{meson}(T, \mu_B)$$

Quasiparticle thermodynamics

- **Idea: treat partons as dynamical quasiparticles.**

Propagator with effective mass M and width γ :

$$G(\omega, \mathbf{p}) = \frac{-1}{\omega^2 - \mathbf{p}^2 - M^2 + 2i\gamma\omega} = \frac{-1}{\omega^2 - \mathbf{p}^2 - \Sigma}$$
$$A(\omega, \mathbf{p}) = \frac{2\gamma\omega}{(\omega^2 - \mathbf{p}^2 - M^2)^2 + 4\gamma^2\omega^2}$$

- **Grand canonical potential in propagator representation:**

$$\beta\Omega[D, S] = \frac{1}{2}\text{Tr}[\ln D^{-1} - \Pi D] - \text{Tr}[\ln S^{-1} + \Sigma S] + \Phi[D, S]$$

with selfenergies

$$\frac{\delta\Phi}{\delta D} = \frac{1}{2}\Pi \quad \frac{\delta\Phi}{\delta S} = -\Sigma$$

$\Phi[D, S]$ **has no contribution to entropy or density.**

Effective coupling

Effective coupling carries nonperturbative informations

- Use Lattice EoS to define the coupling:

$$g^2(s/s_{SB}) = g_0 \left(\left(\frac{s}{s_{SB}} \right)^b - 1 \right)^d$$

- Equation of state

Thermodynamic consistency:

$$P(T) = \int_0^T S(T') dT'$$

$$E = TS - P + \mu N$$

- Small chemical potentials

Scaling Hypothesis:

$$g^2(T, \mu_B) = g^2(T^*/T_c(\mu_B))$$

$$T^* = \sqrt{T^2 + \frac{\mu^2}{\pi^2}} \quad T_c(\mu) = T_c \sqrt{1 - \alpha \mu^2}$$

$$\alpha \approx 8.79 \text{ GeV}^{-2}$$

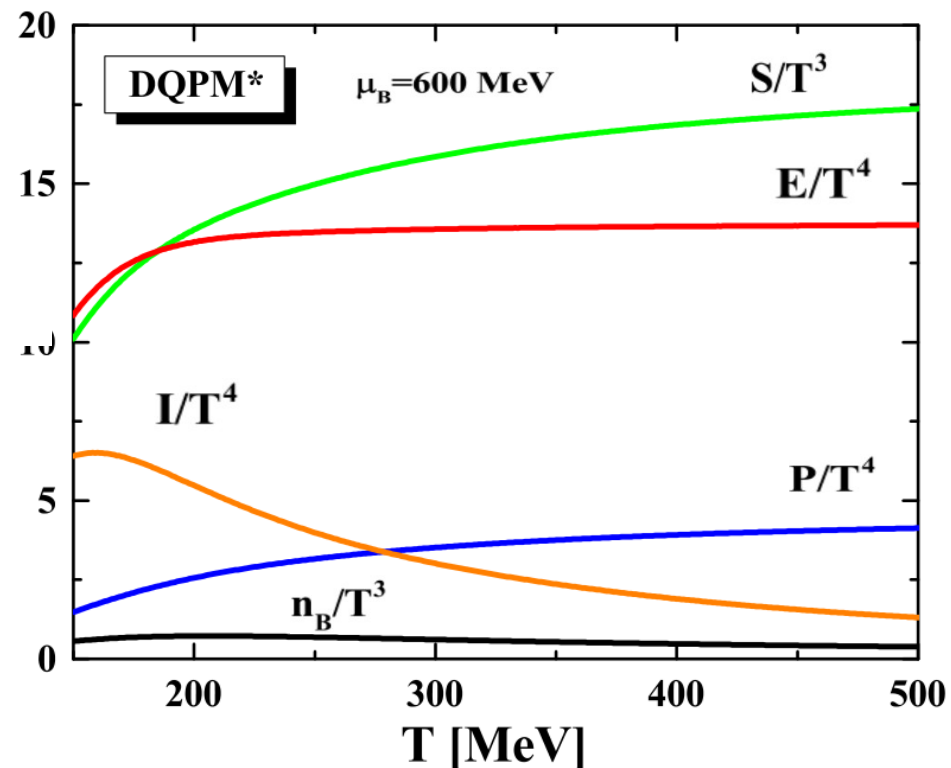
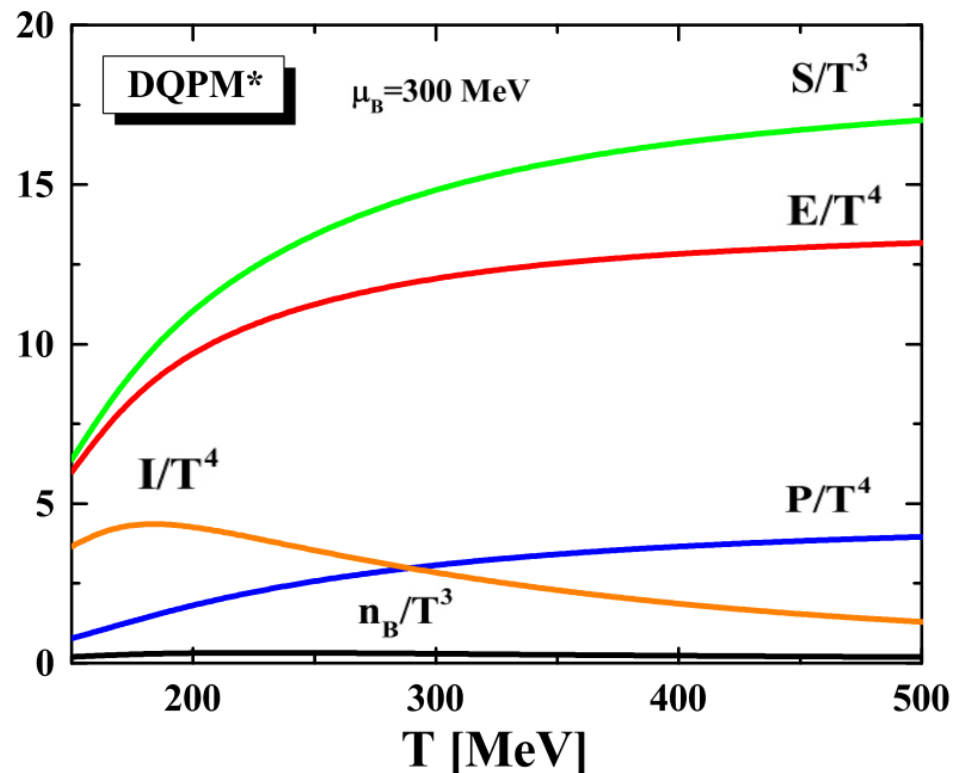
Consistent with lattice curvature

$$\kappa_{DQPM} \approx 0.0122 \quad \kappa = 0.013(2)$$

EoS at finite μ

- The effective coupling defines the EoS at arbitrary chemical potential:

$$P(T, \mu_B) = P(T, 0) + \int_0^{\mu_B} n_B(T, \mu) d\mu$$

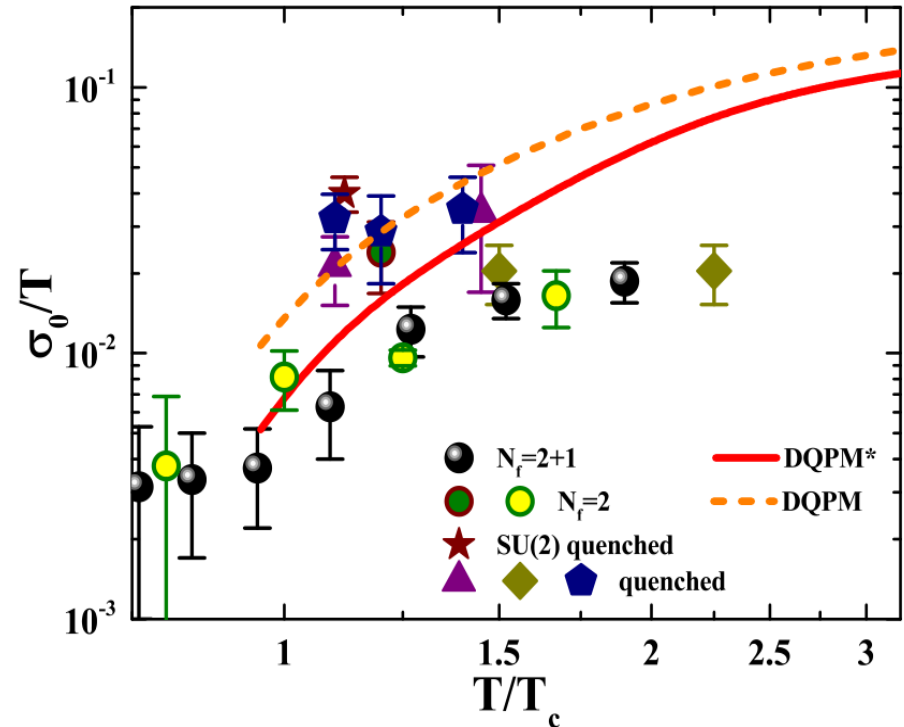


Transport coefficients

- The width so far is not well fixed by the EoS.
- Use transport coefficients

Electric conductivity in relaxation time approach:

$$\sigma_e(T, \mu_q) = \sum_{f, \bar{f}}^{u, d, s} \frac{e_f^2 n_f^{\text{off}}(T, \mu_q)}{\bar{\omega}_f(T, \mu_q) \bar{\gamma}_f(T, \mu_q)}$$



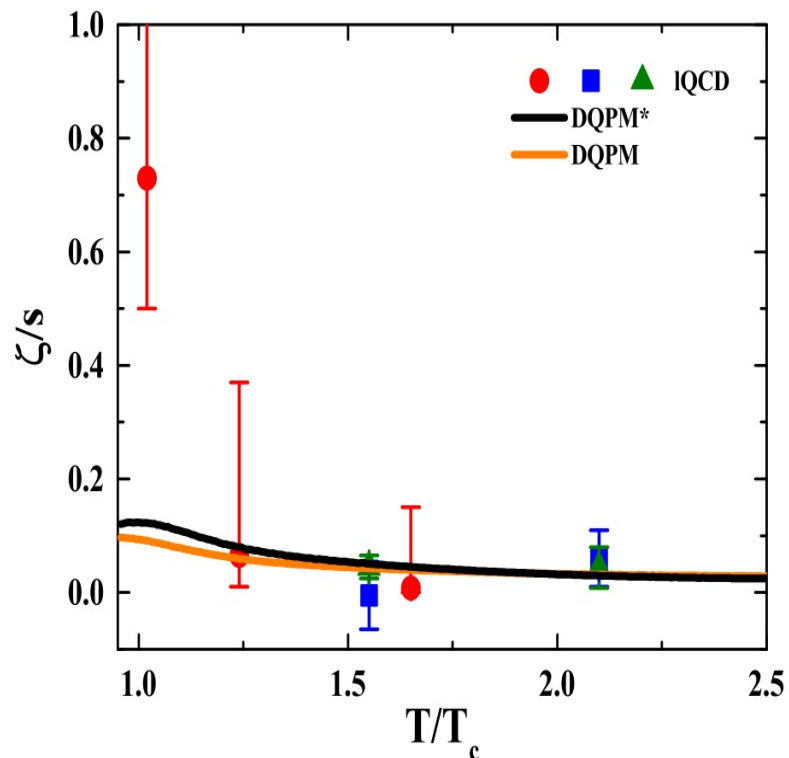
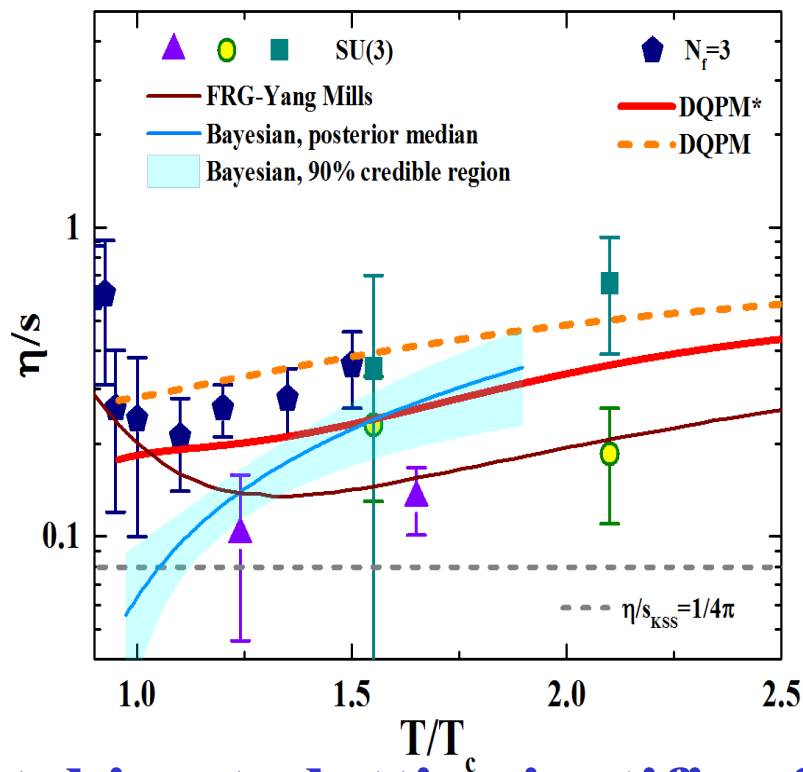
Conductivity probes only the quark width γ_f
since gluons carry no electric charge!

Transport coefficients

- Viscosities probe the whole system!

Shear viscosity decreases flow anisotropies in HIC.

Bulk viscosity acts against the expansion of the fireball.



Matching to lattice justifies functional form of the widths

Phase boundary

HICs create a partonic medium.

The correct transition condition is important for the understanding of heavy-ion collisions.

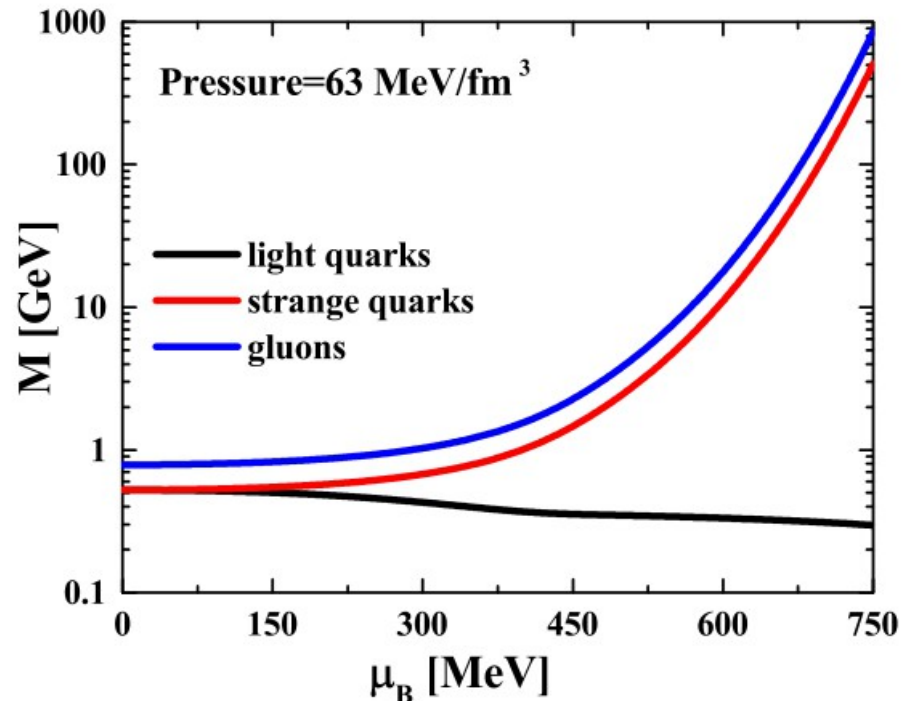
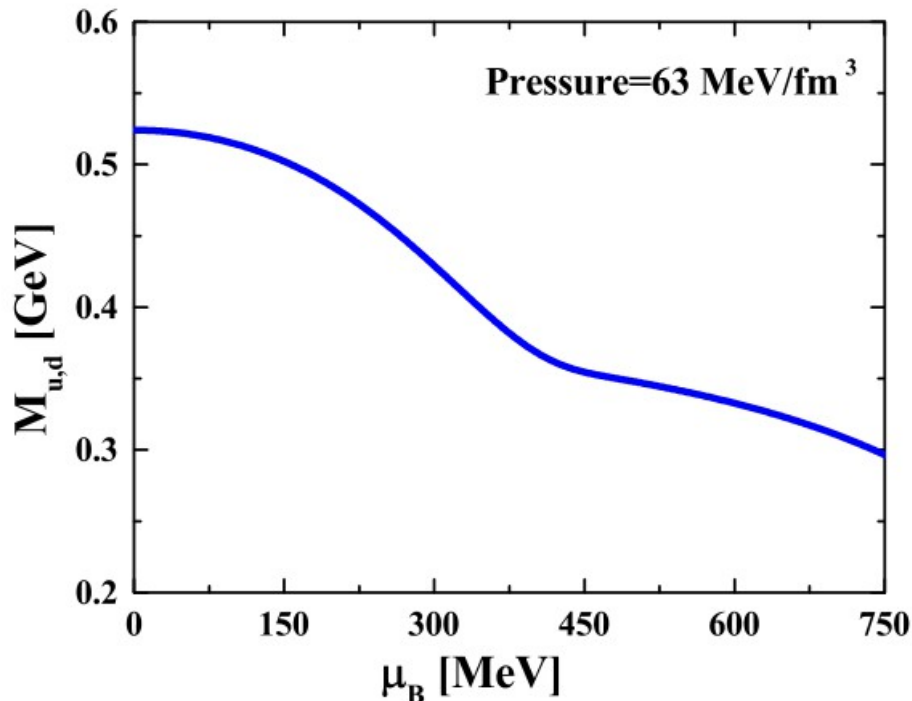
- PHSD and other transport approaches use constant energy density
- Chem. freeze-out at constant thermodynamics
- Transition in neutron stars similar to HIC
- HIC are a microcanonical system with conserved energy, baryon number etc.
- QCD phase diagram is a grand canonical system
- Transition at constant pressure

Effective masses

Light quark mass decreases as intended
 \Rightarrow chiral symmetry restoration

Strange quark and gluon mass increase dramatically!

Light quark mass changes behavior
 \Rightarrow Boundaries split up again

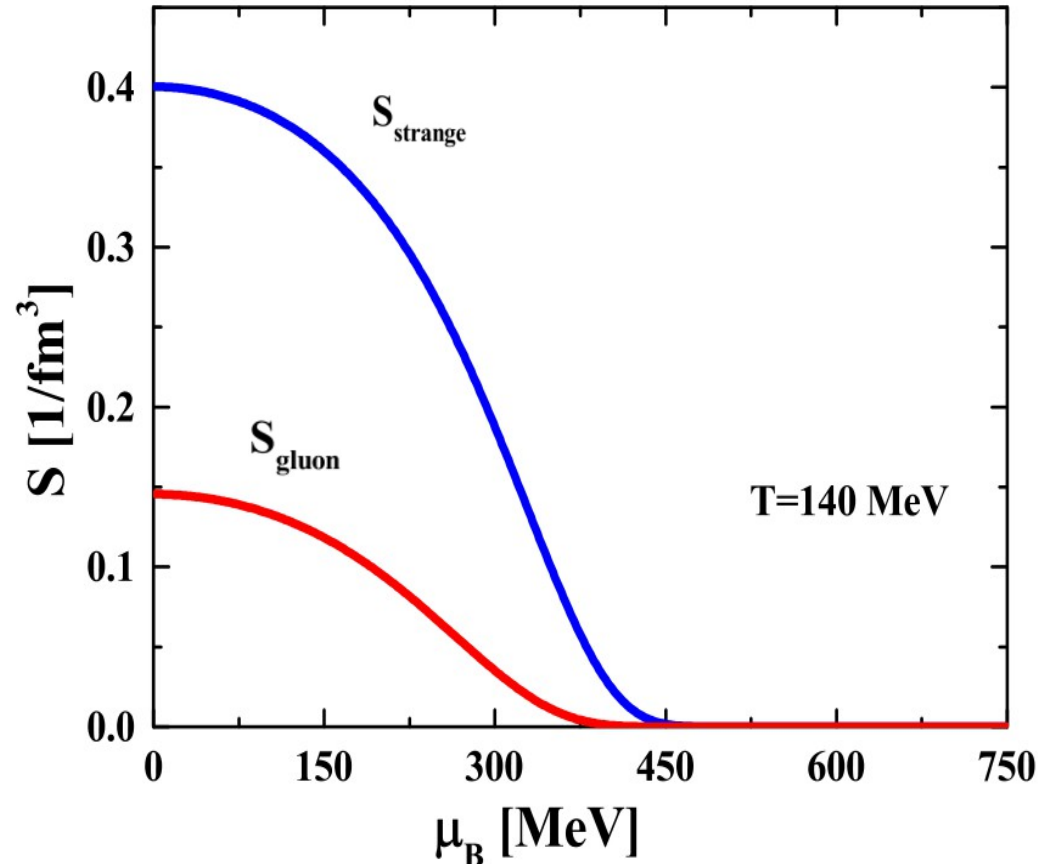


Pure light system

- Strange & gluon masses influence thermodynamics:

Entropy vanishes
with increasing mass.

No more contributions
to the EoS from s-
quarks and gluons \Rightarrow



We have a pure light quark system at $\mu_B > 450$ MeV!

Maxwell equation

We separate the Maxwell equation into contributions from the individual particle species:

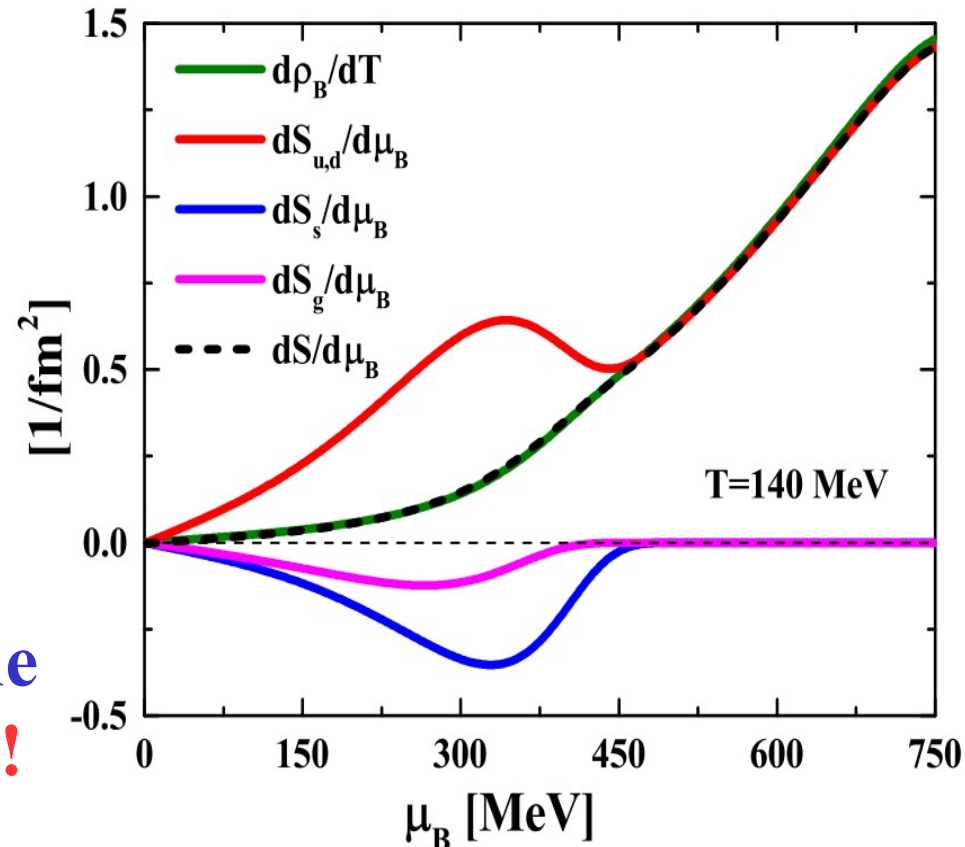
$$\frac{\partial \rho_{u,d}}{\partial T} = \frac{\partial S_{u,d}}{\partial \mu_B} + \frac{\partial S_s}{\partial \mu_B} + \frac{\partial S_g}{\partial \mu_B}$$

Left: only light quarks

Right: all partons

$\partial S_{u,d}/\partial \mu_B$ is very large,
 $\partial \rho_{u,d}/\partial T$ can't counter it.

Strange quark and gluon contributions have to become negative => **Masses increase!**



Phase boundary

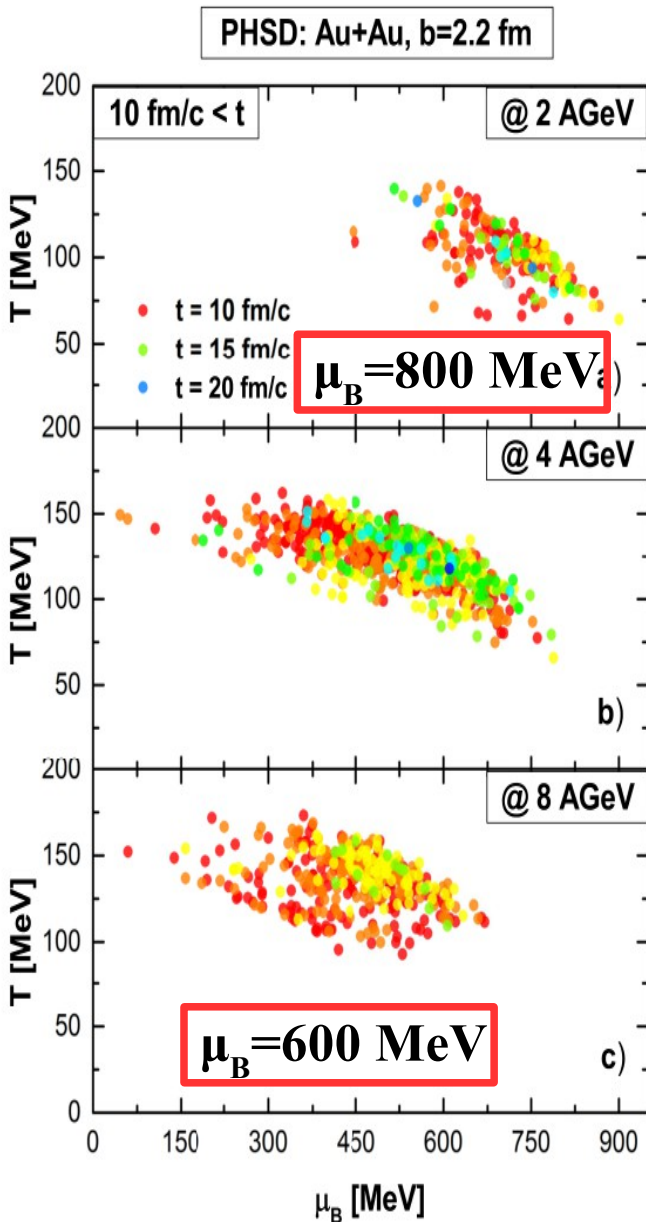
Decrease of the light quark mass has to be counter balanced by an increase in the strange quark and gluon mass.

Strange quarks and gluons will eventually disappear from the system, leaving only light quarks.

The light quark mass becomes the only remaining parameter in the theory. Its behavior as a function of T and μ_B can not be changed and is determined by the Maxwell equation!

We can not extend the phase boundary to larger μ_B via Maxwell relations!

HIC at low \sqrt{s}



No partons at low \sqrt{s}

Without partonic phase no deconfinement transition.

