Two-Point One Loop Fermionic Amplitudes in Constant Homogeneous Magnetic Field

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Outline

Introduction: General Case of Two-Point Correlator

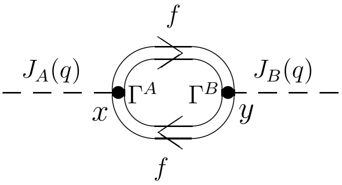
[M. Yu. Borovkov et al., Phys. At. Nucl. 62 (1999) 1601]

- Lagrangian density of local fermion interaction

$$\mathcal{L}_{\rm int}(x) = \left[\bar{f}(x)\Gamma^A f(x)\right] J_A(x)$$

- J_A — generalized current (photon, neutrino current, etc.)
- Γ_A — any of γ-matrices from the set $\{1,\, \gamma_5,\, \gamma_\mu,\, \gamma_\mu\gamma_5, \sigma_{\mu\nu} = i\left[\gamma_\mu, \gamma_\nu\right]/2\}$
- Interaction constants are included into the current J_A

Introduction: General Case of Two-Point Correlator



- Two-point correlation function of general form

$$\Pi_{AB} = \int d^4X \, \mathrm{e}^{-i(qX)} \, \mathrm{Sp}\, \{ S_{\mathrm{F}}(-X) \, \Gamma_A \, S_{\mathrm{F}}(X) \, \Gamma_B \}$$

- $S_{\mathrm{F}}(X)$ — Lorentz-invariant part of exact fermion propagator
- $X^\mu = x^\mu - y^\mu$ — integration variable
- Correlations of scalar, pseudoscalar, vector and axial-vector currents were studied by Borovkov et al. [Phys. At. Nucl. 62 (1999) 1601]
- Consider correlations of a tensor current with the other ones

Propagator in Constant Homogenious Magnetic Field

- Dirac equation in an external electromagnetic field

$$\Big[i\,\hat{\partial} - e\,Q_f\,\hat{A}(\mathbf{r},t) - m_f\Big]\,\Psi(\mathbf{r},t) = 0$$

- Q_f and m_f are the relative charge and mass of the fermion
  $\hat{\partial} = \partial_\mu\,\gamma^\mu, \qquad \hat{A} = A_\mu\,\gamma^\mu$
- Pure constant homogeneous magnetic field: $\mathbf{B} = (0,0,B)$
- Four-potential (in Lorentz-covariant form): $A_\mu(x) = -F_{\mu\nu}x^\nu$
- $F_{\mu\nu}$ — strength tensor of external electromagnetic field
- Equation for fermion propagator in the magnetic field

$$\Big[i\,\hat{\partial} - e\,Q_f\,\hat{A}(x) - m_f\Big]\,G_{\mathrm{F}}(x,y) = \delta^{(4)}(x-y)$$

- Use the Fock-Schwinger method for its solution

Basic Tensors in Presence of Magnetic Field

- Minkowski space filled with external magnetic field is divided into two subspaces:
  - Euclidean with the metric tensor $\Lambda_{\mu\nu} = (\varphi\varphi)_{\mu\nu}$; plane orthogonal to the field strength vector
  - Pseudo-Euclidean with the metric tensor $\tilde{\Lambda}_{\mu\nu} = (\tilde{\varphi}\tilde{\varphi})_{\mu\nu}$
  - Metric tensor of Minkowski space $g_{\mu\nu} = \tilde{\Lambda}_{\mu\nu} - \Lambda_{\mu\nu}$
- Dimensionless tensor of the external magnetic field and its dual

$$\varphi_{\alpha\beta} = \frac{F_{\alpha\beta}}{B}\,, \qquad \tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\,\varepsilon_{\alpha\beta\rho\sigma}\varphi^{\rho\sigma}$$

- Arbitrary four-vector $a^\mu = (a_0, a_1, a_2, a_3)$ can be decomposed into two orthogonal components

$$a_\mu = \tilde{\Lambda}_{\mu\nu}a^\nu - \Lambda_{\mu\nu}a^\nu = a_{\|\mu} - a_{\perp\mu}$$

- For the scalar product of two four-vectors one has

$$(ab) = (ab)_{\|} - (ab)_{\perp}$$

$$(ab)_{\|} = (a\tilde{\Lambda}b) = a^\mu\tilde{\Lambda}_{\mu\nu}b^\nu, \quad (ab)_{\perp} = (a\Lambda b) = a^\mu\Lambda_{\mu\nu}b^\nu$$

Propagator in the Fock-Schwinger Representation

- General representation of the propagator [Itzikson & Zuber]

$$G_{\rm F}(x,y) = {\rm e}^{i\Omega(x,y)}\, S_{\rm F}(x-y)$$

- Lorentz non-invariant phase factor

$$\Omega(x,y) = -eQ_f \int_y^x d\xi^\mu \left[A_\mu(\xi) + \frac{1}{2} F_{\mu\nu}(\xi - y)^\nu\right]$$

- In two-point correlation function phase factors canceled

$$\Omega(x,y) + \Omega(y,x) = 0$$

- Lorentz-invariant part of the fermion propagator ($\beta = eB|Q_f|$)

$$\begin{aligned} S_{\rm F}(X) &= -\frac{i\beta}{2(4\pi)^2} \int_0^\infty \frac{ds}{s^2} \Big\{ (X\widetilde{\Lambda}\gamma)\cot(\beta s) - i(X\widetilde{\varphi}\gamma)\gamma_5 - \\ &- \frac{\beta s}{\sin^2(\beta s)}(X\Lambda\gamma) + m_f s\,[2\cot(\beta s) + (\gamma\varphi\gamma)] \Big\} \times \\ &\times \exp\left(-i\left[m_f^2 s + \frac{1}{4s}(X\widetilde{\Lambda}X) - \frac{\beta\cot(\beta s)}{4}(X\Lambda X)\right]\right) \end{aligned}$$

Orthogonal Basis Motivated by Magnetic Field

- Correlators having rank non-equal to zero, should be decomposed in some orthogonal set of vectors
- In magnetic field, such a basis naturally exists

$$b_\mu^{(1)} = (q\varphi)_\mu, \qquad b_\mu^{(2)} = (q\tilde{\varphi})_\mu$$
$$b_\mu^{(3)} = q^2\,(\Lambda q)_\mu - (q\Lambda q)\,q_\mu, \quad b_\mu^{(4)} = q_\mu$$

- Arbitrary vector a_μ can be presented as

$$a_\mu = \sum_{i=1}^{4} a_i\,\frac{b_\mu^{(i)}}{(b^{(i)}b^{(i)})}, \qquad a_i = a^\mu b_\mu^{(i)}$$

- Arbitrary tensor $T_{\mu\nu}$ can be similarly decomposed

$$T_{\mu\nu} = \sum_{i,j=1}^{4} T_{ij}\,\frac{b_\mu^{(i)}b_\nu^{(j)}}{(b^{(i)}b^{(i)})\,(b^{(j)}b^{(j)})}, \qquad T_{ij} = T^{\mu\nu}b_\mu^{(i)}b_\nu^{(j)}$$

Correlator of Pseudoscalar and Tensor Currents

- Correlator of pseudoscalar and tensor currents is rank-2 tensor
- From six non-trivial coefficients in the basis decomposition, three ones only are independent
- Integral representation of coefficients in tensor decomposition

$$\Pi_{ij}(q^2, q_\perp^2, \beta) = \frac{1}{16\pi^2} \int_0^\infty \frac{dt}{t} \int_0^1 du \, Y_{ij}(q^2, q_\perp^2, \beta; t, u) \times$$

$$\times \exp\left\{ -i \left[m_f^2 t - \frac{q_\parallel^2}{4} \, t \, (1 - u^2) + q_\perp^2 \, \frac{\cos(\beta t u) - \cos(\beta t)}{2\beta \sin(\beta t)} \right] \right\}$$

- Integration variables: $t = s_1 + s_2, \; u = (s_1 - s_2)/(s_1 + s_2)$
- Relation among momentum squared: $q_\parallel^2 = q^2 + q_\perp^2$
- Coefficients in tensor decomposition

$$Y_{12}^{(\mathrm{PT})} = -Y_{21}^{(\mathrm{PT})} = -i \, \beta t \, q_\parallel^2 \, q_\perp^2 \, \frac{\sin(\beta t u)}{\sin(\beta t)} \, [\mathrm{ctg}(\beta t) - u \, \mathrm{ctg}(\beta t u)]$$

Integrands of Pseudoscalar-Tensor Correlator

- The other four coefficients in the decomposition

$$Y_{23}^{(\mathrm{PT})}(q^2, q_\perp^2, \beta; t, u) = -Y_{32}^{(\mathrm{PT})}(q^2, q_\perp^2, \beta; t, u)$$
$$= \frac{i}{2}\,\beta\, q_\parallel^2 q_\perp^2 \,\left\{4\left[m_f^2 t + i\right] - q_\parallel^2 t\, R(\beta; t, u)\right\}$$
$$Y_{24}^{(\mathrm{PT})}(q^2, q_\perp^2, \beta; t, u) = -Y_{42}^{(\mathrm{PT})}(q^2, q_\perp^2, \beta; t, u)$$
$$= -\frac{i}{2}\,\beta\, q_\parallel^2 \left\{4\left[m_f^2 t + i\right] - q_\parallel^2 t\left(1 - u^2\right) - q_\perp^2\, t\, R(\beta; t, u)\right\}$$

- Auxiliary function was introduced

$$R(\beta; t, u) = 1 - u^2 + \frac{2}{\sin^2(\beta t)}\left[\cos(\beta t)\cos(\beta t u) + u \sin(\beta t)\sin(\beta t u) - 1\right]$$

- Other correlators and their decomposition are under derivation

Correlators in Crossed-Field Limit

- Correlators in electromagnetic crossed field can be obtained from the ones calculated in magnetic field after pure field parameter $\beta^2 = e^2 Q_f^2 (FF)/4$ is neglected
- Quantities calculated in the crossed field are completely determined by dynamical parameter: $\chi_f^2 = e^2 Q_f^2 \,(qFFq)$
- Crossed-field limit is valid for an ultrarelativistic particle moving in the direction transverse to the field strength in relatively weak magnetic field
- As basic vectors, it is convenient to use the following set:

$$b_\mu^{(1)} = \frac{eQ_f}{\chi_f}\,(qF)_\mu, \qquad b_\mu^{(2)} = \frac{eQ_f}{\chi_f}\,(q\tilde{F})_\mu$$

$$b_\mu^{(3)} = \frac{eQ_f^2}{\chi_f\sqrt{q^2}}\left[q^2\,(qFF)_\mu - (qFFq)\,q_\mu\right], \quad b_\mu^{(4)} = \frac{q_\mu}{\sqrt{q^2}}$$

Results for Correlators in the Crossed-Field Limit

- Integral representation of coefficients in tensor decomposition

$$\Pi_{ij}(q^2,\chi_f) = \frac{1}{16\pi^2}\int\limits_0^\infty \frac{dt}{t}\int\limits_0^1 du\, Y_{ij}(q^2,\chi_f;t,u)$$
$$\times \exp\left\{-i\left[\left(m_f^2 - \frac{q^2}{4}(1-u^2)\right)t + \frac{1}{48}\chi_f^2(1-u^2)^2\right]\right\}$$

- Integrands of coefficients in pseudoscalar-tensor correlator

$$Y_{12}^{(\mathrm{PT})} = \frac{i}{3}\chi_f^2\, t^2\, u\,(1-u^2)$$
$$Y_{23}^{(\mathrm{PT})} = \frac{i\chi_f}{2\sqrt{q^2}}\left\{4\left[m_f^2\, t + i\right] + \frac{1}{4}\chi_f^2 t^3\left(1-u^2\right)^2\right\}$$
$$Y_{24}^{(\mathrm{PT})} = \frac{i\chi_f}{2\sqrt{q^2}}\left\{4\left[m_f^2\, t + i\right] + q^2 t\left(1-u^2\right) + \frac{1}{4}\chi_f^2 t^3\left(1-u^2\right)^2\right\}$$

- Similar results are obtained for scalar-tensor correlator

Applications of Correlators

- Polarization operator is related with correlator of two vector currents
- Models beyond the Standard Model can effectively generate the Pauli Lagrangian density

$$\mathcal{L}_{\rm AMM}(x) = -\frac{\mu_f}{4}\left[\bar{f}(x)\sigma_{\mu\nu}f(x)\right]F^{\mu\nu}(x)$$

- After combining with the QED Lagrangian, it contributes to the photon polarization operator
- Contribution linear in the fermion AMM μ_f is determined by correlator of vector and tensor currents
- Its influence on photon requires detail discussion

Three-Point Correlators

- Technique we are developed can be extended for calculation of three-point correlators
- My collaborators have such an experience when they calculated axion-two-photon vertex in crossed and magnetic field configurations
- The result by Skobelev obtained later differs from ours but a reason remains unclear and this problem requires to be resolved
- Some other three-point vertecies are also of importance in applications

Conclusions

- Two-point correlators in presence of constant homogeneous external magnetic field are considered
- This analysis extended the previous one by inclusion of tensor currents into consideration
- Study of correlators of tensor fermionic current with other ones allows to investigate possible effects due to anomalous magnetic moment of fermion
- Computer technique developed for two-point correlators is planned to be applied for three-point ones