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Theoretical study of the halo nucleus of ^{11}Be

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Overview:

- *The aim of investigation, the relevance of topic*
- *Halo nuclei*
- *Numerical methods of solving stationary SE*
- *Results: energy*
- *The splitting of the energy levels of ^{11}Be due to the influence of an external magnetic field*
- *Conclusion*

- **The aim of work**

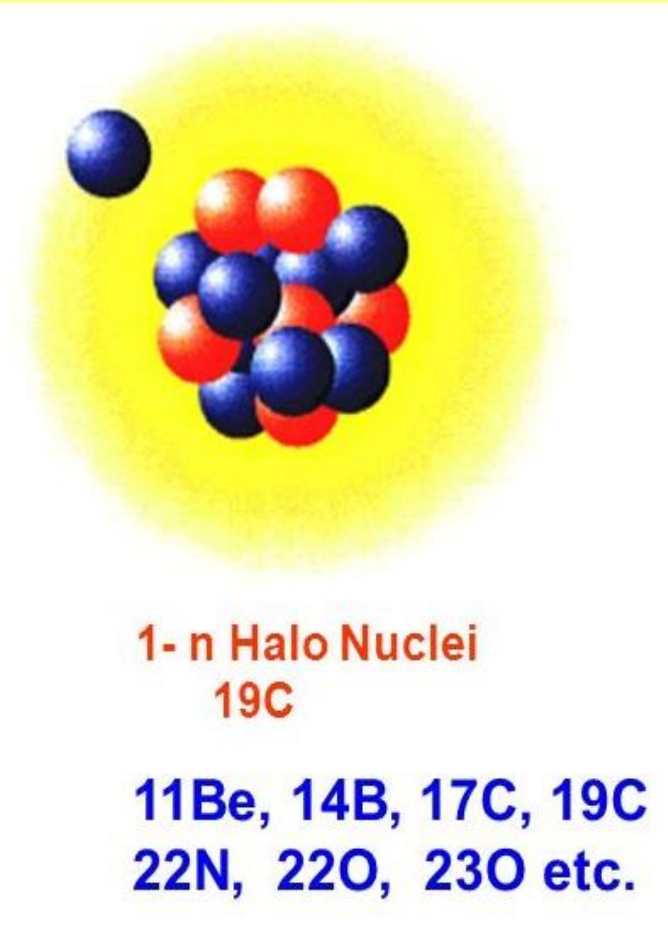
Theoretical study of the processes of Coulomb breakup of halo nucleus- ^{11}Be in the framework of a non-stationary quantum-mechanical approach.

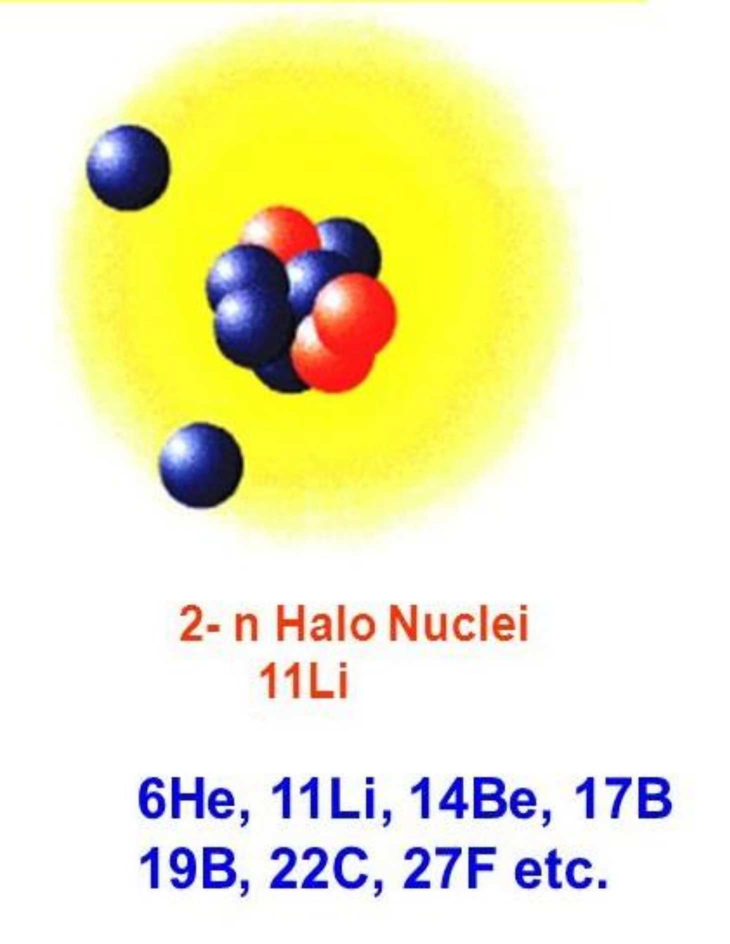
- **Relevance of the research topic:**

A theoretical investigation of the halo nuclei is relevant with the planned experiments on the study of light nuclei on radioactive beams.
Coulomb breakup is one of the main tools for studying the halo nucleus.
The breakup cross section provides useful information about the structure of the halo.
The halo is one of the most intensively studied objects in modern low-nucleus physics. A characteristic feature of halo nuclei physics is correlations between the mechanism of nuclear reaction and structure.

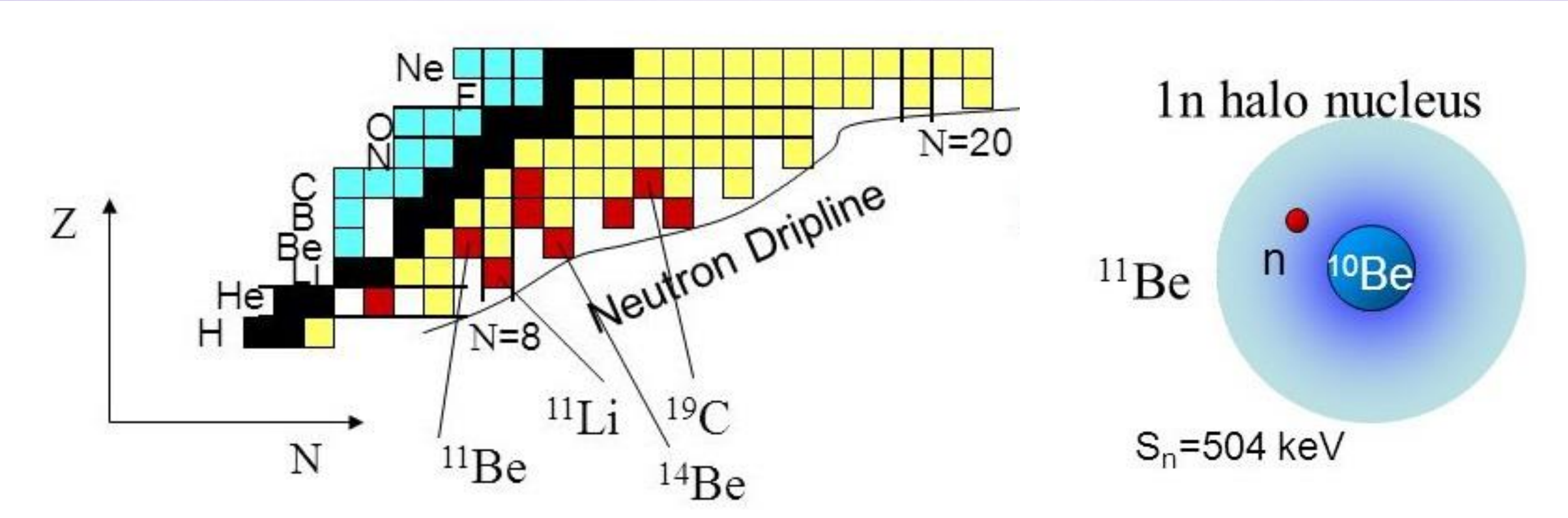
HALO

The neutron halo effect is caused by the presence of weakly bound states of neutrons located near the continuum. The small value of the binding energy of a neutron (or a group of neutrons) and the short-range nature of nuclear forces lead to the tunneling of neutrons into the outer peripheral region over large distances from the core of the nucleus.





Among the halo nuclei, the ^{11}**Be nucleus** is of particular importance, since the relative simplicity of its structure allows for more accurate theoretical studies. In fact, the bound states of the ^{11}**Be nucleus** can be described quite well as a ^{10}**Be core** and a weakly bound **neutron**. With a good approximation, the decay can be regarded as a transition from a two-particle bound state to a continuum due to a changing Coulomb field



Stationary Schrodinger equation:

$$H\psi_{Nlm} = E_N \psi_{Nlm} \qquad (1)$$

with boundary conditions: $\begin{cases} \psi_{Nlm}(r=0) = const; \\ \psi_{Nlm}(r \to \infty) = 0 \end{cases}$

The Hamiltonian of the interaction: $$H_0(r) = -\frac{\hbar^2}{2\mu}\Delta + V_{cf}(r) \qquad (2)$$

$\mu = \frac{m_n \cdot m_c}{M}$ –reduced mass;

$$\psi_{Nlm}(r) = R_{Nl}(r) Y_{lm}(\theta, \varphi) \qquad (3)$$

the radial SE: $$\left[-\frac{\hbar^2}{2\mu}\Delta + \frac{\hbar^2 l(l+1)}{2\mu r^2} + V_{cf}(r)\right] R_l(r) = E\, R_l(r) \qquad (4)$$

internal interaction: $$V_{cf}(r) = V_0(r) + \boldsymbol{LI} V_{LI}(r) \qquad (5)$$

Wood–Saxon potential: $V_0(r) = -V_l\, f(r, R_0, a)$

where $$f(r, R_0, a) = \left[1 + exp\left(\frac{r - R_0}{a}\right)\right]^{-1} \qquad (5')$$

Spin-orbital interaction: $$V_{LI}(r) = V_{LS} \frac{1}{r} \frac{d}{dr} f(r, R_0, a) \qquad (6)$$

[P.Capel, D.Baye and V. S. Melezhik, Phys. Rev. C **68**, 014612 (2003).]

Parameters of potential

$V_{l=0}$ (MeV)	$V_{l>0}$ (MeV)	V_{LS} (MeV fm^2)	a (fm)	R_0 (fm)
59.5	40.5	32.8	0.6	2.669

[V. S. Melezhik and D. Baye, Phys. Rev. C **59**, 3232 (1999).]

Wood-Saxon potential:

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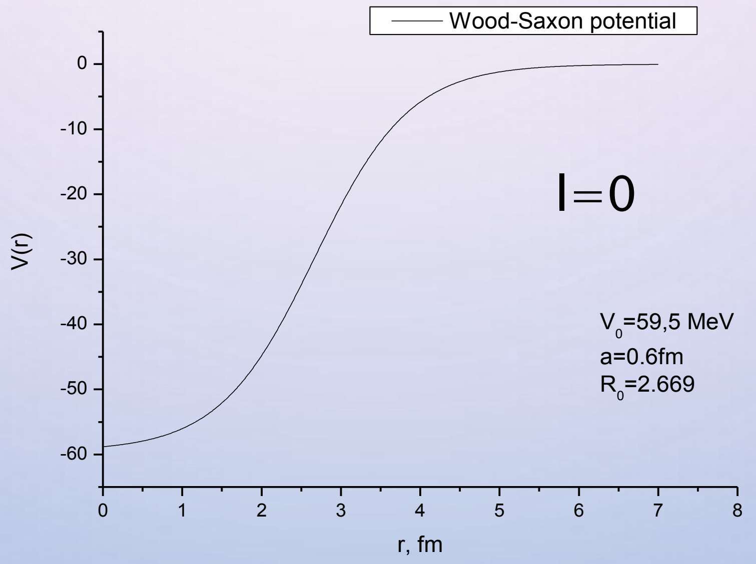


Here V_l is the depth of the Woods-Saxon potential, a is the diffuseness, and R_0 is the radius of the ^{11}Be (R_0 = 1.2$A^{1/3}$ fm). The standard value V_{LS} is used for the potential depth *ls* for the p-shell core

Numerical methods of solving stationary SE

1. Inverse iteration method in the subspace

$$\begin{cases} \hat{A}\,\vec{R} = E\vec{R} \\ (\widehat{A} - \hat{I}E^{(0)})\,\vec{R}^{(i)} = \vec{R}^{(i-1)} \\ E = E_0 + \dfrac{\mathbf{1}}{\hat{R}^{(i)},\hat{R}^{(i-1)}} \end{cases}, \quad i = \overline{1, i_{max}} \qquad (7)$$

where E_0 – initial approximation, $\hat{R}^{(0)}$ – initial vector, and the calculated vector $\hat{R}^{(i)}$ is normalized at each iteration $\hat{R}(r) = \widehat{\varphi}^{(i_{max})}$

Accuracy of the method:

$\Delta_i = \left|E^{(i)} - E^{(i-1)}\right| < 10^{-6}$

or

$\delta_i < 10^{-6}$:

$$(\widehat{A} - \hat{I}E^{(i)})\,R^{(i)} = \delta_i$$

2. *Sweep method*

The solution will be sought in the form:

$$\bar{R}_{j-1} = \alpha_{j-1}\bar{R}_j + \beta_{j-1} \qquad (8)$$

$$\bar{R}_j = \alpha_j{}'\bar{R}_{j+1} + \beta_j{}'$$

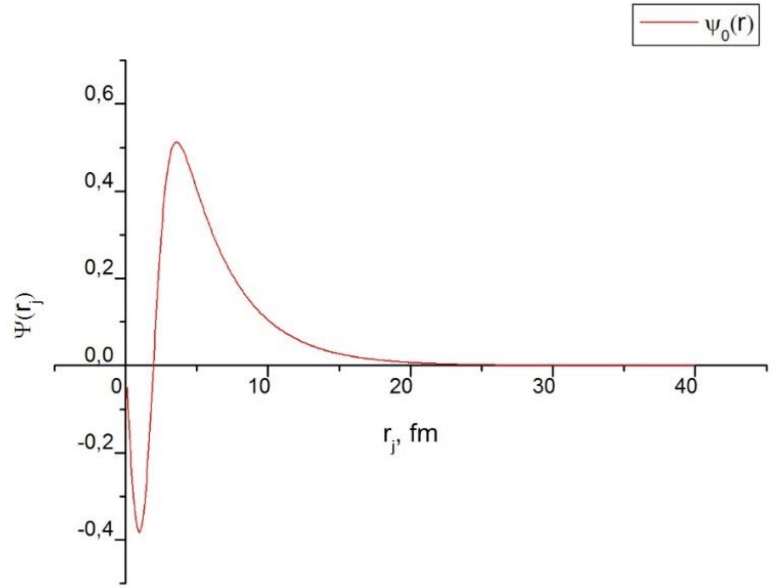
3. The second-order differential can be simplified using the finite-difference method:

$$\frac{d^2}{dr^2}\,(R_j^{(1)}) = \frac{R_{j+1}^{(1)} - 2R_j^{(1)} + R_{j-1}^{(1)}}{(\Delta r)^2} \qquad (9)$$

RESULTS:

Energy

J^π	l	E_{exp} (MeV) [1]	E_{th} (MeV)
$\frac{1}{2}^+$	0	−0.503	−0.5013
$\frac{1}{2}^-$	1	−0.183	−0.1844



Radial WF of 2s-state (l=0)

Convergence of the computational scheme for uniform radial grid (l=0)

N_r	Δr	E, l=0
500	0.08	−0.501317
1000	0.04	−0.505251
2000	0.02	−0.507165
4000	0.01	−0.508109

[1] F.Aizenberg-Selove, Nucl.Phys.A506,1 (1990)

The splitting of the ground state energy levels of ^{11}Be due to the influence of an external magnetic field

The radial Schrödinger equation adding an external field ΔV_μ:

$$\left[\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2\, l(l+1)}{2\mu r^2} + V(r) + \Delta V_\mu\right] R_l(r) = E R_l(r) \quad (10)$$

$$\Delta \mathrm{V}_\mu = \mathrm{B} \cdot \mu_\mathrm{n} \cdot \hat{\mathrm{S}}_n, \qquad (11)$$

here B – is the strength of the magnetic field, μ_n – is the magnetic moment of the neutron, $\hat{S}_n$ – the projection of the spin on the axis.

$$\hat{s_z} = \pm\frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \qquad (12)$$

Wood–Saxon form: $\mathrm{V_0(r)} = -\mathrm{V_l} \times \left[1 + \exp\left(\frac{\mathrm{r}-\mathrm{R_0}}{\mathrm{a}}\right)\right]^{-1}$

Gauss form of potential $\quad V(r) = V_0 e^{-\left(\frac{r}{r_0}\right)^2} = V_0 e^{-gr^2}$

here $V_0 = -59.5$ MeV, the potential width $g = \frac{1}{r_0^2} = 0.117\, fm^{-2}$.

The energy shifts of the ground state of ^{11}Be due to the influence of an external magnetic field

R_m=8 M=200	$\Delta E_{pert}(B_z)$ perturbation	$\Delta E_{num}(B_z)$ Gauss num.	$\Delta E_{num}(B_z)$ WS num.	$\Delta E_{pert}(B_z)$ perturbation	$\Delta E_{num}(B_z)$ Gauss num.	$\Delta E_{num}(B_z)$ WS num.
B (Gauss)	m_s=+1/2 spin projection			m_s= − 1/2 spin projection		
0.1	0.0003	0.0003	0.0003	−0.0003	−0.0003	−0.0003
1	0.0030	0.0030	0.0030	−0.0030	−0.0030	−0.0030
10	0.0300	0.0301	0.0301	−0.0300	−0.0300	−0.0300
100	0.3008	0.3008	0.3008	−0.3008	−0.3008	−0.3008
200	0.6016	0.6016	0.6016	−0.6016	−0.6016	−0.6016
300	0.9024	0.9025	0.9025	−0.9024	−0.9025	−0.9025
400	1.2033	1.2033	1.2033	−1.2033	−1.2033	−1.2033
500	1.5041	1.5041	1.5041	−1.5041	−1.5041	−1.5041
1000	3.0082	3.0082	3.0082	−3.0082	−3.0082	−3.0082
2000	6.0165	6.0165	6.0165	−6.0165	−6.0165	−6.0165

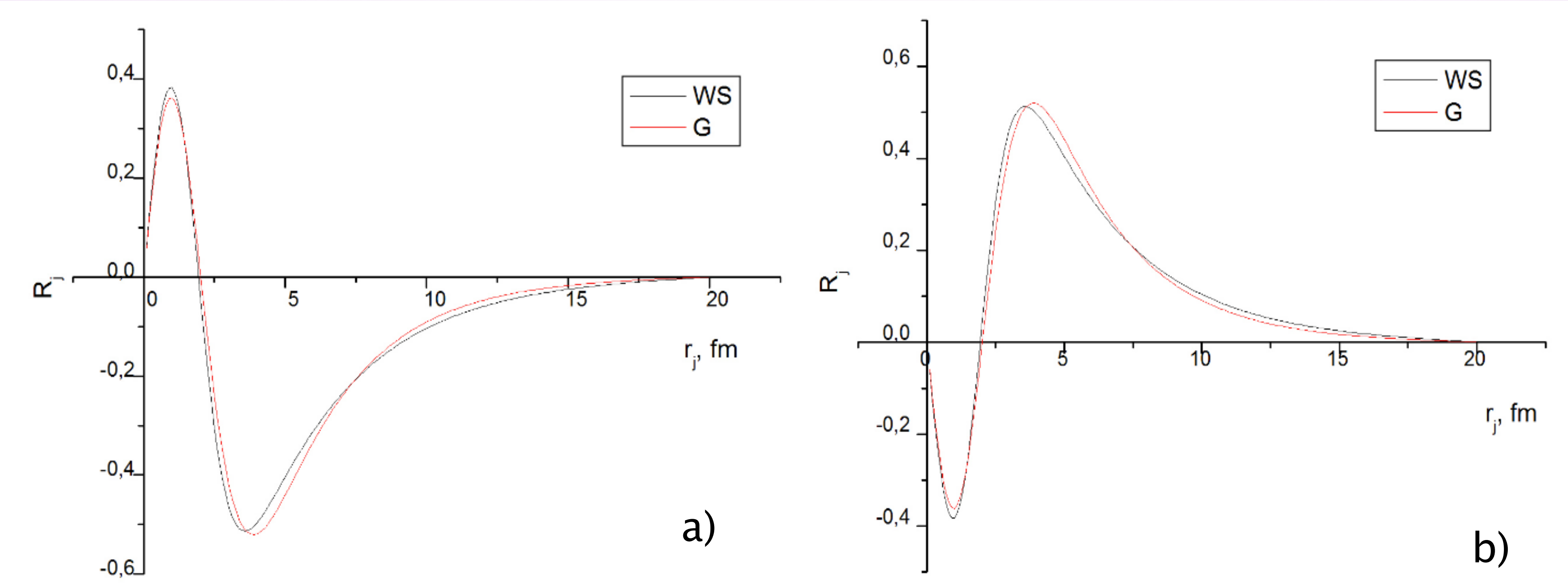
The level shifts are defined as (numerically):

$$\Delta E_{m=\frac{1}{2}} = \langle R_{lm}^{(r)} | \frac{1}{2} \cdot B \cdot \mu_n | R_{lm}^{(r)} \rangle$$
$$\Delta E_{m=-\frac{1}{2}} = \langle R_{lm}^{(r)} | -\frac{1}{2} \cdot B \cdot \mu_n | R_{lm}^{(r)} \rangle$$

The energy shifts in perturbation theory are calculated as:

$$\Delta E_{\frac{1}{2}} = \int_0^\infty R_0(r) \Delta V_{\frac{1}{2}}(r)\, R_0(r) dr$$
$$\Delta E_{-\frac{1}{2}} = \int_0^\infty R_0(r) \Delta V_{-\frac{1}{2}}(r)\, R_0(r) dr$$

Radial wave functions:



a) when the spin is directed upwards (+1/2) and b) when the spin is directed downward (-1/2). Black is denoted to the Woods-Saxon (WS) potential, red line is Gauss (G).

Conclusion and further research

- In this work the energy levels of the ^{11}Be nucleus were reproduced by numerical methods as in [1,2].
- The ^{11}Be nucleus is regarded as a neutron halo consisting of a ^{10}Be core and one neutron. The internal interaction between the core and fragment includes Woods–Saxon potential and spin-orbit terms[1,2].
- Also, the energy level shifts were calculated taking into account the influence of the magnetic field, using two different potentials: the Woods–Saxon and Gauss forms for comparison. The numerical results coincide with the analytical solution, and the first order of perturbation theory is chosen as the analytical one.

- This work is the initial stage on theoretical study of the breakup of halo nuclei within quantum-mechanical approach. The next point is to numerically solve the time-dependent SE by using the solution of stationary SE as an initial condition, when the system is in its ground state.

[1] V. S. Melezhik and D. Baye, Phys. Rev. C **59**, 3232 (1999).
[2] P.Capel, D.Baye and V. S. Melezhik, Phys. Rev. C **68**, 014612 (2003).]

THANK YOU FOR YOUR ATTENTION