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ELECTROCOUPLINGS FOR LOW LYING NUCLEON RESONANCES FROM A LIGHT FRONT QUARK MODEL

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1. Introduction

Last decade has been marked by a significant progress in the experimental study of low-lying baryon resonances – radial/orbital nucleon excitations with $J^P = \frac{1}{2}^{\pm}, \frac{3}{2}^{\pm}, \frac{5}{2}^{\pm}$ (CLAS/CLAS12 Collaborations) that initiates interest in calculation of electrocouplings of baryons at large Q^2.

There are many theoretical approaches to the problem: Lattice QCD, DSE and BS equations, Light-front QCD, AdS/QCD, starting from the first principles, but rough estimates can be made on the basis of a light-front quark model.

It implies the construction of a good basis of light-front quark configurations possessing a definite value of the orbital (L) angular momentum and satisfying the Pauli exclusion principle (both tasks are nontrivial at LF).

Light-front quark wave functions were successfully used by many authors for description nucleon form factors and transition amplitudes

(1) before polarized electron data: F.Schlumpf, PRD 47, 4114; S.Capstick and B.D.Keister, PRD 51, 3598; I.G.Aznauryan, PLB 316, 391; F.Cardarelli *et al*, PLB 371, 7

(2) as well as after these (with taken into account new high-quality data): S.Capstick *et al*, J.Phys.Conf. 69, 012016; I.G.Aznauryan and V.D.Burkert, PRC 85, 055202; G.Ramalho and K.Tsushima, PRD 81,074020; V.E.Lyubovitskij *et al*, PRD 89, 054033.

However in these works the data up to $Q^2 \lesssim 3\text{-}4$ GeV^2 were only available. Now the CLAS12 plans measurements up to 12 GeV^2, and thus the calculation should be extended on this Q^2 region.

In our recent work (PRD 89, 054033) we have generalized our earlier non-relativistic approach to the Roper resonance electroproduction at $Q^2 < 4\,\mathrm{GeV}^2$ (PRD 84, 014004) by going to more high Q^2 in terms of light-front quark configuration.

Our approach is to fit parameters of light-front quark configurations to the elastic nucleon form factors at large Q^2 (up to 30 GeV^2) and further to use these parameters in the basis of excited (L=0,1) nucleon states to calculate the transition form factors at large Q^2 up to 12 GeV^2.

At this point we have run into an unexpected difficulty: the calculation in terms of excited relativistic quark configurations overestimates the transition amplitudes $N + \gamma^* \rightarrow N^*$ (at least for N^* with L=0,1), while the elastic $N + \gamma^* \rightarrow N$ form factors (calculated with the same parameters) are in a good agreement with the data.

Possibly, this implies that along with the quark core, other (more soft) degrees of freedom (e.g. Fock states beyond $|qqq\rangle$) should be taken into consideration in the case of radially/orbitally excited baryons. At high Q^2 the contribution of such soft components to the form factors will be quickly dying out, and only the contribution of the quark core survives.

Hence the weight of the quark core in the resonance wave function should be reduced, and its contribution to transition form factors at large Q^2 will decrease.

It was firstly shown by E. Santopinto et al, J.P.G 24, 753 in the nonrelativistic quark model and by I.G. Aznauryan and V. Burkert, PRC85, 055292 – in terms of a light front quark model.

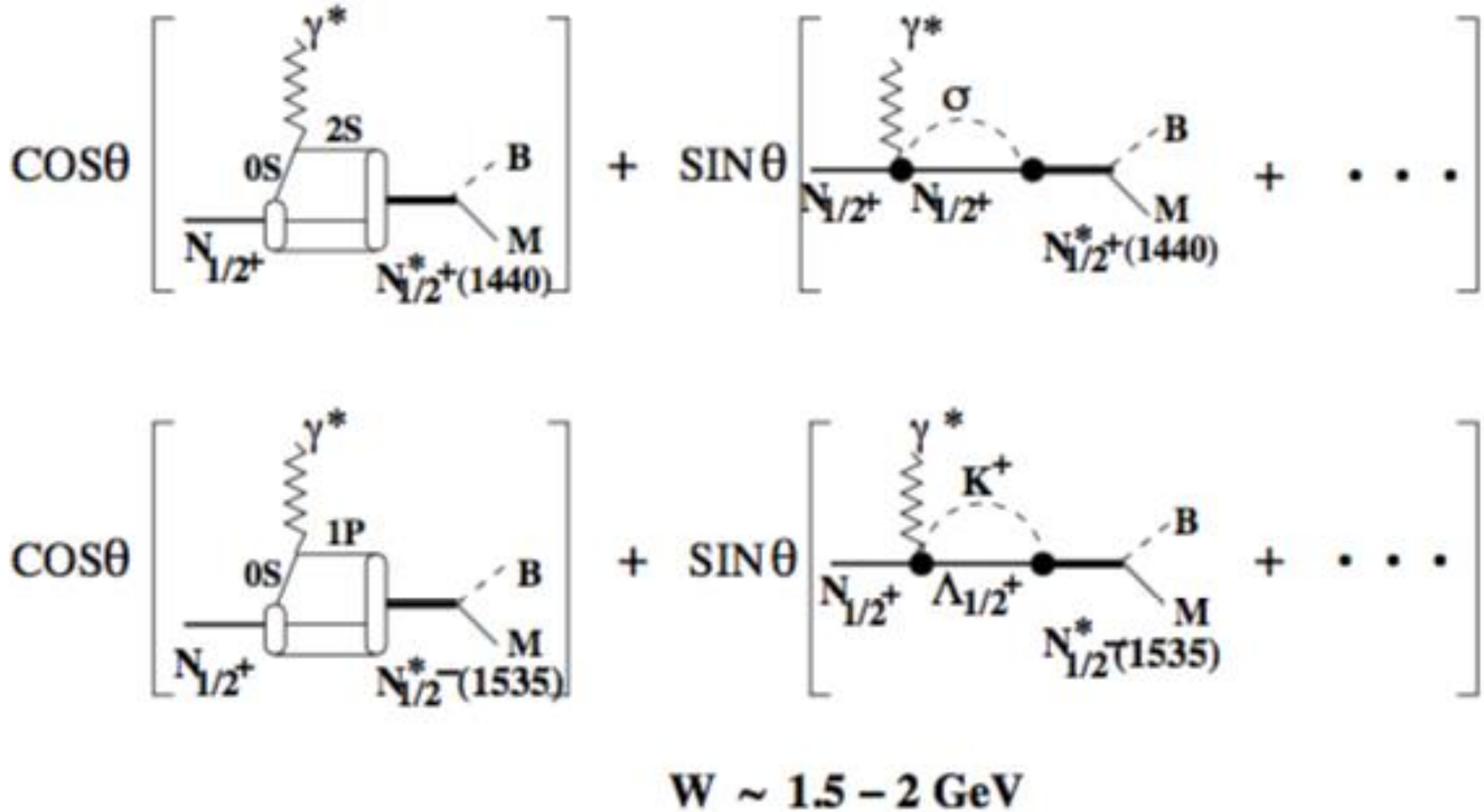
In general, contributions of Fock states beyond $|qqq\rangle$ necessarily exist for relativistic dynamics, and may be described either in the quark-gluon $(|qqq\,q\bar{q}\rangle, \ldots)$ or hadron $(|MB\rangle, \ldots)$ bases.

Each basis is presumably complete, so $|qqq\,q\bar{q}\rangle + |MB\rangle$ raises issues of double counting. A complementarity between the hadron and quark bases (specifically for higher Fock states) is observed in the data and called “duality”, but its origins remains unclear.

In our work we consider the lightest nucleon resonances $N^*(1440)/N^*(1535)$ as mixed states of the radially/orbitally excited quark configurations sp^2/s^2p ("quark core") and the “hadron molecules” (loosely bound states $N\sigma/\Lambda K$) as higher Fock states.

In the case of the Roper resonance such solution of the problem is almost evident: the inner structure of Roper cannot be adequately described in terms of the constituent quark degrees of freedom (the quark model fails to explain the observed mass and a large decay width of the Roper resonance).

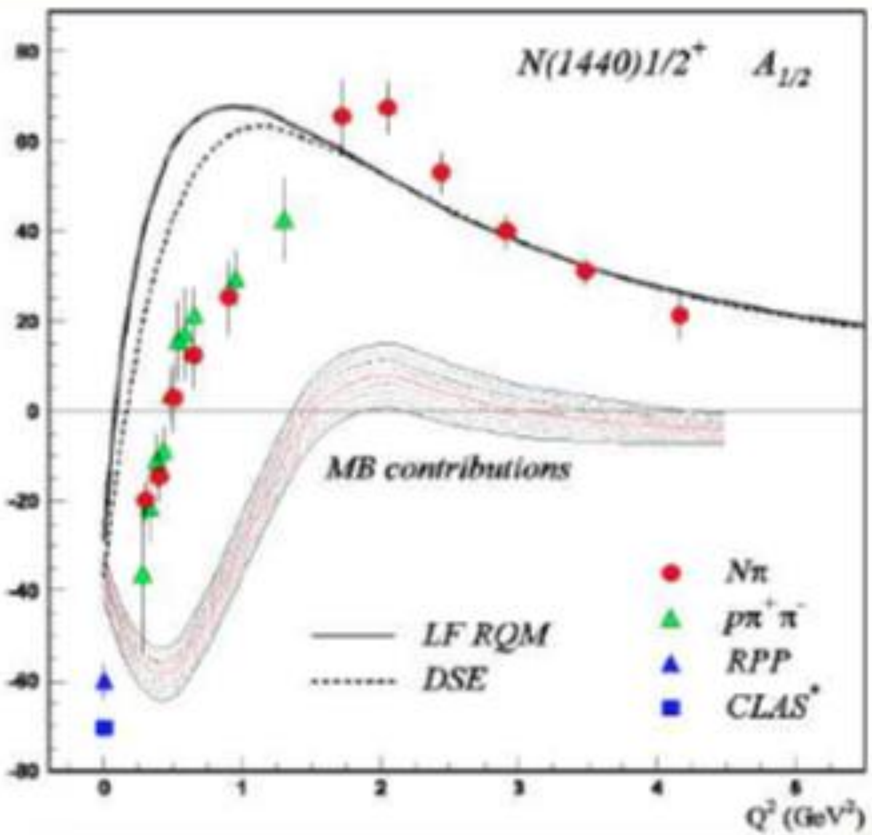
The electroproduction model. Baryons with $L = 0, 1$.



The parameter θ should be adjusted to optimize the description of the helicity amplitude $A_{1/2}$ only.

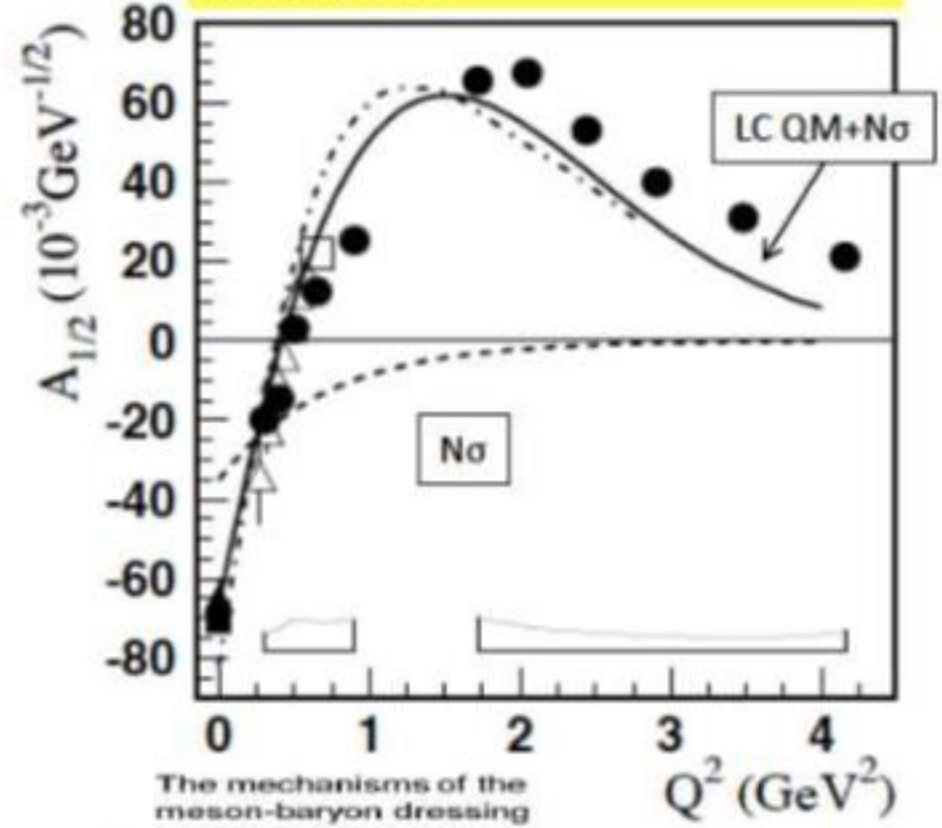
2. Description of low and moderate Q^2 data

We found that at the value of $\cos\theta \approx 0.8$ this model correlates well with the recent CLAS data. But we used non-relativistic h.o. quark configurations. Such calculations would be senseless at $Q^2 \gtrsim 3\text{-}4\ \mathrm{GeV}^2$.

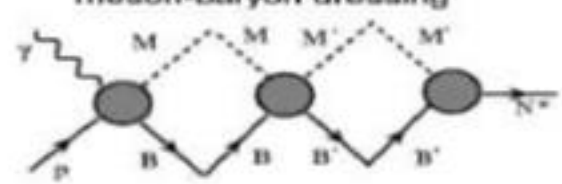


LF RQM-Light Front relativistic quark model: V.D. Burkert, I.G. Aznauryan, Phys . Rev. C85, 055202 (2012); Phys. Rev. C95, 065207 (2017).

I.T. Obukhovsky et al., Phys. Rev. D89, 014032 (2012).

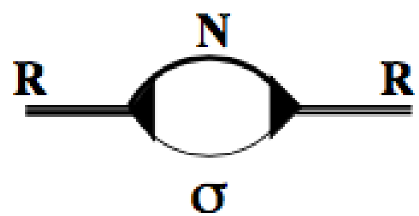


The mechanisms of the meson-baryon dressing



Hadron molecules are described in terms of relativistic- and gauge-invariant models.

The hadron loop



$\Sigma_{N\sigma}$ gives a negative contribution to the mass of the Roper resonance, and the $RN\sigma$ coupling constant $g_{RN\sigma}$ is defined by the 'compositeness condition'

$$Z_R \equiv 1 - \frac{d}{d\not{p}}\Sigma_{N\sigma}(\not{p})|_{\not{p}=m_R} = 0,$$

i.e. the elementary particle R has a zero weight in the hadron molecule.

We use effective Lagrangians (Dubna group: G. Efimov, M. Ivanov, V. Lyubovitskij) for description of non-local RNσ and NNσ interactions, e.g.

$$\mathcal{L}_{str}(x) = g_{RN\sigma}\bar{R}(x)\int d^4y\,\Phi_R(y^2)N(x+\alpha y)\sigma(x-(1-\alpha)y), \quad \alpha = \frac{M_\sigma}{m_N+M_\sigma},$$

and h.o. Gaussians as Fourier transforms of $\Phi_N(y^2)$ and $\Phi_R(y^2)$

$$\tilde{\Phi}_N(k_E^2) = exp(-\frac{k_E^2}{\Lambda^2}) \quad \text{and} \quad \tilde{\Phi}_R(k_E^2) = (1-\lambda\frac{k_E^2}{\Lambda^2})exp(-\frac{k_E^2}{\Lambda^2})$$

with the orthogonality condition $\int\tilde{\Phi}_R(k_E^2)\tilde{\Phi}_N(k_E^2)d^4k_E = 0$.

The electromagnetic interaction term for this non-local vertex

$$\mathcal{L}_{em}^{(1)} = g_{RN\sigma}\bar{R}(x)\int dy\,\Phi_R(y^2)e^{-ieI(x+\alpha y,x,P)}N(x+\alpha y)\sigma(x-(1-\alpha)y) + h.c.$$

is generated when the non-local Lagrangian are gauged with a gauge field exponential $e^{-ieI(x+\alpha y,x,P)}$ where

$$I(y,x,P) = \int_x^y dz_\mu A^\mu(z), \quad P \text{ is the path of integration}$$

S.Mandelstam, Ann.Phys. 19, 1 (1963); J.Terning, Ph.Rev. D44, 887 (1991)

The full Lagrangian of electromagnetic interaction

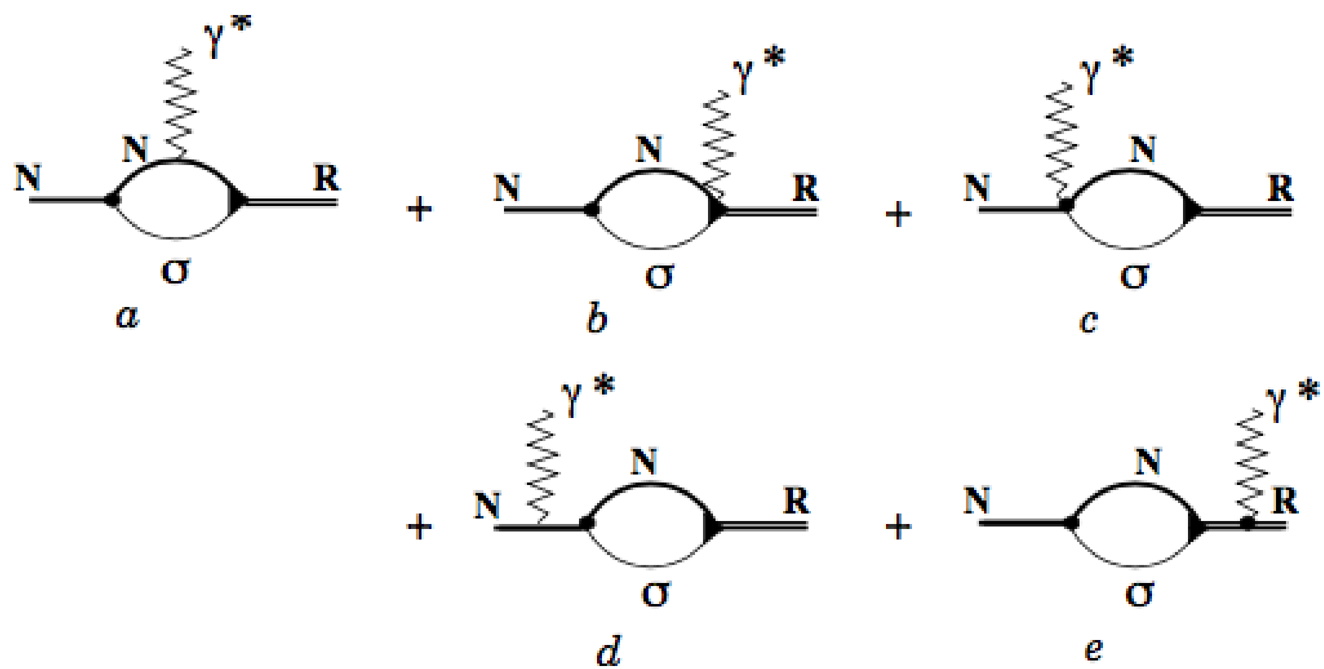
$$\mathcal{L}_{em} = \mathcal{L}_{em}^{(1)} + \mathcal{L}_{em}^{(2)}$$

includes also the standard term

$$\mathcal{L}_{em}^{(2)} = e_B \bar{B}(x) \not{A}(x) B(x), \quad B = N, R$$

obtained by minimal substitution $\partial^\mu B \to (\partial^\mu - e_B A^\mu) B$

Only the total sum of the first order diagrams (including the contact terms $\mathcal{L}_{em}^{(1)}$) satisfies the gauge invariance



3. High Q^2 . Nucleon ground state at the light front.

At high $Q^2 \gtrsim$ 3-4 Gev^2 the contribution of soft components of the baryon (the meson cloud, "molecular" admixtures, etc.) to transition form factors falls off by comparison with the "quark core" contribution. Hence, only the quark contribution should be considered at high Q^2.

However, the form factors defined by a Gaussian as a quark core wave function are also quickly dying out at $Q^2 \gtrsim$ 3-4 GeV^2.

A possible alternative to the Gaussian wave function are:
- many Gaussians complemented by polinomial factors (Capstick, 2007);
- a pole-like w.f.,
- a model with the running quark mass (Aznauryan and Burkert, PRC 85, 055202), following the QCD predictions; etc.

We have chosen a pole-like form of the w.f.

Pole-like form of the nucleon ground state wave function Φ_0

$$\Phi_0(\xi, \eta, k_\perp, K_\perp) = \frac{\mathcal{N}_0}{(1 + M_0^2/\beta^2)^\gamma}$$

with

$$M_0^2 = \frac{m_q^2 + k_\perp^2}{\eta\xi(1-\xi)} + \frac{\eta m_q^2 + K_\perp^2}{\eta(1-\eta)}$$

was firstly fitted to the elastic nucleon form factors by Schlumpf (PRD47, 4114) with γ =3.5 and the scale parameter $\beta \approx 2m_q$.

Here $k_\perp$, $K_\perp$ are relative transverse moments in pairs "1-2" and "(12)-3" respectively (the light cone invariants),
the longitudinal LF variables are $x_1 = \xi\eta,\ x_2 = (1-\xi)\eta,\ x_3 = 1 - \eta$,
and M_0 is the mass of the free $3q$ system (m_q is the quark mass).

Such form is as yet unjustified, but it should be noted that at least in the meson sector the pole-like form of the pion $\bar{q}q$ w.f.

$$f_\pi \varphi_\pi(x, k_\perp^2) = \frac{9}{4\pi^2} \frac{1}{\left(1 + \frac{k_\perp^2}{4m_q^2 x(1-x)}\right)^\gamma}, \quad \gamma = 1 - 2$$

was recently reconstructed starting from the Bethe-Salpeter wave function (C.D. Roberts, arXiv:1509.02925) projected onto the light front (L.Chang *et al*, PRL110, 132001).

The nucleon pole-like w.f. $\Phi_0(\xi, \eta, k_\perp, K_\perp)$ looks like a generalization of the pion $\bar{q}q$ w.f. for the case of the $3q$ system.

(Recall that each diquark in the colorless $3q$ state is equivalent to an antiquark, i.e. the w.f. of each $q - (qq)$ pair in the nucleon should be similar to the $\bar{q}q$ w.f. of the pion).

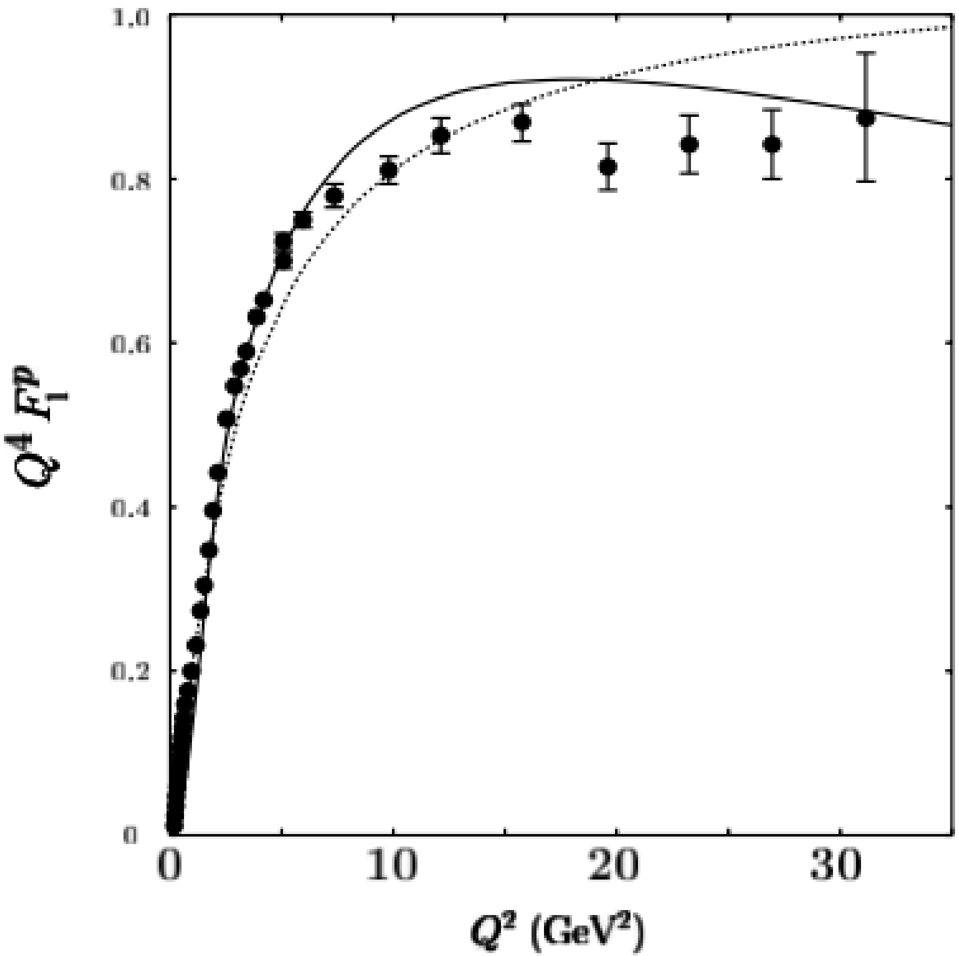
Starting from the "+" component of quark current on the light front

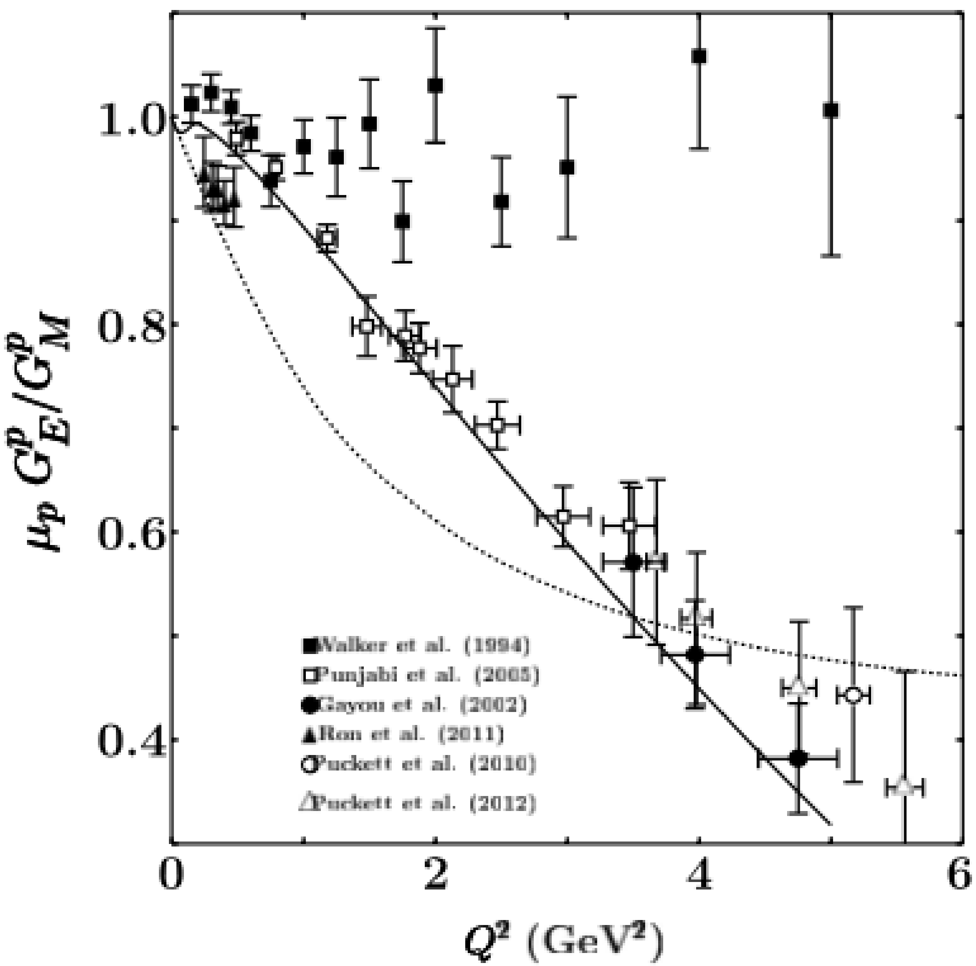
$$I^{(i)+} = e^{(i)} \left(I f_1 + i\, \hat{n}_z \cdot [\sigma^{(i)} \times q_\perp] f_2 \right),$$

(without quark form factors, but with a small anomalous quark magnetic moment $\varkappa_q$, i.e. $f_1 = 1$, $f_2 = \varkappa_q$),
we have fitted the free parameters to the modern data on nucleon form factors (only preserving the characteristic values found by Schlumpf: $\gamma \approx 3.5$ and $\beta \approx 2m_q$).

With the values $\gamma = 3.51$, $m_q = 0.251$ GeV, $\varkappa_u = -0.0028$, $\varkappa_u = 0.0224$, $\beta_u = 0.579$ GeV, $\beta_d = 0.5$ GeV
we have obtained a not so bad description of the elastic nucleon form factors within a full measured range $0 \leq Q^2 \leq 32$ GeV^2 including data on the ratio G_E/G_M at $Q^2 \lesssim$ 6-8 GeV^2.
We also compared our fit with results of another approach, in which an effective light-front wave function was derived from the matching of soft-wall AdS/QCD and light-front QCD. (Th. Gutsche, V.E. Lyubovitskij, I. Schmidt, A. Vega, PRD 89, 054033)

I.T. Obukhovsky et al, Phys. Rev. D 89, 014032 (2014)





——— LFQM .... AdS/QCD (V.E. Lyubovitskij et al., PRD 89)

The pole-like w.f. is also good for the data at $Q^2 \to 0$:

Таблица 1: Electromagnetic properties of nucleons in LF quark models

Quantity	LFQM	AdS/QCD	Data
μ_p (in n.m.)	2.820 (2.820)	2.793	2.793
μ_n (in n.m.)	-1.920 (-1.920)	-1.913	-1.913
μ_u (in n.m.)	3.720 (3.720)	3.673	1.673
μ_d (in n.m.)	-1.020 (-1.020)	-1.033	-2.033
r_E^p (fm)	0.871 (0.872)	0.789	0.8921 ± 0.0073
$\langle r_E^2 \rangle^n$ (fm^2)	-0.014 (-0.022)	-0.108	-0.1161 ± 0.0022
r_M^p (fm)	0.883 (0.872)	0.757	$0.777 \pm 0.013 \pm 0.010$
r_M^n (fm)	0.898 (0.893)	0.773	$0.862^{+0.009}_{-0.008}$
r_E^u (fm)	0.867 (0.866)	0.754	0.8589 ± 0.0107
r_E^d (fm)	0.855 (0.846)	0.638	0.7507 ± 0.0094
r_M^u (fm)	0.875 (0.832)	0.749	0.7288 ± 0.0151
r_M^d (fm)	0.938 (0.949)	0.815	1.0582 ± 0.0434

4. High Q^2 electroproduction.

Quark configurations at LF, Melosh transformations and Pauli principle.

For description of $J^P = 1/2^{\pm}, 3/2^{\pm}, 5/2^{\pm}$ baryons electroproduction we need a good basis of quark configurations at light front with fixed values of orbital (L) and total (J=L+S) angular momenta at L=0,1,2. Here we follow the known works of Yu.Shirokov (50-th), M.Ternt'ev-L.Kondratyuk (70-th) and B.Keister-W.Polyzou (90-th).

We start from non-relativistic shell-model configurations and change the h.o. wave functions for light-front ones (Gaussian or pole-like) dependent on the relativistic relative moments $\boldsymbol{k}$, $\boldsymbol{K}$ and expressed in light-front invariants $\boldsymbol{k}_{\perp}$, $\boldsymbol{K}_{\perp}$ and $\boldsymbol{\xi}, \boldsymbol{\eta}$.

At this stage, as usual, there are problems with boosts (in the instant form of dynamics) or rotations (at the light front). In both cases generators of transformations depend on the dynamics. Choosing the light front basis one can represent boosts as kinematical operators independent on dynamics. But in that case there are problems with representation of rotations for interacting particles

Difficulties in representation of rotations can partially be resolved going to the rest frame for definition of the total angular moment of the system, $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$, and returning to the moving Breit frame using kinematical light-front boosts. Then Melosh transformations for the quark spins should be used.

The transition matrix element $N \to N^*$ can be symbolically written as

$$\mathcal{M}_{N\to N^*} = 3\langle\psi_{N^*}|D^{-1}(k_1')D^{-1}(k_2')D^{-1}(k_3')\, I^{(3)+}D(k_3)D(k_2)D(k_1)|\psi_N\rangle\,,$$

where $D(k_i)$ is the Melosh transformation for spin s_i of the i-th quark

$$D(k_i) := D^{\frac{1}{2}}_{\mu_i'\mu_i}(\theta_{Mel}) = \langle s_i\mu_i'|\frac{m_q + k_{iz} + i\hat{n}_z\cdot[\vec{\sigma}_i \times \vec{k}_{i\perp}]}{\sqrt{(m_q + k_{iz})^2 + k_{i\perp}^2}}|s_i\mu_i\rangle$$

and ψ_N (ψ_{N^*}) is the nucleon (resonance) state vector described by the three-quark configuration at light front.

In the case of 70^- resonances (e.g. $N^*(1535)$ with S=1/2, T=1/2) the LF three-quark configuration $s^2p[21]_X$ at the rest frame reads (the Clebsch-Gordon coeficients for adding $L+S$ are omitted)

$$|\psi_N^*\rangle = \sqrt{\frac{1}{2}}|s^2p[21]_X y_X^{(1)}\rangle\,|[21]_{ST} y_{ST}^{(1)}\rangle + \sqrt{\frac{1}{2}}|s^2p[21]_X y_X^{(2)}\rangle\,|[21]_{ST} y_{ST}^{(2)}\rangle$$

Here we use orbital states with given Yamanouchi symbols $y_X^{(n)}$, n=1,2 (the normalisation factors are omitted)

$$|s^2p[21]_X y_X^{(1)}, \mu_L\rangle = |\vec{K}|Y_{1\mu_L}(\hat{K})\Phi_0(M_0),$$

$$|s^2p[21]_X y_X^{(2)}, \mu_L\rangle = |\underline{\vec{k}}|Y_{1\mu_L}(\hat{\underline{k}})\Phi_0(M_0).$$

These wave functions differ from respective non-relativistic shell-model configurations since they imply relativistic relative momenta $\vec{K}$ and $\vec{k}$ instead of non-relativistic ones.

For example, the Z component of $\vec{K}$ at the LF is

$$K_Z := \frac{1}{2}(K^+ - K^-) = \frac{1}{2}\left[(1-\eta)M_0 - \frac{K_\perp^2 + m_q^2}{(1-\eta)M_0}\right],$$

and in the Breit frame the initial and final transverse momenta of the 3-rd quark are $\vec{K}'_\perp = \vec{K}_\perp + \eta\vec{q}_\perp$.

Moreover, contrary to the standard expression for $y^{(2)}$ component usually written in terms of functions $|\vec{k}|Y_{1\mu_L}(\hat{k})$ we use here the spherical functions $|\underline{\vec{k}}|Y_{1\mu_L}(\underline{\hat{k}})$ dependent on a modified momentum

$$\underline{\vec{k}}_\perp = \vec{k}_\perp + ((1-2\xi)/2)\vec{K}_\perp.$$

Such substitution is required to satisfy the Pauli exclusion principle if the $y^{(1)}$ component is defined as $|\vec{K}|Y_{1\mu_L}(\hat{K})$.

5. Electroproduction of the lightest nucleon resonances.

With this technique we have calculated the transition helicity amplitudes for electroproduction of the lightest nucleon resonances with $L=0,1$.

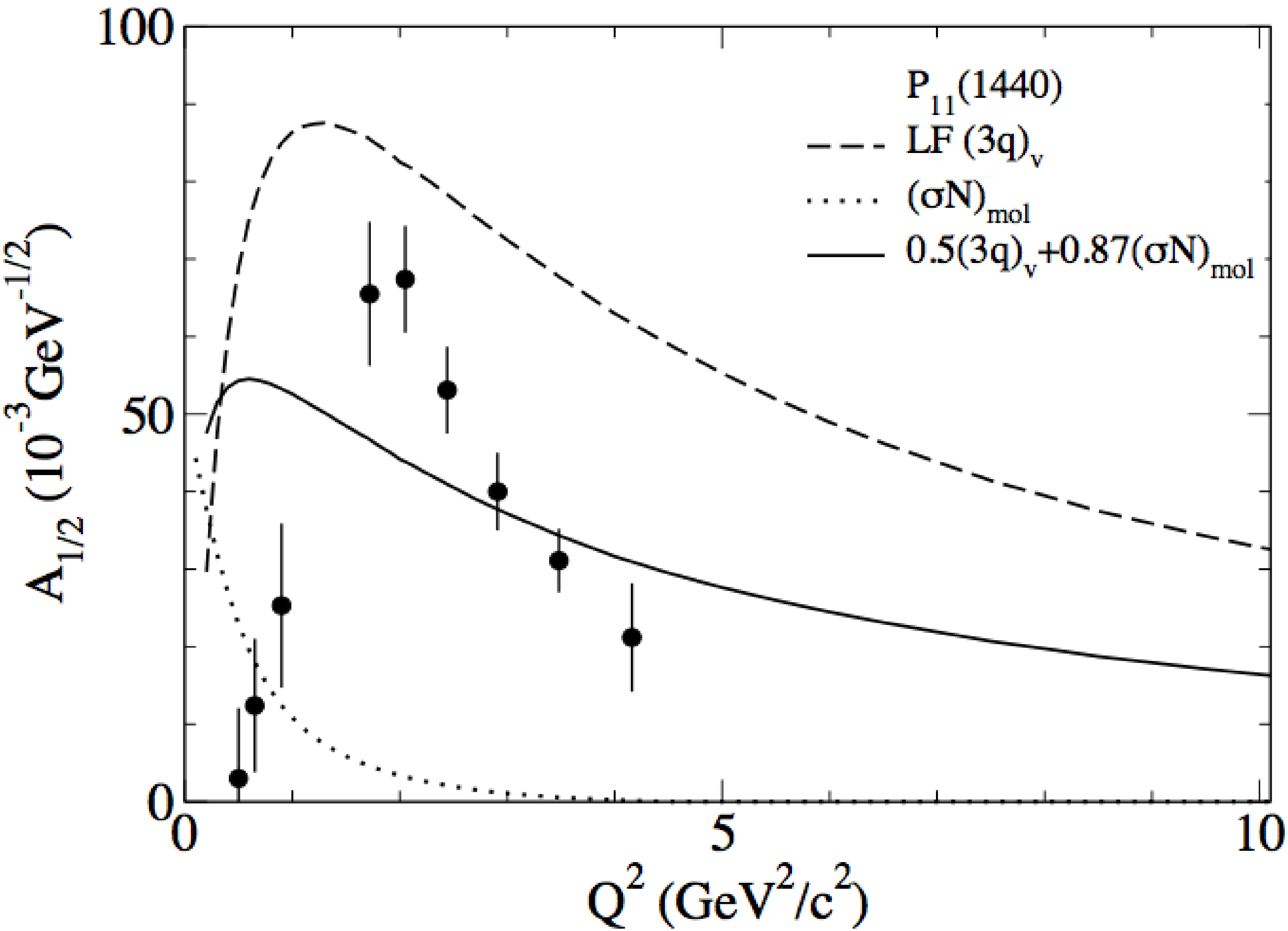
At $L=0$ (Roper resonance) we used the light-front analogue of the radially excited quark configuration for the case a pole-like ground state Φ_0:

$$\Phi_2 = \mathcal{N}_2 \left(1 - C_R \frac{M_0^2}{\beta^2}\right) \Phi_0, \qquad \langle \Phi_2 | \Phi_0 \rangle = 0.$$

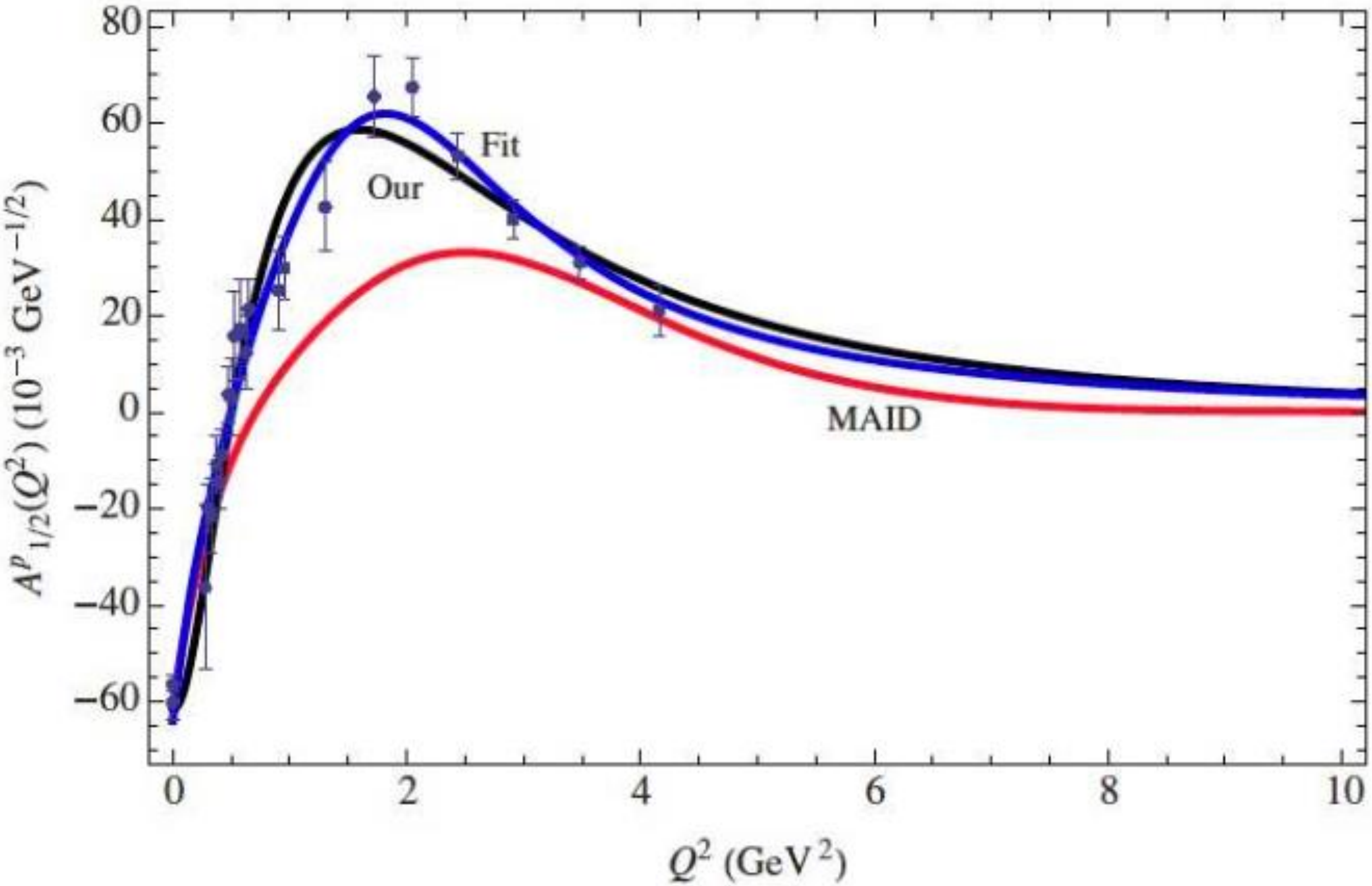
After comparison with the data it turns out that the quark-core weight ($\cos\theta$) in the mixed model $R = \cos\theta(3q)^* + \sin\theta|N+\sigma\rangle$ should be reduced: from $\cos\theta \simeq 0.7$-0.8 of the nonrelativistic (Gaussian) model to the value of $\cos\theta \approx 0.5$ of the relativistic (pole-like) model.

At the same time the nucleon elastic form factors were successfully described without reducing the quark core weight.

I.T. Obukhovsky et al, Phys. Rev. D 89, 014032 (2014)



Th. Gutsche, V.E. Lyubovitskij, and I. Schmidt, Phys. Rev. D 97, 054011 (2018), Electromagnetic structure of nucleon and Roper in soft-wall AdS/QCD



It is also characteristic of the resonances with $L=1$. The corresponding quark configurations of negative parity

$$|s^2p[21]_X J(L{=}1, S{=}1/2)\rangle, \quad J = 1/2, 3/2$$
$$|s^2p[21]_X J(L{=}1, S{=}3/2)\rangle, \quad J = 1/2, 3/2, 5/2,$$

represent the "quark core" for lightest resonances:

$$S_{11}(1535) \to N^*_{1/2^-}(3q) = \ a_1|J{=}\tfrac{1}{2}(S{=}\tfrac{1}{2})\rangle + b_1|J{=}\tfrac{1}{2}(S{=}\tfrac{3}{2})\rangle$$
$$S_{11}(1650) \to N'^*_{1/2^-}(3q) = -b_1|J{=}\tfrac{1}{2}(S{=}\tfrac{1}{2})\rangle + a_1|J{=}\tfrac{1}{2}(S{=}\tfrac{3}{2})\rangle$$

$$D_{13}(1520) \to N^*_{3/2^-}(3q) = \ a_3|J{=}\tfrac{3}{2}(S{=}\tfrac{1}{2})\rangle + b_3|J{=}\tfrac{3}{2}(S{=}\tfrac{3}{2})\rangle$$
$$D_{13}(1720) \to N'^*_{3/2^-}(3q) = -b_3|J{=}\tfrac{3}{2}(S{=}\tfrac{1}{2})\rangle + a_3|J{=}\tfrac{3}{2}(S{=}\tfrac{3}{2})\rangle$$

$$D_{15}(1675) \to N^*_{5/2^-}(3q) = |J{=}\tfrac{5}{2}(S{=}\tfrac{3}{2})\rangle$$

In our approach (pole-like w.f.) at large Q^2 the contribution of "quark core" to the electroproduction of resonances overestimates the data excluding $D_{15}(1675)$ where this contribution is suppressed according to the Moorhouse selection rule.

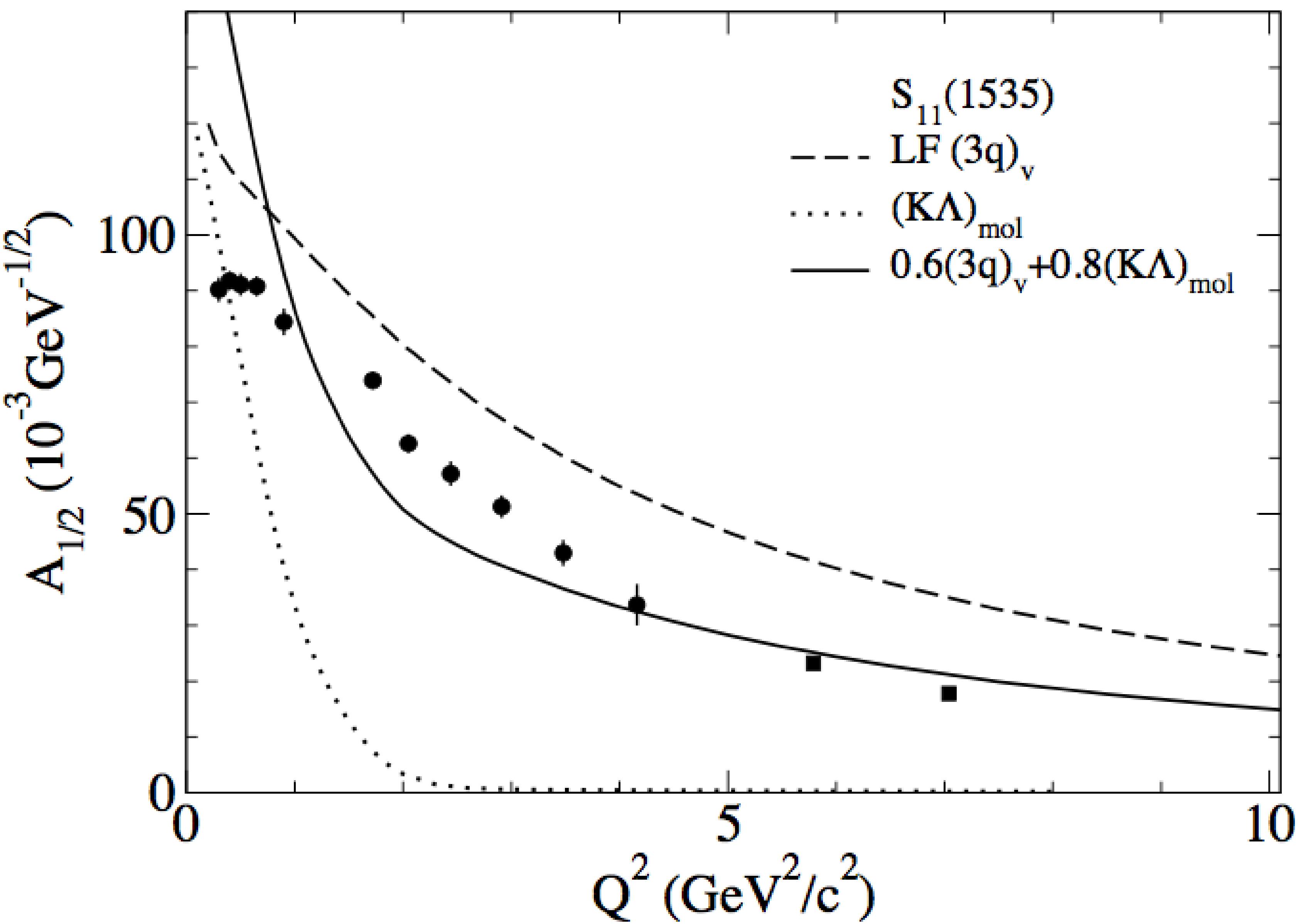
It would be instructive to describe these resonances in terms of the same scheme as we successfully used for the Roper resonance, i.e. as mixed states of excited quark configurations $(3q)^*$ and hadron molecules:

$$P_{11}(1440) = \cos\theta_0 N^*_{1/2^+}(3q) + \sin\theta_0 |N_{1/2^+} + \sigma\rangle,$$

$$S_{11}(1535) = \cos\theta_1 N^*_{1/2^-}(3q) + \sin\theta_1 |\Lambda_{1/2^+} + K^+\rangle.$$

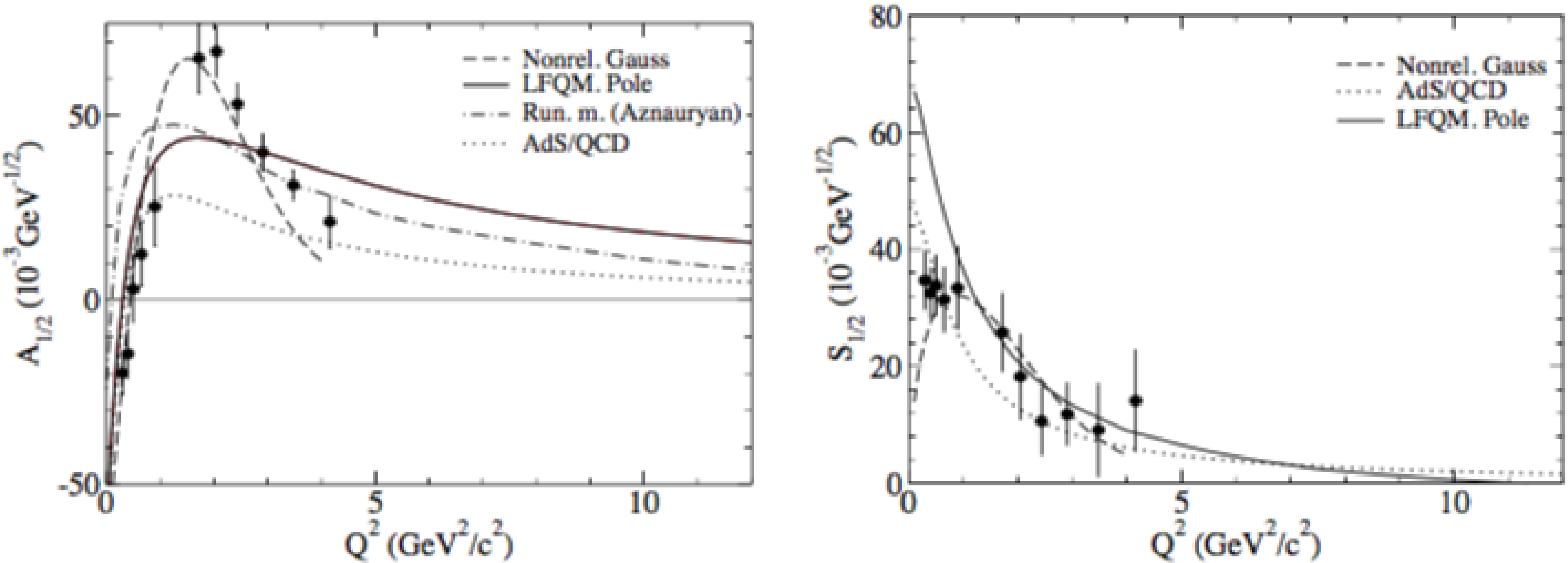
$$S_{11}(1650) = \cos\theta_1' N'^*_{1/2^-}(3q) + \sin\theta_1' |B + M\rangle, \quad . \quad . \quad . \text{ etc.}$$

I.T. Obukhovsky et al, EPJ 138, 04003 (2017)



Such situation is also characteristic of other relativistic models recently developed. It could be considered as evidence of a large role of soft components (the meson cloud or hadron molecular states) in resonances as compared to the ground state of the nucleon.

$$\cos\theta = 0.57:$$



Transverse ($A_{1/2}$) and longitudinal ($S_{1/2}$) helicity amplitudes for electro-production of the Roper resonance at high Q^2 in different approaches

6. Summary.

1. The lightest nucleon resonances are described at light front as mixed states of the $3q$ cluster possessing a definite value of the inner orbital momentum $L = 0{,}1$ and a hadron molecular state, $N + \sigma$ or $\Lambda + K$.

2. Nucleon elastic form factors are successfully described in a large interval of Q^2 by the lowest light-front quark configuration without any hadron-molecular admixtures.

3. But in the case of nucleon resonances the quark core overestimates the transition amplitudes at large Q^2, and thus the weight of quark core in the resonance should be relatively small because of the considerable weight of higher Fock states.