Off-shell initial state effects in Drell-Yan process at hadron colliders.

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Outline.

Motivation

- The study of leading twist Collinear Parton Distribution Functions (PDFs) and Transverse Momentum Dependent (TMD) PDFs of quarks and antiquarks in nucleon is main task of Spin Physics Experiments at NICA-SPD with polarized proton and deuteron beams.
- Appropriate *Factorization* should be chosen accordingly to the kinematics of the observable under consideration.
- We should define correctly *Factorization Formula, PDFs and Hard Amplitudes* taking into account the **Gauge Invariance** as a principal condition in QED/ QCD.

Motivation

Collinear PDFs ⇔ Collinear Factorization, Collinear Parton Model (CPM)

$$\sigma^{\rm CPM} = \int dx_1 \int dx_2 \; f_q(x_1,\mu) f_{\bar q}(x_2,\mu) \hat\sigma_{\rm CPM}(x_1,x_2,\mu) + O\left(\Lambda^2/\mu^2\right) \quad (1)$$

DGLAP resums large logarithms

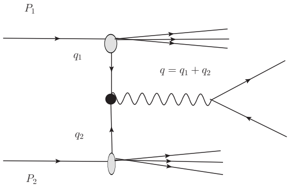
$$\sim \log\left(\mu^2/\Lambda^2\right) \quad (2)$$

TMD PDFs ⇔ TMD Factorization, TMD Approach (TMDA).

$$\begin{aligned}\sigma^{\rm TMD} \;&=\; \int dx_1 \int d\mathbf{q}_{1T} \int dx_2 \int d\mathbf{q}_{2T} \; F_q(x_1,\mathbf{q}_{1T},\mu) F_{\bar q}(x_2,\mathbf{q}_{2T},\mu) \times \\ &\times\;\; \hat\sigma_{\rm CPM}(x_1,x_2,\mu)\; \delta^{(2)}\left(\mathbf{q}_{1T}+\mathbf{q}_{2T}-\mathbf{p}_T\right) + O\left(\Lambda^2/\mu^2,\; \mathbf{p}_T^2/\mu^2\right) \quad (3)\end{aligned}$$

Large logarithmic contributions enhanced by $\log(\mu^2/\Lambda^2)$ and $\log^n(\mu^2/\mathbf{p}_T^2)$ $(n=1,2)$ are taken into account in the TMD PDFs.

Motivation



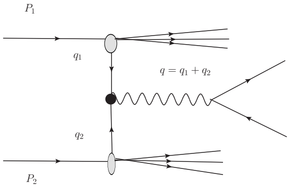
Sudakov (Light-cone) decomposition:
$n_-^\mu = 2P_1^\mu/\sqrt{S}$, $n_+^\mu = 2P_2^\mu/\sqrt{S}$, $n_\pm n_\pm = 0$, $n_\pm n_\mp = 2$:

$$q^\mu = \frac{1}{2}\left(q^+ n_-^\mu + q^- n_+^\mu\right) + q_T^\mu,$$

where $q^\pm = (n_\pm q) = q^0 \pm q^3$, $n_\pm q_T = 0$, and

$$kq = \frac{1}{2}(k^+ q^- + k^- q^+) - \mathbf{k}_T \mathbf{q}_T.$$

Motivation



$$q_1^\mu = \frac{1}{2}\left(q_1^+ n_-^\mu + q_1^- n_+^\mu\right) + q_{1T}^\mu,$$

and

$$q_2^\mu = \frac{1}{2}\left(q_2^+ n_-^\mu + q_2^- n_+^\mu\right) + q_{2T}^\mu,$$

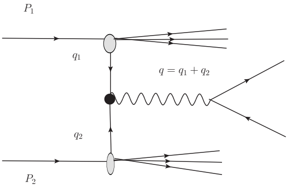
In *CPM, TMD and Multi-Regge-Kinematics (***MRK***)*:

$$q_1^+ \gg q_1^-$$

and

$$q_2^- \gg q_2^+$$

Motivation



$$q_1^\mu \simeq \frac{1}{2} q_1^+ n_-^\mu + q_{1T}^\mu, \quad q_1^2 \simeq q_{1T}^2 = -\mathbf{q_{1T}}^2$$

and

$$q_2^\mu \simeq \frac{1}{2} q_2^+ n_+^\mu + q_{2T}^\mu, \quad q_2^2 \simeq q_{2T}^2 = -\mathbf{q_{2T}}^2$$

- We should deal with **off-shell initial-state** partons.
- The **Gauge Invariance** of QED/QCD amplitudes becomes under the question.

Motivation

In **CPM**:
$q^2_{1,2} = q^2_{1,2T} = {\mathbf{q}_{1,2T}}^2 = 0 \Rightarrow$, and one has Gauge Invariance of hard amplitudes

In **TMD**: Virtualities of initial partons ($|q^2_{1,2}|$) are neglected, **BUT** small transverse momenta ($|q^2_{1,2T}| = {\mathbf{q}_{1,2T}}^2 \ll \mu^2$) are included in TMD PDFs.

In case of DY process we study in TMD only region $|\mathbf{q}_T| \ll Q$

Parton Reggeization Approach (PRA) is a hybrid scheme (CPM, TMD, **MRK**), which combines gauge-invariant matrix elements with off-shell (**Reggeized**) partons in the initial state with the unintegrated PDFs resumming doubly-logarithmic corrections $\sim \log^2(\mathbf{q}_T^2/Q^2)$.
The cornerstones of **PRA** are *Lipatov's Effective Theory of Reggeized partons* and *modified-MRK-Factorization of hard processes.*

$q^2_{1,2} = q^2_{1,2T} \neq 0$, but the **Gauge Invariance** is present!.
In DY-pair production, **PRA** can be used at the **arbitrary** values of $|\mathbf{q}_T|/Q$.

LO PRA framework

LO PRA framework

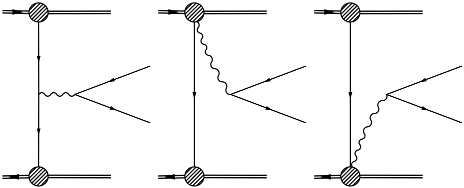
See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. **D96** 096019 (2017)] for details.

- **The aim** of **PRA** is to improve the description of **multi-scale** correlational observables ($|\mathbf{q}_T| \sim Q$) in comparison with the *fixed-order CPM* calculations.
- **The wider task** is to understand the role of transverse momentum in Initial-State Radiation (ISR) and put it under theoretical control at the level of quantum field theory at all transverse momenta, $0 \leq |\mathbf{q}_T| \leq Q$.
- To provide predictions with controllable accuracy and understand our formalism better we can go to NLO in **PRA**.

1. M. Nefedov and V. Saleev, "DIS structure functions in the NLO approximation of the Parton Reggeization Approach," EPJ Web Conf. **158** (2017) 03011.
2. M. Nefedov and V. Saleev, "On the one-loop calculations with Reggeized quarks," Mod. Phys. Lett. A **32** (2017) no.40, 1750207.

LO PRA framework

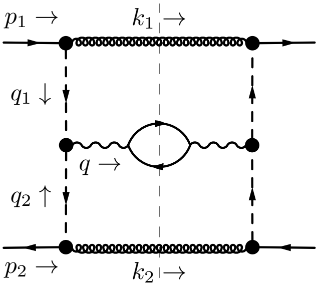
In order for **Gauge Invariance to be satisfied**, in addition to the annihilation diagram, one must also take into account the *direct interaction of the photon with the proton or its fragments*:



- Such a consideration in the general case does not lead to a simple factorization formula.
- However, in the **multi-Regge limit**, factorization is possible, it is **MRK** factorization.

Derivation of the LO factorization formula

See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. **D96** 096019 (2017)] for details.



Auxiliary hard CPM subprocess:

$$q(p_1) + \bar{q}(p_2) \rightarrow g(k_1) + l\bar{l}(P_{\mathcal{A}}) + g(k_2),$$

Modified MRK approximation: $z_{1,2}$ and $\mathbf{q}_{T1,2}^2$ – arbitrary ($q_1^+ = z_1 p_1^+$, $q_2^- = z_2 p_2^-$):

$$\overline{|\mathcal{M}|^2}_{\rm mMRK} \simeq \frac{4g_s^4}{q_1^2 q_2^2} P_{qq}(z_1) P_{qq}(z_2) \frac{\overline{|\mathcal{A}_{PRA}|^2}}{z_1 z_2},$$

where $q_{1,2}^2 = -\mathbf{q}_{T1,2}^2/(1-z_{1,2})$, has correct **collinear** and **Multi-Regge** limits!

Conjecture: mMRK approximation is reasonable zero-approximation for exact $\overline{|\mathcal{M}|^2}$ *away from collinear limit.*

Factorization formula

Substituting the $\overline{|\mathcal{M}|^2}_{\rm mMRK}$ to the factorization formula of CPM and changing the variables we get:

$$d\sigma = \int\limits_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \tilde{\Phi}_q(x_1, t_1, \mu^2) \int\limits_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \tilde{\Phi}_{\bar{q}}(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{\rm PRA},$$

where $x_1 = q_1^+/P_1+$, $x_2 = q_2^-/P_2^-$, $\tilde{\Phi}(x, t, \mu^2)$ – "tree-level" **unintegrated PDFs**, the partonic cross-section in PRA is:

$$d\hat{\sigma}_{\rm PRA} = \frac{\overline{|\mathcal{A}_{PRA}|^2}}{2Sx_1x_2} \cdot (2\pi)^4 \delta \left(\frac{1}{2} \left(q_1^+ n_- + q_2^- n_+ \right) + q_{T1} + q_{T2} - P_{\mathcal{A}} \right) d\Phi_{\mathcal{A}}.$$

Note the usual **flux-factor** Sx_1x_2 for **off-shell** initial-state partons.

LO unintegrated PDF

The "tree-level" unPDF:

$$\tilde{\Phi}_q(x,t,\mu^2) = \frac{1}{t}\frac{\alpha_s}{2\pi}\int_x^1 dz\; P_{qq}(z)\cdot\frac{x}{z}f_g\left(\frac{x}{z},\mu^2\right).$$

contains collinear divergence at $t \to 0$ and IR divergence at $z \to 1$.
In the "dressed" unPDF collinear divergence is regulated by **Sudakov formfactor** $T(t,\mu^2)$:

$$\Phi_i(x,t,\mu^2) \quad = \quad \frac{T_i(t,\mu^2)}{t}\times\frac{\alpha_s(t)}{2\pi}\int_x^1 dz\; \theta_z^{\text{cut}} P_{ij}(z)\frac{x}{z}f_j\left(\frac{x}{z},t\right)$$

where: $\theta_z^{\text{cut}} = \theta\left((1-\Delta_{KMR}(t,\mu^2))-z\right)$, and the Kimber-Martin-Ryskin(KMR) **cut condition** [KMR, 2001]:

$$\Delta_{KMR}(t,\mu^2) = \frac{\sqrt{t}}{\sqrt{\mu^2}+\sqrt{t}},$$

follows from the **rapidity ordering** between the last emission and the hard subprocess.

LO unintegrated PDF

$$\Phi_i(x,t,\mu^2) = \frac{T_i(t,\mu^2)}{t} \times \frac{\alpha_s(t)}{2\pi} \int\limits_x^1 dz\, \theta_z^{\mathrm{cut}} P_{ij}(z) \frac{x}{z} f_j\left(\frac{x}{z}, t\right)$$

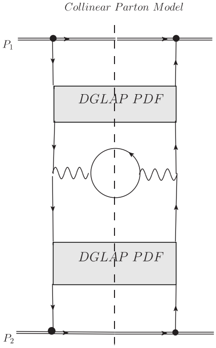
$$\simeq \boxed{\frac{\partial}{\partial t}\left[T_i(t,\mu^2)\cdot x f_i(x,t)\right]} \leftarrow \text{derivative form of unPDF}$$

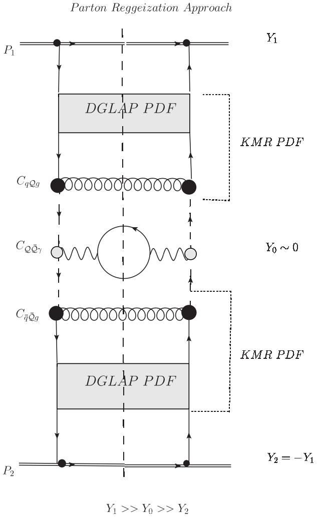
$\Rightarrow$ **LO normalization condition:** (since $T(0,\mu^2,x)=0$ and $T(\mu^2,\mu^2,x)=1$)

$$\boxed{\int\limits_0^{\mu^2} dt\; \Phi_i(x,t,\mu^2) = x f_i(x,\mu^2)} \leftarrow \text{Holds approximately for standard KMR formula!}$$

The normalization can be made exact by introducing the x-dependence to Sudakov factor.

General Scheme of PRA

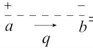
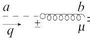
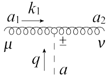
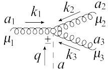


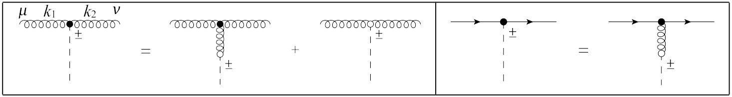


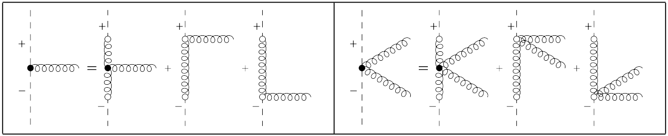
Gauge-invariant off-shell amplitudes

$\overline{|\mathcal{A}_{\rm PRA}|^2}$ is obtained from Lipatov's **gauge-invariant effective theory for MRK processes in QCD** [Lipatov 1995; Antonov, Lipatov, Kuraev, Cherednikov 2005].

Some Feynman rules for **Reggeized gluons:**

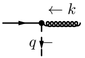
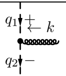
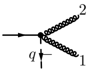
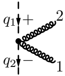
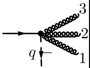
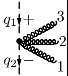
 $= \frac{-i\delta_{ab}}{2q^2}$	 $= (-iq^2) n^{\mp}_{\mu} \delta_{ab}$
	$g_s f_{aa_1a_2} \left(n^{\mp}_{\mu} n^{\mp}_{\nu}\right) \frac{q^2}{k_1^{\mp}}$
	$ig_s^2 \left(n^{\mp}_{\mu_1} n^{\mp}_{\mu_2} n^{\mp}_{\mu_3}\right) \frac{q^2}{k_3^{\mp}} \left[\frac{f_{aba_1} f_{ba_2a_3}}{k_1^{\mp}} + \frac{f_{aba_2} f_{ba_1a_3}}{k_2^{\mp}}\right]$





Gauge-invariant off-shell amplitudes

Feynman rules for Reggeized quarks [Lipatov, Vyazovsky, 2001]; [Fadin, Sherman, 1976]:

	$ig_sT^a\left(\gamma_\mu+\hat q\frac{n^+_\mu}{k^+}\right)$		$ig_sT^a\left(\gamma_\mu+\hat q_2\frac{n^+_\mu}{k^+}+\hat q_1\frac{n^-_\mu}{k^-}\right)$
	$ig_s^2(n^+_{\mu_1}n^+_{\mu_2})\hat q\left[\frac{T^{a_1}T^{a_2}}{k_1^+(k_1+k_2)^+}+\frac{T^{a_2}T^{a_1}}{k_2^+(k_1+k_2)^+}\right]$		$ig_s^2\left[\hat q_2(n^+_{\mu_1}n^+_{\mu_2})\left(\frac{T^{a_1}T^{a_2}}{k_1^+(k_1+k_2)^+}+\frac{T^{a_2}T^{a_1}}{k_2^+(k_1+k_2)^+}\right)-\hat q_1(n^-_{\mu_1}n^-_{\mu_2})\left(\frac{T^{a_2}T^{a_1}}{k_1^-(k_1+k_2)^-}+\frac{T^{a_1}T^{a_2}}{k_2^-(k_1+k_2)^-}\right)\right]$
	$ig_s^3\hat q(n^+_{\mu_1}n^+_{\mu_2}n^+_{\mu_3})\left[\frac{T^{a_1}T^{a_2}T^{a_3}}{k_1^+(k_1+k_2)^+(k_1+k_2+k_3)^+}+(1\leftrightarrow2\leftrightarrow3)\right]$		
	$ig_s^3\left[\hat q_2(n^+_{\mu_1}n^+_{\mu_2}n^+_{\mu_3})\left(\frac{T^{a_1}T^{a_2}T^{a_3}}{k_1^+(k_1+k_2)^+(k_1+k_2+k_3)^+}+(1\leftrightarrow2\leftrightarrow3)\right)+\hat q_1(n^-_{\mu_1}n^-_{\mu_2}n^-_{\mu_3})\left(\frac{T^{a_3}T^{a_2}T^{a_1}}{k_1^-(k_1+k_2)^-(k_1+k_2+k_3)^-}+(1\leftrightarrow2\leftrightarrow3)\right)\right]$		

Reggeized amplitudes in high-energy phenomenology

The first use of Lipatov's effective vertices for Reggeized gluons:

P. Hagler, R. Kirschner, A. Schafer, L. Szymanowski and O. Teryaev, "Heavy quark production as sensitive test for an improved description of high-energy hadron collisions," Phys. Rev. D **62** (2000) 071502.

The first use for Reggeized quarks:

V. A. Saleev, "Prompt photon photoproduction at HERA within the framework of the quark Reggeization hypothesis," Phys. Rev. D **78** (2008) 114031

V. A. Saleev, "Deep inelastic scattering and prompt photon production within the framework of quark Reggeization hypothesis," Phys. Rev. D **78** (2008) 034033

The first use for description of DY process:

M. A. Nefedov, N. N. Nikolaev and V. A. Saleev, "Drell-Yan lepton pair production at high energies in the Parton Reggeization Approach," Phys. Rev. D **87** (2013) no.1, 014022

Structure functions for DY process in PRA

Structure functions for DY process in PRA

$$\frac{d\sigma}{dQ^2 dq_T^2 dy d\Omega} = \frac{\alpha^2}{64\pi^3 S Q^4} L_{\mu\nu} W^{\mu\nu},$$

where y is the rapidity of virtual photon (or l^+l^- lepton pair), $d\Omega = d\phi d\cos\theta$ is the spatial angle of producing positive lepton in the rest frame of virtual photon.

The convolution of hadronic and leptonic tensors reads as a sum of contributions of the so-called helicity structure functions $W_{T,L,\Delta\Delta}$ or $F_{UU}^{1,2,\cos 2\phi}$:

$$\begin{aligned} \frac{d\sigma}{dQ^2 dq_T^2 dy d\Omega} &= \frac{\alpha^2}{64\pi^3 S Q^2}\Big[W_T\cdot(1+\cos^2\theta) + W_L\cdot(1-\cos^2\theta) + \\ &+ W_{\Delta\Delta}\cdot\sin^2\theta\cos 2\phi\Big]. \end{aligned}$$

$$\begin{aligned} \frac{d\sigma}{dx_A dx_B d^2\mathbf{q}_T d\Omega} &= \frac{\alpha^2}{4Q^2}\Big[F_{UU}^{(1)}\cdot\left(1+\cos^2\theta\right) + F_{UU}^{(2)}\cdot\left(1-\cos^2\theta\right) + \\ &+ F_{UU}^{(\cos 2\phi)}\cdot\sin^2\theta\cos(2\phi)\Big], \end{aligned}$$

were $x_{A,B} = Qe^{\pm y}/\sqrt{S}$ and $F_{UU}^{1,2,\cos 2\phi} = W_{T,L,\Delta\Delta}/(2\pi)^4$

Structure functions for DY process in PRA

In the analysis of experimental data, the angular distribution of leptons is represented in terms of two sets of the angular coefficients:

$$\frac{dN}{d\Omega} = (1+\cos^2\theta) + A_0\left(\frac{1}{2}-\frac{3}{2}\cos^2\theta\right) + \frac{A_2}{2}\sin^2\theta\cos 2\phi$$

and

$$\frac{dN}{d\Omega} = \frac{4}{\lambda+3}\left(1+\lambda\cos^2\theta + \frac{\nu}{2}\sin^2\theta\cos 2\phi\right),$$

with the normalization

$$\int\left(\frac{dN}{d\Omega}\right)d\Omega = \frac{16\pi}{3}.$$

One set consists of the coefficients

$$A_0 = \frac{W_L}{W_{TL}}, \quad A_2 = \frac{2W_{\Delta\Delta}}{W_{TL}}, \quad W_{TL} = W_T + W_L/2,$$

and the other one is defined by

$$\lambda = \frac{2-3A_0}{2+A_0}, \quad \nu = \frac{2A_2}{2+A_0}.$$

Structure functions for DY process in PRA

The squared amplitude of the subprocess $(Q_i\bar{Q}_i \to l^+l^-)$:

$$\overline{|M(Q_i\bar{Q}_i \to l^+l^-)|^2} = \frac{16\pi^2}{3Q^4}\alpha^2 e_i^2 L^{\mu\nu} w_{\mu\nu}^{PRA},$$

where the partonic tensor of Reggeized quarks reads [Nefedov, Nikolaev, Saleev, 2013]:

$$\begin{aligned} w_{\mu\nu}^{PRA} &= x_1x_2\Big[-Sg^{\mu\nu} + 2(P_1^\mu P_2^\nu + P_2^\mu P_1^\nu)\frac{(2x_1x_2S - Q^2 - t_1 - t_2)}{x_1x_2S} + \\ &+ \frac{2}{x_2}(q_1^\mu P_1^\nu + q_1^\nu P_1^\mu) + \frac{2}{x_1}(q_2^\mu P_2^\nu + q_2^\nu P_2^\mu) + \\ &+ \frac{4(t_1 - x_1x_2S)}{Sx_2^2}P_1^\mu P_1^\nu + \frac{4(t_2 - x_1x_2S)}{Sx_1^2}P_2^\mu P_2^\nu\Big]. \end{aligned}$$

We define the quark helicity structure functions $w_{T,L,\Delta\Delta}$ respectively to the hadron helicity structure functions $W_{T,L,\Delta\Delta}$. Upon direct calculations we obtain

$$w_T^{PRA} = Q^2 + \frac{(\mathbf{q}_{1T} + \mathbf{q}_{2T})^2}{2}, \quad w_L^{PRA} = (\mathbf{q}_{1T} - \mathbf{q}_{2T})^2, \quad w_{\Delta\Delta}^{PRA} = \frac{(\mathbf{q}_{1T} + \mathbf{q}_{2T})^2}{2}$$

Structure functions for DY process in PRA

The helicity structure functions $W^{PRA}_{T,\dots}$ or $F^{1,\dots}_{UU}$ at the fixed values of variables S, Q, q_T, y can be presented via corresponding Reggeized quark helicity functions $w^{PRA}_{T,\dots}$:

$$W^{PRA}_{T,\dots}(S,Q,q_T,y) = \frac{8\pi^2 S}{3Q_T^4}\int dt_1 \int d\phi_1 \sum_q \Phi^p_q(x_1,t_1,\mu^2)\Phi^p_{\bar{q}}(x_2,t_2,\mu^2)w^{PRA}_{T,\dots},$$

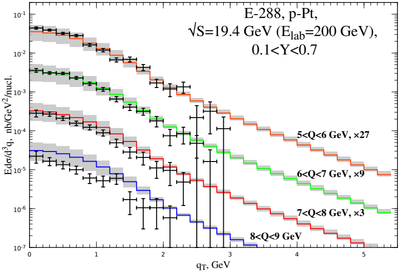
where $Q_T^2 = Q^2 + q_T^2$.
For the calculation of cross-sections, the π^2-resummation K-factor is included:

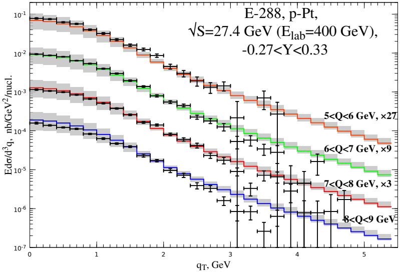
$$K = \exp\left(C_F \frac{\alpha_s(\mu_K^2)}{2\pi}\pi^2\right),$$

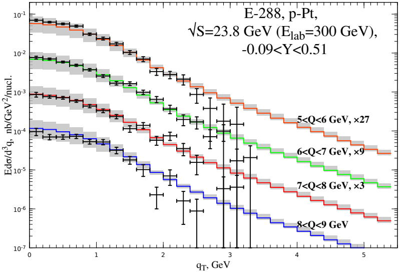
with $\mu_K^2 = Q^{2/3}Q_T^{4/3}$, and $\mu_F = Q_T$. The theoretical uncertainty was obtained by independent variation of this scales. Typical values of K-factor are 1.3 – 1.8.

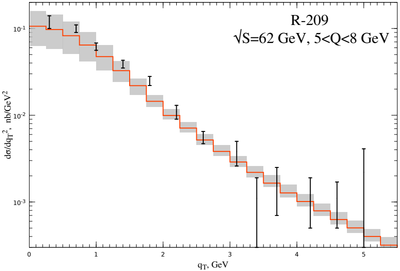
Spectra and angular coefficients for DY process in PRA

Spectra and angular coefficients for DY process in PRA

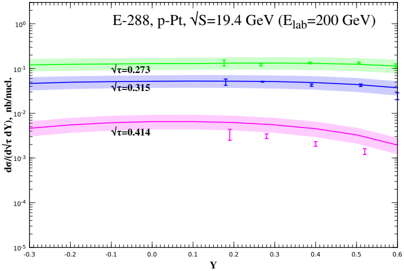


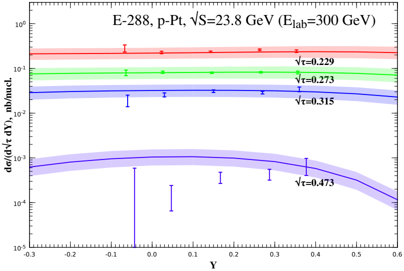


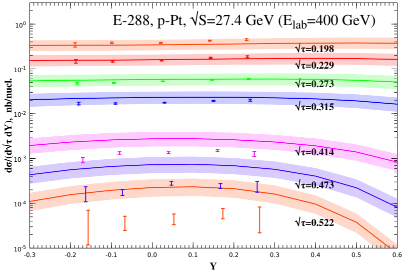




Spectra and angular coefficients for DY process in PRA



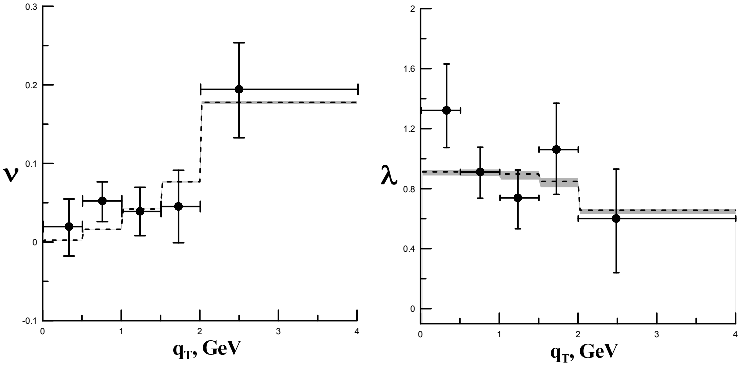




$$\tau = \frac{Q^2}{S} = x_A \times x_B$$

EMC-effect is not included in calculations

Spectra and angular coefficients for DY process in PRA



Angular coefficients ν and λ as function of q_T. The histogram corresponds to LO calculation in PRA. The data are from NuSea Collaboration ($E_{lab} = 800$ GeV, $\sqrt{S} = 39$ GeV, $4.5 < Q < 15$ GeV, $0 < x_F < 0.8$).

Predictions for NICA

Predictions for NICA

Predictions of TMD-factorization:

$$F_{UU}^{(1)} \sim \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2} \cdot f_1^q(x_1, q_{T1}) f_1^{\bar{q}}(x_2, q_{T2}) + O\left(\frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{(2)} \sim O\left(\frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{(\cos 2\phi)} \sim \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2} \cdot h_1^{\perp q}(x_1, q_{T1}) h_1^{\perp \bar{q}}(x_2, q_{T2}) \times$$

$$\times \quad \frac{2(\mathbf{q}_T\mathbf{q}_{T1})(\mathbf{q}_T\mathbf{q}_{T2}) - \mathbf{q}_T^2(\mathbf{q}_{T1}\mathbf{q}_{T2})}{\mathbf{q}_T^2\Lambda^2} + O\left(\frac{q_T^2}{Q^2}\right),$$

where $f_1^q(x, q_T)$ and $h_1^{\perp q}(x, q_T)$ are *unpolarized and Boer-Mulders* leading twist TMD PDFS.

Predictions for NICA

Predictions of PRA:

$$F_{UU}^{(1)} \sim \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2} \cdot f_1^q(x_1, q_{T1}) f_1^{\bar{q}}(x_2, q_{T2}) + O\left(\frac{q_T^2}{Q^2}\right),$$

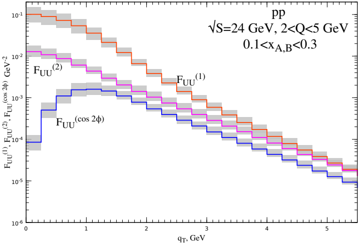
$$F_{UU}^{(2)} \sim \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2} \cdot f_1^q(x_1, q_{T1}) f_1^{\bar{q}}(x_2, q_{T2}) \sim O\left(\frac{q_T^2}{Q^2}\right),$$

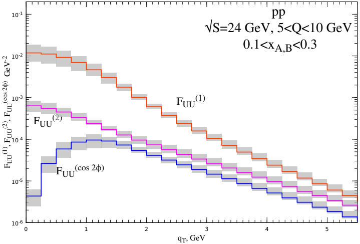
$$F_{UU}^{(\cos 2\phi)} \sim \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2} \cdot f_1^q(x_1, q_{T1}) f_1^{\bar{q}}(x_2, q_{T2}) \sim F_{UU}^{(2)} \sim O\left(\frac{q_T^2}{Q^2}\right),$$

The **Gauge Invariance** formalism leads to reduction of number of independent TMD PDFS or form-factors.

The Boer-Mulders leading twist TMD PDFs are absent in the PRA.

Predictions for NICA





Predictions of PRA:

$$\frac{F_{UU}^{(2)}}{F_{UU}^{(1)}} \sim \frac{\Lambda^2}{Q^2}, \quad q_T \ll Q$$

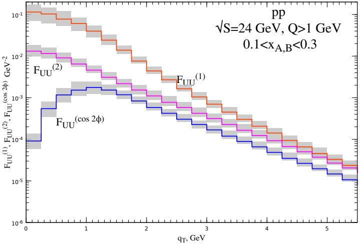
where Λ – hadronic scale.

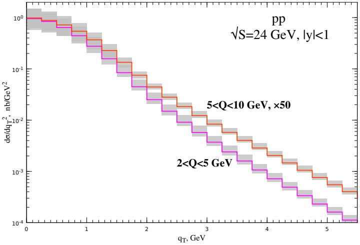
$$\frac{F_{UU}^{(\cos 2\phi)}}{F_{UU}^{(1)}} \sim \frac{q_T^2}{Q^2}, \quad F_{UU}^{(\cos 2\phi)}(q_T = 0) \equiv 0,$$

In case of large q_T **PRA** predicts

$$F_{UU}^{(1)} \simeq \frac{1}{2} F_{UU}^{(2)} \simeq F_{UU}^{(\cos 2\phi)}, \quad q_T \gg Q$$

Predictions for NICA





Predictions of PRA:

Observation of sizable $F_{UU}^{(\cos 2\phi)}$ at small q_T would be very difficult in the formalism, which combines **Factorization** and **QED/QCD Gauge Invariance!**

Conclusions and Plans

Conclusions

- PRA can be applied for study of DY process at energy range of NICA as well as LHC
- PRA takes into account effects of transverse momenta and off-shell effects in the initial state **in a Gauge-Invariant way.**
- PRA can be applied at all values of q_T/Q instead of TMD ($q_T \ll Q$) or CPM ($q_T \sim Q \gg \Lambda$)
- PRA predicts some $O\,(q_T/Q)$ quantities at $q_T << Q$, such as $F_{UU}^{(2)}$ and $F_{UU}^{(\cos 2\phi)}$.

Plans

- In PRA, we can do NLO calculations for DY process at the all q_T/Q, and specially, at the $q_T \leq Q$.
- Nuclear effects and proton polarization effects can be included.

Thank you for your attention!