The effect of final state interactions in two-pion and $K\overline{K}$ transitions of charmonia and bottomonia

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Outline

- **Introduction**
- **The effect of final state interactions in two-pion and $K\overline{K}$ transitions of charmonia and bottomonia**
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- **Conclusions and Discussion**

Introduction

The achievements in hadron spectroscopy of last years are related mainly to heavy mesons including charmonia and bottomonia. Therefore, there is a problem of studying structure of these mesons and their interaction for exploring nonperturbative QCD. There were expressed thoughts that for these aims two-pion transitions of bottomonia are suitable (see, e.g. [Yu.A.Simonov, A.I.Veselov, PR D**79** (2009) 034024] and references therein). Clearly, first it is necessary to investigate contribution of final-state interactions in decay spectra of these mesons.

Earlier we analyzed three-particle decays of charmonia and bottomonia with vector and two pseudoscalar mesons in the final state. These are data of the Crystal Ball, DM2, Mark II, Mark III, BES II, ARGUS, CLEO, CUSB, Belle, and *BaBar* Collaborations on decays $J/\psi \to \phi(\pi\pi, K\overline{K})$, $\psi(2S) \to J/\psi\pi\pi$, and $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n$, $m = 2,3,4,5$, $n = 1,2,3$). In these decays it is reasonable to expect that two pseudoscalar mesons are formed in state with quantum numbers $I^G(J^{PC}) = 0^+(0^{++})$, whereas final vector meson remains a spectator.

For studying final-state effects, pseudoscalar-meson interactions should be considered with allowing for coupled channels. In fact this implies combined analysis of data on indicated decays of charmonia and bottomonia and on isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$. In that way we move forward explanation of dipion mass spectra in decays of these quarkonia and also obtain additional model-independent information for scalar mesons. The latter is important because problem of interpretation of scalar mesons is faraway to be solved completely [C.Patrignani *et al.* (PDG), Chin.Phys. C**40** (2016) 100001]. Importance of studying properties of scalar mesons is related to obvious fact that a comprehension of these states is necessary in principle for most profound topics concerning the QCD vacuum, because these sectors affect each other especially strongly due to possible "direct" transitions between them.

We were shown earlier [Yu.S.Surovtsev *et al.*, EPJ Web of Conferences, Vol.138 (2017) 01025] and confirmed in present investigation (with additional data) [Yu.S.Surovtsev *et al.*, PR D**97** (2018) 014009], that dipion and $K\overline{K}$ mass spectra in above-indicated decays of quarkonia are explained by unified mechanism.

This mechanism is related to contributions of the $\pi\pi$, $K\overline{K}$ and $\eta\eta$ coupled channels, including their interference, and does not depend on layout of quarkonia in mass spectra – below or above the $D\overline{D}$ threshold for charmonia and the $B\overline{B}$ threshold for bottomonia.

The present analysis has allowed us to confirm our earlier conclusions about the scalar-isoscalar mesons obtained when considering the three-channel $\pi\pi$ scattering and decays $J/\psi \to \phi(\pi\pi, K\overline{K})$ [Yu.S.Surovtsev *et al.*, PR D**89** (2014) 036010].

Role of individual f_0 resonances in making up shape of dipion mass distributions in the charmonia and bottomonia decays is considered.

Analyzing decay $X(4260) \to J/\psi\ \pi^+\pi^-$, we obtained additional information on charmonium $X(4260)$ the quantum numbers of which, indicated in the PDG tables, are $I^G(J^{PC}) = ?^?(1^{--})$.

Thanks to specific features of coupled-channel formalism we have obtained predictions for amplitudes of the $\eta\eta$ and $K\overline{K}$ scattering and of transitions $\eta\eta \to \pi\pi$ and $\eta\eta \to K\overline{K}$, which are used in our calculations and almost or entirely unknown from experiment.

The effect of final state interactions in two-pion and $K\overline{K}$ transitions of charmonia and bottomonia

Considering three-channel $\pi\pi$ scattering, we shall deal with three reactions $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$, because it was shown [Yu.S.Surovtsev *et al.*, PR D**86** (2012) 116002; J.Phys. G: Nucl.Part.Phys. **41** (2014) 025006] that this is a minimal number of coupled channels needed for obtaining correct values of f_0 parameters. Data for three-channel $\pi\pi$ scattering were taken from many papers (Refs. are in our paper [Yu.S.Surovtsev *et al.*, PR D**89** (2014) 036010]).

For decay $J/\psi \to \phi\pi^+\pi^-$, data were taken from Mark III, DM2 and BES II Collaborations; for $\psi(2S) \to J/\psi(\pi^+\pi^- \text{ and } \pi^0\pi^0)$ — from Mark II and Crystal Ball(80) (Refs. also in [Yu.S.Surovtsev *et al.*, PR D**89** (2014) 036010]). For $\Upsilon(2S) \to \Upsilon(1S)(\pi^+\pi^- \text{ and } \pi^0\pi^0)$ data were used from ARGUS [H.Albrecht *et al.*, PL **134B** (1984) 137 (1984)], CLEO [D.Besson *et al.*, PR D**30** (1984) 1433], CUSB [V.Fonseca *et al.*, NP B**242** (1984) 31], and Crystal Ball [D.Gelphman *et al.*, PR D**32** (1985) 2893]; for $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$ and $\Upsilon(3S) \to \Upsilon(2S)(\pi^+\pi^-, \pi^0\pi^0)$ — from CLEO [F.Butler *et al.*, PR D**49** (1994) 40; D.Cronin-Hennessy *et al.*, PR D**76** (2007) 072001];

for $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi^+\pi^-$ — from *BaBar* [B.Aubert *et al.*, PRL **96** (2006) 232001] and Belle [A.Sokolov *et al.*, PR D**75** (2007) 071103; A.Bondar *et al.*, PRL **108** (2012) 122001];
for $\Upsilon(5S) \to \Upsilon(1S, 2S, 3S)\pi^+\pi^-$ — also from Belle Collaboration.

The used formalism for calculating dimeson mass distributions in quarkonia decays is analogous to the one proposed in Ref. [D.Morgan, M.R.Pennington, PR D**48** (1993) 1185, 5422] for the decays $J/\psi \to \phi(\pi\pi, K\overline{K})$ and $V' \to V\pi\pi$ $(V = \psi, \Upsilon)$ but with allowing for also amplitudes of transitions between the $\pi\pi$, $K\overline{K}$ and $\eta\eta$ channels in decay formulas. There was assumed that the pseudoscalar-meson pairs in the final state have zero isospin and spin. Only these pairs of pseudoscalar mesons undergo final state interactions whereas the final vector meson remains a spectator.

The decays amplitudes F are related with scattering amplitudes T_{ij} $(i, j = 1 - \pi\pi, 2 - K\overline{K}, 3 - \eta\eta)$ as follows:

$$F\big(J/\psi \to \phi\pi\pi\big) = \frac{1}{\sqrt{3}}\,\Big[c_1(s)\,T_{11} + \Big(\frac{\alpha_2}{s-\beta_2} + c_2(s)\Big)\,T_{12} + c_3(s)\,T_{13}\Big],$$

$$F\big(J/\psi \to \phi K\overline{K}\big) = \frac{1}{\sqrt{2}}\,\Big[c_1(s)\,T_{21} + c_2(s)\,T_{22} + c_3(s)\,T_{23}\Big],$$

$$F\big(\psi(2S) \to \psi(1S)\pi\pi\big) = \frac{1}{\sqrt{3}}\,\Big[d_1(s)\,T_{11} + d_2(s)\,T_{12} + d_3(s)\,T_{13}\Big],$$

$$F\big(\Upsilon(mS) \to \Upsilon(nS)\pi\pi\big) = \frac{1}{\sqrt{3}}\,\Big[e_1^{(mn)}\,T_{11} + e_2^{(mn)}\,T_{12} + e_3^{(mn)}\,T_{13}\Big],$$

$$m > n,\;\; m = 2,3,4,5,\;\; n = 1,2,3$$

where $c_i = \gamma_{i0} + \gamma_{i1}s$, $d_i = \delta_{i0} + \delta_{i1}s$ and $e_i^{(mn)} = \rho_{i0}^{(mn)} + \rho_{i1}^{(mn)}s$; indices m and n correspond to $\Upsilon(mS)$ and $\Upsilon(nS)$, respectively. The free parameters α_2, β_2, γ_{i0}, γ_{i1}, δ_{i0}, δ_{i1}, $\rho_{i0}^{(mn)}$ and $\rho_{i1}^{(mn)}$ depend on couplings of J/ψ, $\psi(2S)$ and the $\Upsilon(mS)$ to channels $\pi\pi$, $K\overline{K}$ and $\eta\eta$. The pole term in front of T_{21} is approximation of possible ϕK states, not forbidden by OZI rules. The numbers in front of square brackets are coefficients of the vector addition of two isospins $I^{(1)}$ and $I^{(2)}$ $\big(I^{(1)}I^{(2)}I_3^{(1)}I_3^{(2)}\big|II_3\big)$ where I and I_3 are the total isospin and its third component.

The explicit form of relevant coefficient of the vector addition is

$$\left(I^{(1)}I^{(2)}I_3^{(1)}, -I_3^{(1)}\middle|00\right) = (-1)^{I^{(2)}-I_3^{(1)}}\frac{\delta_{I^{(1)}I^{(2)}}}{\sqrt{2I^{(2)}+1}}.$$

Then inserting numerical values of pion and kaon isospins, we obtain the corresponding coefficients.

The amplitudes T_{ij} are expressed through the S-matrix elements

$$S_{ij} = \delta_{ij} + 2i\sqrt{\rho_1\rho_2}T_{ij}$$

where $\rho_i = \sqrt{1 - s_i/s}$, s is the invariant total energy of pseudoscalar meson pair, squared, and s_i is the reaction threshold. The S-matrix elements are taken as the products

$$S = S^{bgr}S^{res}$$

where S^{res} represents the contribution of resonances, S^{bgr} is the background part. The 3-channel S-matrix is determined on the 8-sheeted Riemann surface. The matrix elements S_{ij} have right-hand cuts along the real axis of s complex plane, starting with the channel thresholds s_i $(i = 1, 2, 3)$, and the left-hand cuts related to crossed channels.

The S^{res}-matrix elements are parameterized on uniformization plane of the $\pi\pi$-scattering S-matrix element by poles and zeros which represent resonances. However with help of a simple mapping, a function, determined on the 8-sheeted Riemann surface, can be uniformized only on torus. This is unsatisfactory for our purpose. Therefore, we neglected influence of the lowest ($\pi\pi$) threshold branch-point (however, unitarity on the $\pi\pi$ cut is allowed for). This means consideration of the nearest to the physical region semi-sheets of the Riemann surface. In fact, we construct a 4-sheeted model of the initial 8-sheeted Riemann surface approximating it in accordance with our approach of a consistent account of the nearest singularities on all the relevant sheets. In the uniformizing variable used [Yu.S.Surovtsev, P.Bydžovský, V.E.Lyubovitskij, PR D**85** (2012) 036002)]

$$w = \frac{\sqrt{(s-s_2)s_3} + \sqrt{(s-s_3)s_2}}{\sqrt{s(s_3-s_2)}} \qquad (s_2 = 4m_K^2 \text{ and } s_3 = 4m_\eta^2)$$

we have allowed for the $K\overline{K}$- and $\eta\eta$-threshold branch-points and left-hand branch point at $s=0$.

The S-matrix elements S_{ij} are parameterized using the Le Couteur-Newton relations [K.J.Le Couteur, Proc.R.London, Ser. A **256** (1960) 115; R.G.Newton, J.Math.Phys. **2** (1961) 188]. On the w-plane, we have derived for S^{res}:

$$S_{11} = \frac{d^*(-w^*)}{d(w)}, \qquad S_{22} = \frac{d(-w^{-1})}{d(w)}, \qquad S_{33} = \frac{d(w^{-1})}{d(w)},$$

$$S_{11}S_{22} - S_{12}^2 = \frac{d^*(w^{*-1})}{d(w)}, \qquad S_{11}S_{33} - S_{13}^2 = \frac{d^*(-w^{*-1})}{d(w)},$$

$$S_{22}S_{33} - S_{23}^2 = \frac{d(-w)}{d(w)}.$$

The $d_{res}(w)$ is the Jost matrix determinant, which now is free from any branch-points:

$d_{res}(w) = w^{-\frac{M}{2}} \prod_{r=1}^{M}(w + w_r^*)$ (M is number of resonance zeros).

S_{res} is the main (model-independent) contribution of resonances, given by the poles and zeros (resonance pole clusters)

D.Krupa, V.A.Meshcheryakov, Yu.S.Surovtsev, NC A**109** (1996) 281.

The possible remaining small (model-dependent) contributions of resonances and influence of channels which are not taken explicitly into account in the uniformizing variable are included in the background part S_B. For the background part

$$d_B = \exp[-i\sum_{n=1}^{3}\frac{\sqrt{s-s_n}}{2m_n}(\alpha_n + i\beta_n)],$$

$$\alpha_n = a_{n1} + a_{n\sigma}\frac{s-s_\sigma}{s_\sigma}\theta(s-s_\sigma) + a_{nv}\frac{s-s_v}{s_v}\theta(s-s_v),$$

$$\beta_n = b_{n1} + b_{n\sigma}\frac{s-s_\sigma}{s_\sigma}\theta(s-s_\sigma) + b_{nv}\frac{s-s_v}{s_v}\theta(s-s_v)$$

where s_σ is the $\sigma\sigma$ threshold; s_v is the combined threshold of the $\eta\eta',\ \rho\rho,\ \omega\omega$ channels. The resonance zeros w_r and background parameters were fixed by fitting to data on processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$. We have obtained practically zero background of the $\pi\pi$ scattering in the scalar-isoscalar channel. This is important because a reasonable and simple description of the background should be a criterion for correctness of approach.

Furthermore, this shows that consideration of the left-hand branch-point at $s = 0$ in the uniformizing variable solves partly a problem of some approaches (see, e.g., N.N.Achasov, G.N. Shestakov, PR D**49** (1994) 5779) that the wide-resonance parameters are strongly controlled by non-resonant background.

The very simple description of the $\pi\pi$-scattering background confirms well our assumption $\underline{S = S_B S_{res}}$ and also that representation of multi-channel resonances by pole clusters on the uniformization plane is good and quite sufficient.

If the resonance part of amplitude is taken as

$$T^{res} = \sqrt{s}\,\Gamma_{el}/(m_{res}^2 - s - i\sqrt{s}\,\Gamma_{tot}),$$

for the mass and total width, one obtains

$$m_{res} = \sqrt{\mathrm{E}_r^2 + (\Gamma_r/2)^2} \quad \text{and} \quad \Gamma_{tot} = \Gamma_r,$$

where the pole position $\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r/2$ must be taken on sheets II, IV, VIII, depending on the resonance classification (details in [Yu.S.Surovtsev *et al.*, PR D**89** (2014) 036010]).

Table: The masses and total widths of the f_0 resonances.

	$f_0(600)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0'(1500)$	$f_0(1710)$
m_{res}[MeV]	693.9±10.0	1008.1±3.1	1399.0±24.7	1495.2±3.2	1539.5±5.4	1733.8±43.2
Γ_{tot}[MeV]	931.2±11.8	64.0±3.0	357.0±74.4	124.4±18.4	571.6±25.8	117.6±32.8

Further we studied role of individual f_0 resonances in forming energy dependence of amplitudes of reactions $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$. We switched off only those resonances [$f_0(500)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$], removal of which can be somehow compensated by correcting the background to have more-or-less acceptable description of three-channel $\pi\pi$ scattering. Therefore, we considered description of three-channel $\pi\pi$ scattering more for two cases [Yu.S.Surovtsev *et al.*, PR D**92** (2015) 036002]:

- First, when leaving out a minimal set of f_0 mesons consisting of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$, which is sufficient to achieve a description of processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ with a total $\chi^2/\text{ndf} \approx 1.20$.
- Second, from above-indicated three mesons only $f_0(500)$ can be switched off while still obtaining a reasonable description of three-channel $\pi\pi$ scattering (though with an appearance of the pseudo-background) with a total $\chi^2/\text{ndf} \approx 1.43$.

Allowing for all considered f_0-resonances [$f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0'(1500)$ and $f_0(1710)$], we have achieved satisfactory description of three-channel $\pi\pi$ scattering with a total $\chi^2/\text{ndf} \approx 1.16$.

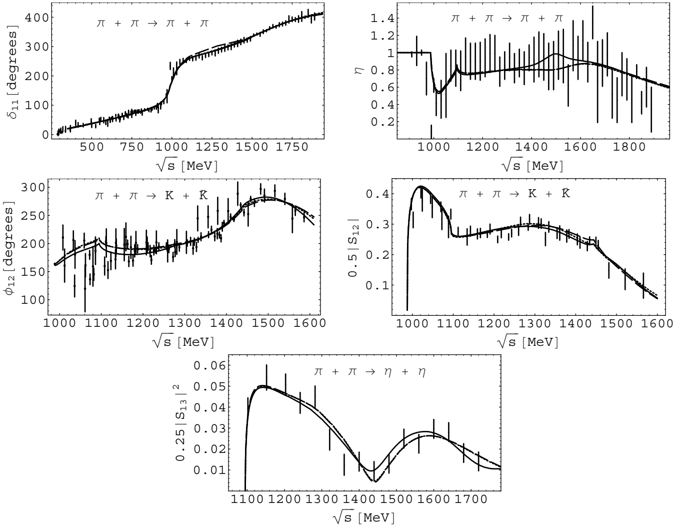

Figure: The phase shifts and moduli of the S-matrix elements. The solid lines correspond to contribution of all relevant f_0-resonances; the dotted, of $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of $f_0(980)$ and $f_0'(1500)$.

One can see that the curves are quite similar in all three cases.

Further we carried out a combined analysis of data on processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and on decays of charmonia — $J/\psi \to \phi(\pi\pi, K\overline{K})$, $\psi(2S) \to J/\psi\,\pi\pi$, $X(4260) \to J/\psi\ \pi^+\pi^-$ — and of bottomonia — $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n$, $m = 2, 3, 4, 5$, $n = 1, 2, 3$). The expression

$$N|F|^2\sqrt{(s-s_1)[m_\psi^2 - (\sqrt{s}-m_\phi)^2][m_\psi^2 - (\sqrt{s}+m_\phi)^2]}$$

for decays $J/\psi \to \phi(\pi\pi, K\overline{K})$ and analogues relations for $\psi(2S) \to \psi(1S)\pi\pi$ and $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ give di-meson mass distributions. N (normalization to experiment), determined in the analysis, and obtained free parameters in equations for amplitudes F of quarkonia decays, depending on couplings of J/ψ, $\psi(2S)$, $X(4260)$ and $\Upsilon(mS)$ to channels $\pi\pi$, $K\overline{K}$ and $\eta\eta$, can be found in [Yu.S.Surovtsev *et al.*, PR D**97** (2018) 014009].

Satisfactory combined description of all considered processes is obtained with the total $\chi^2/\text{ndf} = 764.417/(739 - 119) \approx 1.31$;
for the $\pi\pi$ scattering, $\chi^2/\text{ndf} \approx 1.14$;
for $\pi\pi \to K\overline{K}$, $\chi^2/\text{ndf} \approx 1.65$;
for $\pi\pi \to \eta\eta$, $\chi^2/\text{ndp} \approx 0.88$;
for decays $J/\psi \to \phi(\pi^+\pi^-, K\overline{K})$, $\chi^2/\text{ndf} \approx 1.26$
for $\psi(2S) \to J/\psi(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndf} \approx 2.74$;
for $\Upsilon(2S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndf} \approx 1.07$;
for $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndf} \approx 1.08$,
for $\Upsilon(3S) \to \Upsilon(2S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndf} \approx 0.71$,
for $\Upsilon(4S) \to \Upsilon(1S)(\pi^+\pi^-)$, $\chi^2/\text{ndf} \approx 0.46$,
for $\Upsilon(4S) \to \Upsilon(2S)(\pi^+\pi^-)$, $\chi^2/\text{ndp} \approx 0.20$,
for $\Upsilon(5S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndf} \approx 1.39$,
for $\Upsilon(5S) \to \Upsilon(2S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndf} \approx 1.10$,
for $\Upsilon(5S) \to \Upsilon(3S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndf} \approx 0.87$,
for $X(4260) \to J/\psi\ \pi^+\pi^-$, $\chi^2/\text{ndf} \approx 1.23$.

In all next figures for quarkonia decays the solid lines correspond to contribution of all relevant f_0-resonances; the dotted, of $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of $f_0(980)$ and $f_0'(1500)$.

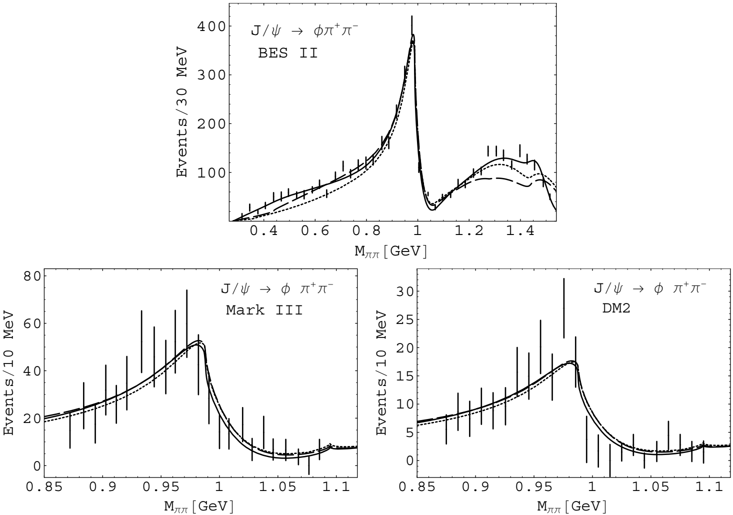

Figure: The decays $J/\psi \to \phi\pi^+\pi^-$.

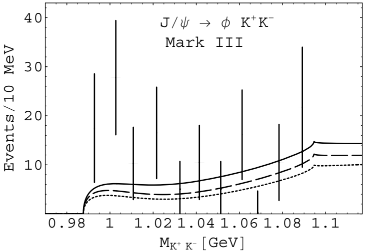

Figure: The decays $J/\psi \to \phi K^+ K^-$.

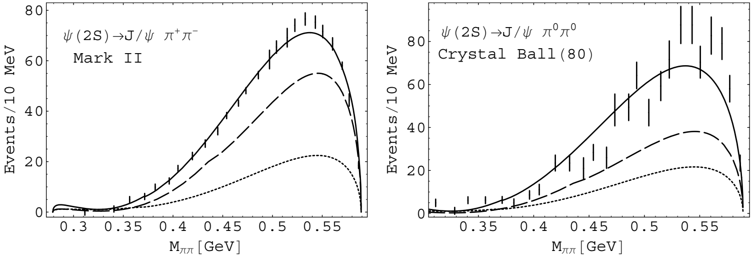

Figure: The decays $\psi(2S) \to J/\psi(\pi^+\pi^-, \pi^0\pi^0)$.

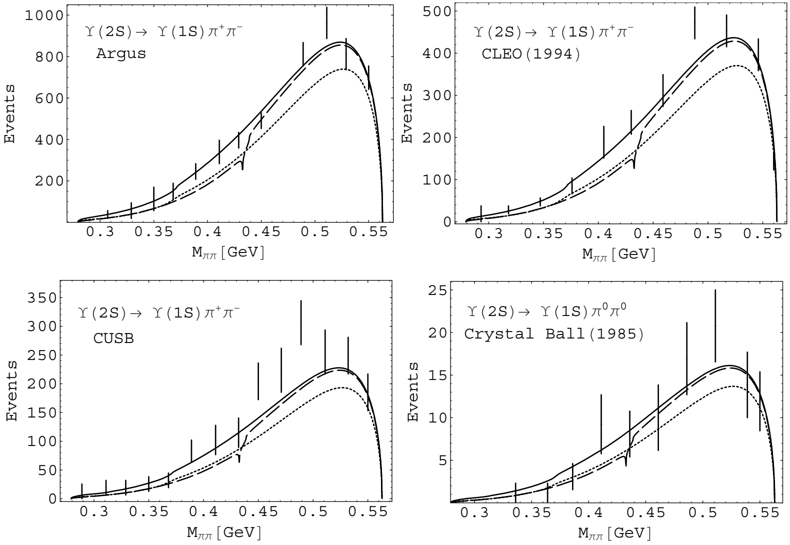

Figure: The decays $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$.

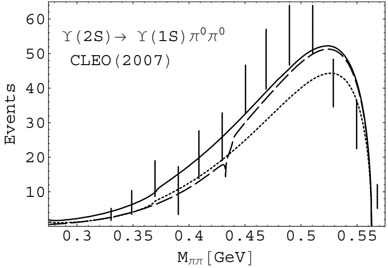

Figure: The decays $\Upsilon(2S) \to \Upsilon(1S)\pi^0\pi^0$.

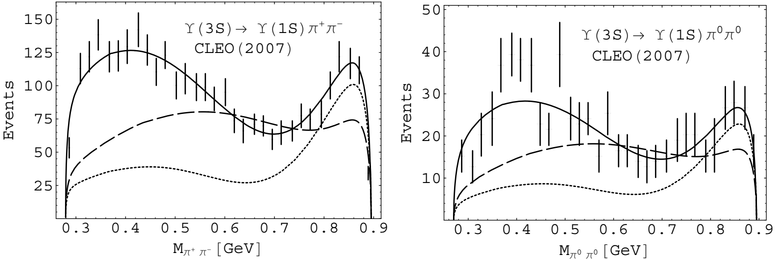

Figure: The decays $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$

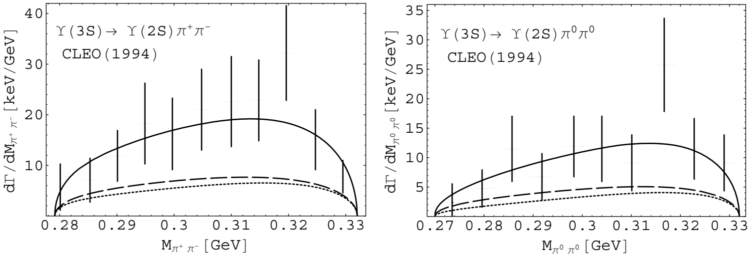

Figure: The decays $\Upsilon(3S) \to \Upsilon(2S)(\pi^+\pi^-, \pi^0\pi^0)$.

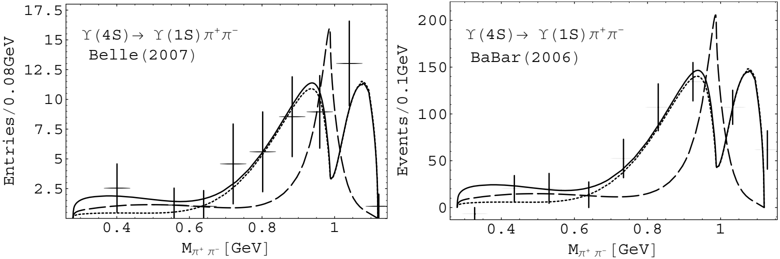

Figure: The decays $\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$.

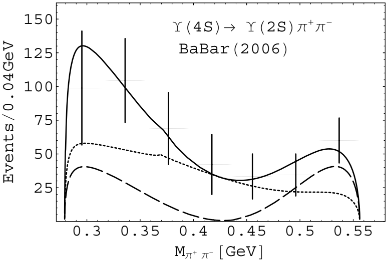

Figure: The decays $\Upsilon(4S) \to \Upsilon(2S)\pi^+\pi^-$.

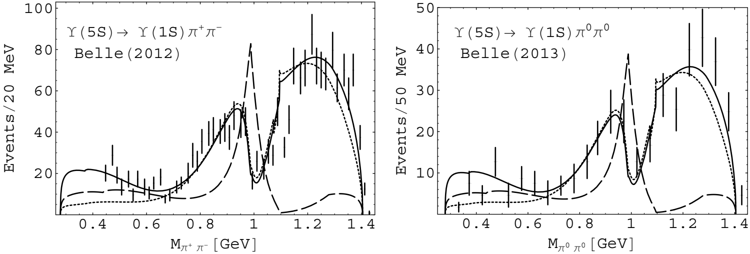

Figure: The decays $\Upsilon(5S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$.

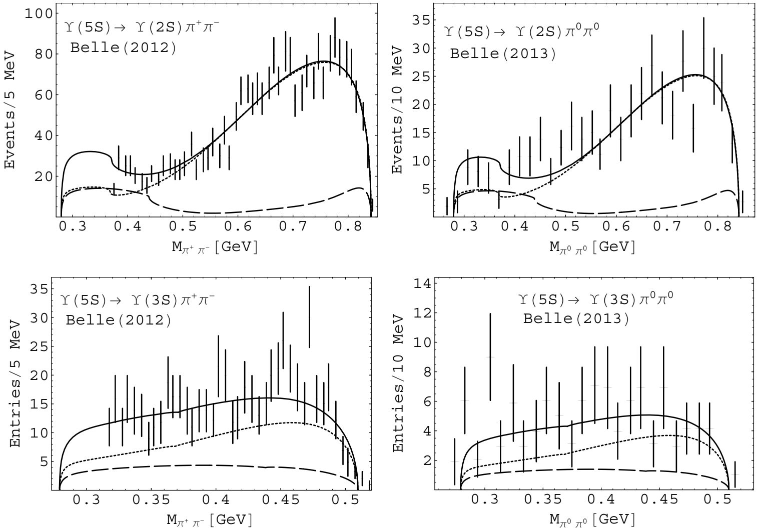

Figure: The decays $\Upsilon(5S) \to \Upsilon(2S,3S)(\pi^+\pi^-, \pi^0\pi^0)$.

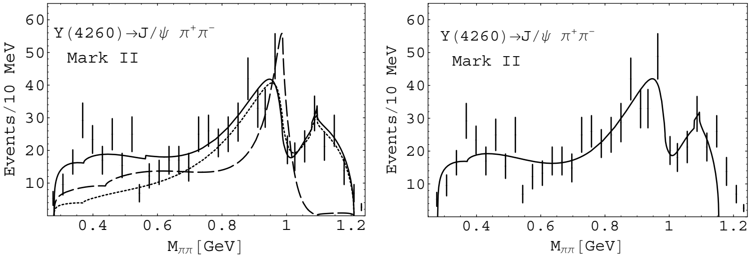

Figure: The decay $Y(4260) \to J/\psi\ \pi^+\pi^-$.

When considering decay of charmonium $X(4260)$ (sometimes indicated as $Y(4260)$) to $J/\psi\ \pi^+\pi^-$, we used data of Belle Collaboration [Z.Q.Liu *et al.*, PRL **110** (2013) 252002].
In the PDG tables for $X(4260)$ one indicates the quantum numbers $I^G(J^{PC}) =?^?(1^{--})$ and mass $m = 4251 \pm 9$ MeV. However, this analysis shows that used data correspond to decay of charmonium with mass 4.3102 GeV (left-hand picture), not with 4.251 GeV (right-hand one).

Furthermore, since we have shown that basic forms of dipion mass spectra of charmonia and bottomonia pion-pion transitions are explained by the unified mechanism, one can think that characteristic pictures of mass spectra of analogous charmonia and bottomonia transitions are similar, of course, with allowing for distortions due to the phase space volume. To some extent this assumption is supported by comparison of corresponding experimental data, e.g., for $\psi(2S) \to J/\psi(1S)(\pi^+\pi^-, \pi^0\pi^0)$ and $\Upsilon(2S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$. Then one can see that basic forms of dipion mass spectra of decay $\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$ and of the charmonium $X(4260)$ are similar. This can be some indication that $X(4260)$ is a third radial excitation of $J/\psi(1S)$, i.e. the $4S$ state with mass 4.3102 GeV.

In view of success in describing considered processes it is worth to show obtained predictions for amplitudes of $\eta\eta$ and $K\overline{K}$ scattering and of transitions $\eta\eta \to \pi\pi$ and $\eta\eta \to K\overline{K}$ which are used in our calculations. They are almost or entirely unknown from experiment.

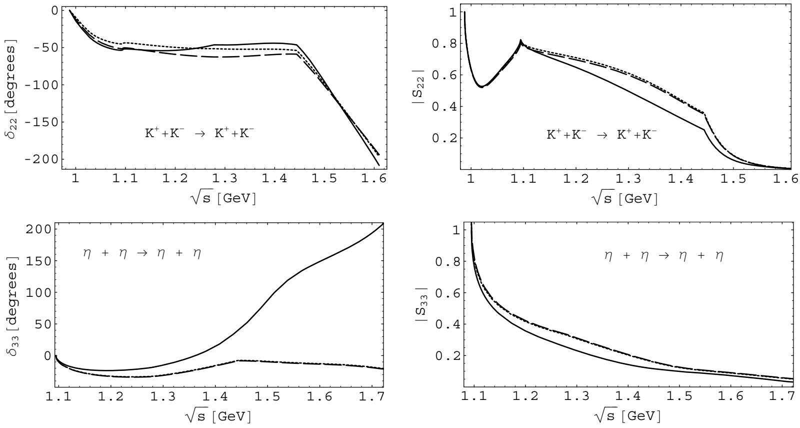

Figure: The phase shifts and moduli of the S-matrix elements in the S-wave $K\overline{K}$ and $\eta\eta$ scattering.

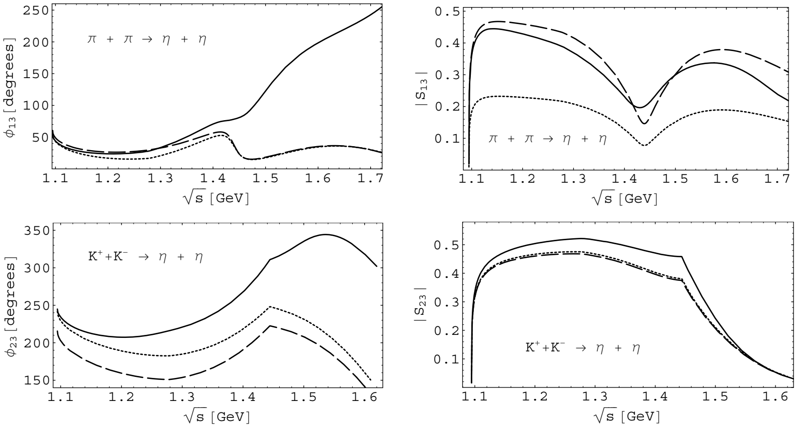

Figure: The phase shifts and moduli of the S-matrix elements of S-wave processes $\pi\pi \to \eta\eta$ and $K\overline{K} \to \eta\eta$.

Conclusions and Discussion

- The combined analysis was performed for data on isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and on decays of charmonia — $J/\psi \to \phi(\pi\pi, K\overline{K})$, $\psi(2S) \to J/\psi\,\pi\pi$, $Y(4260) \to J/\psi\ \pi^+\pi^-$ — and of bottomonia — $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n$, $m = 2, 3, 4, 5$, $n = 1, 2, 3$) from the ARGUS, Crystal Ball, CLEO, CUSB, DM2, Mark II, Mark III, BES II, *BaBar*, and Belle Collaborations.

- It is shown that dipion and $K\overline{K}$ mass spectra in above-indicated decays of charmonia and dipion mass spectra of bottomonia are explained by unified mechanism which is related to contributions of $\pi\pi$, $K\overline{K}$ and $\eta\eta$ coupled channels including their interference. It is shown that in the final states of these decays (except $\pi\pi$ scattering) contribution of coupled processes, e.g., $K\overline{K}, \eta\eta \to \pi\pi$, is important even if these processes are energetically forbidden.

- When analyzing decay $Y(4260) \to J/\psi\ \pi^+\pi^-$, dipion spectrum of which is published in Ref.[Z.Q.Liu *et al.*(Belle Collaboration), PRL **110** (2013) 252002], it is obtained some indication that charmonium $X(4260)$ is a third radial excitation of $J/\psi(1S)$, i.e. the $4S$ state with mass 4.3102 GeV.

- It was very useful to consider a role of individual f_0 resonances in contributions to dipion mass distributions. E.g., it is seen that sharp dips about 1 GeV in the $\Upsilon(4S,5S) \to \Upsilon(1S)\pi^+\pi^-$ are related with $f_0(500)$ contribution to interfering amplitudes of $\pi\pi$ scattering and $K\overline{K},\eta\eta \to \pi\pi$ processes. Namely consideration of this role of $f_0(500)$ allows us to conclude on the sharp dip about 1 GeV in dipion mass spectrum of $\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$ where scarce data do not permit to do that conclusion yet, unlike $\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$.
- Also, a manifestation of $f_0(1370)$ is turned out to be interesting and unexpected. First, in description of $\pi\pi$ spectrum of the $J/\psi \to \phi\pi\pi$, second large peak in the 1.4-GeV region can be naively imagined as related to contribution of $f_0(1370)$. However, we have shown that this is not right – constructive interference between contributions of $\eta\eta$ and $\pi\pi$ and $K\overline{K}$ channels plays the main role in formation of the 1.4-GeV peak. This is in agreement with our earlier conclusion that $f_0(1370)$ has a dominant $s\bar{s}$ component.

- On the other hand, it turned out that $f_0(1370)$ contributes considerably in the near-$\pi\pi$-threshold region of many dipion mass distributions, especially making the threshold bell-shaped form of dipion spectra in decays $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ $(m > n, m = 3, 4, 5, n = 1, 2, 3)$. This fact, first, confirms existence of $f_0(1370)$ (up to now there is no firm conviction if it exists or not). Second, that exciting role of this meson in making the threshold bell-shaped form of dipion spectra can be explained as follows: $f_0(1370)$, being predominantly $s\bar{s}$ state [Yu.S.Surovtsev *et al.*, PR D**89** (2014) 036010] and practically not contributing to $\pi\pi$-scattering amplitude, influences noticeably $K\overline{K}$ scattering; e.g., it was shown that $K\overline{K}$-scattering length is very sensitive to whether this state does exist or not [Yu.S.Surovtsev *et al.*, EPJ A**15** (2002) 409]. The interference of contributions of $\pi\pi$-scattering amplitude and analytically-continued $\pi\pi \to K\overline{K}$ and $\pi\pi \to \eta\eta$ amplitudes leads to the observed results.
- The convincing combined description of mentioned processes (without any change of f_0 parameters) confirmed all our earlier results on scalar mesons [Yu.S.Surovtsev *et al.*, PR D**89** (2014) 036010]; the most important results are:

1. Confirmation of $f_0(500)$ with a mass of about 700 MeV and a width of 930 MeV (pole on sheet II is $514.5 \pm 12.4 - i(465.6 \pm 5.9)$ MeV).
2. Indication for $f_0(980)$ (pole on sheet II is $1008.1 \pm 3.1 - i(32.0 \pm 1.5)$ MeV) to be neither a $q\bar{q}$ state nor $K\overline{K}$ molecule, but possibly bound $\eta\eta$ state.
3. Indication for $f_0(1370)$ and $f_0(1710)$ to have a dominant $s\bar{s}$ component.
4. Indication for existence of two states in the 1500-MeV region: $f_0(1500)$ ($m_{res} \approx 1495$ MeV, $\Gamma_{tot} \approx 124$ MeV) and $f_0'(1500)$ ($m_{res} \approx 1539$ MeV, $\Gamma_{tot} \approx 574$ MeV). The $f_0'(1500)$ is interpreted as a glueball taking into account its biggest width among enclosing states [V.V.Anisovich *et al.*, NP Proc.Suppl. A**56** (1997) 270].