1S-2S energy shift in muonic hydrogen

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Outline

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2. Calculation of the 1S-2S enegry shift in μp
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   - Vacuum polarization in the second order perturbation theory
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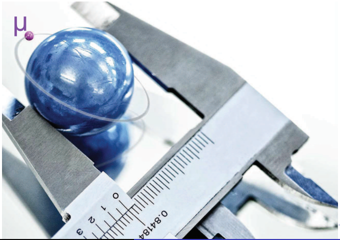
CREMA (Charge Radius Experiment with Muonic Atoms)

Task: to measure Lamb Shift, fine and hyperfine structure in light muonic atoms (muonic hydrogen, muonic deuterium, ions of muonic helium, muonic atoms with Z$\geq$ 3); to determine charge radii of the proton, deuteron, helion, alpha-particle and other light nuclei with the accuracy 0.0005 fm.

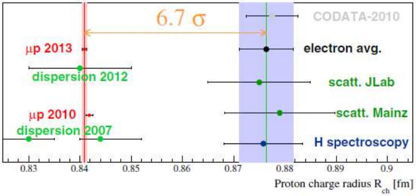


Proton Radius Puzzle

The proton rms charge radius measured with

muons: 0.8409 ± 0.0004 *fm* (CREMA, Lamb Shift in μp)
R. Pohl et.al., Ann. Rev. Nucl. Part. Sci. **63**(175) 2013

electrons: 0.8775 ± 0.0050 *fm* (electron scattering)
P.J. Mohr et.al., J. Phys. Chem. Ref. Data **45**(4) 2016



Measurement of 2S-4P transition frequency in H

The proton rms charge radius from measurement of Rydberg constant in ordinary hydrogen

$$0.8335 \pm 0.0095 \quad fm$$

A. Beyer et.al., Science **358**(6359) 2017

Measurement of 1S-3S transition frequency in H

The proton rms charge radius from measurement 1S-3S transition in hydrogen

$$0.877 \pm 0.013 \quad fm$$

H. Fleurbaey et.al., PRL **120**(183001) 2018

Proton radius, main results

Group	Proton radius
CREMA	0.8409 ± 0.0004 *fm*
CODATA	0.8775 ± 0.0050 *fm*
A. Beyer	0.8335 ± 0.0095 *fm*
H. Fleurbaey	0.877 ± 0.013 *fm*

The discrepancy is still unknown.

Future experiments with light muonic atoms

- Hyperfine splitting in μp and $\mu^3 He$
- Lamb shift $(2P - 2S)$ in μLi and μBe
- $1S - 2S$ transition in H-like He^+ ions
- $1S - 2S$ transition in μp

R. Pohl, J. Phys. Soc. of Japan **85**(091003) 2016
R. Pohl et.al., arXiv:1808.07240v1 2018

Big fine structure in μp

$$1S - 2S$$

In electronic hydrogen

$$\Delta\nu_{1S-2S} = 2466\,061\,413\,187\,018(11)\ Hz$$

A. Beyer et.al., PRL **110**(230801) 2013

Isotopic shift hydrogen-deuterium

$$\Delta\nu = 670\,994\,334.64(15)\ kHz$$

A. Huber et.al., PRL **80**,468 (1998)

Fine structure of energy levels

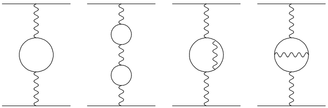
$$E_n = m_1 + m_2 - \frac{\mu^2 (Z\alpha)^2}{2n^2} - \frac{\mu (Z\alpha)^4}{2n^3} \left[1 - \frac{3}{4n} + \frac{\mu^2}{4 m_1 m_2 n} \right]$$

M.I. Eides et.al., Phys.Rep. **342**,62(2001)

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\Delta E = \begin{cases} 1S: & 1\ 043\ 927\ 924\ 269.9985\ meV \\ 2S: & 1\ 043\ 929\ 820\ 665.7786\ meV \end{cases}.$$

One- and two-loop vacuum polarization in 1γ interaction



$$H_B=\frac{\mathbf{p}^2}{2\mu}-\frac{Z\alpha}{r}-\frac{\mathbf{p}^4}{8m_1^3}-\frac{\mathbf{p}^4}{8m_2^3}+\frac{\pi Z\alpha}{2}\left(\frac{1}{m_1^2}+\frac{\delta_I}{m_2^2}\right)\delta(\mathbf{r})-$$

$$-\frac{Z\alpha}{2m_1m_2r}\left(\mathbf{p}^2+\frac{\mathbf{r}(\mathbf{r}\mathbf{p})\mathbf{p}}{r^2}\right)+\frac{Z\alpha}{r^3}\left(\frac{1}{4m_1^2}+\frac{1}{2m_1m_2}\right)(\mathbf{L}\boldsymbol{\sigma}_1).$$

$$\psi_{100}(r)=\frac{W^{3/2}}{\sqrt{\pi}}e^{-Wr},\ \psi_{200}(r)=\frac{W^{3/2}}{2\sqrt{2\pi}}e^{-\frac{Wr}{2}}\left(1-\frac{Wr}{2}\right).$$

$$W=\mu Z\alpha.$$

One-loop vacuum polarization

$$V^C_{VP}(r) = \frac{\alpha}{3\pi}\int_1^\infty d\xi\rho(\xi)\left(-\frac{Z\alpha}{r}e^{-2m_e\xi r}\right), \rho(\xi) = \frac{\sqrt{\xi^2-1}(2\xi^2+1)}{\xi^4},$$

$$\Delta E_{VP}(1S) = -\frac{4\mu\alpha(Z\alpha)^2}{3\pi}\int \rho(\xi)d\xi x e^{-x\left(2+\frac{2m_e\xi}{W}\right)}dx =$$

$$= -1898.8300\ meV$$

$$\Delta E_{VP}(2S) = -\frac{\mu(Z\alpha)^2\alpha}{6\pi}\int_1^\infty \rho(\xi)d\xi\int_0^\infty xdx\left(1-\frac{x}{2}\right)^2 e^{-x\left(1+\frac{2m_e\xi}{W}\right)} =$$

$$= -219.5840\ meV$$

Two-loop vacuum polarization

$$V^C_{VP-VP}(r) = \frac{\alpha^2}{9\pi^2}\int_1^\infty \rho(\xi)d\xi \int_1^\infty \rho(\eta)d\eta \times .$$

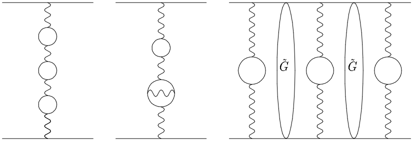
$$\times \left(-\frac{Z\alpha}{r}\right)\frac{1}{(\xi^2-\eta^2)}\left(\xi^2 e^{-2m_e\xi r} - \eta^2 e^{-2m_e\eta r}\right)$$

$$\Delta E_{VP-VP} = \begin{cases} 1S: & -1.7915\ meV \\ 2S: & -0.2426\ meV \end{cases}.$$

$$\Delta V^C_{2-loop\ VP} = -\frac{2}{3}\frac{Z\alpha}{r}\left(\frac{\alpha}{\pi}\right)^2\int_0^1 \frac{f(v)dv}{(1-v^2)}e^{-\frac{2m_e r}{\sqrt{1-v^2}}}.$$

$$\Delta E_{2-loop\ VP} = \begin{cases} 1S: & -12.6145\ meV \\ 2S: & -1.4112\ meV \end{cases}.$$

Three-loop vacuum polarization



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$$V^C_{VP-VP-VP}(r) = -\frac{Z\alpha}{r}\frac{\alpha^3}{(3\pi)^3}\int_1^\infty \rho(\xi)d\xi \int_1^\infty \rho(\eta d\eta \int_1^\infty \rho(\zeta)d\zeta \times$$

$$\times\left[e^{-2m_e\zeta r}\frac{\zeta^4}{(\xi^2-\zeta^2)(\eta^2-\zeta^2)} + e^{-2m_e\xi r}\frac{\xi^4}{(\zeta^2-\xi^2)(\eta^2-\xi^2)} + e^{-2m_e\eta r}\frac{\eta^4}{(\xi^2-\eta^2)(\zeta^2-\eta^2)}\right],$$

$$V^C_{VP-2-loop\ VP} = -\frac{4\mu\alpha^3(Z\alpha)}{9\pi^3}\int_1^\infty \rho(\xi)d\xi \int_1^\infty \frac{f(\eta)d\eta}{\eta}\frac{1}{r(\eta^2-\xi^2)}\left(\eta^2 e^{-2m_e\eta r} - \xi^2 e^{-2m_e\xi r}\right),$$

$$\Delta E_{3VP,1\gamma} = \begin{cases} 1S: & -0.0138\ meV \\ 2S: & -0.0032\ meV \end{cases}.$$

There are some other 3-loop contributions
T. Kinoshita and M. Nio, PRL **62**, 3240 (1999); PRD **60**,053008 (1999)
S.G. Karshenboim et.al., JETP Lett. **92**,9 (2010); PRA **81**, 060501 (2010)

Green's functions

$$\tilde{G}(1S) = -\frac{Z\alpha\mu^2}{\pi} e^{x_1+x_2} g_{1S}$$

$$g_{1S} = \frac{1}{2x_>} - \ln(2x_>) - \ln(2x_<) + Ei(2x_<) + \frac{7}{2} - 2C - (x_1+x_2) + \frac{1-e^{2x_<}}{2x_<}$$

$$\tilde{G}(2S) = -\frac{Z\alpha\mu^2}{4x_1x_2} e^{-\frac{x_1+x_2}{2}} \frac{1}{4\pi} g_{2S}(x_1,x_2),$$

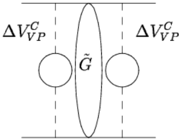
$$g_{2S}(x_1,x_2) = 8x_< - 4x_<^2 + 8x_> + 12x_<x_> - 26x_<^2x_> +$$

$$+2x_<^3x_> - 4x_>^2 - 26x_<x_>^2 + 23x_<^2x_>^2 -$$

$$-x_<^3x_>^2 + 2x_<x_>^3 - x_<^2x_>^3 + 4e^x(1-x_<)(x_>-2)x_> + 4(x_<-2)x_<(x_>-2)x_> \times$$

$$\times[-2C + Ei(x_<) - \ln(x_<) - \ln(x_>)],$$

Two-loop VP effects in the SOPT



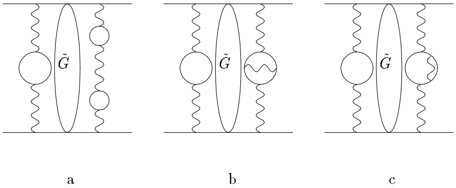
$$< \psi|\Delta V^C_{VP}\tilde{G}\Delta V^C_{VP}|\psi >$$

This contribution is of order

$$\alpha^2(Z\alpha)^2$$

$$\Delta E^{VP,VP}_{SOPT} = \begin{cases} 1S: & -2.0343\ meV \\ 2S: & -0.1532\ meV \end{cases}$$

Three-loop VP effects in the SOPT



These contributions are of order

$$\alpha^3 (Z\alpha)^2$$

$$\Delta E^{3VP}_{SOPT} = \begin{cases} 1S: & -0.0180\ meV \\ 2S: & -0.0022\ meV \end{cases}$$

One-loop VP corrections to the Breit Hamiltonian

$$\Delta V^B_{VP}(r) = \frac{\alpha}{3\pi}\int_1^\infty \rho(\xi)d\xi \sum_{i=1}^4 \Delta V^B_{i,VP}(r),$$

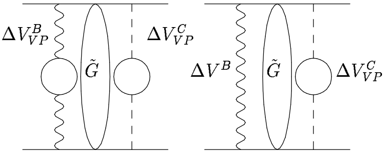
$$\Delta V^B_{1,VP} = \frac{Z\alpha}{8}\left(\frac{1}{m_1^2}+\frac{\delta_I}{m_2^2}\right)\left[4\pi\delta(\mathbf{r}) - \frac{4m_e^2\xi^2}{r}e^{-2m_e\xi r}\right],$$

$$\Delta V^B_{2,VP} = -\frac{Z\alpha m_e^2\xi^2}{m_1 m_2 r}e^{-2m_e\xi r}(1-m_e\xi r),$$

$$\Delta V^B_{3,VP} = -\frac{Z\alpha}{2m_1 m_2}p_i\frac{e^{-2m_e\xi r}}{r}\left[\delta_{ij}+\frac{r_i r_j}{r^2}(1+2m_e\xi r)\right]p_j,$$

$$\Delta E^B_{VP} = \begin{cases} 1S: & 0.1671\ meV \\ 2S: & 0.0249\ meV \end{cases},$$

Relativistic and VP corrections in SOPT



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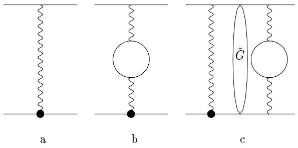
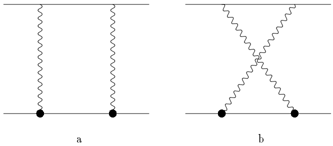
$$2 < \psi|\Delta V^B \tilde{G} \Delta V^C_{VP}|\psi > .$$

$$\Delta E^B_{VP,SOPT} = \begin{cases} 1S: & -0.6746\ meV \\ 2S: & -0.0456\ meV \end{cases},$$

Nuclear structure corrections in 1γ and 2γ interactions





$$r_N = 0.8409 \pm (0.0004)\ fm$$

$$\Delta E_{str} = -\frac{\mu^3 (Z\alpha)^4}{12} < r_N^2 >= \begin{cases} 1S: \ 29.3994\ meV \\ 2S: \ 3.6749\ meV \end{cases}.$$

$$\Delta E^{2\gamma}_{str}(nS) = -\frac{\mu^3 (Z\alpha)^5}{\pi n^3}\delta_{l0}\int_0^\infty \frac{dk}{k} V(k), \Delta E^{2\gamma}_{str} = \begin{cases} -0.1590\ meV \\ -0.0199\ meV \end{cases}.$$

Nuclear structure corrections in 1γ and 2γ interactions

$$V(k)=\frac{2(F^2-1)}{m_1m_2}+\frac{8m_1[-F(0)+4m_2^2F'(0)]}{m_2(m_1+m_2)k}+\frac{k^2}{2m_1^3m_2^3}\times$$

$$\times\left[2(F^2-1)(m_1^2+m_2^2)-F^2m_1^2\right]+\frac{\sqrt{k^2+4m_1^2}}{2m_1^3m_2(m_1^2-m_2^2)k}\times$$

$$\times\left\{k^2\left[2(F^2-1)m_2^2-F^2m_1^2\right]+8m_1^4F^2+\frac{16m_1^4m_2^2(F^2-1)}{k^2}\right\}-$$

$$-\frac{\sqrt{k^2+4m_2^2}m_1}{2m_2^3(m_1^2-m_2^2)k}\left\{k^2\left[2(F^2-1)-F^2\right]+8m_2^2F^2+\frac{16m_2^4(F^2-1)}{k^2}\right\}.$$

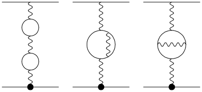
Nuclear structure and one-loop VP correction in the first and second orders PT

$$\Delta V^{VP}_{str}(r) = \frac{2}{3}\pi Z\alpha < r_N^2 > \frac{\alpha}{3\pi}\int_1^\infty \rho(\xi)d\xi \left[\delta(\mathbf{r}) - \frac{m_e^2\xi^2}{\pi r}e^{-2m_e\xi r}\right].$$

$$\Delta E^{VP}_{str} = \begin{cases} 1S: & 0.1925\ meV \\ 2S: & 0.0120\ meV \end{cases}.$$

$$\Delta E^{VP}_{str,SOPT} = \begin{cases} 1S: & 0.1316\ meV \\ 2S: & 0.0167\ meV \end{cases}.$$

Nuclear structure and two-loop VP correction



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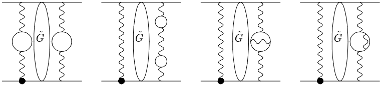

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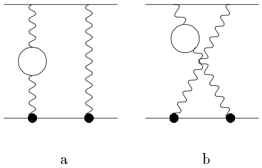
$$\Delta V_{str}^{VP-VP}(r) = \frac{2}{3} Z\alpha < r_N^2 > \left(\frac{\alpha}{3\pi}\right)^2 \int_1^\infty \rho(\xi) d\xi \int_1^\infty \rho(\eta) d\eta \times$$

$$\times \left[\pi\delta(\mathbf{r}) - \frac{m_e^2}{r(\xi^2-\eta^2)}\left(\xi^4 e^{-2m_e\xi r} - \eta^4 e^{-2m_e\eta r}\right)\right],$$

$$\Delta V_{str}^{2-loop\ VP}(r) = \frac{4}{9} Z\alpha < r_N^2 > \left(\frac{\alpha}{\pi}\right)^2 \int_0^1 \frac{f(v)dv}{1-v^2}\left[\pi\delta(\mathbf{r}) - \frac{m_e^2}{r(1-v^2)} e^{-\frac{2m_e r}{\sqrt{1-v^2}}}\right].$$

$$\Delta E_{str}^{VP,VP} = \begin{cases} 1S: 0.0124\ meV \\ 2S: \ 0.0009\ meV \end{cases}.$$

Nuclear structure and VP in 2γ interaction



$$\Delta E^{2\gamma}_{str,VP}(nS) = -\frac{2\mu^3\alpha(Z\alpha)^5}{\pi^2 n^3}\int_0^\infty kV(k)dk\int_0^1\frac{v^2(1-\frac{v^2}{3})dv}{k^2(1-v^2)+4m_e^2},$$

$$\Delta E^{2\gamma}_{str,VP} = \begin{cases} 1S:\ 0.-0030\ meV \\ 2S:\ -0.0004\ meV \end{cases}.$$

Results

Energy of the levels

$$\Delta E = \begin{cases} 1S: & 1\,043\,927\,924\,269.9985 \; meV \\ 2S: & 1\,043\,929\,820\,665.7786 \; meV \end{cases}.$$

Results

Contribution	1S, meV	2S, meV
1-loop VP in 1γ of order $\alpha(Z\alpha)^2$	-1898.8300	-219.5840
2-loop VP in 1γ of order $\alpha^2(Z\alpha)^2$	-14.4060	-1.6538
3-loop VP in 1γ of order $\alpha^3(Z\alpha)^2$	-0.0158	-0.0052
Relativistic an VP in the first order PT $\alpha^3(Z\alpha)^2$	0.1671	0.0249

Results

Contribution	1S, meV	2S, meV
Relativistic an VP in the second order PT $\alpha^3(Z\alpha)^2$	-0.6746	-0.0456
2-loop VP in the second order PT $\alpha^2(Z\alpha)^2$	-2.0343	-0.1532
3-loop VP in the second order PT $\alpha^3(Z\alpha)^2$	-0.0180	-0.0022

Results

Contribution	1S, meV	2S, meV
Nuclear structure of order $(Z\alpha)^4$	29.3994	3.6749
Nuclear structure (2γ) of order $(Z\alpha)^5$	-0.1590	-0.0199
Nuclear structure and VP (2γ) of order $\alpha(Z\alpha)^5$	-0.0030	-0.0004
Nuclear structure and 1-loop VP of order $\alpha(Z\alpha)^4$	0.3241	0.0287
Nuclear structure and 2-loop VP of order $\alpha^2(Z\alpha)^4$	0.0124	0.0009

Results

Total result

$$\Delta E(2S - 1S) = 1\ 898\ 064.2650\ meV$$

Conclusions

- The effects of vacuum polarization led to the modification of the Breit two-particle interaction operator and give the corrections in the energy spectra up to the fifth order in α
- The nuclear structure effects are expressed in terms of proton radius in the leading order $(Z\alpha)^4$ and $(Z\alpha)^5$ for the one-loop amplitudes by means of the nucleus electromagnetic form factors.
- Relativistic and vacuum polarization effects are significant for energy spectra of light muonic atoms, their order is $\alpha^3(Z\alpha)^2$

Thanks you for attention!

Results

Contribution	1S, meV	2S, meV
Radiative corrections of order $\alpha(Z\alpha)^5$	0.0353	0.0043
Rad and VP of order $\alpha^2(Z\alpha)^4$	0.0167	0.0023
Recoil corrections of order $(Z\alpha)^5$	0.3109	0.0419
Nuclear polarization of order $(Z\alpha)^5$	-0.1291	-0.0161
Hadronic VP	-0.0864	-0.0108

Results

Contribution	1S, meV	2S, meV
Muonic self-energy and muonic VP of order $\alpha(Z\alpha)^4$	5.1267	0.6442