Proton Size Puzzle: Thick or Thin?

"Thick and Thin" famous Anton Chekhov novel

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Introduction
Experimental data
Our work
Conclusions

1) Phys.Part.Nucl.Lett. 14 (2017) 857
2) Phys.Lett. B776 (2018) 105; 3) EPJA 54 (2018) arXiv:1804.09749 [hep-ph]

In 2010 the CREMA (Charge Radius Experiments with Muonic Atoms) Collaboration measured very precisely the Lamb shift of muonic hydrogen. It has opened the new era of the precise investigation of the spectrum of simple atoms.

In the new experiments by this Collaboration with muonic deuterium and ions of muonic helium a charge radii of light nuclei were obtained with very high precision.

For muonic hydrogen and muonic deuterium it was shown that obtained values of the charged radii are significantly different from those which were extracted from spectra of electronic atoms and in the scattering of the electrons with nuclei and were recommended for using by CODATA, so-called, “PROTON CHARGE RADIUS PUZZLE”.

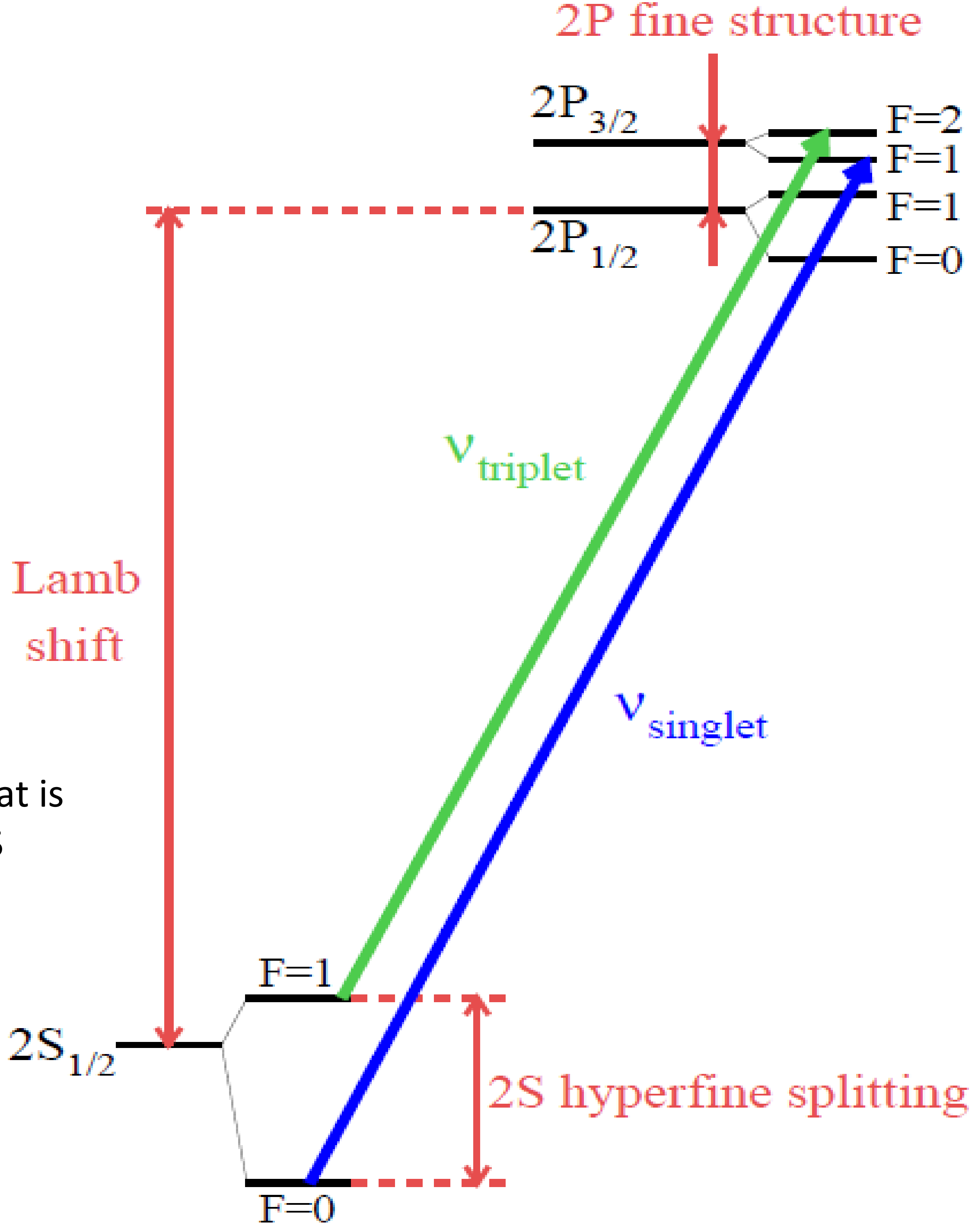
Several experimental groups plan to measure the hyperfine structure of various muonic and electronic atoms with more high precision.

One can consider experiments with muonic atoms as a smoking gun for:

Precise measurements of the proton charge radius

Test of the Standard Model with greater accuracy

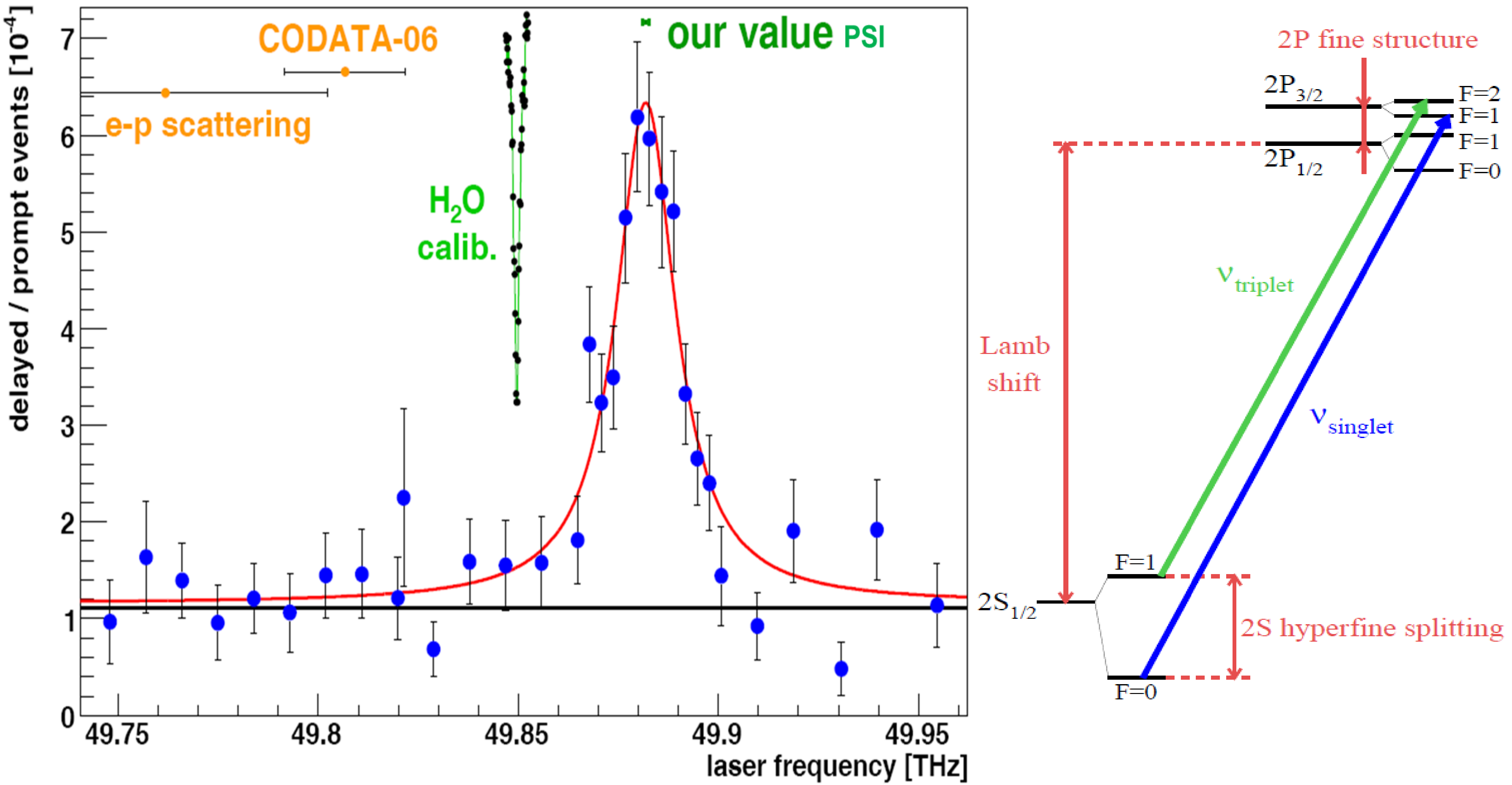
and, possibly, to reveal the source of previously unaccounted interactions between the particles forming the bound state in QED.



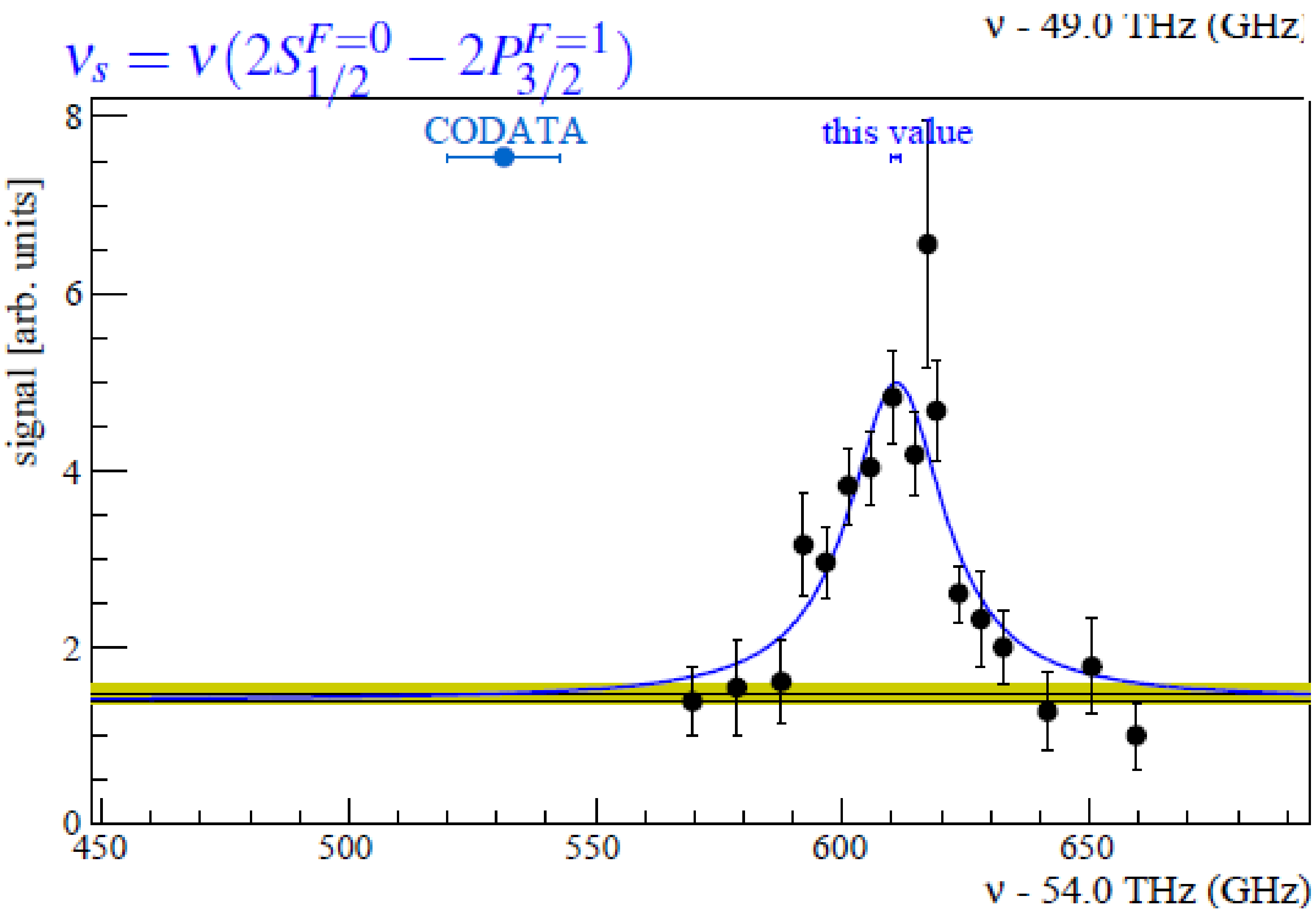
The HFS requires the spin-spin coupling that is the interaction between the nuclear spin S and the lepton total angular momentum j, where F = j + S is the atom total angular momentum

$$\nu_t = \nu(2S_{1/2}^{F=1} - 2P_{3/2}^{F=2})$$

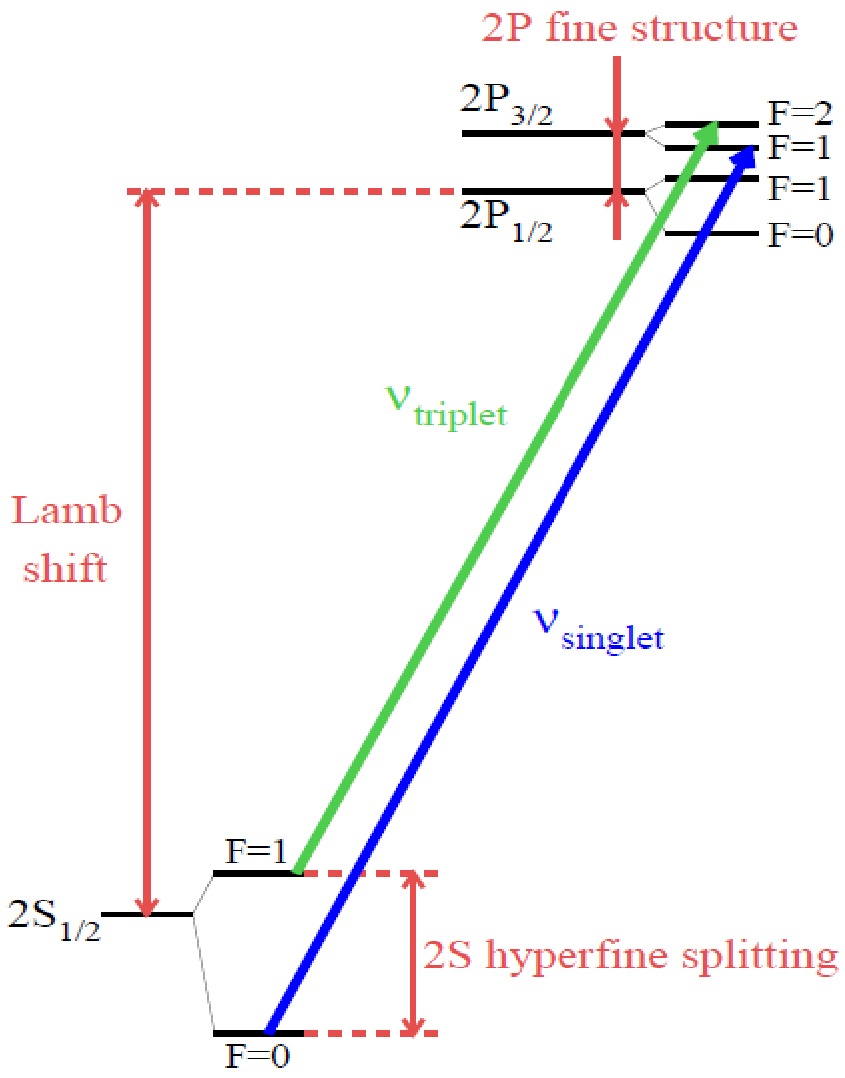
CREMA-10-13



CREMA-13



$\nu_s = \nu(2S_{1/2}^{F=0} - 2P_{3/2}^{F=1})$
ν - 49.0 THz (GHz)
CODATA
this value
signal [arb. units]
0
2
4
6
8
450
500
550
600
650
ν - 54.0 THz (GHz)



2P fine structure
$2P_{3/2}$
F=2
F=1
F=1
F=0
$2P_{1/2}$
Lamb shift
$\nu_{triplet}$
$\nu_{singlet}$
F=1
$2S_{1/2}$
F=0
2S hyperfine splitting

Two transitions measured

$$\nu_T = 49881.35(\ 65)\ \text{GHz}$$
$$\nu_S = 54611.16(1.05)\ \text{GHz}$$

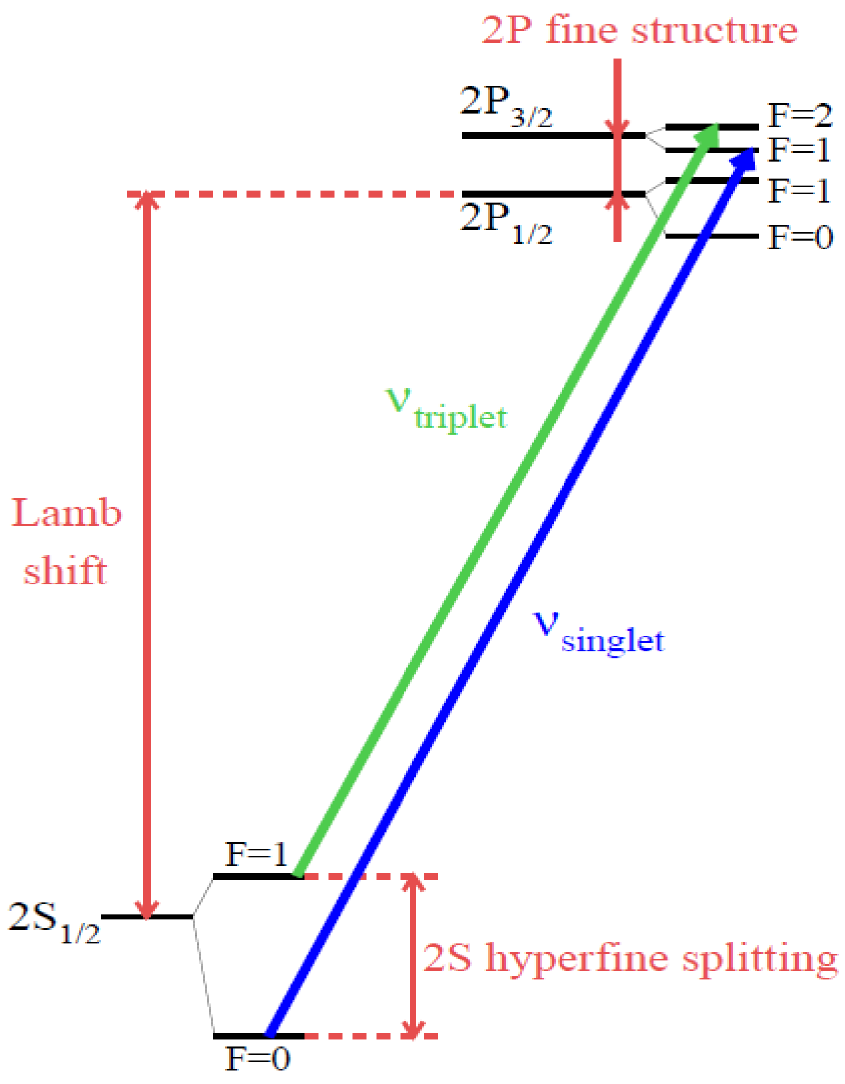
From these two transition measurements, one can independently deduce both the Lamb shift $\Delta E_L = \Delta E(2P_{1/2} - 2S_{1/2})$ and the 2S-HFS splitting (ΔE_{HFS}) by the linear combinations

$$\frac{1}{4}h\nu_s + \frac{3}{4}h\nu_t = \Delta E_L + 8.8123(2)\text{meV}$$

$$h\nu_s - h\nu_t = \Delta E_{HFS} - 3.2480(2)\text{meV}$$

Then one get

$$\Delta E_{LS}^{exp} = 202.3706(23)\ \text{meV}$$
$$\Delta E_{HFS}^{exp} = 22.8089(51)\ \text{meV}$$



Proton Size Puzzle

$$r_p = 0.8751(61)\,\mathrm{fm}\ (\mathrm{CODATA\text{-}2014})$$

From spectrum of electronic atoms and the scattering of the electrons with nuclei

Lamb shift in muonic Hydrogen (μp) (a proton orbited by a negative muon)

muon mass $m_\mu \approx 200 \times m_e$

Bohr radius $r_\mu \approx 1/200 \times r_e$

μ inside the proton: $200^3 \approx 10^7$

μp ***has much smaller Bohr radius compared to electronic hydrogen and so is much more sensitive to the finite size of the proton***

$$\Delta E_{\mathrm{LS}}^{\mathrm{exp}} = 202.3706(23)\ \mathrm{meV}$$

CREMA experiment **Nature 2010; Science 2013**
(0.05% precision)

$$\Delta E_{\mathrm{LS}} = 206.0668(25) - 5.2275(10)\langle r_p^2\rangle + O\left(\langle r_p^2\rangle^{3/2}\right)[\mathrm{meV}]$$

Theory summary: Antognini et al. AnnPhys 2013
(2% effect)

$$r_p = 0.84087(39)\,\mathrm{fm}\ (\mathrm{CREMA\ coll.\ Antognini\ et\ al.,\ 2013})$$

5.6 σ deviation!

$$r_E^2 = \int d^3r\, r^2\, \rho_E(r)$$

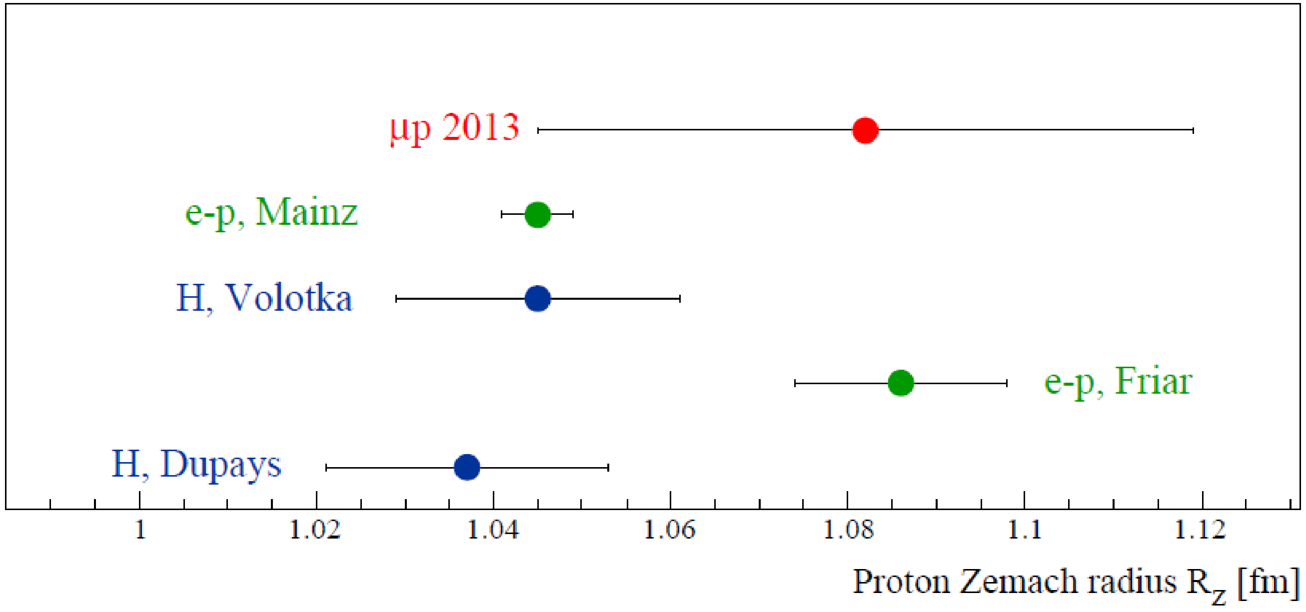
Zemach radius

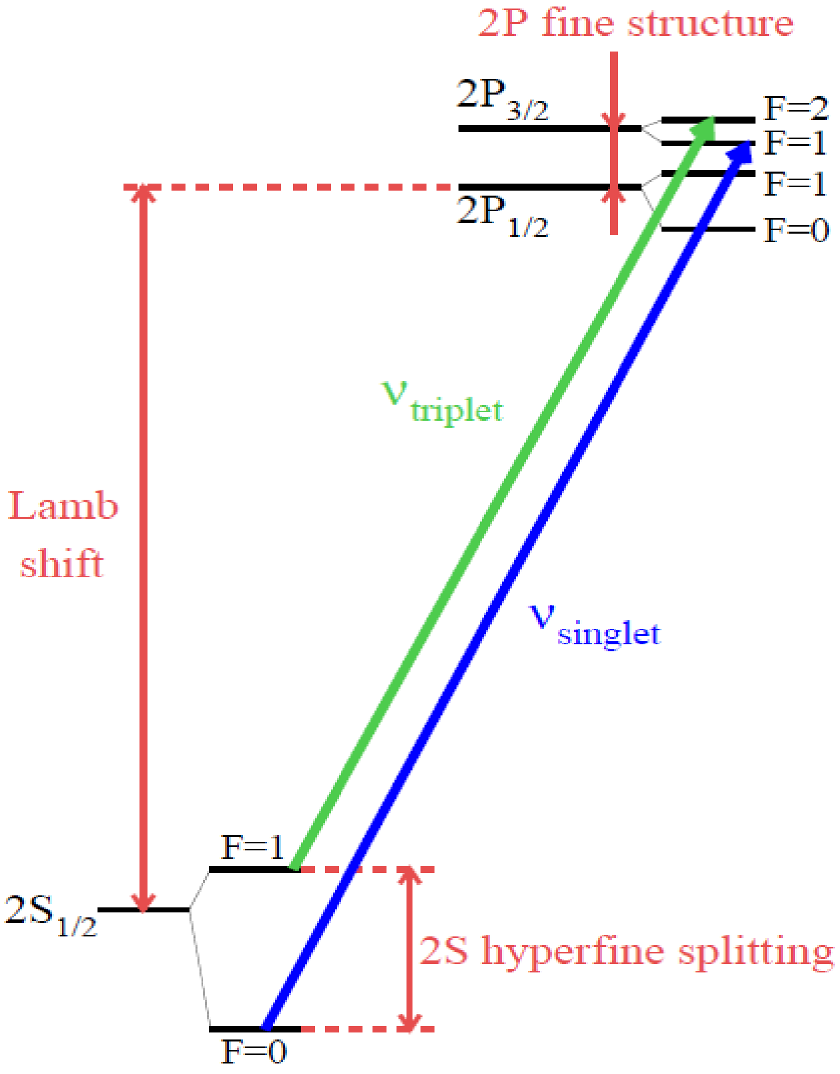
$$\Delta E^{\text{exp}}_{\text{2S-HFS}} = 22.8089(51)\ \text{meV}$$

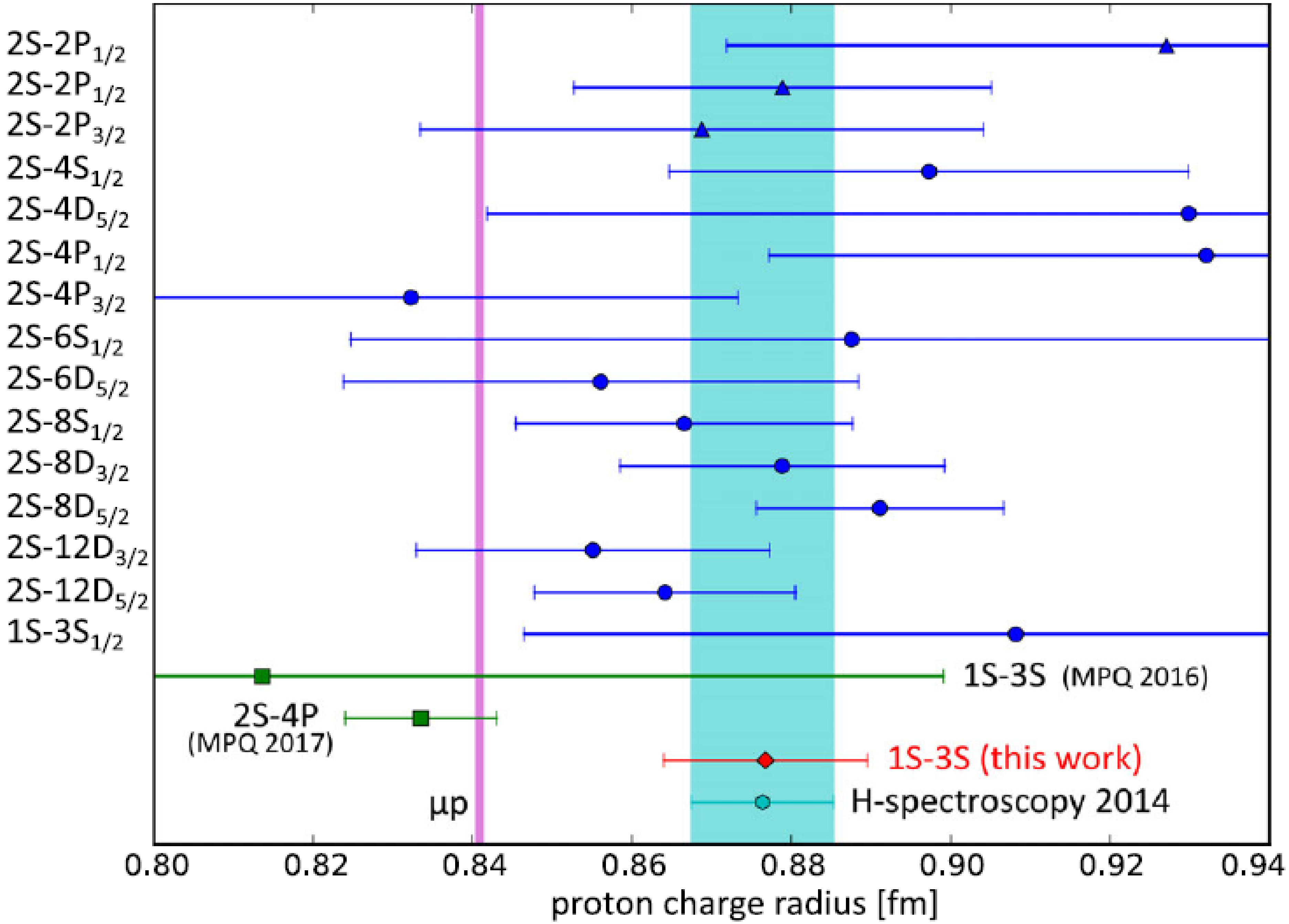
$$\Delta E^{\text{th}}_{\text{2S-HFS}} = 22.9843(30) - 0.1621(10) r_Z\ \ \text{meV}$$

$$r_Z = \int d^3r \int d^3r'\ r\, \rho_E(r)\, \rho_M(r - r')$$

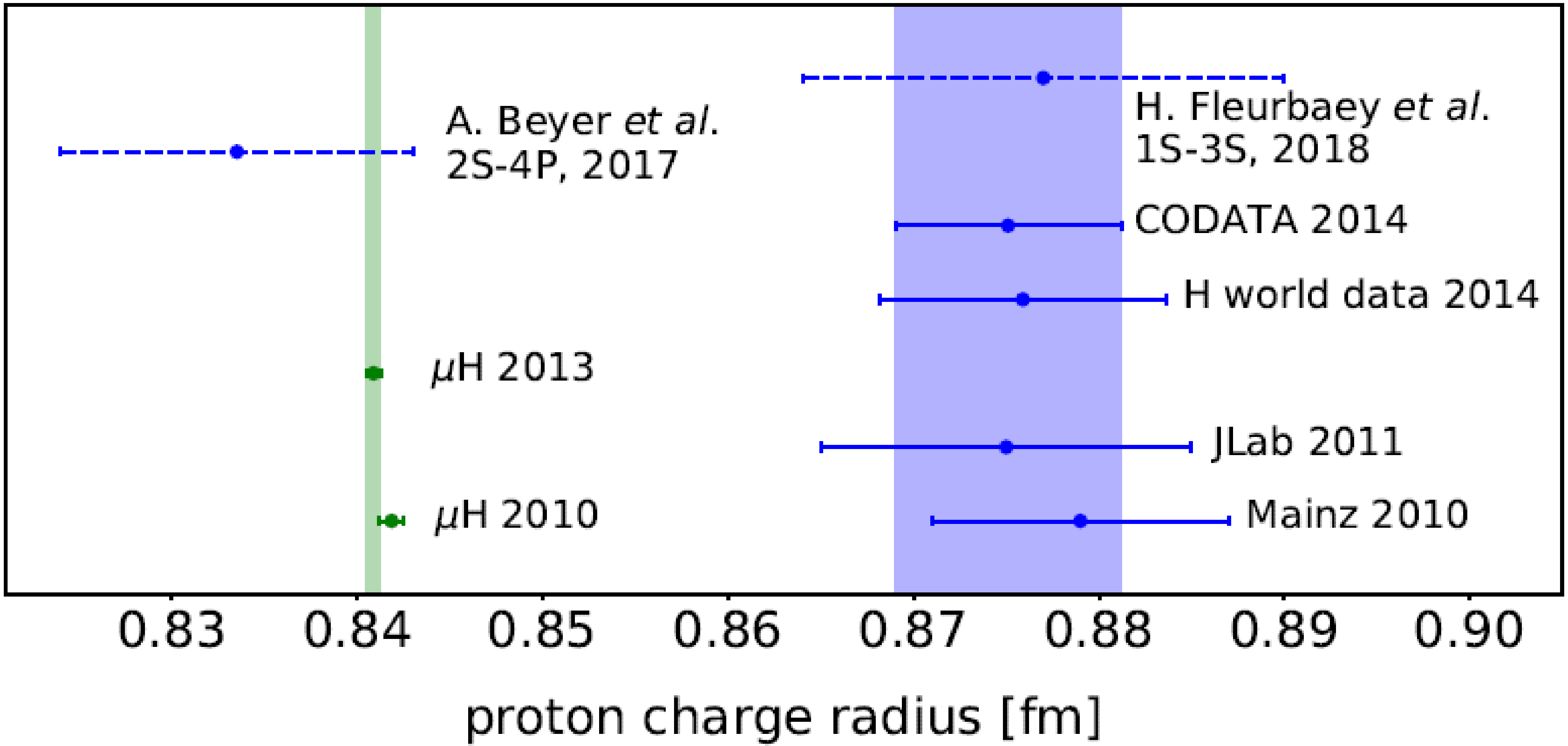
$$r_Z = 1.082(37)\ \text{fm (CREMA coll. Antognini et al., 2013)}$$







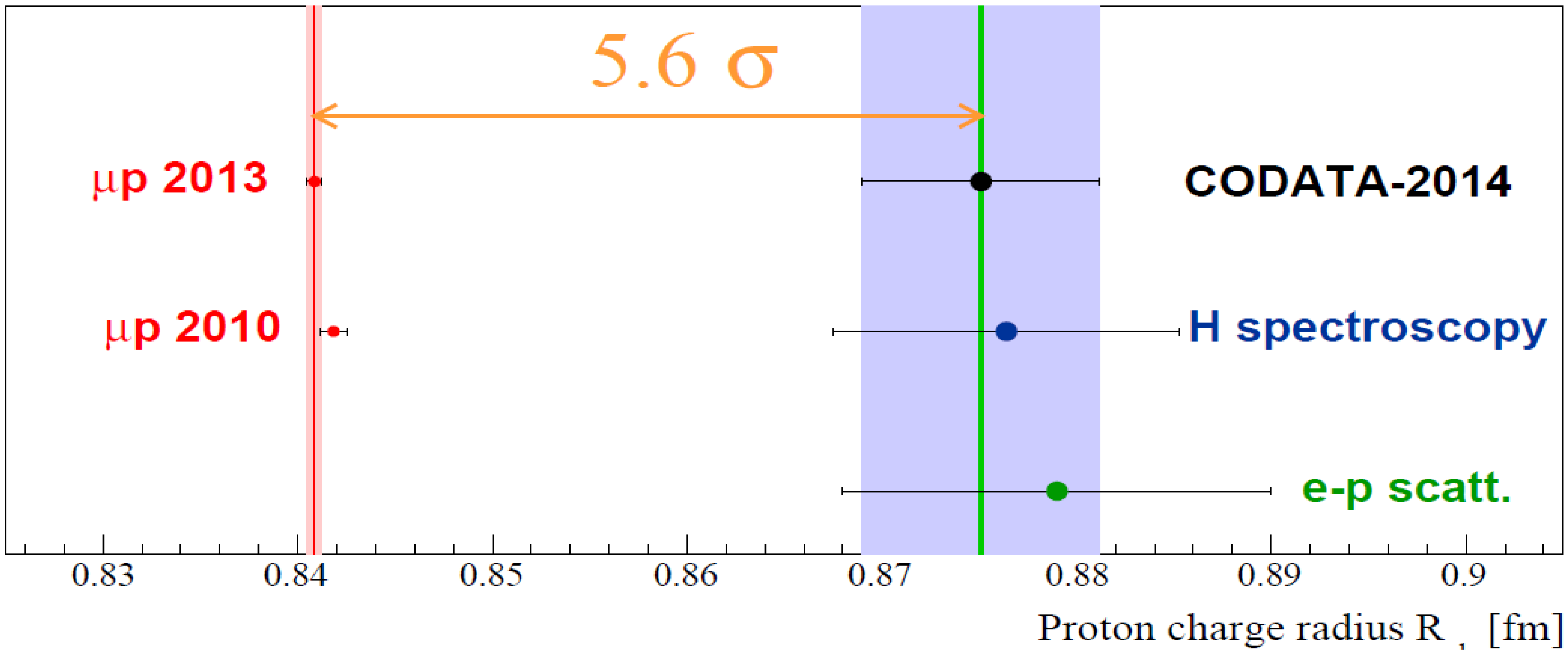
2S-2P1/2
2S-2P1/2
2S-2P3/2
2S-4S1/2
2S-4D5/2
2S-4P1/2
2S-4P3/2
2S-6S1/2
2S-6D5/2
2S-8S1/2
2S-8D3/2
2S-8D5/2
2S-12D3/2
2S-12D5/2
1S-3S1/2
1S-3S (MPQ 2016)
2S-4P
(MPQ 2017)
1S-3S (this work)
H-spectroscopy 2014
μp
0.80
0.82
0.84
0.86
0.88
0.90
0.92
0.94
proton charge radius [fm]


A. Beyer et al.
2S-4P, 2017
H. Fleurbaey et al.
1S-3S, 2018
CODATA 2014
H world data 2014
μH 2013
JLab 2011
μH 2010
Mainz 2010
0.83
0.84
0.85
0.86
0.87
0.88
0.89
0.90
proton charge radius [fm]

The proton rms charge radius measured with

electrons: 0.8751 ± 0.0061 fm

muons: 0.8409 ± 0.0004 fm

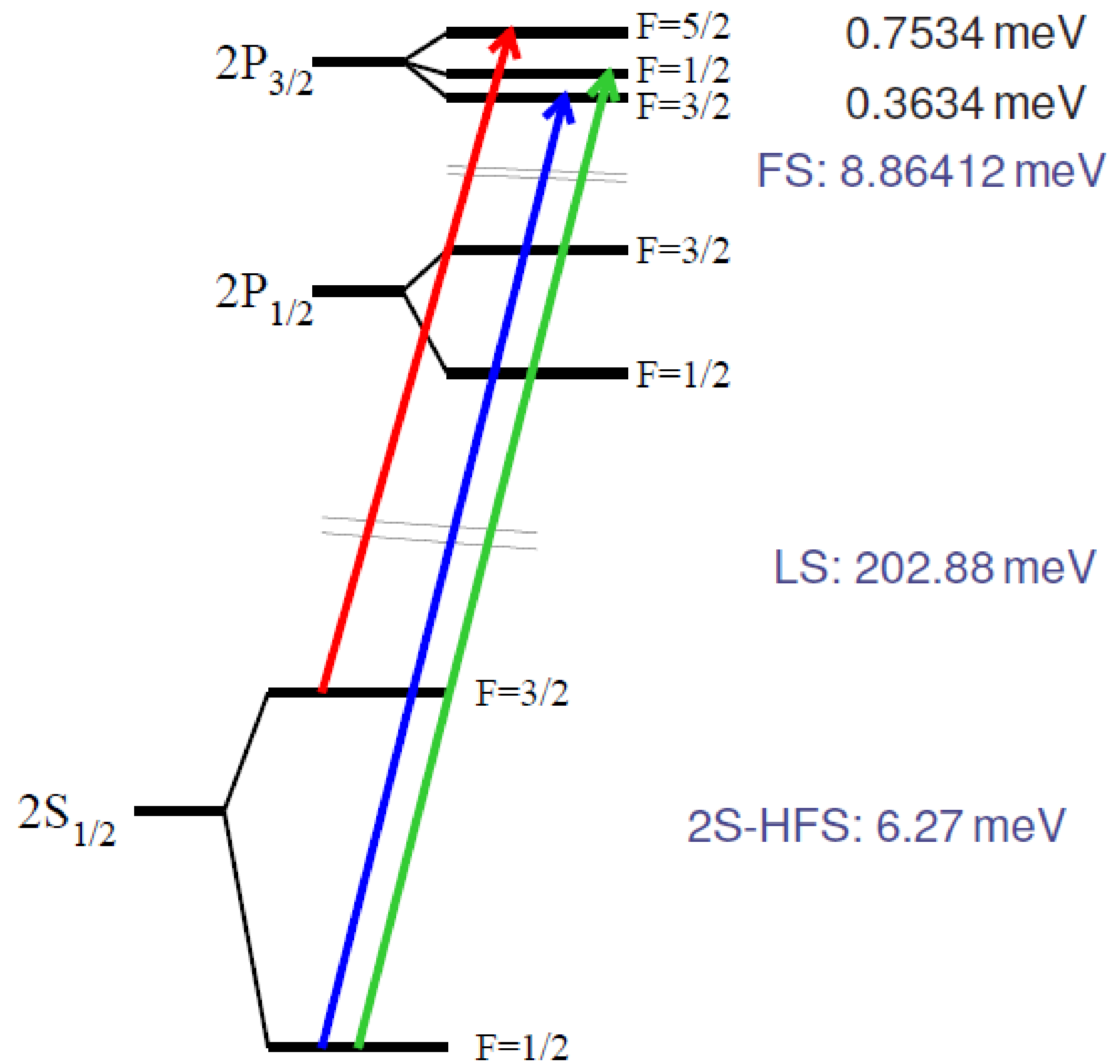


r_p is 7 σ smaller than CODATA-2010

4.0 σ smaller than r_p (H spectrosopy)

Muonic deuterium

CREMA-16



Deutron charge radius

$$\Delta E_{\mathrm{LS}}^{\mathrm{exp}} = 202.8785(34)\ \mathrm{meV}$$

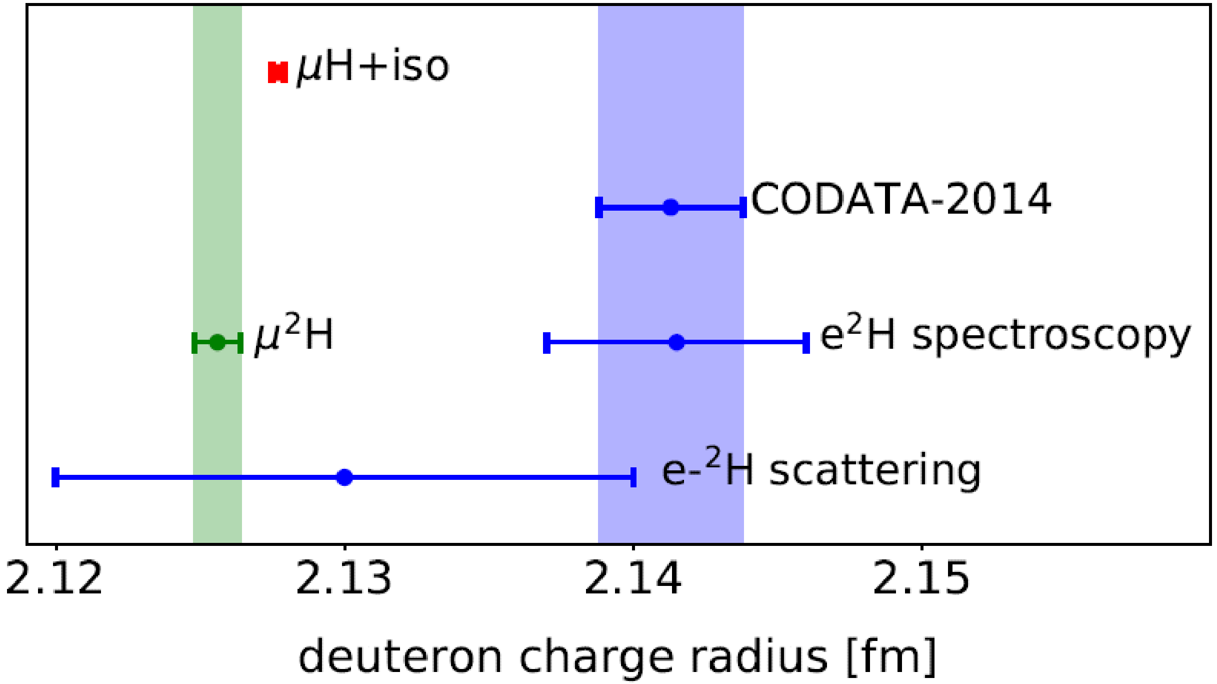
$$\Delta E_{\mathrm{LS}}^{\mathrm{th}} = 228.7766(10)\ \mathrm{meV\ (QED)} + 1.7096(200)\ \mathrm{meV\ (TPE)} - 6.1103(3)\, r_{\mathrm{d}}^2\ \mathrm{meV/fm^2}$$

$$\mu\mathrm{D} \Rightarrow r_{\mathrm{d}} = 2.12562(78)\ \mathrm{meV}\ \ (\mathrm{CREMA\ 2016})$$

H/D isotope shift: $r_{\mathrm{d}}^2 - r_{\mathrm{p}}^2 = 3.82007(65)\ \mathrm{fm}^2$

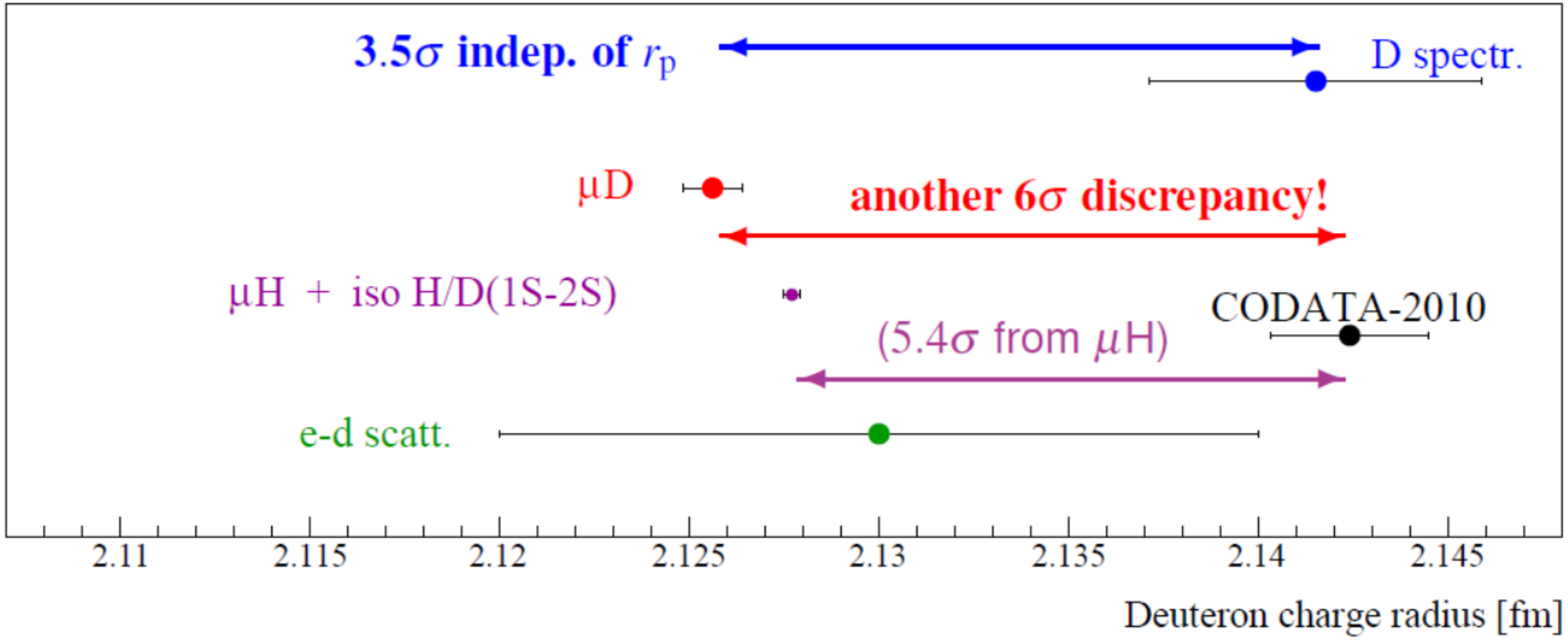
r_{p} from μH gives $r_{\mathrm{d}} = 2.12771(22)\ \mathrm{fm} \leftarrow 5.4\sigma$ from r_{p}

CODATA 2014 $r_{\mathrm{d}} = 2.14130(250)\ \mathrm{fm}$



r_{d} is 7.5 σ smaller than CODATA-2010 (99% correlated with r_{p} !)

3.5 σ smaller than r_{d} (D spectrosopy)



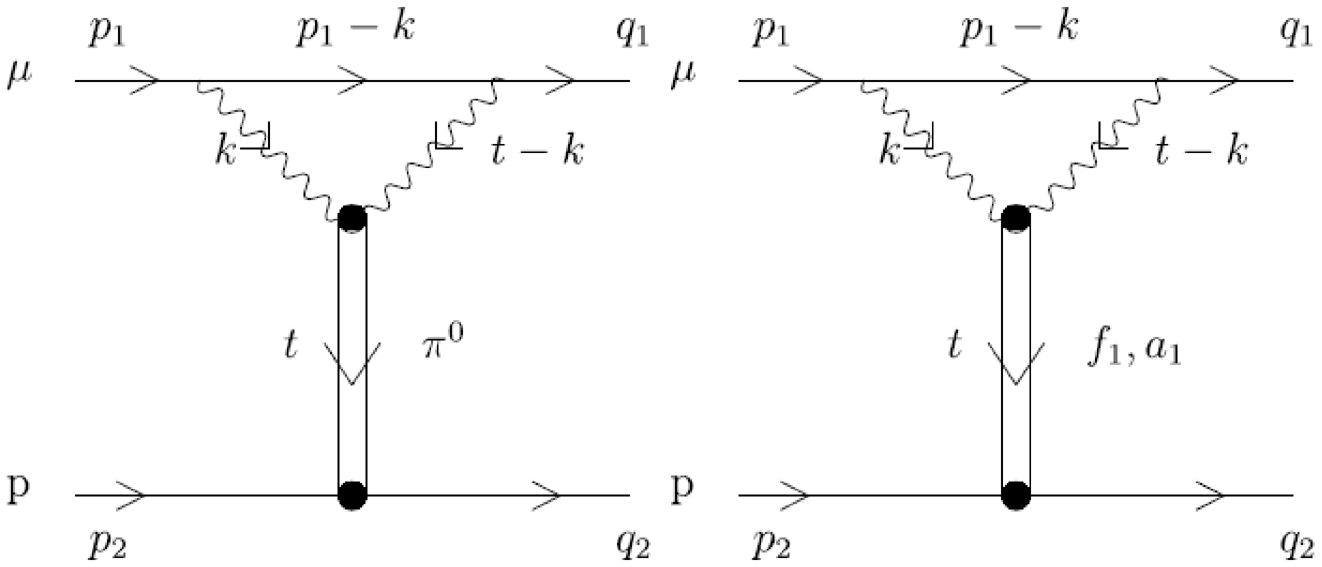
3.5σ indep. of r_p
D spectr.
μD
another 6σ discrepancy!
μH + iso H/D(1S-2S)
CODATA-2010
(5.4σ from μH)
e-d scatt.
2.11
2.115
2.12
2.125
2.13
2.135
2.14
2.145
Deuteron charge radius [fm]

The leading contribution to the hyperfine splitting (HFS) in muonic hydrogen is coming from one-photon exchange [AP Martynenko, RN Faustov 2004]

$$\Delta V_B^{hfs} = \frac{8\pi\alpha\mu_p}{3m_\mu m_p}(\mathbf{S}_p\mathbf{S}_\mu)\delta(\mathbf{r}) - \frac{\alpha\mu_p(1+a_\mu)}{m_\mu m_p r^3}\left[(\mathbf{S}_p\mathbf{S}_\mu) - 3(\mathbf{S}_p\mathbf{n})(\mathbf{S}_p\mathbf{n})\right] +$$

$$\frac{\alpha\mu_p}{m_\mu m_p r^3}\left[1+\frac{m_\mu}{m_p}-\frac{m_\mu}{2m_p\mu_p}\right](\mathbf{L}\mathbf{S}_p)$$

We calculate further the contribution to HFS coming from light pseudoscalar and axial-vector meson exchanges



Pseudosclar mesons

The effective vertex of the interaction of the PS mesons and virtual photons can be expressed in terms of the transition form factors

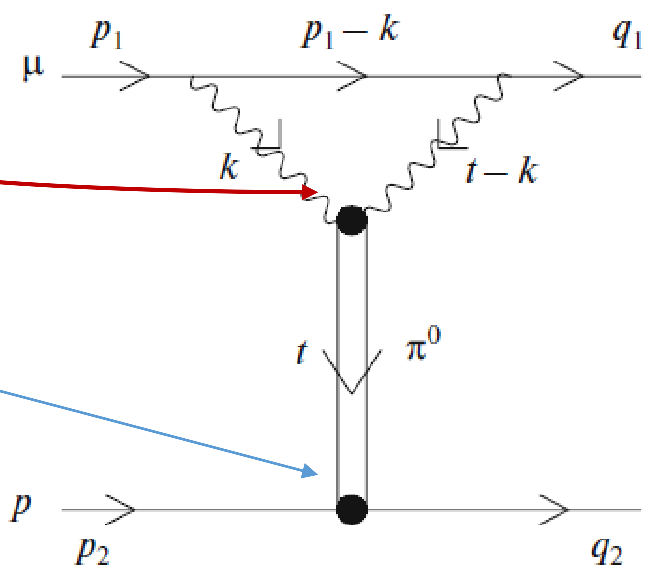
$$A\left(\gamma^*_{(q_1,\epsilon_1)}\gamma^*_{(q_2,\epsilon_2)} \to \pi_{(p)}\right) = ie^2\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu q_1^\rho q_2^\sigma \frac{1}{4\pi^2 F_\pi} F_{\pi\gamma^*\gamma^*}\left(p^2; q_1^2, q_2^2\right)$$

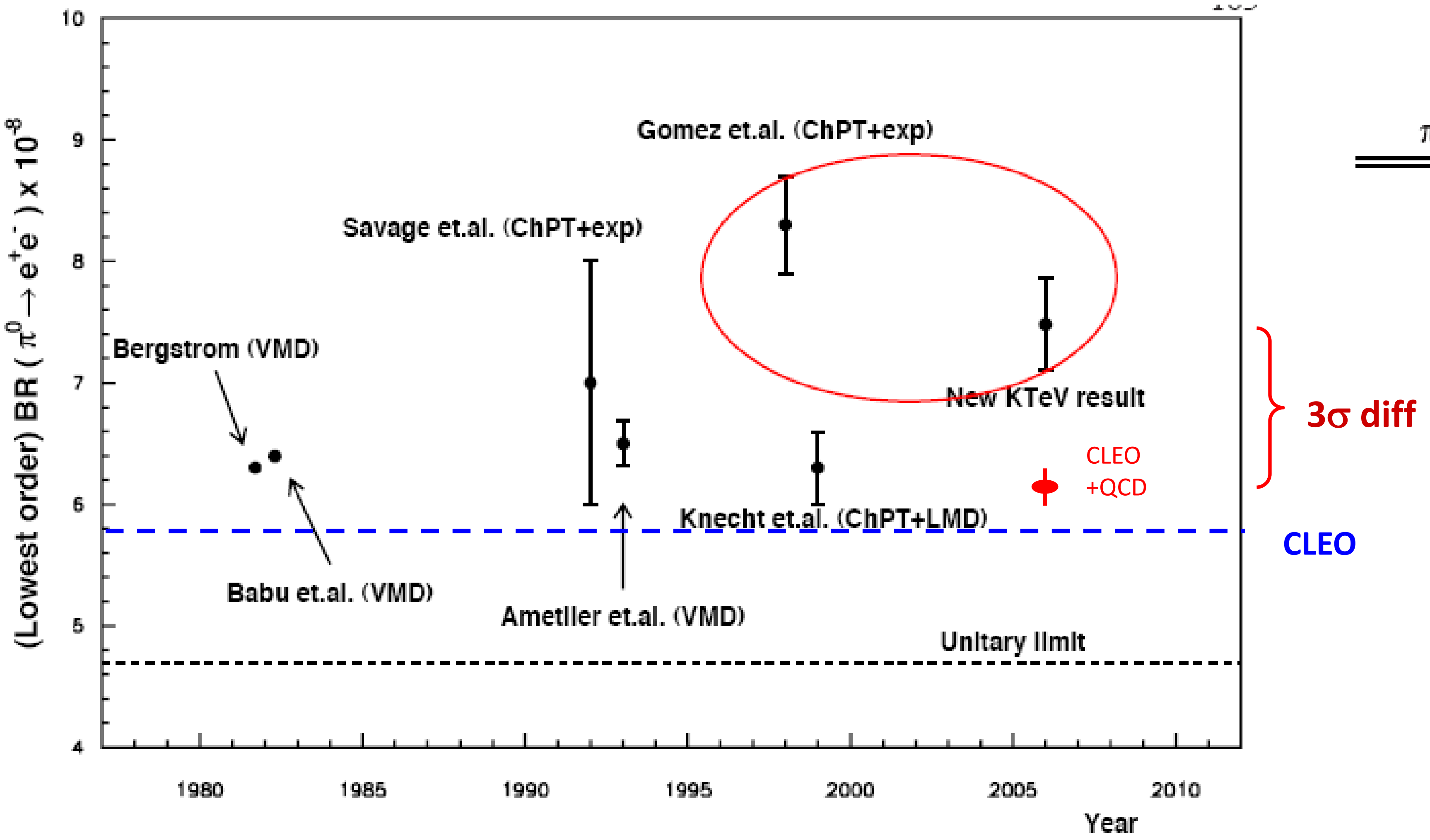
As a result, the hyperfine part of the potential of the one-pion interaction of a muon and a proton in the S-state takes the form

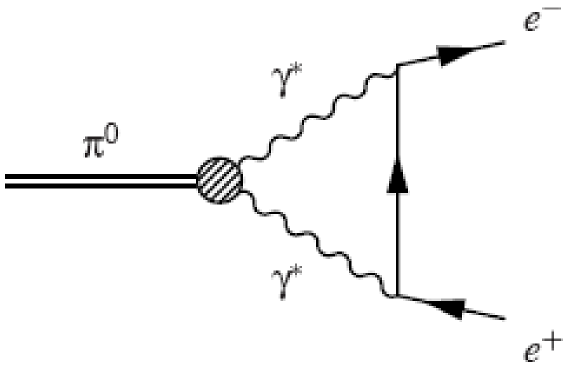
$$\Delta V^{hfs}(\mathbf{p},\mathbf{q}) = \frac{\alpha^2}{6\pi^2}\frac{g_p}{m_p F_\pi}\frac{(\mathbf{p}-\mathbf{q})^2}{(\mathbf{p}-\mathbf{q})^2+m_\pi^2}\mathcal{A}(t^2),$$

where

$$\mathcal{A}(t^2) = \frac{2i}{\pi^2 t^2}\int d^4k \frac{t^2k^2-(tk)^2}{k^2(k-t)^2(k^2-2kp_1)} F_{\pi\gamma^*\gamma^*}(k^2,(k-t)^2).$$







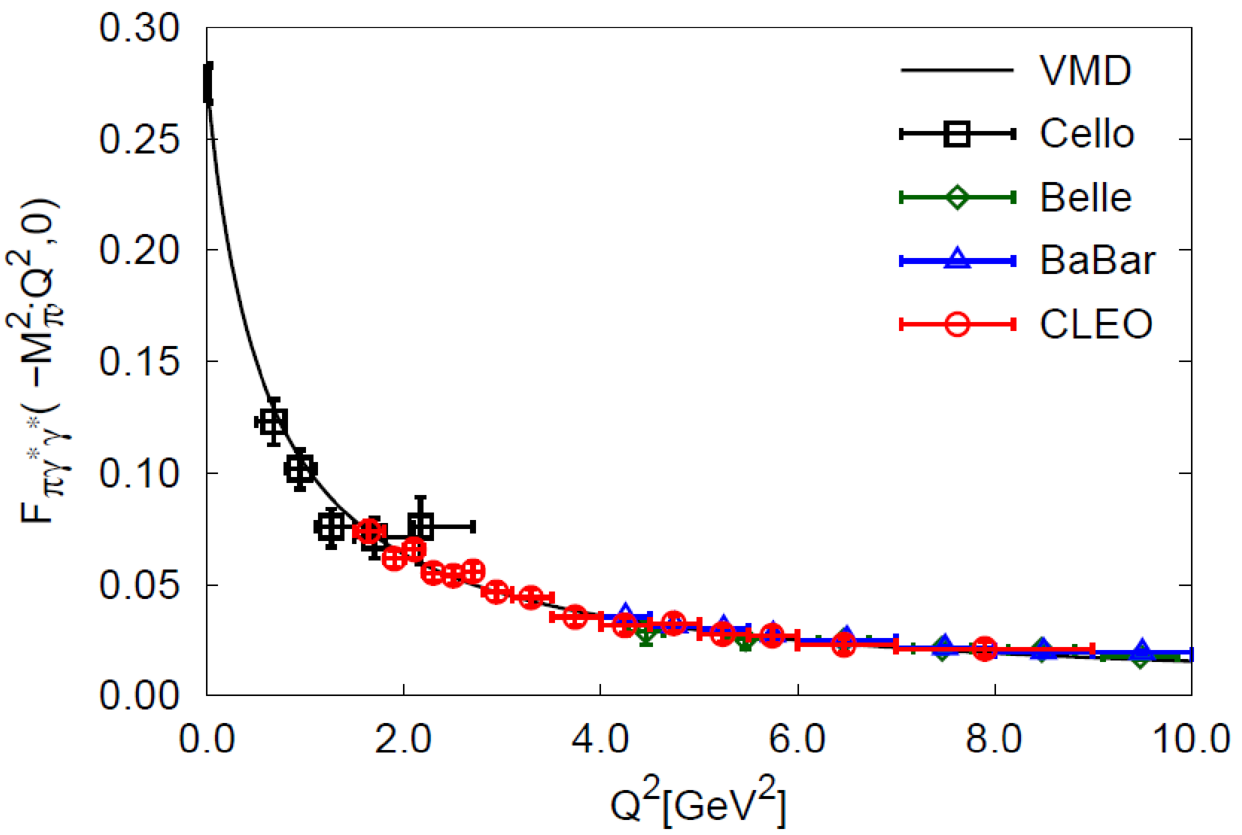
$$B^{\text{theory}}_{\pi\to e^+e^-} = (6.2 \pm 0.1)\cdot 10^{-8}$$

Which is ***3.3σ below data!!***

$$B^{\text{KTeV}}_{\pi\to e^+e^-} = (7.48 \pm 0.38)\cdot 10^{-8}$$

A. E. Dorokhov and M. A. Ivanov, Phys. Rev. D **75** (2007) 114007

Upper block: two-gamma transition FF into PS meson



$$F_{\pi\gamma\gamma^*}\left(Q^2,0\right)=\frac{\Lambda_\pi^2}{\Lambda_\pi^2+k^2},\quad \Lambda_\pi^{CLEO}=776\pm 22\text{ MeV},$$

$$F_{\pi\gamma^*\gamma^*}\left(Q^2,Q^2\right)=\frac{\overline{\Lambda}_\pi^2}{\overline{\Lambda}_\pi^2+k^2},\quad \overline{\Lambda}_\pi^{CLEO+QCD}=499\pm 50\text{ MeV}$$

Calculating the matrix elements with wave functions of 1S , 2S and 2P $_{1/2}$ states, we obtain the corresponding contributions to the HFS spectrum (we use data from CLEO on TFF)

$$\Delta E^{hfs}(1S)=\frac{\mu^3\alpha^5 g_A}{6F_\pi^2\pi^3}\left\{\mathcal{A}(0)\frac{4W(1+\frac{W}{m_\pi})}{m_\pi(1+\frac{2W}{m_\pi})^2}-\frac{1}{\pi}\int_0^\infty\frac{ds}{s}Im\mathcal{A}(s)\left[1+\frac{1}{4W^2(s-m_\pi^2)}\left(\frac{m_\pi^4}{(1+\frac{m_\pi}{2W})^2}-\frac{s^2}{(1+\frac{\sqrt{s}}{2W})^2}\right)\right]\right\}=-0.0017\ meV,$$

$$\Delta E^{hfs}(2S)=\frac{\mu^3\alpha^5 g_A}{48F_\pi^2\pi^3}\left\{\mathcal{A}(0)\frac{W(8+11\frac{W}{m_\pi}+8\frac{W^2}{m_\pi^2}+2\frac{W^3}{m_\pi^3})}{2m_\pi(1+\frac{W}{m_\pi})^4}-\frac{1}{\pi}\int_0^\infty\frac{ds}{s}Im\mathcal{A}(s)\right.$$

$$\left.\left[1+\frac{1}{(s-m_\pi^2)}\left(\frac{m_\pi^2(2+\frac{W^2}{m_\pi^2})}{2(1+\frac{W}{m_\pi})^4}-\frac{s(2+\frac{W^2}{s})}{2(1+\frac{W}{\sqrt{s}})^4}\right)\right]\right\}=-0.0002\ meV,$$

$$\Delta E^{hfs}_{2P_{1/2}}=\frac{\alpha^7\mu^5 g_A}{288\pi^3F_\pi^2m_\pi^2}\mathcal{A}(0)\frac{\left(9+8\frac{W}{m_\pi}+2\frac{W^2}{m_\pi^2}\right)}{(1+\frac{W}{m_\pi})^4}=0.0004\ \mu eV.$$

Axial-Vector Mesons

The transition from initial state of two virtual photons with four-momenta k1, k2 to an axial vector meson A (JPC = 1++) with the mass M_A for the small values of the relative momenta of particles in the initial and final states and small value of transfer momentum t between muon and proton is

$$T^{\mu\nu\alpha} = 8\pi i\alpha\varepsilon_{\mu\nu\alpha\tau}\left(k_1^\tau k_2^2 - k_2^\tau k_1^2\right)F^2_{AV\gamma^*\gamma^*}\left(M_A^2; k_1^2, k_2^2\right)$$

The final result for the HFS potential is

$$\Delta V_{AV}^{hfs}(\mathbf{p}-\mathbf{q}) = -\frac{32\alpha^2 g_{AVPP}}{3\pi^2(\mathbf{t}^2+M_A^2)}\int i d^4k \frac{(2k^2+k_0^2)}{k^2(k^2-2m_\mu k_0)}F_{AV\gamma^*\gamma^*}(0,k^2,k^2)$$

By using L3 Collaboration data (production of $f_1(1285)$ and $f_1(1420)$), we can parameterize the transition form factor for the case of two photons with equal virtualities

$$F_{AV\gamma^*\gamma^*}\left(M_A^2; k^2, k^2\right) = F_{AV\gamma^*\gamma^*}\left(M_A^2; 0, 0\right)F_{AV}^2\left(k^2\right) \qquad F_{AV}\left(k^2\right) = \frac{\Lambda_A^2}{\Lambda_A^2 + k^2}$$

with the normalization fixed from the decay width of axial-vector meson measured by L3 Collaboration. (Expect improvements From BESIII and BELLEII)

$$\Lambda^{L3}_{f_1(1285)} = 1040 \pm 78 \text{ MeV}, \quad \Lambda^{L3}_{f_1(1420)} = 926 \pm 78 \text{ MeV}$$

Meson-Nucleon Couplings

Within the NJL model from Triangle diagram one has

$$F_{f_1(1285)\gamma^*\gamma^*}\left(M^2_{f_1(1285)};0,0\right)=\frac{5g_{f_1(1285)qq}}{72\pi^2M_q^2}$$

Another couplings are related to each others by using chiral symmetry and SU(6)-model for wave function of the proton

$$g_{f_1(1260)qq}=g_{f_1(1285)qq},$$

$$g_{f_1(1285)pp}=g_{f_1(1285)qq},\quad g_{f_1(1260)pp}=\frac{5}{3}g_{f_1(1285)qq}$$

With Mq=300 MeV one gets

$$g_{f_1(1285)pp}=g_{f_1(1285)qq}=g_{f_1(1260)qq}=3.40\pm1.19,$$

$$g_{f_1(1260)pp}=5.67\pm1.98,\quad g_{f_1(1420)pp}=1.51\pm0.19$$

Our result for HFS interaction can be rewritten ($a=2m_\mu/\Lambda_A$)

$$\Delta V_{AV}^{hfs}(\mathbf{p}-\mathbf{q}) = -\frac{32\alpha^2 g_{AVpp} F^{(0)}_{AV\gamma^*\gamma^*}(0,0,0)}{3\pi^2(\mathbf{t}^2+M_A^2)} I\left(\frac{m_\mu}{\Lambda_A}\right)$$

$$I\left(\frac{m_\mu}{\Lambda_A}\right) = -\frac{\pi^2\Lambda_A^2}{4(1-a_\mu^2)^{5/2}}\left[3\sqrt{1-a_\mu^2} - a_\mu^2(5-2a_\mu^2)\ln\frac{1+\sqrt{1-a_\mu^2}}{a_\mu}\right]$$

Making the Fourier transform and averaging the obtained expression with the wave functions for 1S and 2S states, we obtain the following contribution to hyperfine splitting

$$\Delta E_{AV}^{hfs}(1S) = \frac{32\alpha^5\mu^3\Lambda^2 g_{AVNN} F^{(0)}_{AV\gamma^*\gamma^*}(0,0,0)}{3M_A^2\pi^3\left(1+\frac{2W}{M_A}\right)^2} I\left(\frac{m_\mu}{\Lambda}\right),$$

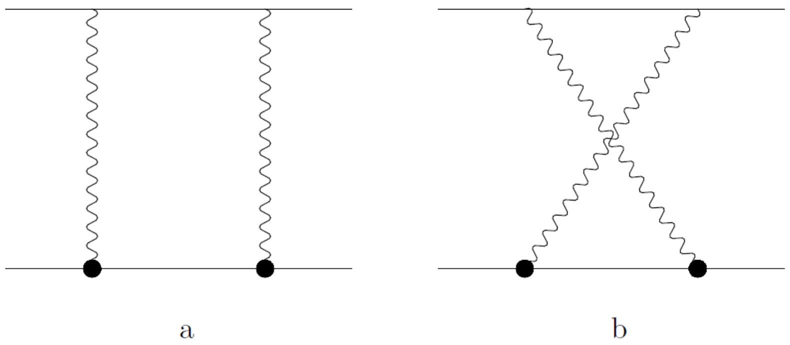
$$\Delta E_{AV}^{hfs}(2S) = \frac{2\alpha^5\mu^3\Lambda^2 g_{AVNN} F^{(0)}_{AV\gamma^*\gamma^*}(0,0,0)\left(2+\frac{W^2}{M_A^2}\right)}{3M_A^2\pi^3\left(1+\frac{W}{M_A}\right)^4} I\left(\frac{m_\mu}{\Lambda}\right)$$

where $W=\mu\,\alpha$ and μ is the reduced mass.

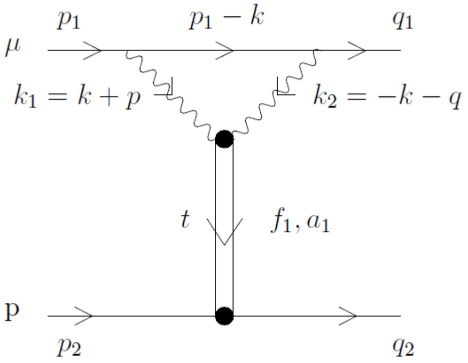
Axial-Vector and Pseudoscalar mesons contribution to HFS of muonic hydrogen

mesons	$I^G(J^{PC})$	Λ_A in MeV	$F^{(0)}_{AV\gamma^*\gamma^*}(0,0)$ in GeV^{-2}	$\Delta E^{hfs}(1S)$ in meV	$\Delta E^{hfs}(2S)$ in meV
$f_1(1285)$	$0^+(1^{++})$	1040	0.266	-0.0093 ± 0.0033	-0.0012 ± 0.0004
$a_1(1260)$	$1^-(1^{++})$	1040	0.591	-0.0437 ± 0.0175	-0.0055 ± 0.0022
$f_1(1420)$	$0^+(1^{++})$	926	0.193	-0.0013 ± 0.0008	-0.0002 ± 0.0001
π^0	$1^-(0^{-+})$	776		-0.0017 ± 0.0001	-0.0002 ± 0.00002
Sum				-0.0560 ± 0.0178	-0.0071 ± 0.0024

Corrections of 2-photon interaction to the fine and hyperfine structure of P-energy levels of mu-hydrogen



$\sim \alpha^6$



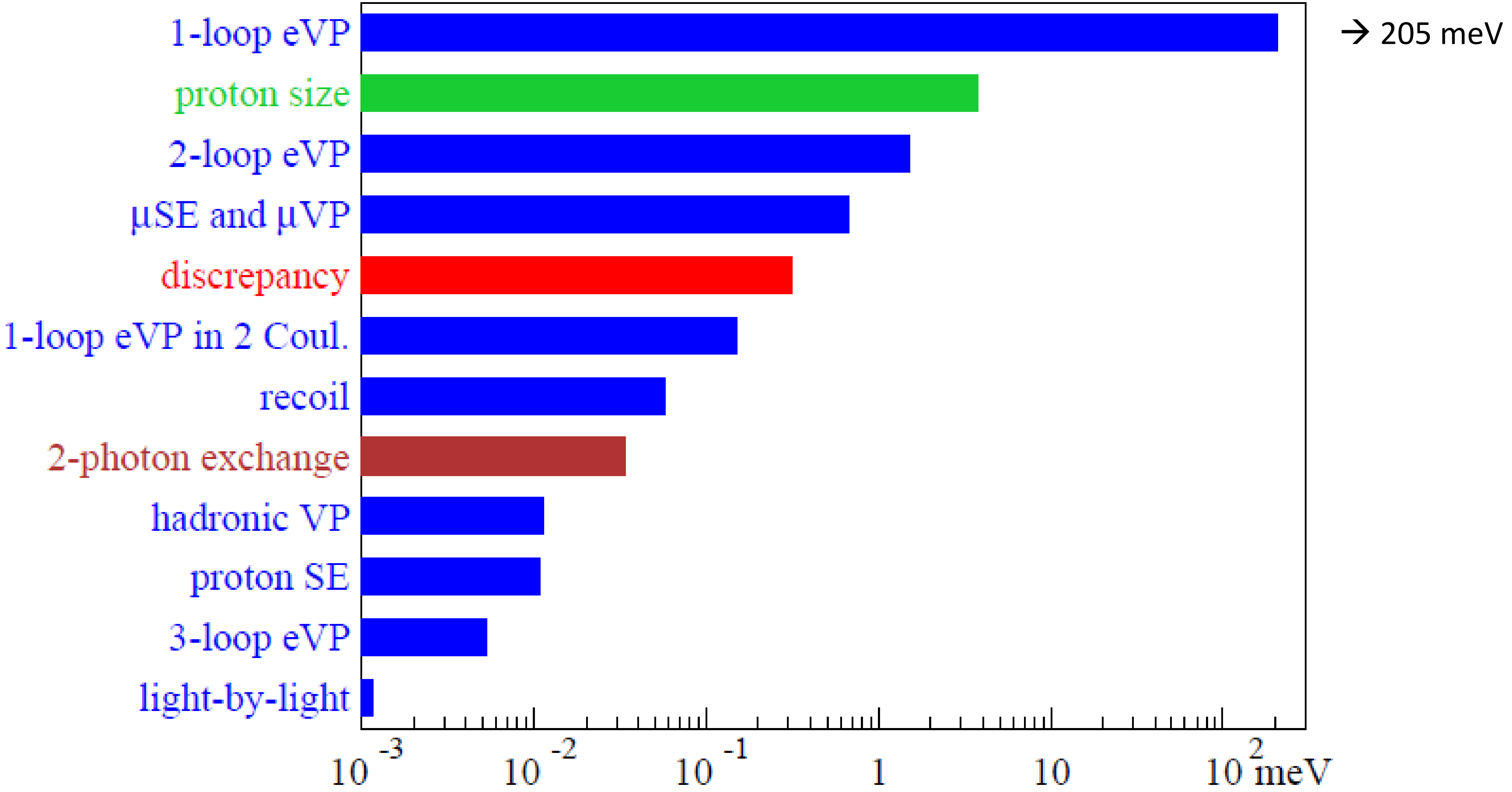
$\sim \alpha^7$

State	Correction of 2γ exchange amplitudes, μeV	Correction of axial meson exchange amplitudes, μeV
$2^1P_{1/2}$	-0.2027	0.0005
$2^3P_{1/2}$	0.0761	-0.0002
$2^3P_{3/2}$	0	-0.00005
$2^5P_{3/2}$	0	0.00003

The corrections from two-photon exchange amplitudes are more significant for a precise comparison with the experimental data because their numerical values are of the order of 0.0001 meV.

Recall that to explain the proton radius puzzle , a contribution of about 0.3 meV is needed.

Hierarchy of contributions to Lamb shift



Conclusions

From CREMA experiment side:

“Proton radius puzzle” is in fact “Z=1 radius puzzle”

$r_p = 0.84087(39)\,\mathrm{fm}$

$r_Z = 1.082(37)\,\mathrm{fm}$

Muonic helium-3 and -4 ions show (preliminary) no big discrepancy

New projects are ongoing, one of them FAMU (Fisica Atomi MUonici) with accuracy 2 ppm

In our theoretical work:

A new **large** contribution to the HFS of muonic hydrogen is found, that Induced by **pseudoscalar** and **axial-vector** couplings to two photon state.

While results do not influence on the proton charge radius, they provide diminishing of the Zemach radius $r_Z = 1.040(37)\,\mathrm{fm}$ (compare with $r_Z = 1.082(37)\,\mathrm{fm}$)

It should be taking into account for the interpretation of the new data on HFS in this atom.

There are still a number of uncertainties in phenomenological input used in our calculations and some other new effects unaccounted by us. (Work is in progress)

Electronic Anomaly (New) 1806 10252

A recent precise determination of the fine structure constant, a , has introduced a new twist to this story. An improvement in the measured mass of atomic Cesium used in conjunction with other known mass ratios and the Rydberg constant leads to the new now most precise value [R. H. Parker, C. Yu, W. Zhong, B. Estey, H. Mueller, Science 360, 191 (2018)]

$$\alpha^{-1}(\mathrm{Cs}) = 137.035999046(27).$$

As a result, comparison of the theoretical prediction of aSMe [KinoshitaNio18] with the existing experimental measurement of aexpe [Gabrielse08,10] now leads to a discrepancy

$$\begin{aligned} \Delta a_e &\equiv a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}} \\ &= [-87 \pm 28\,(\mathrm{exp}) \pm 23\,(\alpha) \pm 2\,(\mathrm{theory})] \\ &\times\ 10^{-14}, \end{aligned}$$

$$\Delta a_e = (-87 \pm 36) \times 10^{-14}\,,$$

which represents a 2.4sigma discrepancy that is opposite in sign from the long standing muon discrepancy previously mentioned. Note that the current discrepancy in Eqs. (2) and (3) results from an improvement in alpha^-1 from 137.035998995(85) which previously [13] gave Delta ae = -130(77) 10^-14 and represented a 1.7sigma effect.
The central value has decreased in magnitude, but its signicance has increased. The errors from the experimental determinations of ae and are now the dominant sources of uncertainty.