Chiral study of the four-quark state $f_0(500)$ in pion-pion and pion-nucleon scattering

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- But we are able to make a strong statement on the nature of low-lying scalar mesons

The scalar meson puzzle

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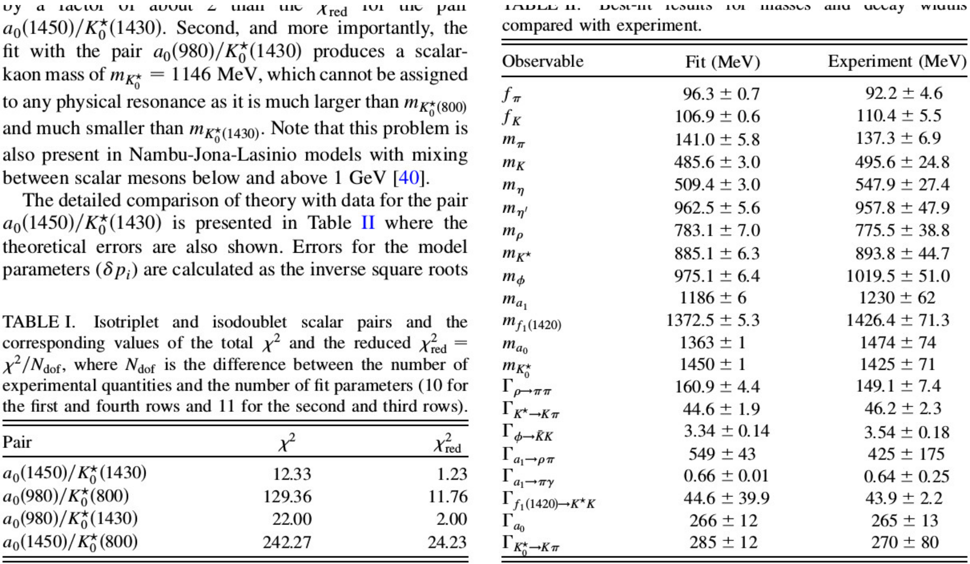
- Parganlija et al. [19] were able to obtain reasonable results for a three-flavor eLSM

Results from Ref. [19]



by a factor of about 2 than the $\chi_{\rm red}$ for the pair $a_0(1450)/K_0^\star(1430)$. Second, and more importantly, the fit with the pair $a_0(980)/K_0^\star(1430)$ produces a scalar-kaon mass of $m_{K_0^\star} = 1146$ MeV, which cannot be assigned to any physical resonance as it is much larger than $m_{K_0^\star(800)}$ and much smaller than $m_{K_0^\star(1430)}$. Note that this problem is also present in Nambu-Jona-Lasinio models with mixing between scalar mesons below and above 1 GeV [40].

The detailed comparison of theory with data for the pair $a_0(1450)/K_0^\star(1430)$ is presented in Table II where the theoretical errors are also shown. Errors for the model parameters (δp_i) are calculated as the inverse square roots

TABLE I. Isotriplet and isodoublet scalar pairs and the corresponding values of the total χ^2 and the reduced $\chi^2_{\rm red} = \chi^2/N_{\rm dof}$, where $N_{\rm dof}$ is the difference between the number of experimental quantities and the number of fit parameters (10 for the first and fourth rows and 11 for the second and third rows).

Pair	χ^2	$\chi^2_{\rm red}$
$a_0(1450)/K_0^\star(1430)$	12.33	1.23
$a_0(980)/K_0^\star(800)$	129.36	11.76
$a_0(980)/K_0^\star(1430)$	22.00	2.00
$a_0(1450)/K_0^\star(800)$	242.27	24.23

TABLE II. Best-fit results for masses and decay widths compared with experiment.

Observable	Fit (MeV)	Experiment (MeV)
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
$m_{K^\star}$	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^\star}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho\to\pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi\to\bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1\to\rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1\to\pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^\star\to K\pi}$	285 ± 12	270 ± 80

Intepretation of the quark content of scalar mesons

- In this paper [19] the assignment of the σ-meson with the lightest scalar resonance $f_0(500)$ leads to a very small decay width $\Gamma_{\sigma\to\pi\pi} < 20$ MeV - in contradiction to the large decay width reported by the PDG, $\Gamma_{\sigma\to\pi\pi} \approx 550$ MeV

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- Furthermore, the pion-pion scattering length a_0^0 turned out to be small - which suggests to extend this model by an additional scalar degree of freedom $\longrightarrow$ Incorporate $f_0(500)$ as potential tetraquark candidate!

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(Pseudo-)scalar meson Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{S}} = \mathrm{Tr}\left[(\partial_\mu \Phi)^{\dagger}(\partial^\mu \Phi) - \mu^2 \Phi^{\dagger}\Phi - \lambda_2 (\Phi^{\dagger}\Phi)^2\right] - \lambda_1 (\mathrm{Tr}[\Phi^{\dagger}\Phi])^2 \\ + c(\det \Phi^{\dagger} + \det \Phi) + h_0 \mathrm{Tr}[\Phi^{\dagger} + \Phi] \end{aligned} \tag{1}$$

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(Axial-)vector meson Lagrangian

$$\begin{aligned}\mathcal{L}_\mathrm{V} = &-\frac{1}{4}\mathrm{Tr}\left[(L^{\mu\nu})^2 + (R^{\mu\nu})^2\right] + \frac{m_1^2}{2}\mathrm{Tr}\left[(L^\mu)^2 + (R^\mu)^2\right] \\ &- i\frac{g_2}{2}\Big(\mathrm{Tr}\big[L_{\mu\nu}[L^\mu, L^\nu]\big] + \mathrm{Tr}\big[R_{\mu\nu}[R^\mu, R^\nu]\big]\Big) \\ &+ g_3\left(\mathrm{Tr}\left[L_\mu L_\nu L^\mu L^\nu + R_\mu R_\nu R^\mu R^\nu\right]\right) + g_4\left\{\mathrm{Tr}\left[(L_\mu)^2(L_\nu)^2\right]\mathrm{Tr}\left[(R_\mu)^2(R_\nu)^2\right]\right\} \\ &+ g_5\,\mathrm{Tr}\left[(R_\mu)^2\right]\mathrm{Tr}\left[(L_\nu)^2\right] + g_6\left\{\mathrm{Tr}\left[(R_\mu)^2\right]\mathrm{Tr}\left[(R_\nu)^2\right] + \mathrm{Tr}\left[(L_\mu)^2\right]\mathrm{Tr}\left[(L_\nu)^2\right]\right\} \end{aligned} \quad (2)$$

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- $R_\mu \sim \bar{q}_\mathrm{R}\gamma_\mu q_\mathrm{R}\,, L_\mu \sim \bar{q}_\mathrm{L}\gamma_\mu q_\mathrm{L}$
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- Vector mesons: $V_\mu = \frac{1}{2}(R_\mu + L_\mu)$, axial-vector mesons: $A_\mu = \frac{1}{2}(L_\mu - R_\mu)$.

More interaction terms

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Interaction between (pseudo-)scalar and (axial-)vector mesons

$$\begin{aligned}\mathcal{L}_{\mathrm{SV}} = {} & i g_1 \mathrm{Tr}\left[\partial_\mu \Phi (\Phi^\dagger L^\mu - R^\mu \Phi^\dagger) - \partial_\mu \Phi^\dagger (L^\mu \Phi - \Phi R^\mu)\right] \\ & + \frac{h_1}{2} \mathrm{Tr}[\Phi^\dagger \Phi] \mathrm{Tr}\left[(L^\mu)^2 + (R^\mu)^2\right] + (g_1^2 + h_2) \mathrm{Tr}\left[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger\right] \\ & - 2(g_1^2 - h_3) \mathrm{Tr}\left[\Phi R_\mu \Phi^\dagger L^\mu\right] \end{aligned} \tag{3}$$

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$\Longrightarrow$ Complete meson Lagrangian given by:

$$\mathcal{L}_{\mathrm{M}} = \mathcal{L}_{\mathrm{S}} + \mathcal{L}_{\mathrm{V}} + \mathcal{L}_{\mathrm{VS}} \tag{4}$$

Two-Flavor Multiplets

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Scalar meson multiplet in two flavors:

$$\Phi = \sqrt{2}\bar{\psi}_j^R \psi_i^L = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma + a_0^0 + i(\eta + \pi^0)}{\sqrt{2}} & a_0^+ + \pi^+ \\ a_0^- + \pi^- & \frac{\sigma - a_0^0 + i(\eta - \pi^0)}{\sqrt{2}} \end{pmatrix} . \tag{5}$$

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Vector- and axial-vector mesons are contained in the right- and left-handed meson matrices:

$$R_\mu = \sqrt{2}\bar{\psi}_j^R \gamma_\mu \psi_i^R = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_\mu + \rho_\mu^0 - f_{1,\mu} - a_{1,\mu}^0}{\sqrt{2}} & \rho_\mu^+ - a_{1,\mu}^+ \\ \rho_\mu^- - a_{1,\mu}^- & \frac{\omega_\mu - \rho_\mu^0 - f_{1,\mu} + a_{1,\mu}^0}{\sqrt{2}} \end{pmatrix} , \tag{6}$$

$$L_\mu = \sqrt{2}\bar{\psi}_j^L \gamma_\mu \psi_i^L = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_\mu + \rho_\mu^0 + f_{1,\mu} + a_{1,\mu}^0}{\sqrt{2}} & \rho_\mu^+ + a_{1,\mu}^+ \\ \rho_\mu^- + a_{1,\mu}^- & \frac{\omega_\mu - \rho_\mu^0 + f_{1,\mu} - a_{1,\mu}^0}{\sqrt{2}} \end{pmatrix} . \tag{7}$$

Baryon Lagrangian in the mirror assignment

$$\begin{aligned}
\mathcal{L}_{\mathrm{mirror}} = {} & \bar{\Psi}_{1L} i\gamma_\mu D^\mu_{1L}\Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D^\mu_{1R}\Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D^\mu_{2R}\Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D^\mu_{2L}\Psi_{2R} \\
& - \hat{g}_1(\bar{\Psi}_{1L}\Phi\Psi_{1R} + \bar{\Psi}_{1R}\Phi^\dagger\Psi_{1L}) - \hat{g}_2(\bar{\Psi}_{2L}\Phi^\dagger\Psi_{2R} + \bar{\Psi}_{2R}\Phi\Psi_{2L}) \\
& - m_0\left(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L}\right) . \qquad (8)
\end{aligned}$$

where $D^\mu_{1/2,\mathrm{R}} = \partial^\mu - ic_{1/2}R^\mu$, $D^\mu_{1/2,\mathrm{L}} = \partial^\mu - ic_{1/2}L^\mu$.

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- In this model a mass term is introduced that gives a contribution to the nucleon mass that does not stem from the chiral condensate
- This is possible by the so-called mirror assignment:

$$\Psi_{1,R/L} \to U_{R/L}\Psi_{1,R/L} \quad \Psi_{2,R/L} \to U_{L/R}\Psi_{2,R/L} \tag{9}$$

Results of Refs. [2, 15]

Table: scattering parameters for $m_{N^*} = 1535\,\mathrm{MeV}$

Parameter	**Value**	**Experiment**
$m_\pi\, a_0^{(-)}$	0.0782	0.0883 ± 0.0014
$m_\pi^3\, a_{1+}^{(-)}$	-0.048	-0.081 ± 0.002
$m_\pi^3\, a_{1-}^{(-)}$	-0.042	-0.013 ± 0.003
$m_\pi^3\, r_0^{(-)}$	0.022	0.007 ± 0.005
$m_\pi\, a_0^{(+)}$	-0.0083	-0.0012 ± 0.0010
$m_\pi^3\, a_{1+}^{(+)}$	0.049	0.130 ± 0.003
$m_\pi^3\, a_{1-}^{(+)}$	-0.093	-0.056 ± 0.010
$m_\pi^3\, r_0^{(+)}$	0.009	-0.06 ± 0.02

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$$m_0 \equiv a\chi_0 + bG_0 + \frac{c_N\varphi^2}{2}$$
- We hope now to kill two birds with on stone: Incorporating the tetraquark and the scalar glueball should affect both mesonic observables (decay width $\Gamma_{f_0(500)\to\pi\pi}$, $\Gamma_{f_0(1370)\to\pi\pi}$, $\Gamma_{f_0(1710)\to\pi\pi}$ and pion-pion scattering lengths) and baryonic observables (pion-nucleon scattering)

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Tetraquark-Quarkonium interaction terms

$$g_\chi\, \chi\,(\det\Phi + \mathrm{h.c.}) \quad \& \quad g_\mathrm{AV}\, \chi(\det R_\mu + \det L_\mu)\,. \tag{10}$$

Noteworthy: These terms contribute to a mass shift

$$g_\chi \chi\,(\sigma^2 + \vec{\pi}^2 - \eta^2 - \vec{a}_0^2) \quad \& \quad g_\mathrm{AV} \chi\,(\vec{\rho}_\mu^2 + \vec{a}_{1,\mu}^2 - \omega_\mu^2 - f_{1,\mu}^2)\,. \tag{11}$$

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- The classical Yang-Mills part of QCD has a dilatation symmetry which is broken at quantum level (trace anomaly)
- This can be modeled by introducing a dilaton field whose excitations are interpreted as the scalar glueball

Incorporating the Glueball

Glueball Lagrangian

The trace anomaly is modeled by the term

$$\mathcal{L}_{\text{trace anomaly}} = -\frac{1}{4}\frac{m_G^2}{\Lambda^2}G^4\left(\ln\left|\frac{G}{\Lambda}\right| - \frac{1}{4}\right) . \tag{12}$$

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On the other hand, all other terms must be dilatation invariant:

$$\begin{aligned}\mathcal{L}_{\text{dil}} = &-\mu^2\frac{G^2}{G_0^2}\mathrm{Tr}\left[\Phi^\dagger\Phi\right] + m_1^2\frac{G^2}{G_0^2}\mathrm{Tr}\left[R_\mu^2 + L_\mu^2\right] \\ &+ 2g_\chi\frac{G}{G_0}\chi\left(\det\Phi + \text{h.c.}\right) - 2g_{\text{AV}}\frac{G}{G_0}\chi(\det R_\mu + \det L_\mu) .\end{aligned} \tag{13}$$

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- For details on the calculation of these observables please refer to paper: https://arxiv.org/abs/1807.03735

Parameters of the fit

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- The Lagrangian contains twelve parameters that are of relevance for our fit: $\mu^2, \lambda_1, \lambda_2, c, m_1^2, h_1 + h_2 \equiv h, h_3, g_\chi, g_{\mathrm{AV}}, m_\chi, M_G, G_0$.

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- We fix the parameters $\lambda_2, h_3, c, \mu^2, m_1^2, g_{\mathrm{AV}}$ by the masses $m_\eta, m_\pi, m_\rho, m_{a_1}, m_{\mathrm{f}_1}, m_\omega$ and the remaining model parameters included in the fit

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- Thus, five parameters $h, g_\chi, m_\chi, M_G, G_0$ need to be fitted in a χ^2 fit

Assignments of the degrees of freedom

Field	**Assignment**	**Masses**
H	$f_0(500)$	475 ± 75 MeV
S	$f_0(1370)$	1350 ± 150 MeV
G'	$f_0(1710)$	1723 ± 5 MeV
a_0	$a_0(1450)$	1474 ± 19 MeV
a_1	$a_1(1260)$	1230 ± 40 MeV
ρ	$\rho(770)$	775.26 ± 0.25 MeV
f_1	$f_1(1285)$	1281.9 ± 0.5 MeV
ω	$\omega(782)$	782.65 ± 0.12 MeV

Table: The masses of the fields as given by the PDG [16].

Results

Param.	Value
g_χ	2.87 ± 0.54 MeV
h	5.12 ± 2.75
M_G	1543 ± 84 MeV
G_0	406 ± 135 MeV
m_χ	534 ± 33 MeV
g_{AV}	-11999 ± 738 MeV
μ^2	$-873 \times 10^3\ \mathrm{MeV}^2$
m_1^2	$733^2\ \mathrm{MeV}^2$
c	$99 \times 10^3\ \mathrm{MeV}^2$
m_σ	$1401\ \mathrm{MeV}$
χ_0	0.24 ± 0.02 MeV

Observ.	Value	Exp. Data
M_H	534 ± 33 MeV	475 ± 75 MeV
M_S	1215 ± 113 MeV	1350 ± 150 MeV
$M_{G'}$	1694 ± 49 MeV	1720 ± 50 MeV
$\Gamma_{H\to\pi\pi}$	504 ± 148 MeV	550 ± 150 MeV
$\Gamma_{S\to\pi\pi}$	479 ± 98 MeV	325 ± 150 MeV
$\Gamma_{G'\to\pi\pi}$	29 ± 7 MeV	29.3 ± 6.5 MeV
$m_\pi a_0^0$	0.210 ± 0.016	0.218 ± 0.02
$m_\pi a_0^2$	-0.027 ± 0.005	-0.046 ± 0.013

Table: The result of the fit where the glueball is included. $\chi^2_{\mathrm{red}} = 1.8$

Mixing

From this fit the following scalar-isoscalar mixing matrix is obtained:

$$O^T = \begin{pmatrix} 1.00 & -0.00 & 0.00 \\ 0.00 & 0.81 & -0.59 \\ 0.00 & 0.59 & 0.81 \end{pmatrix} , \tag{14}$$

which corresponds to the following admixtures of the physical states:

$$f_0(500): \quad 100\%\,\chi, \quad 0\%\,\sigma, \quad 0\%\,G\,, \tag{15}$$

$$f_0(1370): \quad 0\%\,\chi, \quad 65\%\,\sigma, \quad 35\%\,G\,, \tag{16}$$

$$f_0(1710): \quad 0\%\,\chi, \quad 35\%\,\sigma, \quad 65\%\,G\,. \tag{17}$$

Again: Pion-Nucleon scattering

- We have two couplings: tetraquark-nucleon coupling a and glueball-nucleon coupling b; $m_0 = (a\chi_0 + bG_0 + \frac{c_\mathrm{N}}{2}\sigma_0^2)$

Again: Pion-Nucleon scattering

- We have two couplings: tetraquark-nucleon coupling a and glueball-nucleon coupling b; $m_0 = (a\chi_0 + bG_0 + \frac{c_\mathrm{N}}{2}\sigma_0^2)$
- Rewrite $a = \dfrac{m_0 - bG_0 - c_\mathrm{N}/2\sigma_0^2}{\chi_0}$ and fit b

Resuts for the baryon sector

Parameter	**Value**	**Experiment**
$m_\pi\, a_0^{(+)}$	-0.0029	-0.0012 ± 0.0010
$m_\pi^3\, a_{1+}^{(+)}$	0.047	0.133 ± 0.004
$m_\pi^3\, a_{1-}^{(+)}$	-0.092	-0.056 ± 0.010
$m_\pi^3\, r_0^{(+)}$	0.012	-0.06 ± 0.02

Table: Best fit where $a = 0.802$, $b = 1.138$, $c_N = 0$ (here the glueball contribution dominates).

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- Our ambitious goal was to explain both pure meson and meson-baryon phenomenology
- We got reasonable results in the pure meson sector, a_0^0 and $\Gamma_{f_0(500)\to\pi\pi}$ agree now very well with data!
- pion-nucleon scattering parameters still not satisfying

Conclusions and Outlook

- The coupling of the scalar tetraquark to scalar quarkonia is very small, but the coupling to axial-vector mesons is very important → Why?

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    2. Extend model to three-flavor case: This enables a study of all scalar resonances up to 2 GeV
    3. Include Δ resonance, this would contribute both to isospin-even and isospin-odd scattering parameters

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Complete meson lagrangian

$$\Phi \overset{U(2)_R\times U(2)_L}{\Longrightarrow} \Phi' = U_L \Phi U_R^\dagger \,, \qquad R_\mu \overset{U(2)_R\times U(2)_L}{\Longrightarrow} R'_\mu = U_R R_\mu U_R^\dagger \tag{18}$$

$$\begin{aligned}
\mathcal{L}_{meson} = {} & \mathrm{Tr}\left[(D_\mu\Phi)^\dagger(D^\mu\Phi) - \mu^2\Phi^\dagger\Phi - \lambda_2(\Phi^\dagger\Phi)^2\right] - \lambda_1(\mathrm{Tr}[\Phi^\dagger\Phi])^2 + c(\det\Phi^\dagger + \det\Phi) \\
& + h_0\mathrm{Tr}[\Phi^\dagger + \Phi] - \frac{1}{4}\mathrm{Tr}\left[(L^{\mu\nu})^2 + (R^{\mu\nu})^2\right] + \frac{m_1^2}{2}\mathrm{Tr}\left[(L^\mu)^2 + (R^\mu)^2\right] \\
& + \frac{h_1}{2}\mathrm{Tr}[\Phi^\dagger\Phi]\mathrm{Tr}\left[(L^\mu)^2 + (R^\mu)^2\right] + h_2\mathrm{Tr}\left[\Phi^\dagger L_\mu L^\mu\Phi + \Phi R_\mu R^\mu\Phi^\dagger\right] \\
& + 2h_3\mathrm{Tr}\left[\Phi R_\mu\Phi^\dagger L^\mu\right] - i\frac{g_2}{2}\left(\mathrm{Tr}\left[L_{\mu\nu}[L^\mu, L^\nu]\right] + \mathrm{Tr}\left[R_{\mu\nu}[R^\mu, R^\nu]\right]\right) \\
& + g_3\left(\mathrm{Tr}\left[L_\mu L_\nu L^\mu L^\nu + R_\mu R_\nu R^\mu R^\nu\right]\right) + g_4\left\{\mathrm{Tr}\left[(L_\mu)^2(L_\nu)^2\right]\mathrm{Tr}\left[(R_\mu)^2(R_\nu)^2\right]\right\} \\
& + g_5\,\mathrm{Tr}\left[(R_\mu)^2\right]\mathrm{Tr}\left[(L_\nu)^2\right] + g_6\left\{\mathrm{Tr}\left[(R_\mu)^2\right]\mathrm{Tr}\left[(R_\nu)^2\right] + \mathrm{Tr}\left[(L_\mu)^2\right]\mathrm{Tr}\left[(L_\nu)^2\right]\right\} .
\end{aligned} \tag{19}$$