STRUCTURE OF THE MAJORANA'S EQUATION AND ITS PHYSICAL INTERPRETATION

O. S. Kosmachev

JOINT INSTITUTE FOR NUCLEAR RESEARCH
141980, Dubna, Moscow Region, Russia

INTRODUCTION

ON THE CHALLENGES FOR MODERN PARTICLE PHYSICS

LEPTON SECTOR IS KEY PART OF THE ELEMENTARY PARTICLE PHYSICS

SOME CONSEQUENCES OF THE STRUCTURE APPROACH

OUTLOOK AND CONCLUSION

MAIN GOAL OF THIS REPORT

is to answer on the question

WHAT IS THE REAL PHYSICAL SENSE OF MAJORANA EQUATION?
WHY DID THIS QUESTION ARISE AND WHY IS IT NECESSARY TO UNDERSTAND?

ANSWER TO THE FIRST QUESTION BECOME POSSIBLE ON THE BASIS OF THE GROUP-THEORETICAL APPROACH TO ANALYSIS OF THE LEPTON SECTOR AS A WHOLE.

THE SYSTEMATIC ANALYSIS OF CLASSIC WORKS DIRAC (1928), PAULI (1932), MAJORANA (1937) WAS IMPLEMENTED.

The result of the analysis of the Dirac equation:

Dirac's algorithm was formulated, i.e. the sequence of actions for obtain the wave equations of the leptons on the basis of strict fixed five initial assumptions.

The result of the analysis of the equation Pauli:

it is shown that this equation is one of the variants of quartet state. This means a description within the same group two pairs particle-antiparticle.

The result of the analysis of the equation of the Majorana:

it is proved after elimination of errors that this equation describes the doublet state, i.e. a pair of particle-antiparticle. Particles do not have electric charge.

SOME CONSEQUENCES OF THE STRUCTURE APPROACH

1. It is founded possibility to list all types of lepton equations in the framework of the initial assumptions.

2. Structural individuality of each of the lepton equations is established.

3. The primary structural classification of the leptons was up.

4. Simple and natural (without additional assumptions) possibility to describe states that do not exist in free states, but occur in connected states.

5. Foundations of relativistic description of unstable leptons was laid. The structure of their equations indicates internal self-consistency the whole of lepton sector keep within the limits of the proposed algorithm.

6. The existence of massive unstable neutrino is predicted.

7. The existence of "doubles"for $\tau^{\pm}$-leptons is predicted, i.e. $(\tau^{*})^{\pm}$-leptons. These four leptons form a quartet state on the basis of a single group.

In the final result, we obtained a holistic description of the leptonic sector. It contains massive and massless, charged and neutral, stable and unstable leptons.
Among them there is a doublet of stable massive particles. The group of this equation is such that all the matrices are reduced to the real form. We call the corresponding equation - the Majorana equation.
In his work (1937) Majorana made two mistakes. One of them could be corrected, and the second is unacceptably crude.

1. ETTORE MAJORANA , «Nuovo Cimento» (1937. V. 14. P.171-184).
"Poniamo $\psi = U + iV$ e consideriamo le equazioni reali (8') in quanto agiscone sulle U..."
Il vantaggio di questo procedimento rispetto all' interpretazione elementare delle equazioni di Dirac è (come vedremo meglio fra poco) che "non wi è più nessuna ragione di presumere l'esisteza di antineutroni o antineutrini.
Э.МАЙОРАНА, ("Симметричная теория электрона и позитрона") перевод с итальянского, Физика ЭЧАЯ (2003), Т.34, вып.1, стр.242-256, В частности, стр.246, 12 строка снизу:
"Положим $\psi = U + iV$ и рассмотрим действительные уравнения (8'), действующие на U,...."
Далее следует вывод, стр.247, 15 строка снизу:
"Преимуществом такого описания по сравнению с элементарной интерпретацией уравнений Дирака является, как мы увидим вскоре, то, что в нем "нет никаких оснований предполагать существование антинейтронов или антинейтрино."

2. CONDON E.U. and SHORTLEY G.H., The Theory of Atomic Spectra, London, (1935).
"The distinction between Ψ and $\overline{\Psi}$ is more fundamental than that between ordinary complex conjugates; there is no sense in which we can split Ψ into a real and an imaginary part."
Е.КОНДОН, Г.ШОРТЛИ , "Теория атомных спектров"М. 1949, ИЛ стр.20, 3 строка снизу:
"Различие между Ψ и $\overline{\Psi}$ более фундаментально, чем между обычными комплексно-сопряженными величинами; операция разбиения Ψ на вещественную и мнимую части не имеет никакого смысла."

3. DIRAC P.A.M. , The principles of Quantum Mechanics, Oxford, (1958).

"Our bra and ket vectors are complex quantities, since they can be multiplied by complex quantities and are then of the same nature as before, but they are complex quantities of special kind which cannot be split up into real and pure imaginary parts."

П.ДИРАК, "Принципы квантовой механики"ГИ ф.-м. М.: (1960),

стр.39, третья строка сверху: "Наши векторы и со-векторы являются комплексными числами, так как их можно умножать на комплексные числа, после чего их природа не меняется, однако. они являются комплексными величинами особого рода, которые не могут быть разбиты на чисто-вещественную и чисто-мнимую части."

A not excessive set of

INITIAL REQUIREMENTS ,

underlying Dirac algorithm, is following.

1. The equations must be invariant and covariant corresponding to the homogeneous Lorentz transformations taking into account all the four connected components.

2. The equations must be formulated on the basis of irreducible representations of the groups determining every lepton equation

3. Conservation of four-vector of the probability current must be fulfilled and the fourth component of the current must be positively defined.

4. The lepton spin is supposed to be equal to 1/2.

5. Every lepton equation must be reduced to the Klein-Gordon equation.

WHAT IS THE MAJORANA EQUATION?

In our notations it is $D_\gamma(I)$ with:

structural composition of the equation for a doublet of massive neutrinos

$$D_\gamma(I)\colon \boxed{\mathbf{d}_\gamma,\ \mathbf{c}_\gamma,\ \mathbf{f}_\gamma},$$

structural invariant $In[D_\gamma(I)] = 1$.

The explicit form of the operators $\mathbf{a_1, a_2, a_3}$ and $\mathbf{b_1, b_2, b_3}$
for irreducible representations allows us to find the weight numbers for these representations. In the case of the d_γ group such operators are constructed:

$$\begin{array}{ll} \mathbf{H_+ = ia_1 - a_2}, & \mathbf{F_+ = ib_1 - b_2}, \\ \mathbf{H_- = ia_1 + a_2}, & \mathbf{F_- = ib_1 + b_2}, \\ \mathbf{H_3 = ia_3}, & \mathbf{F_3 = ib_3}. \end{array}$$

Weight numbers are the eigenvalue of the operators $\mathbf{H_3 = ia_3}$ and $\mathbf{F_3 = ib_3}$.
IN THE GENERAL CASE EVERY IRREDUCIBLE REPRESENTATION OF PROPER LORENTZ GROUP IS DEFINED BY A PAIR NUMBERS $(\mathbf{l_0, l_1})$. HERE $(\mathbf{l_0})$ – IS A POSITIVE INTEGER OR HALF-INTEGER NUMBER AND $(\mathbf{l_1})$ – IS AN ARBITRARY COMPLEX NUMBER.

THE FIRST WEIGHT NUMBER OF THE $\mathbf{d}_\gamma$ GROUP IS

$$\boxed{\mathbf{l_0 = 1/2}}$$

The unified form of four connected components

We will use contracted form for notation of connected components. It looks for d_γ-group as:

$$\{b_i, b_k\} = 2\delta_{ik}, \quad (i,k = 1,2,3). \tag{1}$$

Lie algebra of d_γ-group is:

$$\begin{array}{lll}
[a_1, a_2] = 2a_3, & [a_2, a_3] = 2a_1, & [a_3, a_1] = 2a_2, \\
[b_1, b_2] = -2a_3, & [b_2, b_3] = -2a_1, & [b_3, b_1] = -2a_2, \\
[a_1, b_1] = 0, & [a_2, b_2] = 0, & [a_3, b_3] = 0, \\
[a_1, b_2] = 2b_3, & [a_1, b_3] = -2b_2, & \\
[a_2, b_3] = 2b_1, & [a_2, b_1] = -2b_3, & \\
[a_3, b_1] = 2b_2, & [a_3, b_2] = -2b_1. &
\end{array}$$

The obtained commutation relations coincide with commutation relations of the infinitesimal matrices of the proper homogeneous Lorentz group. Due to construction of commutation relation, all six operators a_1, a_2, a_3 and b_1, b_2, b_3 have a definite physical meaning.

P-conjugate representation and duality of d_γ-group

The duality means, that d_γ contains apart from $Q_2[a_1, a_2]$ one more group of the eighth order $q_2[a_1, a_2']$. Here $a_2' = a_2 \cdot c, \quad c = \sigma_x\sigma_y\sigma_z$.
Lie algebra is:

$$[a_1, a_2'] = 2a_3', \qquad [a_2', a_3'] = -2a_1, \qquad [a_3', a_1] = 2a_2', \tag{2}$$

where $a_3' \equiv a_1 a_2'$. Let us coll this group quaternion group of the second kind $q_2[a_1, a_2]$. As corollary we have another Lie algebra. We will denote it as f_γ

$$\begin{array}{lll}
[a_1, a_2'] = 2a_3', & [a_2', a_3'] = -2a_1, & [a_3', a_1] = 2a_2', \\
[b_1', b_2'] = -2a_3', & [b_2', b_3'] = 2a_1, & [b_3', b_1'] = -2a_2', \\
[a_1, b_1'] = 0, & [a_2', b_2'] = 0, & [a_3', b_3'] = 0, \\
[a_1, b_2'] = 2b_3', & [a_1, b_3'] = -2b_2', & \\
[a_2', b_3'] = -2b_1', & [a_2', b_1'] = -2b_3', & \\
[a_3', b_1'] = 2b_2', & [a_3', b_2'] = 2b_1'. &
\end{array}$$

The contracted defining relations for f_γ-group take the form

$$\begin{array}{ll}
\{b_1, b_k\}_p = 2\delta_{1k}, & (k = 1, 2, 3), \\
\{b_i, b_k\}_p = -2\delta_{ik}, & (i, k = 2, 3).
\end{array} \tag{3}$$

T-conjugate representation

The contracted defining relations for b_γ-group take the form

$$\{b'_i, b'_k\} = -2\delta_{ik}, \quad (i,k = 1,2,3). \tag{4}$$

Lie algebra of b_γ-group is:

$$\begin{array}{lll}
[a_1, a_2] = 2a_3, & [a_2, a_3] = 2a_1, & [a_3, a_1] = 2a_2, \\
[b'_1, b'_2] = 2a_3, & [b'_2, b'_3] = 2a_1, & [b'_3, b'_1] = 2a_2, \\
[a_1, b'_1] = 0, & [a_2, b'_2] = 0, & [a_3, b'_3] = 0, \\
[a_1, b'_2] = 2b'_3 & [a_1, b'_3] = -2b'_2, & \\
[a_2, b'_3] = 2b'_1, & [a_2, b'_1] = -2b'_3, & \\
[a_3, b'_1] = 2b'_2, & [a_3, b'_2] = -2b'_1, &
\end{array}$$

(PT)-conjugate representation

The contracted defining relations for c_γ-group take the form

$$\begin{array}{ll} \{b_1^*, b_k^*\}_{pt} = -2\delta_{1k}, & (k = 1,2,3), \\ \{b_i^*, b_k^*\}_{pt} = 2\delta_{ik}, & (i,k = 2,3). \end{array} \quad (5)$$

$$\begin{array}{lll} [a_1, a_2'] = 2a_3', & [a_2', a_3'] = -2a_1, & [a_3', a_1] = 2a_2', \\ [b_1^*, b_2^*] = 2a_3', & [b_2^*, b_3^*] = -2a_1, & [b_3^*, b_1^*] = 2a_2', \\ [a_1, b_1^*] = 0, & [a_2', b_2^*] = 0, & [a_3', b_3^*] = 0, \\ [a_1, b_2^*] = 2b_3^* & [a_1, b_3^*] = -2b_2^*, & \\ [a_2', b_3^*] = -2b_1^*, & [a_2', b_1^*] = -2b_3^*, & \\ [a_3', b_1^*] = 2b_2^*, & [a_3', b_2^*] = 2b_1^*. & \end{array}$$

Here (PT)=(P)(T)=(T)(P) means sequential action (P)- and (T)-conjugation.

Now we have complete system of constituents for constructing of lepton wave equations.

Structures of the stable lepton groups.

The Dirac equation — $D_\gamma(II)$: $\boxed{\mathbf{d}_\gamma,\ \mathbf{b}_\gamma,\ \mathbf{f}_\gamma,}$
structural invariant $In[D_\gamma(II)] = -1$.

The equation for a doublet of massive neutrinos — $D_\gamma(I)$: $\boxed{\mathbf{d}_\gamma,\ \mathbf{c}_\gamma,\ \mathbf{f}_\gamma}$,
structural invariant $In[D_\gamma(I)] = 1$.

The equation for a quartet of massless neutrinos —
$D_\gamma(III)$: $\boxed{\mathbf{d}_\gamma,\ \mathbf{b}_\gamma,\ \mathbf{c}_\gamma,\ \mathbf{f}_\gamma}$,
structural invariant $In[D_\gamma(III)] = 0$.

The equation for a massless T-singlet — $D_\gamma(IV)$: $\boxed{\mathbf{b}_\gamma}$,
structural invariant $In[D_\gamma(IV)] = -1$.

The equation for a massless (PT)-singlet — $D_\gamma(V)$: $\boxed{\mathbf{c}_\gamma}$,
structural invariant $In[D_\gamma(V)] = 1$.

Every group related with corresponding equation has
nonrecurrent composition.

Structures of the unstable lepton groups.

Group Δ_1 has the following defining relations:

$$\Gamma_\mu\Gamma_\nu + \Gamma_\nu\Gamma_\mu = 2\delta_{\mu\nu}, \quad (\mu, \nu = 1, 2, 3, 4, 5) \tag{6}$$

As a result we obtain the following composition:

$$\boxed{\Delta_1\{D_\gamma(II), \quad D_\gamma(III), \quad D_\gamma(IV)\}} \qquad In[\Delta_1] = -1.$$

Group Δ_3 has the following defining relations:

$$\begin{array}{ll} \Gamma_s\Gamma_t + \Gamma_t\Gamma_s = 2\delta_{st}, & (s, t = 1, 2, 3, 4), \\ \Gamma_s\Gamma_5 + \Gamma_5\Gamma_s = 0, & (s = 1, 2, 3, 4), \\ \Gamma_5^2 = -I. & \end{array}$$

It follows from here:

$$\boxed{\Delta_3\{D_\gamma(II), \quad D_\gamma(I), \quad D_\gamma(III)\}} \qquad In[\Delta_3] = 0.$$

Structures of the unstable lepton groups.

Group Δ_2 has the following defining relations:

$$\begin{array}{ll} \Gamma_s\Gamma_t + \Gamma_t\Gamma_s = 2\delta_{st}, & (s,t = 1,2,3), \\ \Gamma_s\Gamma_4 + \Gamma_4\Gamma_s = 0, & (s = 1,2,3), \\ \Gamma_4^2 = -I. & \\ \Gamma_u\Gamma_5 + \Gamma_5\Gamma_u = 0, & (u = 1,2,3,4), \\ \Gamma_5^2 = -I. & \end{array}$$

We obtain in this case:

$$\boxed{\Delta_2\{D_\gamma(I), \quad D_\gamma(III), \quad D_\gamma(V)\}} \qquad In[\Delta_2] = 1.$$

All three groups have its own structures.

We see that four conjugate components of Lorentz group allowed to describe different leptons due to complication of structural constituents.

THE LEPTON SECTOR
TAKES A SELECTED POSITION
IN THE PARTICLE PHYSICS.

ALL ELEMENTARY PARTICLES DECAY INTO

STABLE LEPTONS + PHOTONS + PROTONS.

THEREFORE, A BACKWARD PROBLEM ARISES:

IS IT POSSIBLE TO BUILD EVERYTHING ON BASIS OF STABLE LEPTONS AND PHOTONS?

EVIDENT, THIS GREAT PROBLEM IS TIGHTLY CONNECTED TO THE MATTER IN THE EXTREME CONDITIONS.

For this reason, the lepton sector should be strictly based on the fundamental principles.

TOPOLOGICAL AND STRUCTURAL APPROACHES

This aspect of RELATIVISTIC NUCLEAR PHYSICS is an antithesis and addition to the direction that has been successfully developed throughout decades under the overall leadership of the academician A.M. BALDIN.

APPENDICES

RELATIVISTIC NUCLEAR PHYSICS
CONTAINS ALL PHYSICS
KNOWN AND UNKNOWN.

THEREFORE FORWARD —
TO INEVITABLE DISCOVERIES
AND SUCCESSES!

An effective tool for analysis and constructing lepton equations was used, i.e. numerical characteristic of irreducible matrix group.

THEOREM. **If $D_\gamma = \{\gamma_1, ..., \gamma_n\}$ is an irreducible matrix group, then**

$$\mathbf{In}[D_\gamma] = \frac{1}{n}\sum_{i=1}^{n} Sp(\gamma_i^2) = \left\{ \begin{array}{c} 1, \\ -1, \\ 0. \end{array} \right. \quad (7)$$

Here n - is order of the group, $Sp(\gamma_i^2)$ - is a trace of i-matrix squared. $\mathbf{In}[D_\gamma]$ — will be called structural invariant of D_γ-group.

SOME PROPERTIES USEFUL FOR PHYSICAL APPLICATIONS

If $\mathrm{In}[D_\gamma] = 1$, then D_γ-group is equivalent to a group of real matrices.

If $\mathrm{In}[D_\gamma] = -1$, then D_γ-group is equivalent to its complex conjugate ($D_i^* = S^{-1} D_i S$), but not real matrices.

If $\mathrm{In}[D_\gamma] = 0$, then D_γ-group is not equivalent to its complex conjugate (i.e., there does not exist S such that $D_i^* = S^{-1} D_i S$).