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Hadronic contribution of light by light scattering in the energy spectrum of muonic hydrogen.

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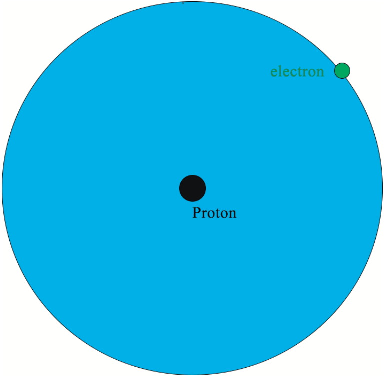
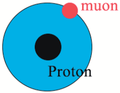
In this work we study the energy spectrum of muonic hydrogen





Regular hydrogen:
electron e^- + proton p

Muonic hydrogen:
muon μ^- + proton p
muon mass $m_\mu \approx 200 m_e$
Bohr radius $r_\mu \approx \frac{1}{200}\ r_e$

Proton radius puzzle

- Earlier measurements of the proton charge radius (spectroscopy and scattering in eH) $\rightarrow r_p = 0.8775(51)$ fm

  P.J. Mohr, D.B. Newell, B.N. Taylor, CODATA recommended values of the funda-mental physical constants: 2014, Rev. Mod. Phys. **88**, 035009 (2016).

- Lamb shift in $\mu H \rightarrow r_p = 0.84184(67)$ fm

  R. Pohl, A. Antognini, F. Nez et al., Nature **466**, 213 (2010)

- Lamb shift in $\mu H \rightarrow r_p = 0.84087(39)$ fm

  A. Antognini et al., Science **339**, 417 (2013)

- 2S-4P transition frequency measurement in $eH \rightarrow r_p = 0.8335(95)$ fm

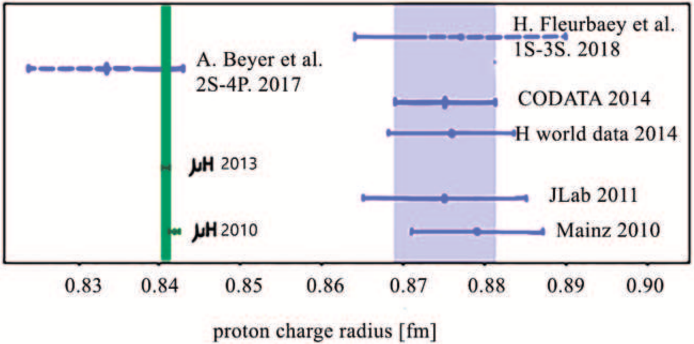
  A. Beyer, et al., Science **358**, 79–85 (2017)

- 1S-3S transition frequency measurement in $eH \rightarrow r_p = 0.877(13)$ fm

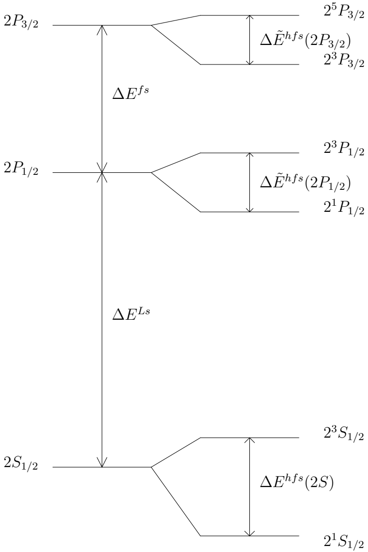
  H. Fleurbaey et al. Phys. Rev. Lett. **120**, 183001 (2018)

Proton radius puzzle

"Proton radius puzzle"is a disagreement between the value of the proton charge radius r_p obtained from experiments involving muonic hydrogen and those based on electron-proton systems.



The aim of the work



One way to overcome the crisis situation is a deeper theoretical analysis of the muonic hydrogen energy spectrum:

- The problem of a more accurate theoretical construction of the particle interaction operator
- The calculation of new corrections in the energy spectrum of muonic hydrogen.

Among the various electromagnetic interactions, the processes of two-photon meson production take a special place. First, they have been studied experimentally for quite a long time, for which a rich material has been accumulated

V. M. Budnev, I. F. Ginzburg, G. V. Meledin and V. G. Serbo, Phys. Rep. **15**, 181 (1975).

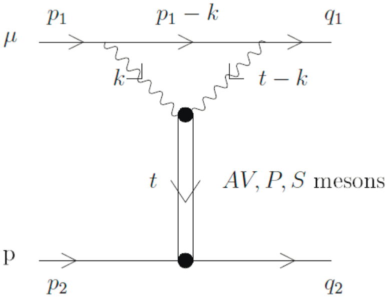
Secondly, with the development of the quark model and nonperturbative methods of quantum chromodynamics, such reactions, as well as the reverse decay processes of mesons into two photons, were constantly in the field of intensive theoretical studies.

A new round of interest in $\gamma + \gamma \to$ *meson* processes is connected with their possible role as a new source of interactions between leptons and nucleons. Since in atomic physics there are precise experiments to measure the fine and hyperfine structure of the spectrum, any new contributions to the particle interaction operator are of interest and can be studied experimentally. The first estimates of the contribution of effective meson exchanges in muonic hydrogen, which have already appeared, show that this contribution can be significant

- F. Hagelstein, V. Pascalutsa, PoS CD15 077 (2016).
- H. Q. Zhou, H. R. Pang, Phys. Rev. A **92**, 032512 (2015).
- N. T. Huong, E. Kou, B. Moussallam, Phys. Rev. D **93**, 114005 (2016).
- H.-Q. Zhou, Phys. Rev. C **95**, 025203 (2017).
- A. E. Dorokhov, N. I. Kochelev, A. P. Martynenko, F. A. Martynenko, and R. N. Faustov, Phys. Part. Nucl. Lett. 14, 857 (2017) ; arXiv:1704.07702 [hep-ph].
- A. E. Dorokhov, N. I. Kochelev, A. P. Martynenko, F. A. Martynenko, and A. E. Radzhabov, Phys. Lett. B **776**, 105 (2018);

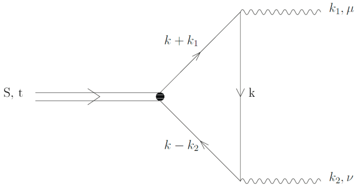
Effective one meson exchange

New direction in the study of the energy spectrum (μp) is connected with processes of two-photon interaction leading to effective one meson exchange



Two-photon exchange between proton and muon by pseudoscalar (P), axial-vector (AV) and scalar (S) meson.

Transition form factor



The general parameterization of scalar meson - two photon vertex function:

$$T_S^{\mu\nu}(t, k_1, k_2) = e^2 \Big\{ A(t^2, k_1^2, k_2^2)(g^{\mu\nu}(k_1 \cdot k_2) - k_1^\nu k_2^\mu) + \quad (1)$$

$$B(t^2, k_1^2, k_2^2)(k_2^\mu k_1^2 - k_1^\mu(k_1 \cdot k_2))(k_1^\nu k_2^2 - k_2^\nu(k_1 \cdot k_2)) \Big\}.$$

- M. K. Volkov, E. A. Kuraev, and Yu. M. Bystritskiy, Phys. Atom. Nucl. **73**, 443 (2010).
- F Giacosa, Th. Gutsche, and V. E. Lyubovitskij, Phys. Rev. D **77**, 034007 (2008).

Correction to the energy level

S-states:

$$\psi_{100}(r) = \frac{W^{3/2}}{\sqrt{\pi}} e^{-Wr}, \ \psi_{200}(r) = \frac{W^{3/2}}{2\sqrt{2\pi}} e^{-Wr/2} \left(1 - \frac{Wr}{2}\right), \ W = \mu Z \alpha, \tag{2}$$

P-states:

$$\Psi_{2P}(\mathbf{p}) = (\varepsilon n_p) R_{21}(p), \quad R_{21}(p) = \frac{128}{\sqrt{3\pi}} \frac{W^{7/2} p}{(4p^2 + W^3)^3} \tag{3}$$

where ε is the polarization vector of the muon orbital motion, $n_p = (0, \mathbf{p}/p)$.
The correction to the energy level is determined in integral form in momentum representation:

$$\Delta E = \int \Psi_{nlm}(\mathbf{q}) \frac{d\mathbf{q}}{(2\pi)^{3/2}} \int \Psi_{nlm}(\mathbf{p}) \frac{d\mathbf{p}}{(2\pi)^{3/2}} \Delta V(\mathbf{p}, \mathbf{q}). \tag{4}$$

The muon-proton interaction amplitude

The muon-proton interaction amplitude via the meson exchange is:

$$i\mathcal{M} = -\frac{\alpha^2 g_s}{\pi^2}\int \frac{d^4k}{k^4(k^2-2m_1k_0)}A(t^2,k^2,k^2)(g^{\mu\nu}(k_1\cdot k_2)-k_1^\nu k_2^\mu)\times \quad (5)$$

$$[\bar{u}(q_1)\gamma_\mu(\hat{p}_1-\hat{k}+m_1)\gamma_\nu u(p_1)][\bar{v}(p_2)v(q_2)]\frac{1}{\mathbf{t}^2+M_s^2},$$

where we set $t = p_1 - q_1 = 0$, because this momentum is small. This leads to the cancelation of the term with the function $B(t^2, k_1^2, k_2^2)$. To obtain the interaction operator we use the projection operators on the S - states with total angular momentum of the atom $F = 0, 1$ which are constructed by means of free wave functions at the rest frame:

$$\hat{\Pi}_{F=0[1]} = u(0)\bar{v}(0)|_{F=0[1]} = \frac{1}{2\sqrt{2}}(1+\gamma_0)\gamma_5[\hat{\varepsilon}] \quad (6)$$

$$\hat{\Pi}^*_{F=0[1]} = v(0)\bar{u}(0)|_{F=0[1]} = \frac{1}{2\sqrt{2}}\gamma_5[\hat{\varepsilon}^*](1+\gamma_0).$$

Projection operators

To calculate P energy levels we introduce on first step the projection operators on the state with total angular momentum of a muon $J = 1/2$:

$$\hat{\Pi}_\tau = u(0)\varepsilon^*_\tau(0)|_{J=1/2} = \frac{1}{\sqrt{3}}\gamma_5(\gamma_\tau - v_\tau)\psi(0), \tag{7}$$

$$\hat{\Pi}^*_\tau = \varepsilon_\tau(0)\bar{u}(0)|_{J=1/2} = \frac{1}{\sqrt{3}}\bar{\psi}(0)(\gamma_\tau - v_\tau)\gamma_5,$$

where $v = (1,0,0,0)$, $\psi(0)$ is the new wave function that describes muon with $J = 1/2$. On the second step we introduce the projection operators on the states with total angular momentum of the atom $F = 0, 1$

$$\hat{\Pi}_{F=0[1]} = \psi(0)\bar{v}(0)|_{F=0[1]} = \frac{1}{2\sqrt{2}}(1+\gamma_0)\gamma_5[\hat{\varepsilon}] \tag{8}$$

$$\hat{\Pi}^*_{F=0[1]} = v(0)\bar{\psi}(0)|_{F=0[1]} = \frac{1}{2\sqrt{2}}\gamma_5[\hat{\varepsilon}^*](1+\gamma_0).$$

The shift of 2S level

Introducing the projection operators we can write the numerator of the amplitude as the trace of all gamma factors. For example, in the case of S-states we have:

$$\mathcal{T}(2S_{F=0}) = Tr\Big[\frac{1+\gamma_0}{2\sqrt{2}}\gamma_5(\hat{p}_2-m_2)(\hat{q}_2-m_2)\gamma_5\frac{1+\gamma_0}{2\sqrt{2}}(\hat{q}_1+m_1)\gamma_\mu(\hat{p}_1-\hat{k}+m_1)\gamma_\nu(\hat{p}_1+m_1)\Big] \tag{9}$$

$$\mathcal{T}(2S_{F=1}) = Tr\Big[\frac{1+\gamma_0}{2\sqrt{2}}\hat{\varepsilon}(\hat{p}_2-m_2)(\hat{q}_2-m_2)\hat{\varepsilon}^*\frac{1+\gamma_0}{2\sqrt{2}}(\hat{q}_1+m_1)\gamma_\mu(\hat{p}_1-\hat{k}+m_1)\gamma_\nu(\hat{p}_1+m_1)\Big] \tag{10}$$

After trace calculation using package Form we obtain:

$$\mathcal{T}_{2S_{F=1}} = \mathcal{T}_{2S_{F=0}} = k^2(3k_0+2m_1) - 2m_1k_0^2. \tag{11}$$

We get that there is no contribution to hyperfine splitting of the S-state. At the same time there is a shift of the level 2S as whole.

Momentum integrals for 2P state

For P-states, the calculation of the trace gives:

$$\mathcal{T}_{2P} = \frac{(\mathbf{pq})}{pq}\left[-\frac{2}{3}m_1 k_0^2 + k^2 k_0 + \frac{2}{3}m_1 k^2\right] + pq\left(-\frac{5}{18}\frac{k_0^2}{m_1} + \frac{1}{4}\frac{k_0 k^2}{m_1^2} - \frac{1}{18}\frac{k^2}{m_1}\right). \quad (12)$$

To get the contribution in energy levels we need to calculate the integrals over the momentum of initial and final states p and q. After the integration in Mathematica we have:

$$\mathcal{J}_{2P}^{(1)} = \int R_{21}(q)\frac{d\mathbf{q}}{(2\pi)^{3/2}}\frac{\frac{\mathbf{pq}}{pq}}{(\mathbf{p}-\mathbf{q})^2 + M_s^2}R_{21}(p)\frac{d\mathbf{p}}{(2\pi)^{3/2}} = \frac{W^5}{4M_s^4}\frac{1}{(1+\frac{W}{M_s})^4}, \quad (13)$$

$$\mathcal{J}_{2P}^{(2)} = \int R_{21}(q)\frac{d\mathbf{q}}{(2\pi)^{3/2}}\frac{\frac{pq}{m_1^2}}{(\mathbf{p}-\mathbf{q})^2 + M_s^2}R_{21}(p)\frac{d\mathbf{p}}{(2\pi)^{3/2}} = \frac{W^5}{8M_s^2 m_1^2}\frac{3 + 4\frac{W}{M_S} + \frac{3W^2}{2M_S^2}}{(1+\frac{W}{M_s})^4}, \quad (14)$$

where we use

$$R_{21}(p) = \frac{128}{\sqrt{3\pi}}\frac{W^{7/2}p}{(4p^2+W^3)^3} \quad (15)$$

Loop momentum integrals

It is also necessary to calculate two integrals over the loop momentum k.

$$\mathcal{I}_1 = 2m_1 \int \frac{d^4(k^2+2k_0^2)}{k^2(k^4+4m_1^2k_0^2)} \frac{\Lambda^4}{(\Lambda^2+k^2)^2}, \tag{16}$$

$$\mathcal{I}_2 = \frac{m_1}{18} \int \frac{d^4k(4k_0^2-k^2)}{k^2(k^4+4m_1^2k_0^2)} \frac{\Lambda^4}{(\Lambda^2+k^2)^2}, \tag{17}$$

where we use the monopole parametrization of the function $A(0,k^2,k^2)$ for each variable. These integrals can be calculated analytically in the Euclidean space:

$$\begin{cases} k^2 \to -(k^E)^2 \\ k_0^2 \to -(k_0^E)^2 \\ k_0 \to ik_0^E \end{cases} , \quad \begin{cases} k_0^E \to kCos(\phi) \\ |\mathbf{k}^E| \to kSin(\phi) \end{cases}$$

After the integration in Wolfram Mathematica we obtain:

$$\mathcal{I}_1 = m_1\frac{\pi^2}{6}\left[-9+36\ln 2+2a_1^2(-7+12\ln 2)-12(3+2a_1^2)\ln a_1\right],$$

$$\mathcal{I}_2 = \frac{\pi^2}{108}m_1[-9+a_1^2(-5+6\ln 2)-6a_1^2\ln a_1], \tag{18}$$

The integrals are presented after an expansion over $a_1 = 2m_1/\Lambda$ up to terms of the second order.

Analytical results for shifts

Taking together intermediate relations we obtain the shift of 2S and 2P-states in the form:

$$\Delta E^{Ls}(2S) = \frac{\alpha^5\mu^3 g_s m_1 A_S}{96\pi M_s^2}\frac{(2+\frac{W^2}{M_S^2})}{(1+\frac{W}{M_S})^4}\Big[-9+36\ln 2+2a_1^2(-7+12\ln 2)-12(3+2a_1^2)\ln a_1\Big]. \tag{19}$$

$$\Delta E^{Ls}(2P) = \frac{\alpha^7\mu^5 g_s A_S}{288\pi m_1 M_s^2(1+\frac{W}{M_s})^4}\Big\{\Big[\frac{3}{4}+\frac{W}{M_S}+\frac{3}{8}\frac{W^2}{M_S^2}\Big]\big[-9+a_1^2(-5+6\ln 2)-6a_1^2\ln a_1\big]- \tag{20}$$

$$\frac{3m_1^2}{M_S^2}\big[-9+36\ln 2+2a_1^2(-7+12\ln 2)-12(3+2a_1^2)\ln a_1\big]\Big\},$$

where parameter $A_S = A(0,0,0)$. For its calculation we use the quark model. The transition form factor parametrization

$$A(0,k^2,k^2) = A(0,0,0)\frac{\Lambda^4}{(k^2+\Lambda^2)^2}$$

Transion form factor $2\gamma \to S$

One of the main quantities that determine the energy shifts is the vertex function, in which two virtual photons are transformed into a scalar meson. In local quark model it is given by the loop integral of the following form:

$$T_S^{\mu\nu} = 4\pi\alpha \int \frac{d^4k}{(2\pi)^4} Tr[\gamma^\mu \frac{(\hat{k}+m_q)}{(k^2-m_q^2)}\gamma^\nu \frac{(\hat{k}-\hat{k}_2+m_q)}{[(k-k_2)^2-m_q^2]}\frac{(\hat{k}+\hat{k}_1+m_q)}{[(k+k_1)^2-m_q^2]}] + (k_1,\mu) \leftrightarrow (k_2,\nu). \quad (21)$$

As noted above, this tensor is determined by two scalar functions $A(t^2,k_1^2,k_2^2)$ and $B(t^2,k_1^2,k_2^2)$. We are interested in the case when the kinematics is $t^2=0$, $k_1^2=k_2^2$ and only the contribution of the function $A(t^2,k_1^2,k_2^2)$ remains. In the local quark model, it has the form:

$$A(t^2,k_1^2,k_2^2) = g_{Sqq}\frac{N_c}{2\pi^2}\mathrm{Tr}_f[\tau_M QQ] I_{S\gamma\gamma}(t^2,k_1^2,k_2^2). \quad (22)$$

For the isoscalar meson (σ) the trace over flavour $\mathrm{Tr}_f[\tau_M QQ]=5/9$,
For the isovector state ($a_0(980)$) $\mathrm{Tr}_f[\tau_M QQ]=1/3$.
The coupling constant of scalar meson with the quarks is $g_{Sqq}=\frac{m_q}{f_\pi}$

Feynman parameterization

The loop momentum integral $I_{S\gamma\gamma}(t^2, k_1^2, k_2^2)$

$$I_{S\gamma\gamma}(t^2, k_1^2, k_2^2) = \int \frac{d^4k}{(2\pi)^4} Tr[\gamma^\mu(\hat{k} + m_q)\gamma^\nu(\hat{k} - \hat{k}_2 + m_q)(\hat{k} + \hat{k}_1 + m_q)] \tag{23}$$

$$\frac{1}{k^2 - m_q^2} \frac{1}{(k - k_2)^2 - m_q^2} \frac{1}{(k + k_1)^2 - m_q^2} + (k_1, \mu) \leftrightarrow (k_2, \nu),$$

can be directly calculated using the Feynman parameterization and intermediate dimensional regularization:

$$\frac{1}{a_1^{\alpha_1} \ldots a_n^{\alpha_n}} = \frac{\Gamma(\sum_1^n \alpha_i)}{\prod_{i=1}^n \Gamma(\alpha_i)} \int_0^1 dx_1 \int_0^{x_1} dx_2 \ldots \int_0^{x_{n-2}} dx_{n-1}$$

$$\frac{(1 - x_1)^{\alpha_1 - 1} \prod_{i=2}^{n-1} (x_{i-1} - x_i)^{\alpha_1 - 1}}{[a_1 + (a_2 - a_1)x_1 + \ldots + (a_n - a_{n-1})x_{n-1}]^{\sum_1^n \alpha_i}}$$

where a_i is denominators of propagators.
These calculations and integration over d^4k can be performed using a package "Feynman parameters and trace"for Wolfram Mathematica.

T. West, Comp. Phys. Comm. **77**, 286 (1993).

After integration over d^4k we obtain:

$$I_{S\gamma\gamma}(t^2,k_1^2,k_2^2)=-\frac{m_q}{(k_1\cdot k_2)}\int_0^1 d\{x_1x_2x_3\}\frac{B+(1-2x_1x_2)(k_1\cdot k_2)+k_1^2x_1^2+k_2^2x_2^2}{B+m_q^2}, \quad (24)$$

$$d\{x_1x_2x_3\}=d(x_1x_2x_3)\delta\left[1-(x_1+x_2+x_3)\right],$$

$$B=-\left(t^2x_1x_2+k_1^2x_1x_3+k_2^2x_2x_3\right), \quad 2(k_1\cdot k_2)=t^2-k_1^2-k_2^2.$$

Setting further our kinematics $t^2=0$, $k_1^2=k_2^2=-k^2$ and calculating integrals over $d\{x_1x_2x_3\}$ we obtain:

$$I_{S\gamma\gamma}(0,k^2,k^2)=\frac{m_q}{k^2}\left(-2+\frac{4m_q^2\ln\left(\frac{\sqrt{k^2}\sqrt{4m_q^2+k^2}+2m_q^2+k^2}{2m_q^2}\right)}{\sqrt{k^2\left(4m_q^2+k^2\right)}}\right) \quad (25)$$

Analytical result for A_S

To get the value of $A_S = A(0,0,0)$ we use the expansion of the integral $I_{S\gamma\gamma}(0,k^2,k^2)$ at small momenta:

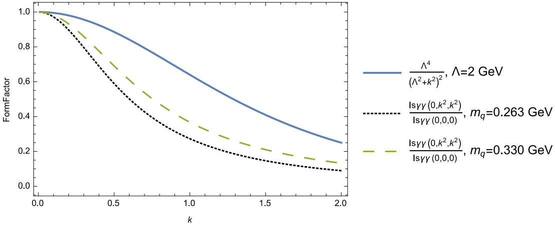
$$I_{S\gamma\gamma}(0,k^2,k^2) \approx -\frac{1}{3m_q} + \frac{k^2}{15m_q^3}. \tag{26}$$

So, for the isoscalar and isovector cases we obtain:

$$A_S^{I=0} = A(0,0,0) = -\frac{1}{2\pi^2 f_\pi}\frac{5}{9}, \quad A_S^{I=1} = A(0,0,0) = -\frac{1}{2\pi^2 f_\pi}\frac{1}{3}. \tag{27}$$

Comparison of form factors

Comparison of the phenomenological transition form factor $\Lambda^4/(\Lambda^2+k^2)^2$ of two virtual photons to scalar meson with the form factor calculated in the local quark model $I_{S\gamma\gamma}(0,k^2,k^2)/I(0,0,0)$.



The form factor $\Lambda^4/(\Lambda^2+k^2)^2$ is usually used for experimental data description.

For a check of value $A(0,0,0)$ we use the results from the papers

M. K. Volkov, E. A. Kuraev, and Yu. M. Bystritskiy, Phys. Atom. Nucl. **73**, 443 (2010).

F Giacosa, Th. Gutsche, and V. E. Lyubovitskij, Phys. Rev. D **77**, 034007 (2008).

in which a calculation of $A_S = A(t^2 = M_s^2, 0, 0)$ was carried out on the basis of quark model. Using the quark-loop amplitude contributing to the decay $S \to \gamma + \gamma$ they presented the decay amplitude:

$$T^{\mu\nu}_{S\gamma\gamma} = -\frac{\alpha g_{\sigma_u}}{\pi m_u}(g^{\mu\nu}(k_1 k_2) - k_1^\nu k_2^\mu) a_{S\gamma\gamma}. \tag{28}$$

The expression for the decay width which is measured in experiment, has the form:

$$\Gamma_{S\gamma\gamma} = \frac{M_s^3 \alpha^2 g^2_{\sigma_u}}{64\pi^3 m_u^2} |a_{S\gamma\gamma}|^2 \tag{29}$$

Taking the experimental value of $\Gamma_{S_{\gamma\gamma}}$ or its theoretical estimate we can find the value of phenomenological constant $a_{S_{\gamma\gamma}}$ and relate it to our parameter A_S. Corresponding numerical values

$$|A_S| = \frac{g_{S_u} a_{S_{\gamma\gamma}}}{4\pi^2 m_u e} \tag{30}$$

for scalar mesons $\sigma(450)$, $\sigma(550)$, $\sigma(600)$, are the following:

$$|A_S(\sigma(0.450))| = 0.28\ GeV^{-1}, \quad |A_S(\sigma(0.550))| = 0.26\ GeV^{-1}, \tag{31}$$

$$|A_S(\sigma(0.600))| = 0.25\ GeV^{-1},$$

where we introduced an additional factor outside the mass shell, based on the assumption

$$A_S(t,0,0) = A_S(t = M_S^2, 0, 0) e^{\frac{t-M_S^2}{M_S^2}}. \tag{32}$$

Numerical results

The experimental value of the mass and the width of the σ meson is not well established. So we make estimations for the different masses of σ meson.

Scalar mesons contribution to the energy spectrum of muonic hydrogen.

S meson	$\Gamma_{\sigma\to\gamma\gamma}$ in keV	A_S, from $\Gamma_{\sigma\to\gamma\gamma}$ in GeV	A_S from quark model in GeV^{-1}	$\Delta E^{ls}(2S)$ in μeV	$\Delta E^{ls}(2P)$ in μeV
$\sigma(450)$	2.18	-0.28	-0.30	-13.7538	0.000023
$\sigma(550)$	3.53	-0.26	-0.30	-11.2657	0.000014
$\sigma(600)$	4.3	-0.25	-0.30	-10.1182	0.000011

Our results are in agreement with the estimate made in

H.-Q. Zhou, arXiv:1608.06460.

The obtained contribution to the Lamb shift is significant and should be used for comparison with experimental data.

Thank you for attention!