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Poincare′-invariant quark model of light mesons

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INTRODUCTION

Electroweak and semileptonic decays of pseudoscalar and vector mesons have always been a convenient tool for approbation of various theoretical models and approaches for studying the structure of hadrons. In this work we present scheme of obtaining parameters of the model, based on point form of Poincare$'$-invariant quantum mechanics (further PiQM): authors using the integral representation of pseudoscalar and vector meson decay constants define the basic parameters for u, d and s - quark sector.

The authors note that the obtained values of the parameters of the model based on the PiQM are used for estimation the observed of $V(P) \to P(V)\gamma$ process with following evaluation the quarks magnetic moments.

OUTLINE

Our aim:

Without using the explicit form of quark-antiquark potential obtain the model parameters for pseudoscalar and vector mesons on the basis of modern experimental data.

Stages of solution:

1. Develop a technique for calculating the matrix elements of electroweak transitions for obtaining the integral representations of the pseudoscalar and vector mesons decay constants within the framework of a point form of PiQM.
2. Using this technique to obtain integral representations of the constant of the pseudoscalar density.
3. Carry out the procedure of fixing the parameters of the model: constituent quark ($u-$,$d-$ and $s-$ quark) masses, parameters of wave functions $\beta^{P,V}_{q\bar{Q}}$.
4. Using these parameters calculate the observed of $V(P) \to P(V)\gamma$ process with following evaluation of the magnetic moments μ_q of the quarks in the framework of PiQM.

Basic features of the model, based on PiQM

In the case of a system of two particles with the masses m_q and $m_{\bar{Q}}$ and, respectively, with 4-momentums $p_1 = (\omega_{m_q}(\mathrm{p}_1), \mathbf{p}_1)$ $p_2 = \left(\omega_{m_{\bar{Q}}}(\mathrm{p}_2), \mathbf{p}_2\right)$ basis in point form of PiQM given by

$$|\mathbf{p_1}, \lambda_1\rangle \otimes |\mathbf{p_2}, \lambda_2\rangle = |\mathbf{p_1}, \lambda_1, \mathbf{p_2}, \lambda_2\rangle \tag{1}$$

and defines a reducible representation of the Poincare group. Using the Clebsch-Gordan decomposition for the Poincare group let's construct irreducible representation that characterizes the entire system: we introduce a full momentum

$$\mathbf{P} = \mathbf{p_1} + \mathbf{p_2} \tag{2}$$

and the relative momentum $\mathbf{k}$ of two particles [1].

[1] Keister, B. D. Relativistic Hamiltonian dynamics in nuclear and particle physics/ B. D. Keister, W. N. Polyzou // Adv.Nucl.Phys.-1991.-Vol. 20.-P. 225-479.

Basic features of the model, based on PiQM

The basis of the two-particle irreducible representation is defined by the quantum numbers of the total momentum of the total angular momentum J with a projection μ, effective mass of noninteracting particles

$$M_0 = M(q\bar{Q}) = \omega_{m_q}(\mathrm{k}) + \omega_{m_{\bar{Q}}}(\mathrm{k}) \ , \ \ \omega_m(\mathrm{k}) = \sqrt{\mathrm{k}^2 + m^2}, \quad (3)$$

where $\mathrm{k} = |\mathbf{k}|$, and two additional numbers that remove the degeneracy of this basis. As a result, the vector of a meson with momentum $\mathbf{Q}$, mass M is given by

$$\left|\mathbf{Q}, J\mu, M(q\bar{Q})\right\rangle = \int \mathrm{d}\mathbf{k} \sqrt{\frac{\omega_{m_q}(\mathrm{p}_1)\,\omega_{m_{\bar{Q}}}(\mathrm{p}_2)}{\omega_{m_q}(\mathrm{k})\,\omega_{m_{\bar{Q}}}(\mathrm{k})\, V_0}} \times$$
$$\times \sum_{\lambda_1,\lambda_2} \sum_{\nu_1,\nu_2} \Omega \left\{ {\ell \atop \nu_1,} {s \atop \nu_2,} {J \atop \mu} \right\} (\theta_k, \phi_k)\, \Phi^J_{\ell\, s}(\mathrm{k}) \times \quad (4)$$
$$\times D^{1/2}_{\lambda_1,\nu_1}(\mathbf{n}_{W_1})\, D^{1/2}_{\lambda_2,\nu_2}(\mathbf{n}_{W_2})\, |\mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2\rangle \, .$$

In (4)

$$\Omega \left\{ {\ell \ \ s \ \ J \atop \nu_1, \nu_2, \mu} \right\} (\theta_k, \phi_k) = \mathbf{C} \left\{ {s_1 \ \ s_2 \ \ s \atop \nu_1, \nu_2, \lambda} \right\} \mathbf{C} \left\{ {\ell \ \ s \ \ J \atop m, \lambda, \mu} \right\} Y_{\ell\, m} (\theta_k, \phi_k) =$$

$$= \mathbf{C} \left\{ {s_1 \ \ s_2 \ \ \ \ s \atop \nu_1, \nu_2, \nu_1 + \nu_2} \right\} \mathbf{C} \left\{ {\ell \qquad\qquad s \qquad J \atop \mu - (\nu_1 + \nu_2), \nu_1 + \nu_2, \mu} \right\} Y_{\ell\, \mu - (\nu_1 + \nu_2)} (\theta_k, \phi_k) , \quad (5)$$

where $\mathbf{C} \left\{ {s_1 \ \ s_2 \ \ s \atop \nu_1, \nu_2, \lambda} \right\}$, $\mathbf{C} \left\{ {\ell \ \ s \ \ j \atop m, \lambda, \mu} \right\}$ – Clebsch-Gordan coefficients of $SU(2)$ group, $Y_{lm}(\theta_k, \phi_k)$ – the spherical functions and $D(\mathbf{n}_W)$ – Wigner rotation function.

Wave function $\Phi^J_{\ell s}(\mathrm{k})$ taking into account the number of colors of quarks N_c is normalized by the condition[2]:

$$\sum_{\ell, s} \int\limits_0^\infty \mathrm{d}\mathrm{k}\, \mathrm{k}^2 \left| \Phi^J_{\ell s}(\mathrm{k}) \right|^2 = N_c . \quad (6)$$

[2] Keister, B. D. Relativistic Hamiltonian dynamics in nuclear and particle physics/ B. D. Keister, W. N. Polyzou // Adv.Nucl.Phys.-1991.-Vol. 20.-P. 225-479.

Leptonic decay in PiQM

The decay constants $P(q\bar{Q}) \to \ell + \nu_\ell$, $\ell \to V(q\bar{Q}) + \bar{\nu}_\ell$ and $V(q\bar{q}) \to \ell + \overline{\ell}$ pseudoscalar and vector mesons with masses M_P and M_V, after removing the element of the Kobayashi-Maskawa matrices, are determined by the expressions[3]:

$$\left\langle 0 \left| \hat{J}^{\mu}_{P}(0) \right| \mathbf{Q}, M_P \right\rangle = \frac{\mathrm{i}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{M_P}(\mathrm{P})}} P^{\mu} f_P, \qquad (7)$$

$$\left\langle 0 \left| \hat{J}^{\mu}_{V}(0) \right| \mathbf{Q}, 1\lambda_V, M_V \right\rangle = \frac{\mathrm{i}}{(2\pi)^{3/2}} \frac{\varepsilon^{\mu}(\lambda_V)}{\sqrt{2\omega_{M_V}(\mathrm{P})}} M_V f_V \qquad (8)$$

where $\varepsilon^{\mu}(\lambda_V)$ - polarization vector of vector meson and f_P, f_V - decay constants of mesons, respectively.

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[3] Andreev, V. V. QCD coupling constant below 1 GeV in the poincare-covariant model / V. V. Andreev // Physics of Particles and Nuclei Letters-2011.-Vol.4 – 347-355 P.

Leptonic decay in PiQM

The matrix element of the current in the quark basis in the case of leptonic decays of pseudoscalar and vector mesons are determined by:

$$\left\langle 0 \left| \hat{J}^{\mu}_{P}(0) \right| \mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2 \right\rangle = \frac{1}{(2\pi)^3} \frac{\bar{v}_{\lambda_2}(\mathbf{p}_2, m_{\bar{Q}})\gamma^{\mu}\gamma^{5} u_{\lambda_1}(\mathbf{p}_1, m_q)}{\sqrt{2\omega_{m_{\bar{Q}}}(\mathrm{p}_2)}\sqrt{2\omega_{m_q}(\mathrm{p}_1)}}, \tag{9}$$

$$\left\langle 0 \left| \hat{J}^{\mu}_{V}(0) \right| \mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2 \right\rangle = \frac{1}{(2\pi)^3} \frac{\bar{v}_{\lambda_2}(\mathbf{p}_2, m_{\bar{Q}})\gamma^{\mu} u_{\lambda_1}(\mathbf{p}_1, m_q)}{\sqrt{2\omega_{m_{\bar{Q}}}(\mathrm{p}_2)}\sqrt{2\omega_{m_q}(\mathrm{p}_1)}}. \tag{10}$$

Substituting the meson state vectors (4) in relations (7) and (8) and using relation (5) for $N_c = 3$, one can obtain integral representations of leptonic decays constants of pseudoscalar and vector mesons :

Leptonic decay in PiQM

$$f_l(m_q, m_{\bar{Q}}, \beta^l_{q\bar{Q}}) = \sqrt{\frac{3}{2}}\frac{1}{\pi}\int\limits_0^\infty dk\, k^2 \Phi(k, \beta^l_{q\bar{Q}}) \sqrt{\frac{W^+_{m_q}(k)\;\; W^+_{m_{\bar{Q}}}(k)}{M_0\;\omega_{m_q}(k)\,\omega_{m_{\bar{Q}}}(k)}} \times$$

$$\times \left(1 + a_l \frac{k^2}{W^+_{m_q}(k)\, W^+_{m_{\bar{Q}}}(k)}\right)\;,\quad l = P, V;\quad a_P = -1, a_V = 1/3\,, \tag{11}$$

where

$$W^{\pm}_m(k) = \omega_m(k) \pm m\,. \tag{12}$$

Note,that the integral representation (11) match with the expressions for the decay constants of pseudoscalar and vector mesons in the instant and front-forms of PiQM [4].

[4] Krutov, A. The radius of the ρ–meson determined from its decay constant/ A. Krutov, A. Polezhaev // Phys. Rev.–2016. – Vol. D93. – P. 036007.

Leptonic decay in PiQM

For fixing the parameters we using the constant of pseudoscalar density, which determined by relation [5]

$$\left\langle 0 \,\middle|\, \bar{Q}\gamma_5 q \,\middle|\, \mathbf{Q}, M_P \right\rangle = -\frac{\mathrm{i}}{(2\pi)^{3/2}} \frac{g_P}{\sqrt{2\,\omega_{M_P}(\mathrm{P})}}. \tag{13}$$

From the QCD relations known, that axial current $\hat{J}^{\alpha}_{P}(x) = \bar{Q}(x)\gamma^{\alpha}\gamma_5 q(x)$ and pseudoscalar density $j^5(x) = \mathrm{i}\ \bar{Q}(x)\gamma_5 q(x)$ are related by

$$\partial_\alpha \hat{J}^{\alpha}_{P}(x) = (\hat{m}_q + \hat{m}_{\bar{Q}}) j^5(x), \tag{14}$$

where $\hat{m}_{q,\bar{Q}}$ – current masses of quarks. Therefore relation (14) is used, as usual, for u and d-quarks (sometimes and for s-quark).

[5] Jaus, W. Consistent treatment of spin 1 mesons in the light-front quark model/ W. Jaus// Phys.Rev. – 2003. – Vol. D67. – P. 094010.

Leptonic decay in PiQM

The equation (14) leads to the fact, that the constants f_P and g_P are related by

$$\left(\hat{m}_q + \hat{m}_{\bar{Q}}\right) g_P = M_P^2 f_P. \tag{15}$$

Using the matrix element of the current in the quark basis (13)

$$\left\langle 0 \left| \hat{j}^5(0) \right| \mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2 \right\rangle = \frac{\mathrm{i}}{(2\pi)^3} \frac{\bar{v}_{\lambda_2}(\mathbf{p}_2, m_{\bar{Q}})\gamma^5 u_{\lambda_1}(\mathbf{p}_1, m_q)}{\sqrt{2\omega_{m_{\bar{Q}}}(\mathrm{p}_2)}\sqrt{2\omega_{m_q}(\mathrm{p}_1)}}, \tag{16}$$

after some calculation we obtain the integral representation decay constants g_P of pseudoscalar meson:

$$g_P\left(m_q, m_{\bar{Q}}, \beta^P_{q\bar{Q}}\right) = \sqrt{\frac{3}{2}\frac{1}{\pi}} \int\limits_0^\infty \mathrm{d}k\, k^2 \Phi\left(k, \beta^P_{q\bar{Q}}\right) \sqrt{\frac{M_0}{\omega_{m_q}(k)\, \omega_{m_{\bar{Q}}}(k)}}$$
$$\times \left(\sqrt{W^+_{m_q}(k)\, W^+_{m_{\bar{Q}}}(k)} + \sqrt{W^-_{m_q}(k)\, W^-_{m_{\bar{Q}}}(k)}\right). \tag{17}$$

Numerical results for input parameters

The values of the constituent masses of light quarks (u, d and s) and the parameters of the wave function β^I_{qQ} can be fixed by means of the experimental values of the decay constants

$$f_{\pi^+}^{(\exp)} = (131.61 \pm 0.17)\ \text{MeV}, \quad f_{K^+}^{(\exp)} = (156.87 \pm 0.78)\ \text{MeV} \tag{18}$$

and the values of the current masses of quarks, obtained with the use of lattice calculations [6]:

$$\hat{m}_u = \left(2.2^{+0.5}_{-0.4}\right)\ \text{MeV}, \quad \hat{m}_d = \left(4.7^{+0.5}_{-0.3}\right)\ \text{MeV}, \quad \frac{(\hat{m}_u + \hat{m}_d)}{2} = \left(3.5^{+0.5}_{-0.2}\right)\ \text{MeV}, \quad \hat{m}_s = \left(95^{+9}_{-3}\right)\ \text{MeV}. \tag{19}$$

The masses of current quarks in $\overline{\text{MS}}$-scheme were calculated on a scale $\mu = 2$ GeV.

[6] Tanabashi, M. The review of particle physics / M.Tanabashi (and others) //Phys. Rev.– 2018.–Vol.D 98.–1898 P.

Numerical results for input parameters

Assuming that the scale of the violation of isotopic symmetry for the constituent u and d-quarks is the same as for current, we get the system of equation

$$\begin{cases} m_d - m_u = \hat{m}_d - \hat{m}_u = (2.5 \pm 0.4)\ \text{MeV} \\ f_P(m_q, m_{\bar{Q}}, \beta^P_{q\bar{Q}}) = f_P^{(\exp)}, \\ \left(\hat{m}_q + \hat{m}_{\bar{Q}}\right) g_P(m_q, m_{\bar{Q}}, \beta^P_{q\bar{Q}}) = f_P^{(\exp)} M_P^2, \end{cases} \tag{20}$$

whose solution, depending on the choice of the trial wave function

$$\Phi^{\text{os}}(\mathrm{k}, \beta) = N_{os} \exp\left[-\frac{\mathrm{k}^2}{2\beta^2}\right], N_{os} = \frac{2}{\pi^{1/4}\,\beta^{3/2}}\,, \tag{21}$$

$$\Phi^{\text{coul}}(\mathrm{k}, \beta) = \frac{N_{coul}}{(1 + \mathrm{k}^2/\beta^2)^2}\,, \quad N_{coul} = 4\sqrt{\frac{2}{\pi\,\beta^3}}\,, \tag{22}$$

$$\Phi^{\text{pl}}(\mathrm{k}, \beta) = \frac{N_{pl}}{(1 + \mathrm{k}^2/\beta^2)^3}\,, \quad N_{pl} = 16\sqrt{\frac{2}{7\pi\,\beta^3}}\,, \tag{23}$$

Numerical results for input parameters

is presented in the following table:

Table: The values of the constituent masses of u,d and s-quarks and parameters of the trial WF $\beta^{P}_{q\bar{Q}}$

WF	m_u, MeV	$\bar{m}_{ud}$, MeV	m_s, MeV	β^P_{ud}, MeV	β^P_{us}, MeV
Φ^{os} – (21)	219.5 ± 9.6	222.0 ± 9.6	417.0 ± 61.2	367.9 ± 25.1	375.5 ± 19.7
Φ^{coul} – (22)	—	—	—	—	—
Φ^{pl} – (23)	234.6 ± 25.7	237.1 ± 25.7	—	557.7 ± 38.2	—

Values for the test oscillator (Gaussian) WF correlate with the different models and approaches [7, 8]
(mark "–" means no solution to the system (20) within the error).

[7] Choi, H.-M. Kaon electroweak form-factors in the light– front quark model/ H.-M. Choi, C.-R. Ji// Phys. Rev. – 1999. – Vol. D59. – P. 034001.

[8] Krutov, A. F. The K-meson form factor and charge radius: linking low-energy data to future Jefferson Laboratory measurements/ A. F. Krutov, S. V. Troitsky, V. E. Troitsky// The European Physical Journal C. – July 2017. – Vol. 77,№ 7. – P. 464

Numerical results for input parameters

The experimental values $f_V^{(\mathrm{exp})}$ from decays $\tau \to V(q\overline{Q}) + \nu_\tau$

$$f_{\rho^+}^{(\mathrm{exp})} = (209.32 \pm 1.51)\ \mathrm{MeV}, \quad f_{K^{*+}}^{(\mathrm{exp})} = (205.33 \pm 6.23)\ \mathrm{MeV}\ . \tag{24}$$

After solving the equation $f_V(m_q, m_{\bar{Q}}, \beta_{q\bar{Q}}^V) = f_V^{(\mathrm{exp})}$ we find the values of the parameters β_{ud}^V and β_{us}^V for oscillator WF (21):

$$\beta_{ud}^V = (311.0 \pm 2.1)\ \mathrm{MeV}, \quad \beta_{us}^V = (313.6 \pm 24.2)\ \mathrm{MeV};. \tag{25}$$

Numerical results for input parameters

Since in our model isotopic symmetry between $u-$ and $d-$ quarks weekly broken, we assume, that

$$\begin{aligned}\beta^V_{uu} &= \beta^V_{ud} - \triangle\beta_{ud},\\ \beta^V_{dd} &= \beta^V_{ud} + \triangle\beta_{ud},\\ \beta^V_{ds} &= \beta^V_{us} + \triangle\beta_{ud},\end{aligned} \tag{26}$$

where $\triangle\beta_{ud} \simeq m_d - m_u = (2.5 \pm 0.2)$ MeV. Using experimental data [9] on ω–meson decays and experimental value for $\theta_V = (31.92 \pm 0.2)^o$–angle into $\ell\bar{\ell}$–pair it's become possible to obtain following value:

$$\beta^V_{ss} = (336.56 \pm 1.38) \ \ \text{MeV}. \tag{27}$$

[9] Amelino-Camelia,G. Physics with the KLOE-2 experiment at the upgraded DAϕNE/ G. Amelino-Camelia, F. Archilli, D. Babusci // Eur. Phys. J.–Vol. C 68–2010.–P.619-681

Numerical results

Since we fixed basic parameters of the model, we can estimate following observed:

Table: Comparing experimental and theoretical values

Decay channel	Exp.value of decay constant, MeV	Theoretical value, MeV
$^{(\dagger)}\pi^+ \to \ell\tilde{\nu}_\ell$	$f_{\pi^+}^{(exp.)}$=131.61±0.17	f_{π^+}=131.61±0.11
$^{(\dagger)}K^+ \to \ell\tilde{\nu}_\ell$	$f_{K^+}^{(exp.)}$=156.87±0.78	f_{K^+}=156.87±0.43
$^{(\dagger)}\tau \to \rho^+\tilde{\nu}_\tau$	$f_{\rho^+}^{(exp.)}$=209.32±1.51	f_{ρ^+}=209.34±0.55
$^{(\dagger)}\tau \to K^{*+}\tilde{\nu}_\tau$	$f_{K^{*+}}^{(exp.)}$=205.33±6.23	$f_{K^{*+}}$=205.33±4.32
$^{(\dagger)}\omega \to \ell\bar{\ell}$	$f_{\omega}^{(exp.)}$=46.82±8.12	f_{ω}=46.82±2.74
$\rho^0 \to \ell\bar{\ell}$	$f_{\rho^0}^{(exp.)}$=156.42 ±14.44	f_{ρ^0}=148.36±3.12
$\phi \to \ell\bar{\ell}$	$f_{\phi}^{(exp.)}$=76.21 ±1.24	f_{ϕ}=76.24±3.72

Note, that theoretical value for f_ρ in a good agreement with [9,10]

$$f_{\rho^0} = \frac{f_{\rho^+}}{\sqrt{2}} = (148.39 \pm 0.04) \quad \text{MeV}. \tag{28}$$

[9] Kokkedee, J. The quark model/ J. Kokkedee. – New York- Amsterdam: W. A. Benjamin Inc., 1969. 239 P.

[10] Okun, L.B. Leptons and quarks / L.B. Okun – Moscow, Science .– FML., 1990. – 346P .

Radiative decays of mesons in PiQM

Matrix element of vector (pseudoscalar) meson decay $V(P) \to P(V)\gamma$ in point form of PiQM is parameterized by relation

$$\langle \mathbf{Q}^{'}, M_P | \hat{J}^{\alpha}(0) | \mathbf{Q}, 1\lambda_V, M_V \rangle = \frac{\mathrm{i}}{(2\pi)^3} g_{VP\gamma} \frac{\epsilon^{\alpha\nu\rho\sigma} \varepsilon_{\nu}(\lambda_V) V_{\rho} V_{\sigma}^{'}}{\sqrt{4 V_0 V_0^{'}}} \sqrt{M_V M_P}, \tag{29}$$

where V and $V^{'}$ – 4–violicities of mesons. Using definition $K^{*}(\lambda_V) = -\mathrm{i}\epsilon^{\alpha\nu\rho\sigma} \varepsilon_{\nu}(\lambda_V) V_{\rho} V_{\sigma}^{'}$ for decay constant we get

$$g_{VP\gamma} = (2\pi)^3 \sqrt{4 V_0 V_0^{'}} \langle \mathbf{Q}^{'}, M_P | \frac{(K^{*}(\lambda_V) \cdot J(0))}{\sqrt{M_V M_P}\,(K(\lambda_V) \cdot K^{*}(\lambda_V))} | \mathbf{Q}, 1\lambda_V, M_V \rangle . \tag{30}$$

Radiative decays of mesons in PiQM

The matrix element of the current in the quark basis for these types of decay is given by

$$\left\langle \mathbf{p}_1^{'}, \lambda_1^{'}, \mathbf{p}_2^{'}, \lambda_2^{'} \left| \hat{J}^{\mu}(0) \right| \mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2 \right\rangle = \frac{1}{(2\pi)^3} \frac{e_q \bar{u}_{\lambda_1^{'}}(\mathbf{p}_1^{'}, m_q^{'}) \Gamma^{\mu} u_{\lambda_1}(\mathbf{p}_1, m_q)}{\sqrt{2\omega_{m_q^{'}}(\mathrm{p}_1^{'})} \sqrt{2\omega_{m_q}(\mathrm{p}_1)}} + \frac{1}{(2\pi)^3} \frac{e_{\bar{Q}} \bar{\upsilon}_{\lambda_2^{'}}(\mathbf{p}_2^{'}, m_{\bar{Q}}^{'}) \Gamma^{\mu} \upsilon_{\lambda_2}(\mathbf{p}_2, m_{\bar{Q}})}{\sqrt{2\omega_{m_{\bar{Q}}^{'}}(\mathrm{p}_2^{'})} \sqrt{2\omega_{m_{\bar{Q}}}(\mathrm{p}_2)}}, \tag{31}$$

where we assuming, that $V(P) \to P(V)\gamma$ decay caused by electromagnetic interaction of quark and photon, so

$$\begin{gathered} m_q = m_q^{'}, \;\; m_Q = m_Q^{'}, \\ \Gamma^{\mu} = F_1(q^2)\gamma^{\mu} + F_2(q^2)\frac{\sigma^{\mu\nu}}{2m_{q,Q}} q_{\nu}. \end{gathered} \tag{32}$$

Radiative decays of mesons in PiQM

In (29) form–factors of quarks $F_1(q^2)$ and $F_2(q^2)$ at

$$q^2 = 0, \quad q = Q - Q'. \tag{33}$$

normalized by

$$F_1(0) + F_2(0) = \mu_{q,\bar{Q}}, \ \mu = \frac{e}{2m}(1+\kappa). \tag{34}$$

After some calculation in the Breit system, where

$$K(\lambda_V) = \frac{\sqrt{\varpi^2 - 1}}{\sqrt{2}}\{0, -\mathrm{i}\lambda_V, 1, 0\}, \ \varpi = \left(V_Q \cdot V_{Q'}\right), \tag{35}$$

one can obtain the integral representation of decay constant:

Radiative decays of mesons in PiQM

$$g_{VP\gamma} = \int \mathrm{dk}\ \mathrm{k}^2\ \Phi(\mathrm{k}, \beta^V_{q\bar{Q}})\Phi(\mathrm{k}, \beta^P_{q\bar{Q}}) \left(e_q f_1(\mathrm{k}, m_q, m_{\bar{Q}}) + \right. \quad (36)$$

$$\left. + \frac{e_q \kappa_q}{2m_q} f_2(\mathrm{k}, m_q, m_{\bar{Q}}) - e_{\bar{Q}} f_1(\mathrm{k}, m_{\bar{Q}}, m_q) - \frac{e_{\bar{Q}}\kappa_{\bar{Q}}}{2m_{\bar{Q}}} f_2(k, m_{\bar{Q}}, m_q) \right),$$

where

$$f_1(\mathrm{k}, m_q, m_{\bar{Q}}) = \frac{1}{3\ \omega_{m_q}(\mathrm{k})} \left(\frac{m_q + m_{\bar{Q}}}{\omega_{m_q}(\mathrm{k}) + \omega_{m_{\bar{Q}}}(\mathrm{k})} + \frac{m_q}{\omega_{m_q}(\mathrm{k})} + 1 \right),$$

$$f_2(\mathrm{k}, m_q, m_{\bar{Q}}) = -\frac{2}{3} \frac{m_q^2 + \omega_{m_q}(\mathrm{k})(m_q + \omega_{m_q}(\mathrm{k}))}{\omega^2_{m_q}(\mathrm{k})}. \quad (37)$$

Numerical stadies of radiative decays of mesons in PiQM

For fixing values of the $u,\ d, and\ s$–quark magnetic moments we use ρ^+, K^{*+} and K^{*0} decay constant values (this choice is due to the fact that the quark structure of these mesons doesn't depend on the mixing angles $\theta_{V,P}$).

Using relations (37) leads to following values of the quarks anomalous magnetic moments [11]:

Table: Quark magnetic moments from $V \to P\gamma$ decays

Quark magnetic moment	PiQM
κ_u	-0.123±0.08
κ_d	-0.088±0.015
κ_s	-0.198 ±0.011

[11] Andreev, V.V., Haurysh V.Yu. Radiative decays of light vector mesons in Poincare-invariant quantum mechanics / V.V. Andreev, V.Yu. Haurysh // Journal of Physics: Conference Series – V.678 –2016.

Mixing scheme of mesons in PiQM

For further calculations let's define the mixing scheme for pseudoscalar and vector mesons by

$$\begin{pmatrix} \phi \\ \omega \end{pmatrix} = U(\phi_V) \begin{pmatrix} \psi_q \\ \psi_s \end{pmatrix}, \quad \begin{pmatrix} \eta \\ \eta\prime \\ G \end{pmatrix} = U(\phi_P, \alpha_G, \phi_G) \begin{pmatrix} \psi_q \\ \psi_s \\ \psi_G \end{pmatrix}, \tag{38}$$

where $\phi_V, \phi_P, \alpha_G, \phi_G-$mixing angles [12,13],

$$\psi_q = \frac{1}{\sqrt{2}} \left(|u\bar{u}\rangle + |d\bar{d}\rangle \right), \ \psi_s = |s\bar{s}\rangle, \tag{39}$$

and ψ_G- glueball component.

We note that similar mixing scheme was used to calculate the lepton constants in (27,28).

[12] Osipov, A.A. The $\pi^0 - \eta - \eta'$ mixing in a generalized multiquark interaction scheme / A.A. Osipov, B. Hiller // Phys.Rev.D–1999.–Vol.93,n.11–P.116005

[13] Thomas, C. E. Composition of the Pseudoscalar η and $\eta\prime$ Mesons/ C. E. Thomas// JHEP. – 2007. – Vol. 10. –P. 026.

Numerical stadies of radiative decays of mesons in PiQM

In (38) matrix $U(\phi_P, \alpha_G, \phi_G)$ defined as

$$U(\phi_P, \alpha_G, \phi_G) = \begin{pmatrix} X_\eta & Y_\eta & Z_\eta \\ X_{\eta'} & Y_{\eta'} & Z_{\eta'} \\ X_G & Y_G & Z_G \end{pmatrix} \tag{40}$$

wherefrom

$$\eta = X_\eta \psi_q + Y_\eta \psi_s + Z_\eta \psi_G, \tag{41}$$
$$\eta' = X_{\eta'} \psi_q + Y_{\eta'} \psi_s + Z_{\eta'} \psi_G.$$

Using (41) and experimental data [14], we also can obtain gluonium $Z_{\eta/\eta'}$–component [15].

[14] Tanabashi, M. The review of particle physics / M.Tanabashi (and others) //Phys. Rev.– 2018.–Vol.D 98.–1898 P.

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Numerical stadies of radiative decays of mesons in PiQM

Using the obtained values of quarks magnetic moments and value $\theta_P = (-9 \pm 2.4)^\circ$, we obtain the following values of the decay wight $V(P) \to P(V)\gamma$ in the framework of the model:

Table: Decay wight $V(P) \to P(V)\gamma$ taking into account quark magnetic moments

$V \to P\gamma$	$\Gamma^{(exp.)}$, keV	Γ, keV
$(\dagger)$ $\rho^+ \to \pi\gamma$	68±7 (fit) 68±7 (average)	68 ± 2
$(\dagger)$ $K^{*+} \to K^+\gamma$	50±5 (fit) 50±5 (average)	50 ± 3
$(\dagger)$ $K^{*0} \to K^0\gamma$	116±10 (fit) 116±9 (average)	116 ± 5
$\omega \to \pi^0\gamma$	713±26 (fit) 788±29 (Achasov)	704 ± 13
$\omega \to \eta\gamma$	3.8±0.4 (fit) 5.4±1.1 (average)	6.8 ± 2.3
$\phi \to \pi^0\gamma$	5.5±0.2 (fit) 5.4±0.5 (Achasov)	5.6 ± 2.1
$\phi \to \eta\gamma$	55.4±1.2 (fit) 58.9±2.9 (Achasov)	55.7 ± 2.5
$\phi \to \eta\prime\gamma$	0.26(fit) 0.4 (Akhmetshin)	0.27 ± 0.14
$\rho^0 \to \pi^0\gamma$	69±9(fit) 77±28(Achasov)	83 ± 4
$\rho^0 \to \eta\gamma$	44±3(fit) 41±2(average)	44.8 ± 3.5
$\eta\prime \to \rho^0\gamma$	56±4(fit) 56±3(average)	56.9 ± 2.3
$\eta\prime \to \omega\gamma$	5.1±0.5(fit) 5.0±0.4(Ablikim)	5.1 ± 0.8

Discussions

Well known, that a naive mixing scheme for$\eta - \eta'$–mesons without glueball component leads to a value of the angle $\theta_P \approx -14^\circ$, which is confirmed in the works, devoted to calculations on lattice QCD, chiral models etc. Nevertheless, the recent tendency to take into account the glueball component leds to the results $-10^\circ \geq \theta_P \geq -20^\circ$: for SU(3)-breaking effects used the value $\theta_P \approx -17^\circ$ [16], in hidden local symmetry model (HLS) obtained $\theta_P \approx -11^\circ$ [17], while the analysis of the experimental data gives the value of the angle $\theta_P \approx -12^\circ$.

The obtained in our work value $\theta_P = (-9.1 \pm 2.4)^\circ$ or $\phi_P = \theta_P + \arctan\sqrt{2} = (54.3 \pm 2.1)^\circ$ lays within reasonable limits compared to other approaches and models. In addition, using the values α_G and ϕ_G (see (38)) one can obtain value $|Z_{\eta'}| = 0.48 \pm 0.12$, which according to the authors is a good result, using global fit for $V(P) \to P(V)\gamma$ decays, was obtained $|Z_{\eta'}| = 0.34 \pm 0.04$ [18] .

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SUMMARY AND DISCUSSIONS

In the course of the work, the technique of calculating the integral representations of the pseudoscalar and vector meson decay constants was presented, as well as the pseudoscalar density constants, on the basis of which the values of the constituent quark masses and the parameters of the wave functions has been obtained. Note, that parameters of the model, based on point form of PiQM, correlated with the values obtained in the light–front and instant form of dynamics.

The authors note that the obtained values of the parameters of the model are used for estimation the observed of $V(P) \to P(V)\gamma$ process with following evaluation the quarks magnetic moments. Proposed calculation technique modified for future research form–factors behavior of the $V \to P\ell^+\ell^-$ decay of vector mesons.

THANK YOU FOR YOUR ATTENTION!

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