Joint Institute for Nuclear
Research

Analytical description of hadron production in hadron-hadron and nuclear-nuclear collisions in the mid rapidity region

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The XXIV International Baldin Seminar on High Energy Physics Problems
"Relativistic Nuclear Physics and Quantum Chromodynamics"

September 17-22, 2018, Dubna

Content

1. Introduction

Almost all theoretical approaches operate the relativistic invariant Mandelstam variables s, t, u to analyze the hadron inclusive spectra in the mid-rapidity region.

However, there is another approach to analyze multiple hadron production in hh and AA collisions at high energies, which operates the four velocities of the initial and final particles.

It is the so called the self-similarity approach, which demonstrates a similarity of inclusive spectra of hadrons produced in pp and AA collisions, as a function of similarity parameter .

In the present work we have used an approach based on the law of similarity.

For example, when planning large expensive hydraulic structures it is necessary to carry out physical modeling. Geometrically, the body of model is made similarly to the nature-body.

As the main parameters of the problem we take the following: *l* – the characteristic size of the body model, l^0 is the size of the nature body, l^0/l is the coefficient of geometric similarity, and *U* is the velocity of the impinging flow, *μ* is the viscosity of the fluid, *ρ* is the fluid density.

These parameters define the system of units: *l* – length, M – mass, T – time, and have the following dimensions:

$$[l] = L, \quad [U] = L\cdot T^{-1}, \quad [\mu] = M\cdot L^{-1}\cdot T, \quad [\rho] = M\cdot L^{-3}.$$

From the defining parameters we can be construct only one dynamic similarity parameter (a dimensionless combination, independent of the choice of measuring units):

$$\Pi = \rho U l/\mu = \mathrm{Re}.$$

This invariant is called the Reynolds number. To provide the similarity it is required to have equality of this parameter for the model and nature.

2. The parameter of self-similarity

Let us briefly present here the main idea of this study. Consider, for example, the production of hadrons 1, 2, etc. in the collision of a nucleus A with a nucleus B:

$$A + B \rightarrow 1 + 2 + \ldots$$

According to this assumption more than one nucleon in the nucleus A can participate in the interaction. The value of N_A is the effective number of nucleons inside the nucleus A, participating in the interaction which is called the cumulative number.

Its values lie in the region of $0 \leq N_A \leq A_A$ (A_A - atomic number of nucleus A). The cumulative area complies with $N_A > 1$.

Of course, the same situation will be for the nucleus B, and one can enter the cumulative number of N_B .

$A + B \rightarrow 1 + \ldots$

For reaction with the production of the inclusive particle 1 we can write the conservation law of four-momentum in the following form:

$$(N_A P_A + N_B P_B - p_1)^2 = (N_A m_0 + N_B m_0 + M)^2$$

where N_A and N_B the number of nucleons involved in the interaction or *the fraction of four momenta transmitted by the nucleus A and the nucleus B*; P_A , P_B , p_1 are four momenta of the nuclei A and B and particle 1, respectively; m_0 is the mass of the nucleon; M is the mass of the particle providing the conservation of the baryon number, strangeness, and other quantum numbers.

For π mesons $m_1 = m_\pi$ and $M = 0$.
For antinuclei and K^- mesons $M = m_1$.
For nuclear fragments $M = - m_1$.
For K^+ mesons $m_1 = m_K$ and $M = m_\Lambda - m_K$, m_Λ is the mass of the Λ barion.

In *A. M. Baldin, A. A. Baldin. Phys. Particles and Nuclei, 29 (3), (1998) 232* the parameter of self-similarity is introduced, which allows one to describe the differential cross section of the yield of a large class of particles in relativistic nuclear collisions:

$$\Pi = \min \tfrac{1}{2}\sqrt{(u_A N_A + u_B N_B)^2}$$

where u_A and u_B are four velocities of the nuclei A and B.

Then the inclusive spectrum of the produced particle 1 in AA collision can be presented as the universal function dependent of the self-similarity parameter:

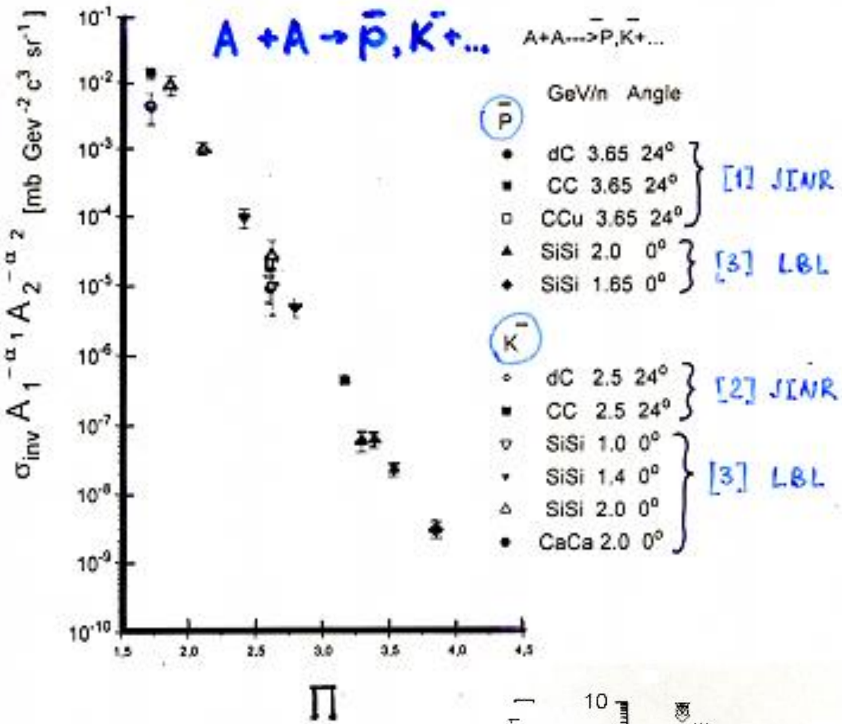
$$E \cdot \frac{d^3\sigma}{dp^3} = C_1 \cdot A_A^{\alpha(N_A)} \cdot A_B^{\alpha(N_B)} \cdot \exp(-\Pi/C_2)$$

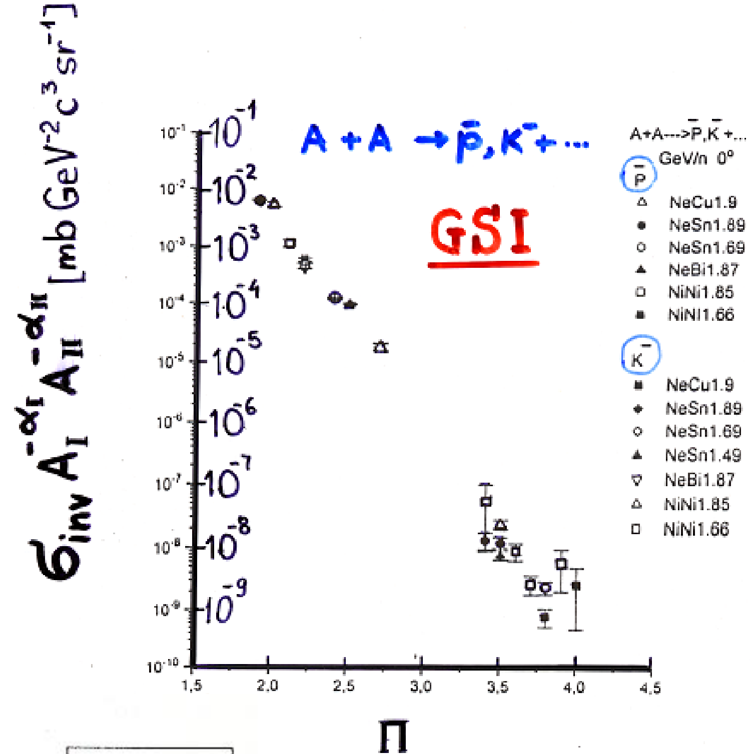
where $\boldsymbol{\alpha(N_A)} = 1/3 + \boldsymbol{N_A}/3$,

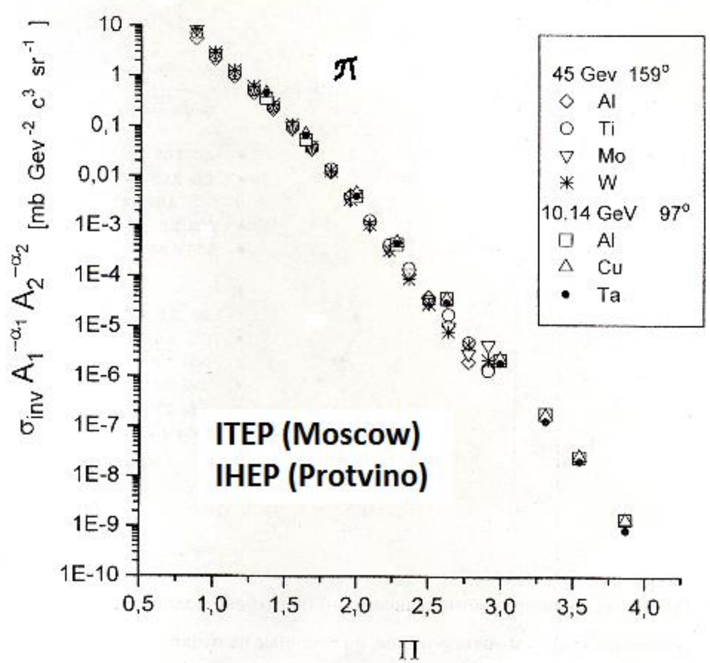
$\boldsymbol{\alpha(N_B)} = 1/3 + \boldsymbol{N_B}/3$,

$\boldsymbol{C_1 = 1.9 \cdot 10^4 \mathrm{mb} \cdot \mathrm{GeV}^{-2} \cdot c^3 \cdot \mathrm{st}^{-1}}$

$\boldsymbol{C_2 = 0.125 \pm 0.002}$.


A +A → p̄, K⁻+...
A+A--->P,K+...
GeV/n Angle
P̄
dC 3.65 24°
CC 3.65 24°
CCu 3.65 24°
[1] JINR
SiSi 2.0 0°
SiSi 1.65 0°
[3] LBL
K⁻
dC 2.5 24°
CC 2.5 24°
[2] JINR
SiSi 1.0 0°
SiSi 1.4 0°
SiSi 2.0 0°
CaCa 2.0 0°
[3] LBL
$\sigma_{inv} A_1^{-\alpha_1} A_2^{-\alpha_2}$ [mb Gev^{-2} c^3 sr^{-1}]
Π


$\sigma_{inv} A_I^{-\alpha_I} A_{II}^{-\alpha_{II}}$ [mb GeV^{-2}c^{3}sr^{-1}]
A + A → p̄, K⁻+ ...
A+A--->P,K +...
GeV/n 0°
GSI
P̄
NeCu1.9
NeSn1.89
NeSn1.69
NeBi1.87
NiNi1.85
NiNi1.66
K⁻
NeCu1.9
NeSn1.89
NeSn1.69
NeSn1.49
NeBi1.87
NiNi1.85
NiNi1.66
Π


π
45 Gev 159°
Al
Ti
Mo
W
10.14 GeV 97°
Al
Cu
Ta
ITEP (Moscow)
IHEP (Protvino)
$\sigma_{inv} A_1^{-\alpha_1} A_2^{-\alpha_2}$ [mb Gev^{-2} c^3 sr^{-1}]
Π

3. Analytical solution for self-similarity parameter*

$$A + B \rightarrow 1 + \ldots \qquad (1)$$

$$(N_A P_A + N_B P_B - p_1)^2 = (N_A m_0 + N_B m_0 + M)^2 \qquad (2)$$

Equation (2) can be written as follows:

$$N_A \cdot N_B - \Phi_A \cdot N_A - \Phi_B \cdot N_B = \Phi_M , \qquad (3)$$

Where relativistic invariant dimensionless values have been introduced:

$$\Phi_A = [(m_1/m_0)\cdot(u_A u_1) + M/m_0]/[(u_A u_B) - 1]$$
$$\Phi_B = [(m_1/m_0)\cdot(u_B u_1) + M/m_0]/[(u_A u_B) - 1]$$
$$\Phi_M = (M^2 - m_1^2)/[2m_0^2((u_A u_B) - 1)].$$

Equation (3) can be written as follows:

$$[(N_A/\Phi_B) - 1]\cdot[(N_B/\Phi_A) - 1] = 1 + [\Phi_M/(\Phi_A\cdot\Phi_B)]. \qquad (4)$$

) Baldin A.M., Malakhov A.I. JINR Rapid Communications, No.1(87)-98, 1998, pp.5-12.

Minimum П is found from the following:

$$d\Pi/dN_A = 0 \quad и \quad d\Pi/dN_B = 0. \qquad (5)$$

Let us introduce the intermediate variables:

$$F_A = [(N_A/\Phi_B) - 1], \qquad F_B = [(N_B/\Phi_A) - 1].$$

From the above we obtain: $F_A \cdot F_B = 1 + \Phi_M/(\Phi_A \cdot \Phi_B)$.

Then (5) is also equal to 0 as

$$d\Pi/dF_A = 0 \quad d\Pi/dF_B = 0.$$

From $\Pi = \min \frac{1}{2}\sqrt{(u_A N_A + u_B N_B)^2}$

we can obtain:

$$4\Pi^2 = N_A^2 + N_B^2 + 2N_A \cdot N_B \cdot (u_A u_B),$$

$$4\Pi^2 = (F_A + 1)^2 \Phi_B^2 + (F_B + 1)^2 \Phi_A^2 + 2\, \Phi_A \cdot \Phi_B (F_A + 1) \cdot (F_B + 1) \cdot (u_A u_B),$$

$$F_B = \alpha / F_A$$

The condition of the minimum $d(4\Pi^2)/dF_A = 0$ gives the equation for F_A:

$$F_A^4 + F_A^3 - (\Phi_A/\Phi_B)^2 \cdot (\alpha^2 + \alpha F_A) + (u_A u_B) \cdot (\Phi_A/\Phi_B) \cdot (F_A^3 - \alpha F_A) = 0$$

or

$$F_A^4 + F_A^3[1 + (u_A u_B)/z] - (\alpha/z) \cdot F_A \cdot [(u_A u_B) + (1/z)] - \alpha^2/z^2 = 0,$$

where $z = \Phi_B/\Phi_A$.

When changing A to B: $z \rightarrow (1/z)$, $F_I \rightarrow (\alpha/F_B)$.

$$(\alpha/F_B)^4 + (\alpha/F_B)^3[1 + (u_A u_B)z] - \alpha z(\alpha/F_B)[(u_A u_B) + z] - \alpha^2 z^2 = 0$$

or

$$F_B^{\ 4} + F_B^{\ 3}[1 + (u_A u_B)z] - z\alpha \cdot F_B \cdot [z + (u_A u_B)] - \alpha^2 z = 0.$$

Previously, we derived a formula for F_A :

$$F_A^{\ 4} + F_A^{\ 3}[1 + (u_A u_B)/z] - (\alpha/z) \cdot F_A \cdot [(u_A u_B) + (1/z)] - \alpha^2/z^2 = 0,$$

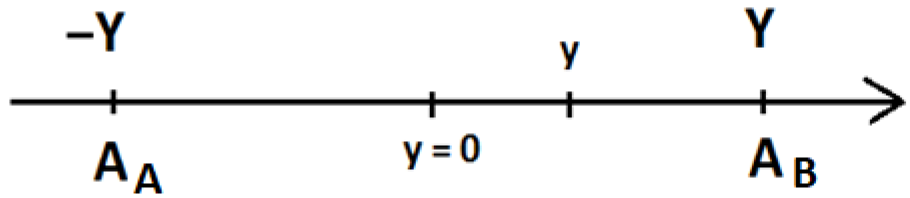
Thus, at $z = 1 \rightarrow F_A = F_B$, $\Phi_A = \Phi_B = \Phi$.

But since $F_A = F_B$, then $(N_A/\Phi - 1) = (N_B/\Phi - 1)$ and $N_A = N_B$.

$$F^2 = \alpha \text{ and } F_A = F_B = \alpha^{½} = [1 + (\Phi_M/\Phi^2)]^{½}.$$

$$N_A = N_B = N = (1+F)\Phi = \{1 + [1 + (\Phi_M/\Phi^2)]^{½}\}\Phi.$$

$$\Pi = ½\,[2N^2 + 2N^2(u_A u_B)]^{½} = (N/\sqrt{2})[1 + (u_A u_B)]^{½} = N \cdot ChY$$



$(u_A u_B) = \mathrm{ch}2Y,$
$(u_A u_1) = (m_{1t}/m_1)\cdot\mathrm{ch}(-Y-y) = (m_{1t}/m_1)\cdot\mathrm{ch}(Y+y).$
$(u_B u_1) = (m_{1t}/m_1)\cdot\mathrm{ch}(Y-y).$
Here m_t is the transverse mass of the particle 1, $m_{1t} = (m_1^2 + p_{1t}^2)^{½}$,
Y - rapidity of interacting nuclei, y – rapidity particle 1.

At y = 0 (in central rapidity region) we obtain:

$(u_A u_1) = (u_B u_1) = (m_{1t}/m_1)\cdot\mathrm{ch}Y, \quad m_{1t} = (m_1^2 + p_{1t}^2)^{½}$

$\Phi = (1/m_0)\cdot(m_{1t}\mathrm{ch}Y + M)\cdot[1/(2\mathrm{sh}^2Y)]$
$\Phi_M = (M^2 - m_1^2)/(4m_0^2\mathrm{sh}^2Y)$

$N = [1 + ((\Phi_M/\Phi^2) + 1)^{½}]\cdot[(m_{1t}/m_0)\mathrm{ch}Y + (M/m_0)]\cdot[1/(2\mathrm{sh}^2Y)].$

$$(u_A u_1) = (m_{1t}/m_1)\cdot ch(-Y-y) = (m_{1t}/m_1)\cdot ch(Y+y).$$
$$(u_B u_1) = (m_{1t}/m_1)\cdot ch(Y-y).$$

$$ch(Y+y) = ch\, Y\cdot ch\, y + sh\, Y\cdot sh\, y.$$
$$(u_A u_1) = (m_{1t}/m_1)\cdot ch(Y+y) = (m_{1t}/m_1)\cdot(chY\cdot chy + shY\cdot shy) \approx (m_{1t}/m_1)\cdot[(1+y^2)\cdot chY + y\cdot shY] \approx$$
$$\approx (m_{1t}/m_1)\cdot(1+y^2)\cdot chY$$
$$e^y \approx 1 + y + y^2/2$$
$$e^{-y} \approx 1 - y + y^2/2$$
$$sh\, y = ½(e^y - e^{-y}) \approx ½(1 + y + y^2/2 - 1 + y - y^2/2) = y$$
$$ch\, y = ½(e^y + e^{-y}) \approx ½(1 + y + y^2/2 + 1 - y + y^2/2) = 1 + y^2$$

$$(u_B u_1) = (m_{1t}/m_1)\cdot ch(Y-y) = (m_{1t}/m_1)\cdot(chY\cdot chy - shY\cdot shy) \approx (m_{1t}/m_1)\cdot[(1+y^2)\cdot chY - y\cdot shY] \approx$$
$$\approx (m_{1t}/m_1)\cdot(1+y^2)\cdot chY$$

$$(u_A u_1) \approx (u_B u_1) \approx (m_{1t}/m_1)\cdot(1+y^2)\cdot chY$$

$$\Phi = \Phi_A = \Phi_B = [(m_1/m_0)\cdot(u_A u_1) + M/m_0]/[(u_A u_B) - 1] \approx$$
$$\approx [(m_1/m_0)\cdot(m_{1t}/m_1)\cdot(1+y^2)\cdot chY + M/m_0]/[ch2Y - 1] =$$
$$= \{(1/m_0)[m_{1t}\cdot(1+y^2)\cdot chY + M]\}\cdot[1/(2sh^2Y)]$$

$$\Phi_M = (M^2 - m_1^2)/(4m_0^2 sh^2 Y)$$

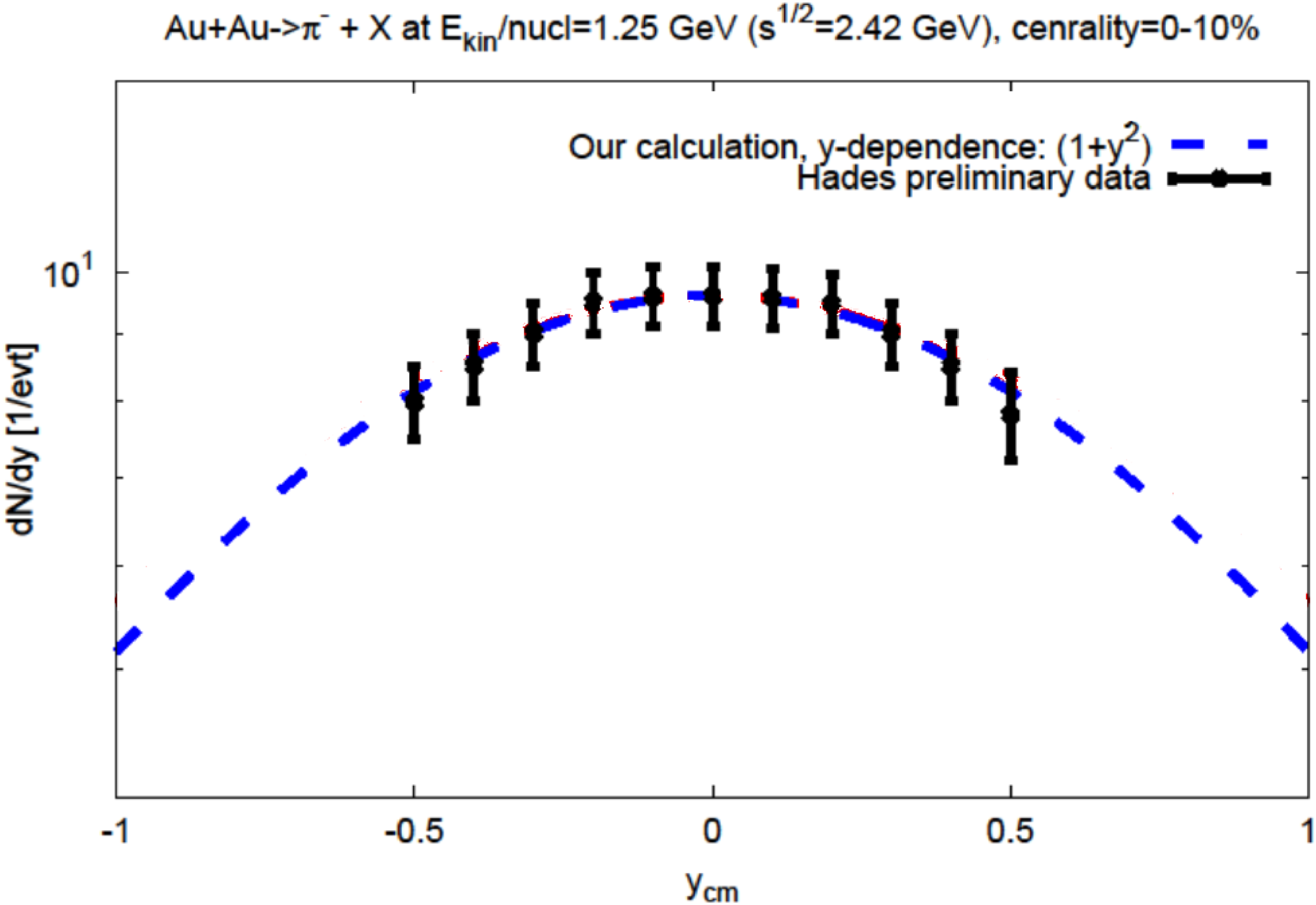
$$\Pi = N \cdot \mathrm{ch}Y$$

$$N = \{1 + [1 + (\Phi_M / \Phi^2)]^{1/2}\}\Phi$$

$$\Phi \approx \{(1/m_0)[m_{1t} \cdot (1+y^2) \cdot \mathrm{ch}Y + M]\} \cdot [1/(2\mathrm{sh}^2 Y)]$$

$$\Phi_M = (M^2 - m_1{}^2)/(4m_0{}^2\mathrm{sh}^2 Y)$$

$$y \ll 1$$



Au+Au->π⁻ + X at E_kin/nucl=1.25 GeV (s^1/2=2.42 GeV), cenrality=0-10%
Our calculation, y-dependence: (1+y²)
Hades preliminary data
10¹
dN/dy [1/evt]
-1
-0.5
0
0.5
1
y_cm

$$(u_A u_1) = (P_A/m_A)(P_1/m_1) = E_A \cdot E_1/m_I \cdot m_1 - p_A \cdot p_1/m_A \cdot m_1 =$$

$$= \cancel{m}_A \cdot chY \cdot m_{1t} \cdot chy/\cancel{m}_A \cdot m_1 + \cancel{m}_A \cdot shY \cdot m_{1t} \cdot shy/\cancel{m}_A \cdot m_1 =$$

$$= (m_{1t}/m_1) \cdot (chY \cdot chy + shY \cdot shy) = (m_{1t}/m_1) \cdot ch(Y+y).$$

Since this equation does not depend on m_A, it is valid for any hadrons and nuclei, for example, for π mesons.

Therefore, we conclude that our approach also works for projectile π mesons.

4. Self-similarity parameter in the central rapidity region

In the mid-rapidity region ($y=0$, y is the rapidity of particle 1) the analytical form for Π was found in ***A. M. Baldin, A. I. Malakhov. JINR Rapid Communications, 1 [87]-98 (1998) 5-12*****.**

In this case N_I and N_{II} are equal to each other: $N_I = N_{II} = N$.

$$N = [1 + (1 + \Phi_\delta / \Phi^2)^{1/2}]\Phi,$$

where

$\Phi = (m_{1t}\, chY + M)/(2m_0 sh^2 Y)$,

$\Phi_\delta = (M^2 - m^2_1)/(4m^2_0 \cdot sh^2 Y)$.

Here m_{1t} is the transverse mass of the particle **1**, $m_{1t} = (m^2_1 + p^2)^{1/2}$, **Y** - rapidity of interacting nuclei.

And then

$$\Pi = N \cdot chY.$$

For baryons we have

$$\Pi_b = (m_{1t}\mathrm{ch}Y - m_1)\mathrm{ch}Y/(m_0\mathrm{sh}^2Y)$$

and for antibaryons

$$\Pi_a = (m_{1t}\mathrm{ch}Y + m_1)\mathrm{ch}Y/(m_0\mathrm{sh}^2Y).$$

The results of calculations for the ratio of the antiproton cross section to the proton one after integration of over dm_{1t} are presented in the following figure.

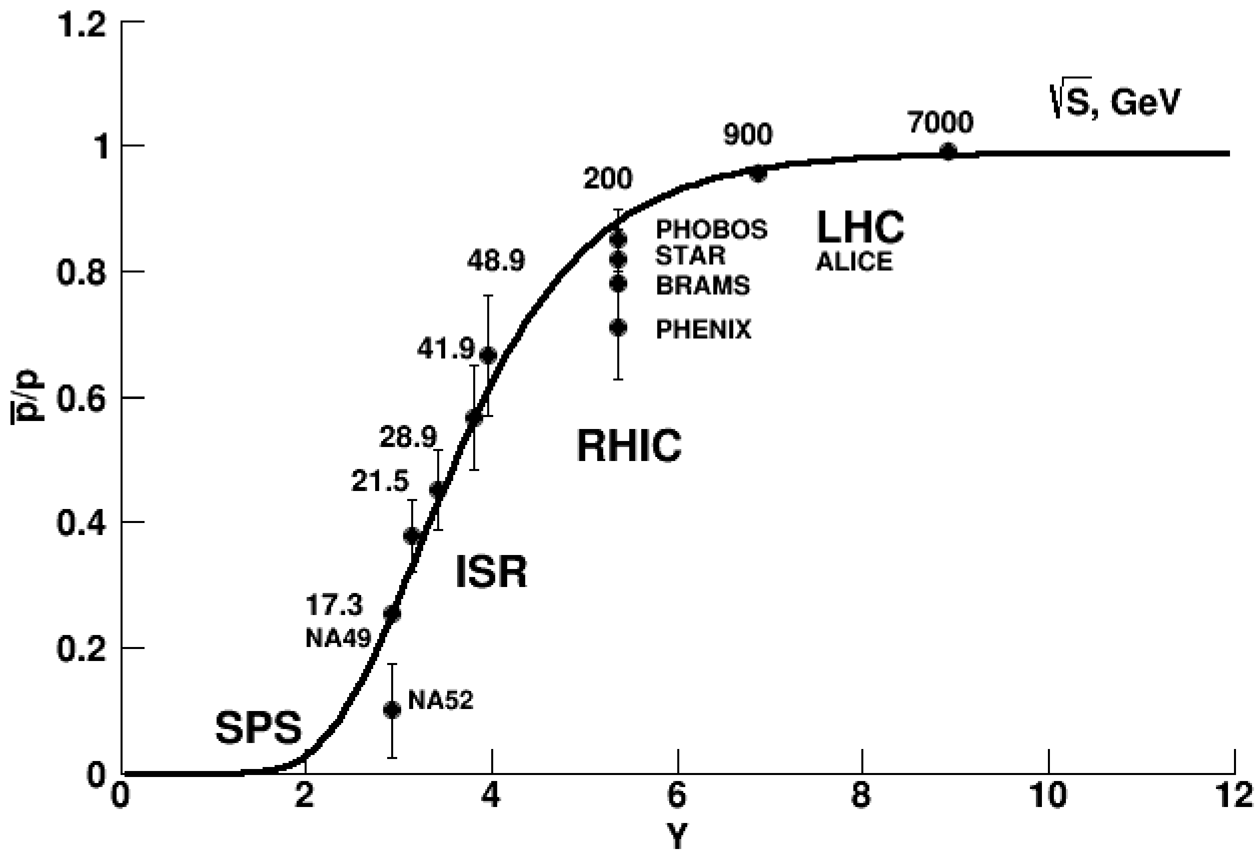
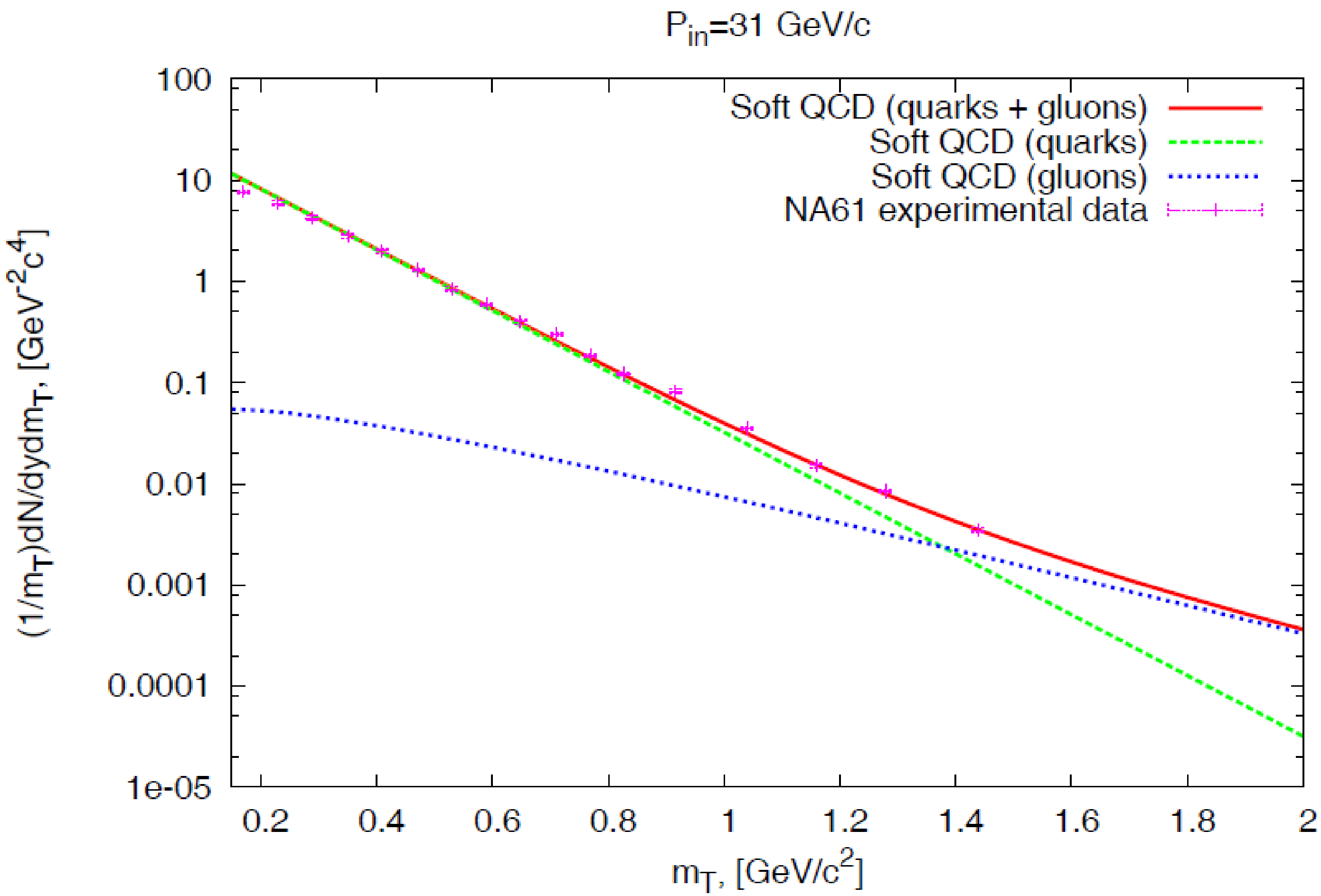
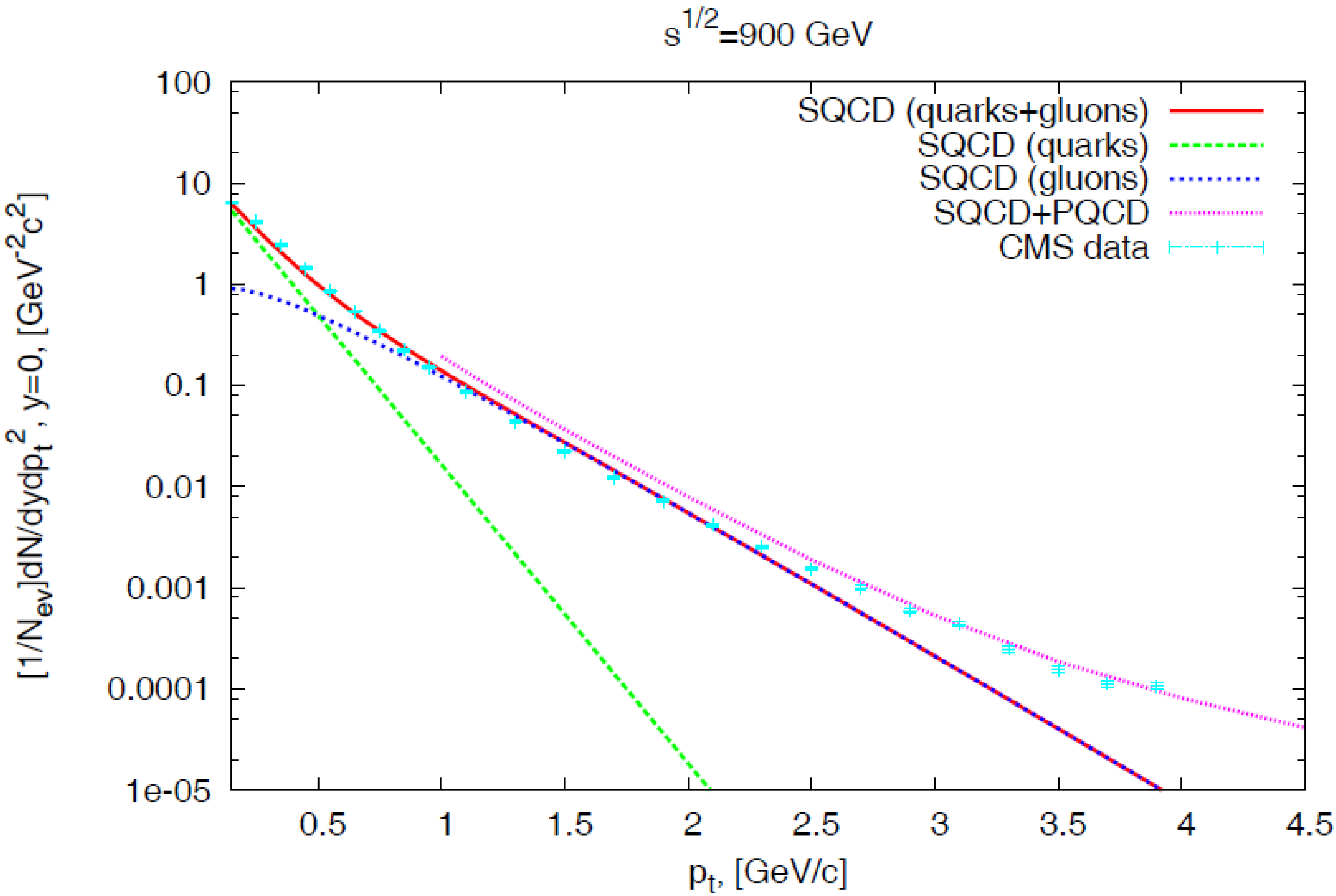


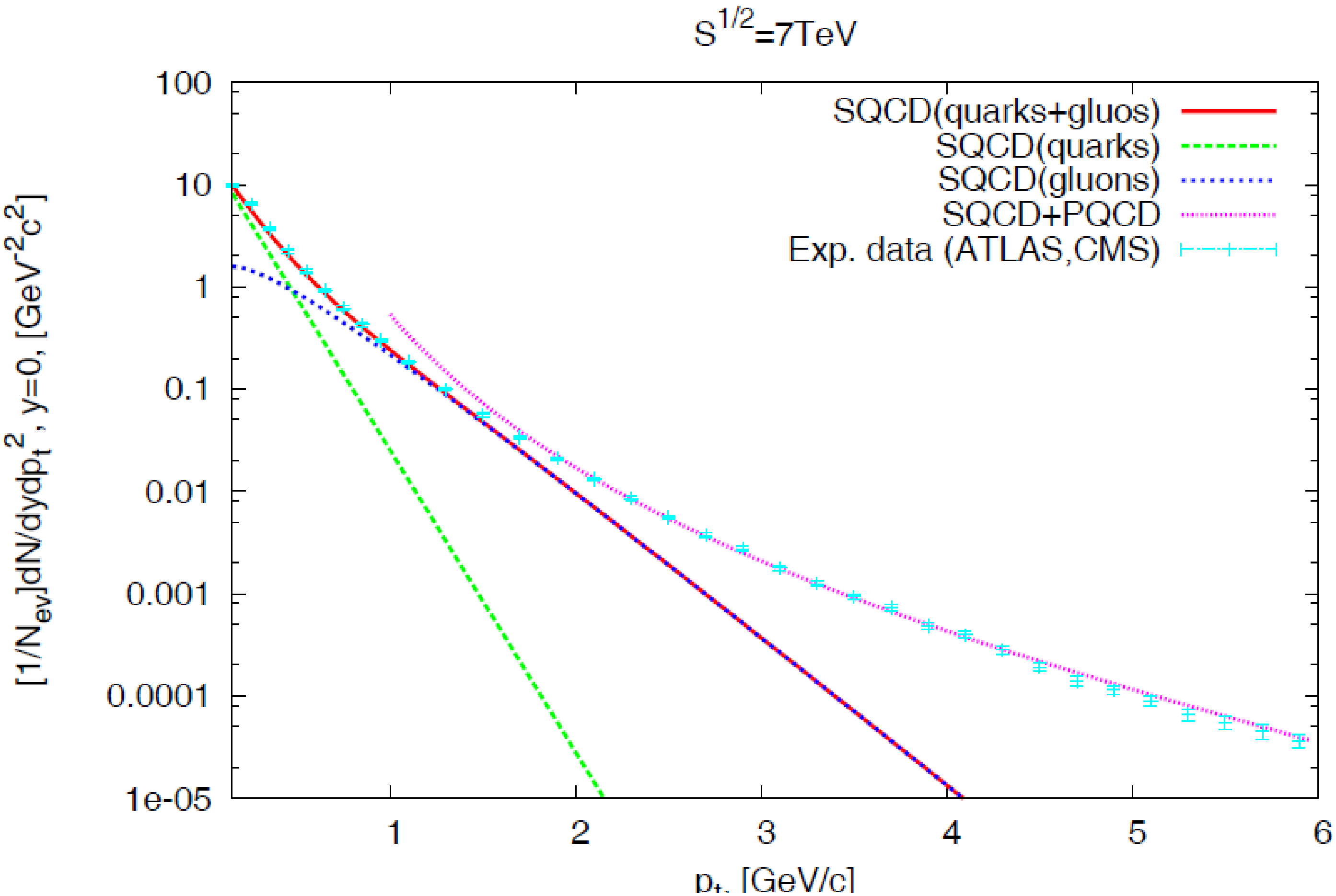
Fig.1. The dependence of ratio of the antiproton cross section to the proton one as a function of initial rapidity **Y** or energy (√**S**, GeV) of the interacting nuclei. The points are the experimental data.



Results of the calculations of the inclusive cross section of hadron production in pp collisions as a function of the transverse mass at the initial momenta Pin = 31 GeV/c. They are compared to the NA61 experimental data from A. A. Abgrall et al. Eur.Phys.J., C74 (2014) 2794.



Results of the calculations of the inclusive cross section of charge hadrons produced in pp collisions at the LHC energies as a function of their transverse momentum p_t at $\sqrt{s}$ =0.9 TeV. The points are the LHC experimental data [***V. Khachatryan, et al. (CMS Collaboration), Phys. Rev. Lett. 105, 022002 (2010)***].



Results of the calculations of the inclusive cross section of charge hadrons produced in pp collisions at the LHC energies as a function of their transverse momentum p_t at $\sqrt{s}$ =7 TeV. The points are the LHC experimental data: ***G. Aad, et al. (ATLAS Collaboration), New J. Phys. 13, 053033 (2011) and V. Khachatryan, et al. (CMS Collaboration), Phys. Rev. Lett. 105, 022002 (2010)***.

G. Lykasov will tell about the results obtained in the framework of our approach in the next report in more detail.

5. Conclusion

1. **The inclusive spectra of the produced hadrons in hadron-hadron and nuclear-nuclear collisions can be presented as the universal function dependent of the self-similarity parameter in analytical form.**

2. **The experimental data are in good agreement with results our calculations in a wide energy range from a few GeV to a few TeV in central rapidity region.**

3. **The use of the self-similarity approach allows us to describe the ratio of the total yields of particles to antiparticles produced in A-A collisions as a function of the energy in the mid-rapidity region and a wide energy range.**

4. **A description of the self-similarity parameter depending on the rapidity in the mid-rapidity region is obtained.**

Thank you for your attention!