Solving Dyson-Schwinger and Bethe-Salpeter equations at zero and finite temperatures

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(nucl-th 1807.10075, PhysPNL 15,2018, PRC 89 (2014), PRC 91 (2015), J. Mod. Phys. 7 (2016), Few Body Syst.49, Few Body Syst.49, Few Body Syst.42…)

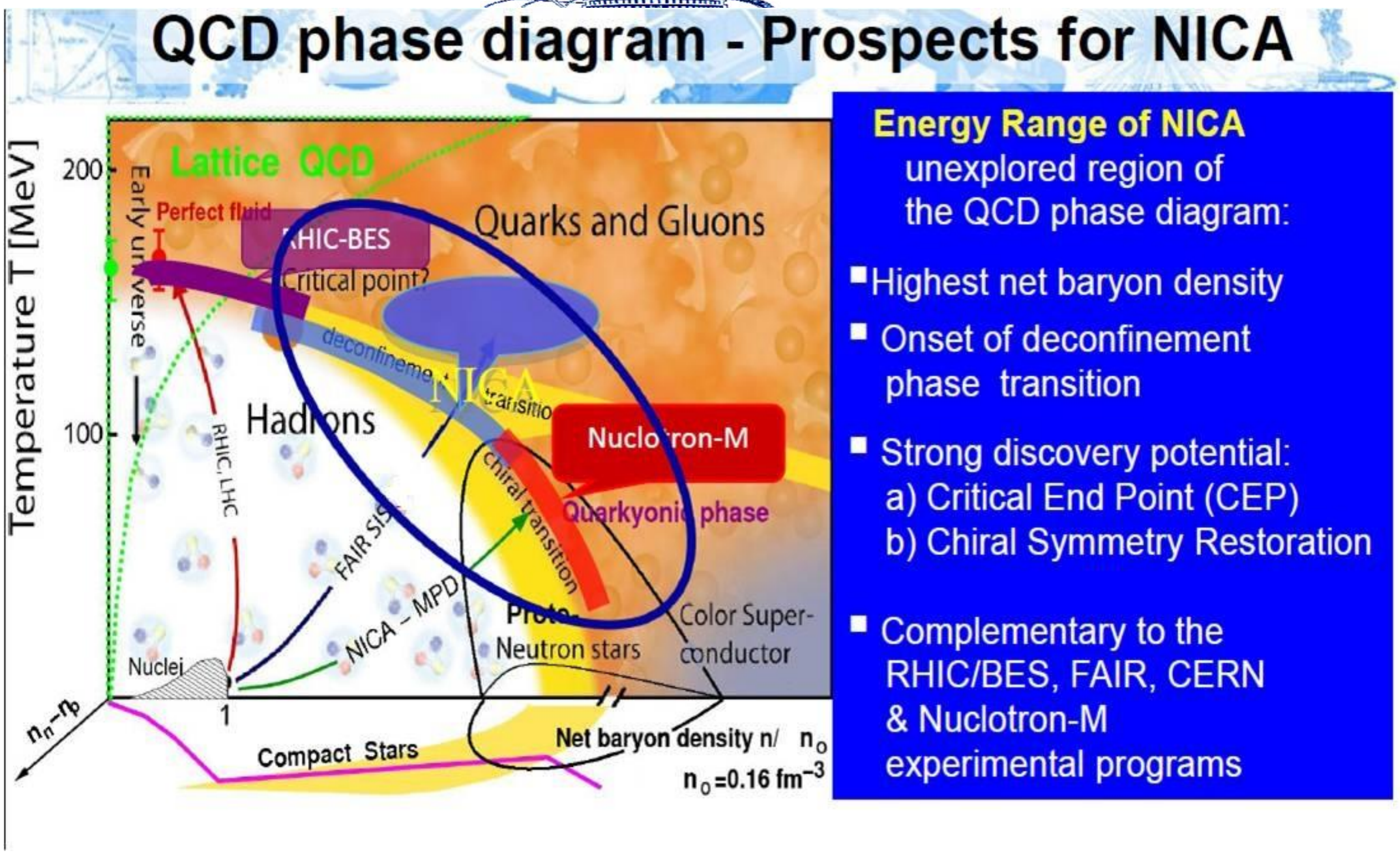
XXIV Baldin Conference

Relativistic Nuclear Physics&QCD

Dubna, September 17-22,2018

MOTIVATION

➢ **QGP signals (GSI (10-40 GeV/N), RHIC ($\sqrt{s_{NN}}$> 200 GeV), NICA ($\sqrt{s_{NN}}$> 4-11 GeV/N)...**



MOTIVATION

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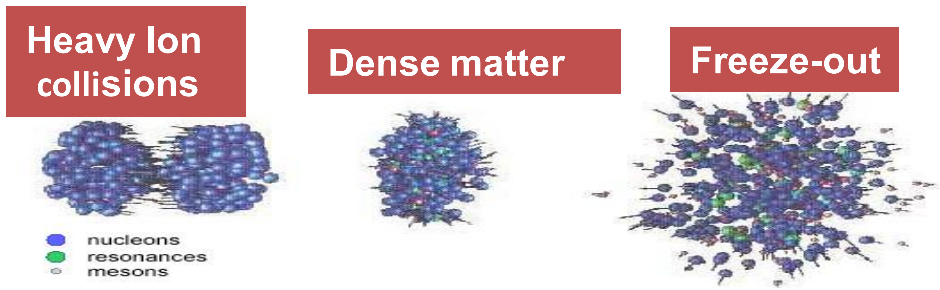
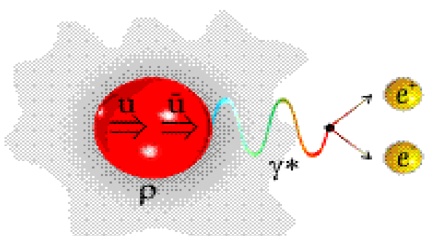
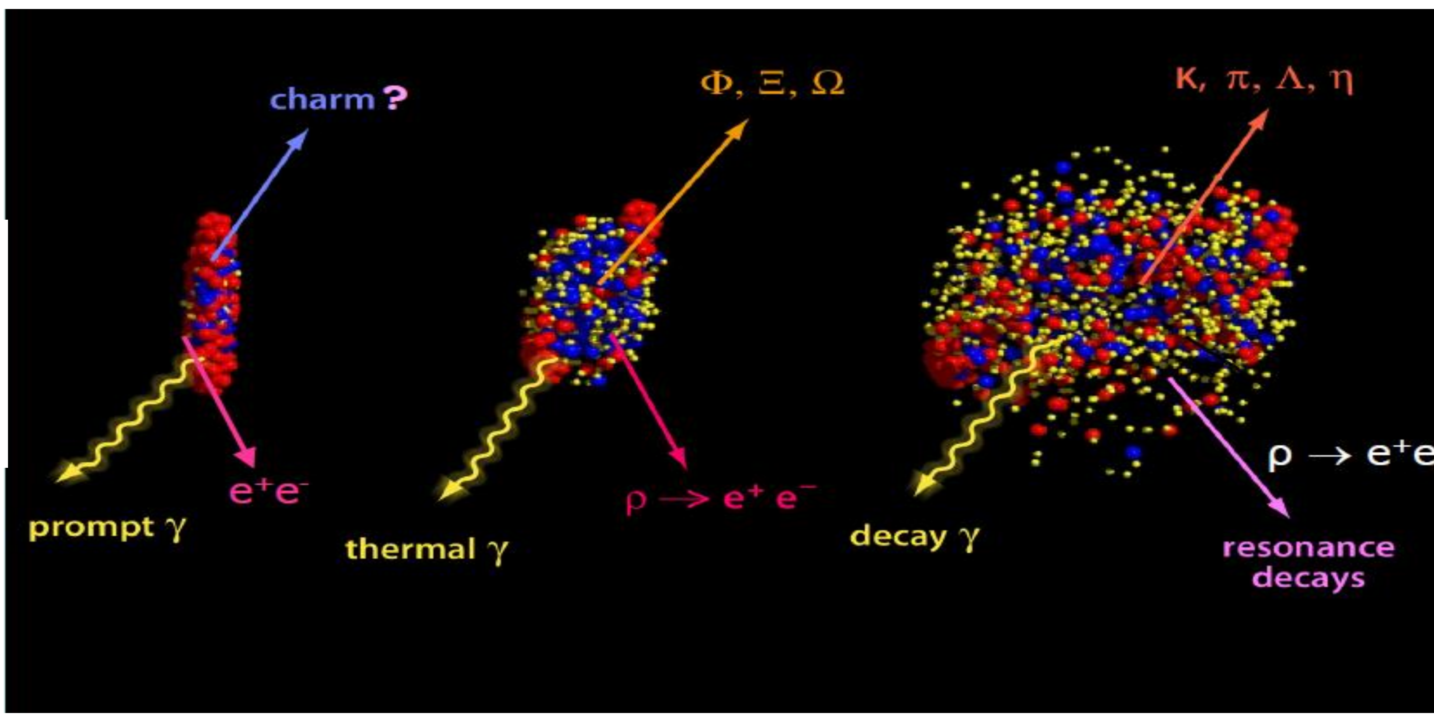
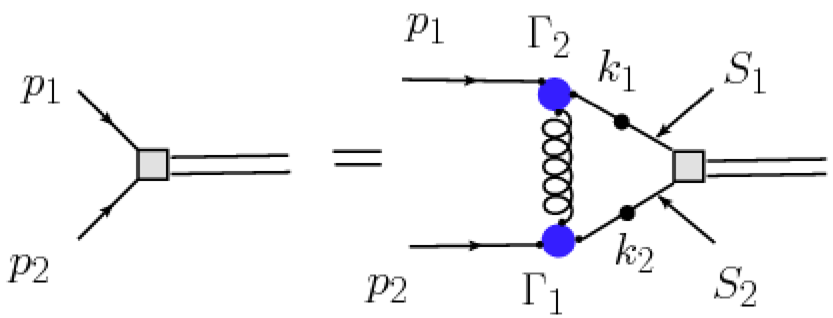


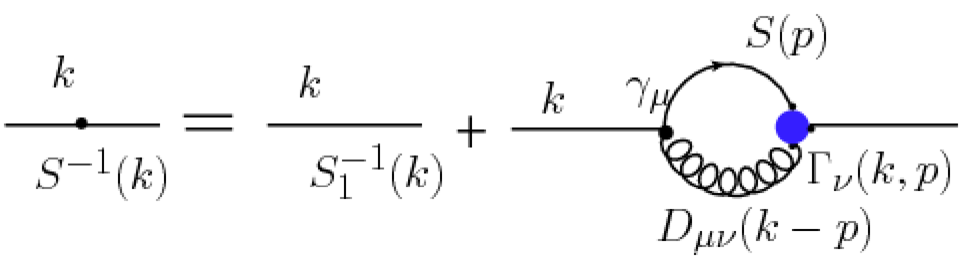




$q\bar{q}$ Bound States: BS & Dyson-Schwinger Equations

Mesons as $q\bar{q}$ bound systems: $\pi^+(\sim 0.140, u\bar{d})$ $\rho^+(\sim 0.77, u\bar{d})$ $K^+(\sim 0.494, u\bar{s})$, $K^{*+}(\sim 0.891, u\bar{s})$, $\eta_c(\sim 2.98, c\bar{c})$, $D^+(1.869, c\bar{d})$ **...**





$$S^{-1}(p) = S_0^{-1}(p) + \frac{4}{3}\int \frac{d^4k}{(2\pi)^4}\left[g^2\mathcal{D}_{\mu\nu}(p-k)\right]\gamma_\mu S(k)\gamma_\nu\,,$$

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)} = \frac{1}{i\gamma\cdot pA(p^2)+B(p^2)} = -i\gamma\cdot p\sigma_V(p^2)+\sigma_S(p^2)$$

Rainbow approximation ($\Gamma_\nu^a(l,p) = \frac{\lambda^a}{2}\gamma_\nu$) + effective model for gluon propagator:

$$\frac{D(q^2)}{q^2} = \frac{4\pi^2}{\omega^6}Cq^2\mathrm{e}^{-q^2/\omega^2} + \frac{8\pi^2\gamma_m}{\ln\left[\tau+\left(1+\frac{q^2}{\Lambda_{QCD}^2}\right)^2\right]}F(q^2)$$

IR

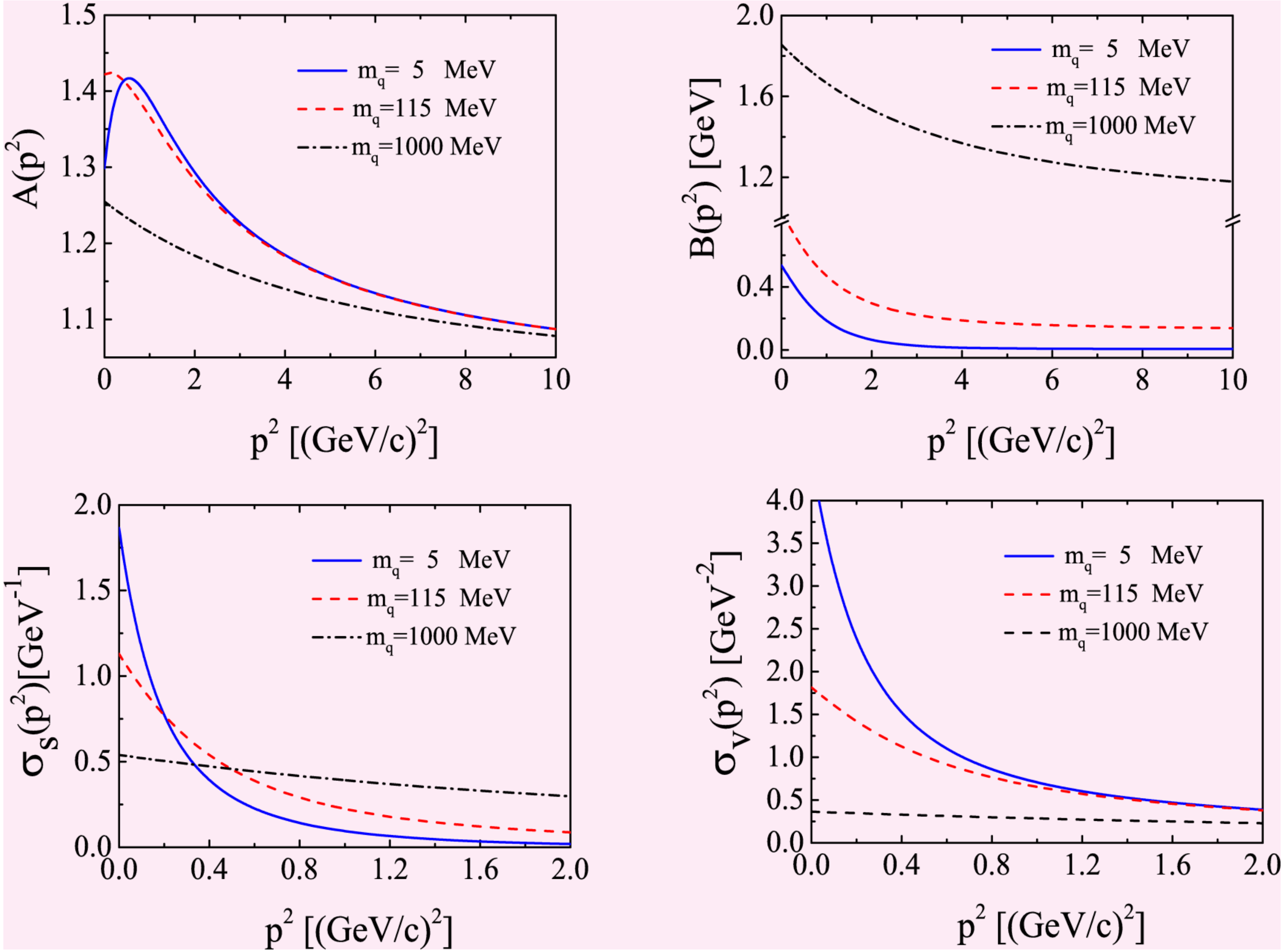
UV

$$\langle\bar{q}q\rangle = -N_c\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[S(p)\right] = -N_c\int\frac{d^4p}{(2\pi)^4}\sigma_s(p)$$

$$A(p) = 1+2C\int dk\frac{k^4}{p}\frac{A(k)}{k^2A^2(k)+B^2(k)}\mathrm{e}^{-(p-k)^2/\omega^2}\left\{\frac{[p^2+k^2+2\omega^2]}{kp}I_2^{(s)}(z)-2I_1^{(s)}(z)\right\},$$

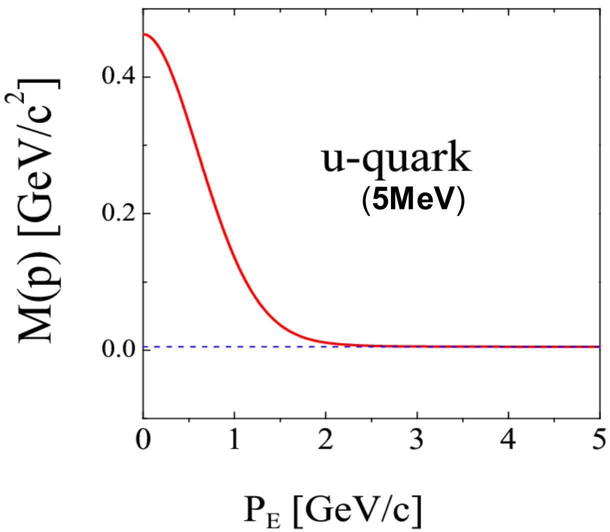
$$B(p) = m_q+2C\int dk k^3\frac{B(k)}{k^2A^2(k)+B^2(k)}\mathrm{e}^{-(p-k)^2/\omega^2}\left\{\frac{[p^2+k^2]}{kp}I_1^{(s)}(z)-2I_2^{(s)}(z)\right\},$$

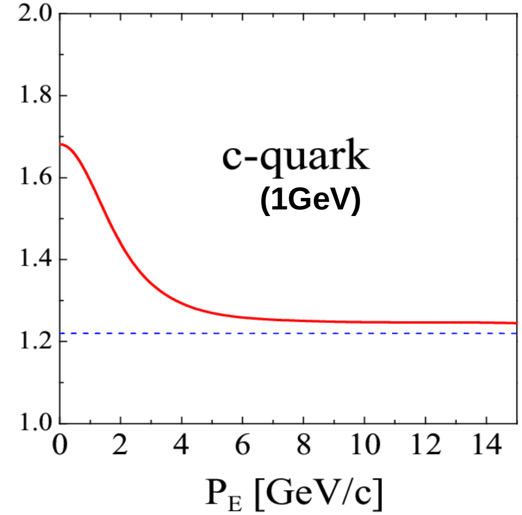
RESULTS



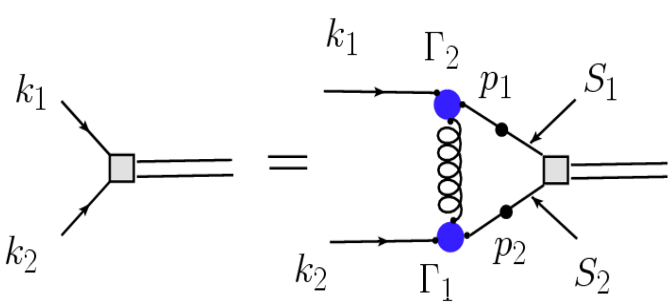
A(p^2)
B(p^2) [GeV]
σ_S(p^2)[GeV^-1]
σ_V(p^2) [GeV^-2]
p^2 [(GeV/c)^2]
m_q= 5 MeV
m_q=115 MeV
m_q=1000 MeV

RESULTS

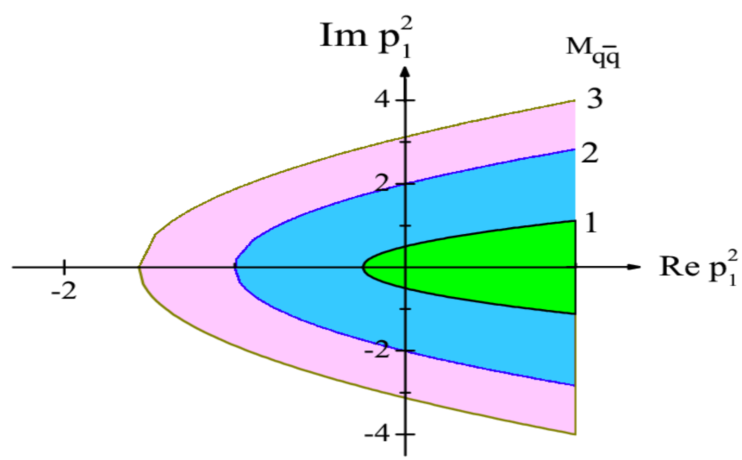




BSE domain

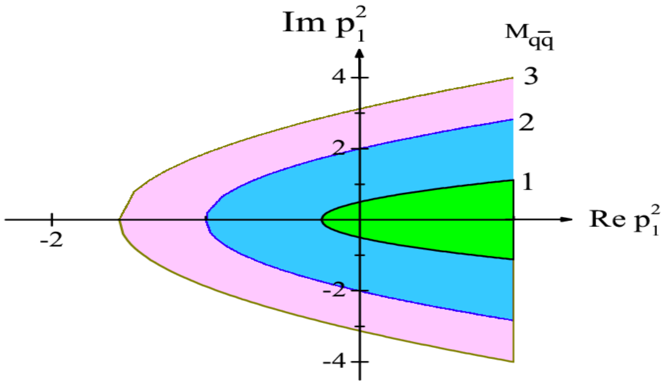


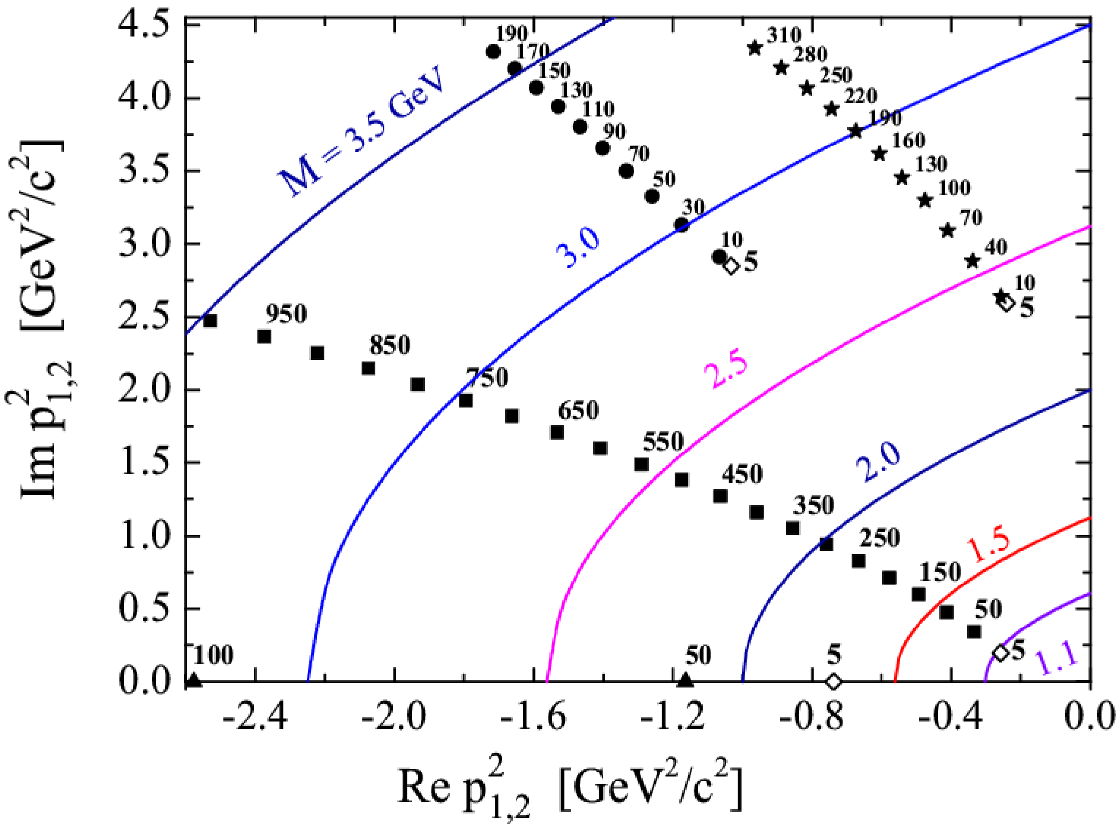
$$p_1^2 = -M_{q\bar{q}}^2/4 + p^2 + i\, M_{q\bar{q}}\, p \, \mathrm{Cos}(\chi)$$



SINGULARITIES OF THE PROPAGATOR FUNCTIONS !!!

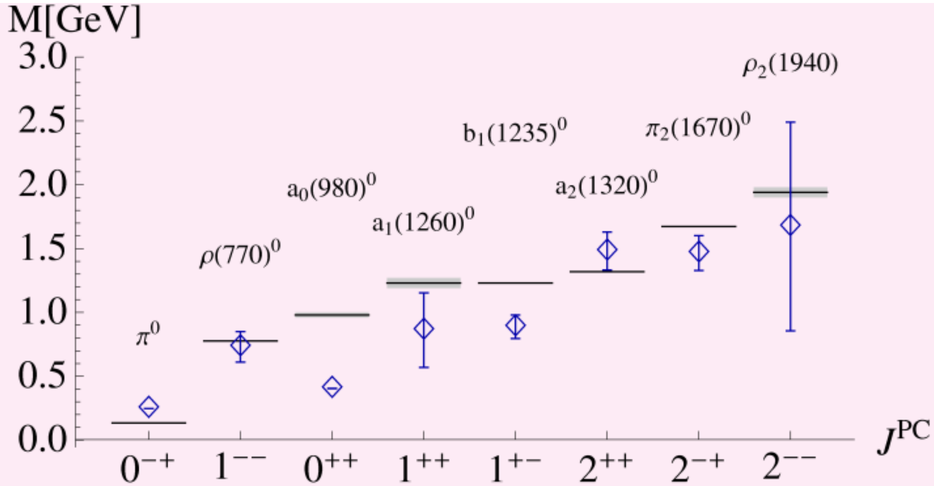
$$p_1^2 = -M_{q\bar{q}}^2/4 + p^2 + i\,M_{q\bar{q}}\, p\,\mathrm{Cos}(\chi)$$



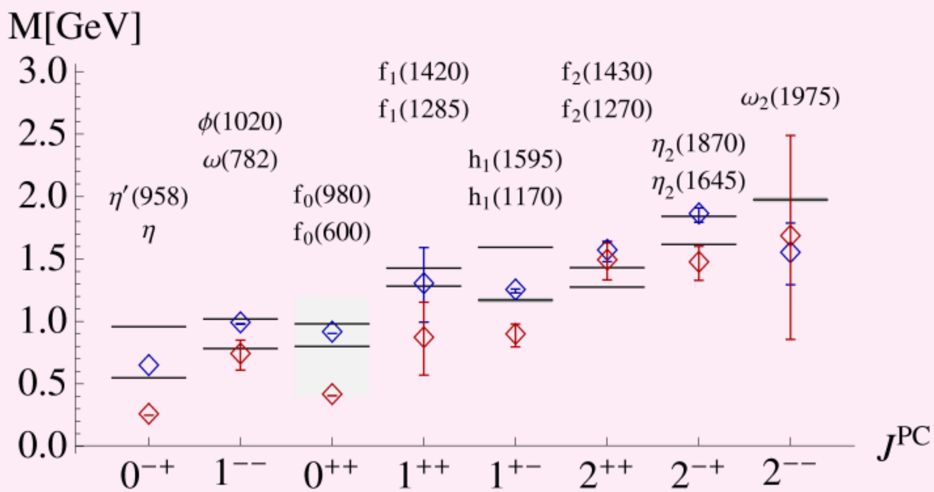


$$\sigma_{s,v}(\tilde{k}^2) = \widetilde{\sigma}_{s,v}(\tilde{k}^2) + \sum_i \frac{\mathrm{res}[\sigma_{s,v}(\tilde{k}^2_{0i})]}{\tilde{k}^2 - k^2_{0i}},$$

$$\widetilde{\sigma}_{s,v}(\tilde{k}^2) = \frac{1}{2\pi i}\oint_\gamma \frac{\widetilde{\sigma}_{s,v}(\xi)}{\xi - \tilde{k}^2}d\xi = \frac{1}{2\pi i}\oint_\gamma \frac{\sigma_{s,v}(\xi)}{\xi - \tilde{k}^2}d\xi.$$



The light-isovector ground-state spectrum compared to experimental data.



The light-isoscalar ground-state spectrum compared to experimental data (LPK& SMD PRC **89** (2014), PRC **91** (2015), Few Body Syst. **49** (2011), arXiv: 1012.5372; M. Blank and A. Krassnigg, PRD **84** (2011)

	experiment (estimates)	calculated († fitted)
$m^{u=d}_{\mu=1\mathrm{GeV}}$	5 - 10 MeV	5.5 MeV
$m^{s}_{\mu=1\mathrm{GeV}}$	100 - 300 MeV	125 MeV
- $\langle \bar{q}q \rangle^0_\mu$	(0.236 GeV)^3	(0.241)^3
m_π	0.1385 GeV	0.138
f_π	0.131 GeV	0.131
m_K	0.496 GeV	0.497
f_K	0.160 GeV	0.155
m_ρ	0.770 GeV	0.742
f_ρ	0.216 GeV	0.207
$m_{K^\star}$	0.892 GeV	0.936
$f_{K^\star}$	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Overview of the results of the model for the meson masses and decay constant, adapted from P. Maris and P. C. Tandy, PRC **60**, 055214 (1999).

INTERMEDIATE SUMMARY

The Dyson–Schwinger-Bethe-Salpeter approach + rainbow approximation with only two-three effective parameters, describe fairly well the vacuum (T=0) properties (masses, decay constants…) of the scalar, pseudoscalar, vector etc., mesons and allow for Poincarè covariant studies of reactions with mesons

Finite Temperatures

The statistical density matrix:

$$\hat{\rho} = \exp\left[-\beta\left(\hat{H} - \mu_i \hat{N}_i\right)\right]$$

Ansamble average of an operator :

$$\langle \hat{A} \rangle = \frac{Tr[\hat{A}\hat{\rho}]}{Tr[\hat{\rho}]} \equiv \frac{Tr[\hat{A}\hat{\rho}]}{Z}$$

$$\mathbf{Z} = \mathbf{Tr}\left[\exp\left\{-\boldsymbol{\beta}\left(\hat{\mathbf{H}} - \boldsymbol{\mu}_{\mathbf{i}}\hat{\mathbf{N}}_{\mathbf{i}}\right)\right\}\right] = \int \mathbf{d}\boldsymbol{\phi}_{\mathbf{a}} \left\langle \boldsymbol{\phi}_{\mathbf{a}} \left| \exp\left\{-\boldsymbol{\beta}\left(\hat{\mathbf{H}} - \boldsymbol{\mu}_{\mathbf{i}}\hat{\mathbf{N}}_{\mathbf{i}}\right)\right\}\right| \boldsymbol{\phi}_{\mathbf{a}} \right\rangle$$

Transition amplitude from a state $|\phi_a\rangle$ to a state $|\phi_b\rangle$ after a time t is $\left\langle \boldsymbol{\phi}_{\mathbf{b}} \left| e^{-\mathbf{i}\hat{\mathbf{H}}\mathbf{t}} \right| \boldsymbol{\phi}_{\mathbf{a}} \right\rangle$, where $|\phi_a\rangle$ is an eigenstate of the field operator $\hat{\phi}(\mathbf{x})$, i.e. $\hat{\boldsymbol{\phi}}(\mathbf{x})|\boldsymbol{\phi}_{\mathbf{a}}\rangle = \boldsymbol{\phi}(\mathbf{x})|\boldsymbol{\phi}_{\mathbf{a}}\rangle$

Imaginary time (Matsubara) formalism $\tau = it$ and periodic condition= after a time $\tau = \beta$ the system comes back to its initial state: $\boldsymbol{\phi}(\mathbf{x}, 0) = \pm\boldsymbol{\phi}(\mathbf{x}, \beta)$

$$Z = \int [d\pi] \int d[\boldsymbol{\phi}] \exp\left\{\int_0^\beta d\tau \int d^3x \left(i\pi\frac{\partial\boldsymbol{\phi}}{\partial\tau} - H(\boldsymbol{\phi}, \pi) + \mu_i N_i(\boldsymbol{\phi}, \pi)\right)\right\}$$

Main difference

1. **Furie transform** ($O(4)$**symmetry lost!!**)

$$\int \frac{d^4x}{(2\pi)^4}\exp(ipx)\boldsymbol{\phi}(p) \longrightarrow T\sum_{n=-\infty}^{\infty}\int \frac{d^3x}{(2\pi)^3}\exp(i\ \mathbf{px}+\omega_n\tau)\boldsymbol{\phi}(\mathbf{p},\omega_n)$$

$\omega_n = \pi(2n+1)T$ **(fermions);** $\omega_n = 2n\pi T$ **(bosons)**

2. **The inverse quark propagator is now parametrized as**

[redacted]

3. **Debye mass (leading order)**

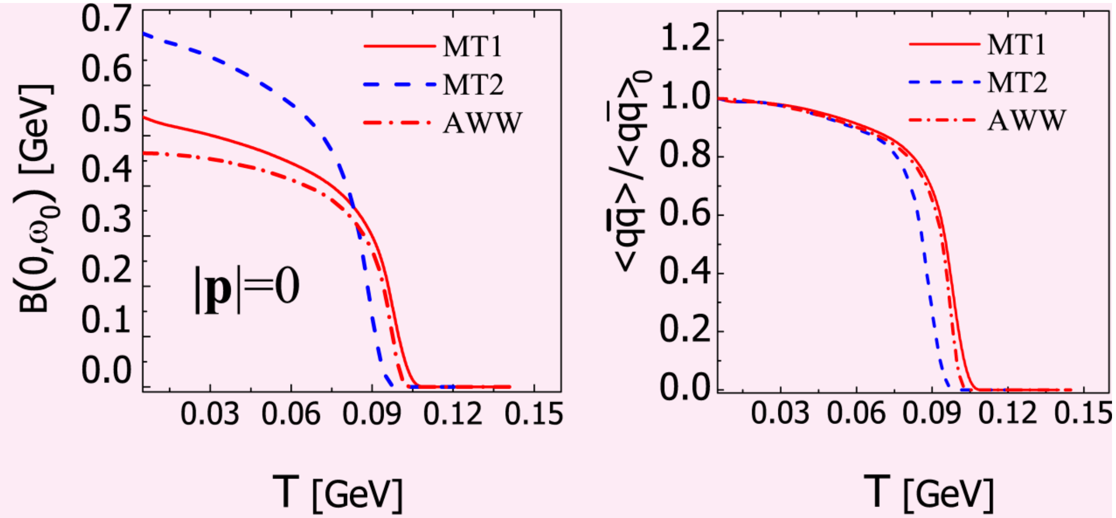
$$m_g^2 = \alpha_s\frac{\pi}{3}\left[2N_c+N_f\right]T^2, \qquad \alpha_s(E) \equiv \frac{g^2(E)}{4\pi} = 2\frac{12\pi}{11N_c-2N_f}$$

$$D^T(\mathbf{q},\Omega_{mn}(T),0) = D_{IR}(\mathbf{q}^2+\Omega_{mn}(T)^2) + D_{UV}(\mathbf{q}^2+\Omega_{mn}(T)^2),$$

$$D^L(\mathbf{q},\Omega_{mn}(T),m_g(T)) = D_{IR}(\mathbf{q}^2+\Omega_{mn}(T)^2+m_g^2(T)) + D_{UV}(\mathbf{q}^2+\Omega_{mn}(T)^2+m_g(T)^2).$$

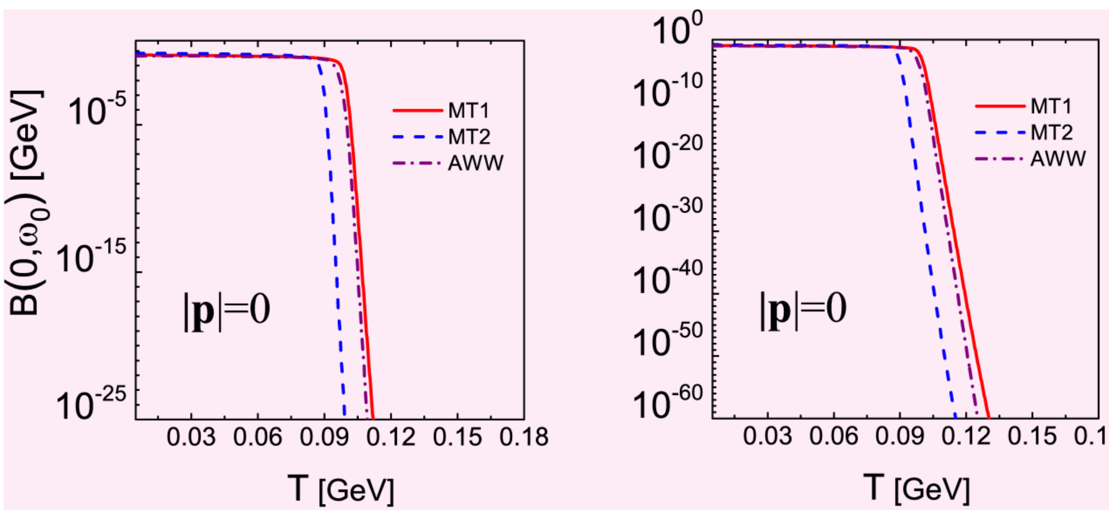
$$
\begin{aligned}
A(\boldsymbol{p}^2,\omega_n^2) &= 1+\frac{4}{3}T\sum_{m=-\infty}^{\infty}\int\frac{d\boldsymbol{k}}{(2\pi)^3}\left\{2\frac{\boldsymbol{qk}}{\boldsymbol{q}^2}\left(1-\frac{\boldsymbol{pk}}{\boldsymbol{p}^2}\right)\sigma_A(\boldsymbol{k},\omega_m)D^T(\boldsymbol{q},\Omega_{nm})\, + \right.\\
&\left[\frac{\boldsymbol{pk}}{\boldsymbol{p}^2}\sigma_A(\boldsymbol{k},\omega_m)+2\frac{\Omega_{mn}}{q^2}\left(1-\frac{\boldsymbol{pk}}{\boldsymbol{p}^2}\right)\omega_m\sigma_C(\boldsymbol{k},\omega_m)-\right.\\
&\left.\left.-2\frac{\Omega_{mn}^2}{q^2}\frac{\boldsymbol{qk}}{\boldsymbol{q}^2}\left(1-\frac{\boldsymbol{pk}}{\boldsymbol{p}^2}\right)\sigma_A(\boldsymbol{k},\omega_m)\right]D^L(\boldsymbol{q},\Omega_{nm},m_g)\right\},\\
B(\boldsymbol{p}^2,\omega_n^2) &= m_q+\frac{4}{3}T\sum_{m=-\infty}^{\infty}\int\frac{d\boldsymbol{k}}{(2\pi)^3}\left[D^L(\boldsymbol{q},\Omega_{nm},m_g)+2D^T(\boldsymbol{q},\Omega_{nm},0)\right]\sigma_B(\boldsymbol{k},\omega_m),\\
C(\boldsymbol{p}^2,\omega_n^2) &= 1+\frac{4}{3}T\sum_{m=-\infty}^{\infty}\int\frac{d\boldsymbol{k}}{(2\pi)^3}\left\{2\frac{\omega_m}{\omega_n}\sigma_C(\boldsymbol{k},\omega_m)D^T(\boldsymbol{q},\Omega_{nm},0)+\right.\\
&\left.\left[-\left(1-2\frac{\Omega_{mn}^2}{q^2}\right)\frac{\omega_m}{\omega_n}\sigma_C(\boldsymbol{k},\omega_m)+2\frac{\mathbf{qk}}{q^2}\frac{\Omega_{nm}}{\omega_n}\sigma_A\right]D^L(\boldsymbol{q},\Omega_{nm},m_g)\right\},
\end{aligned}
$$

$$
\sigma_F(\boldsymbol{k},\omega_m)=\frac{F(\boldsymbol{k},\omega_m)}{\boldsymbol{k}^2A^2(\boldsymbol{k},\omega_m)+\omega_m^2C^2(\boldsymbol{k},\omega_m)+B^2(\boldsymbol{k},\omega_m)} \qquad (1)
$$

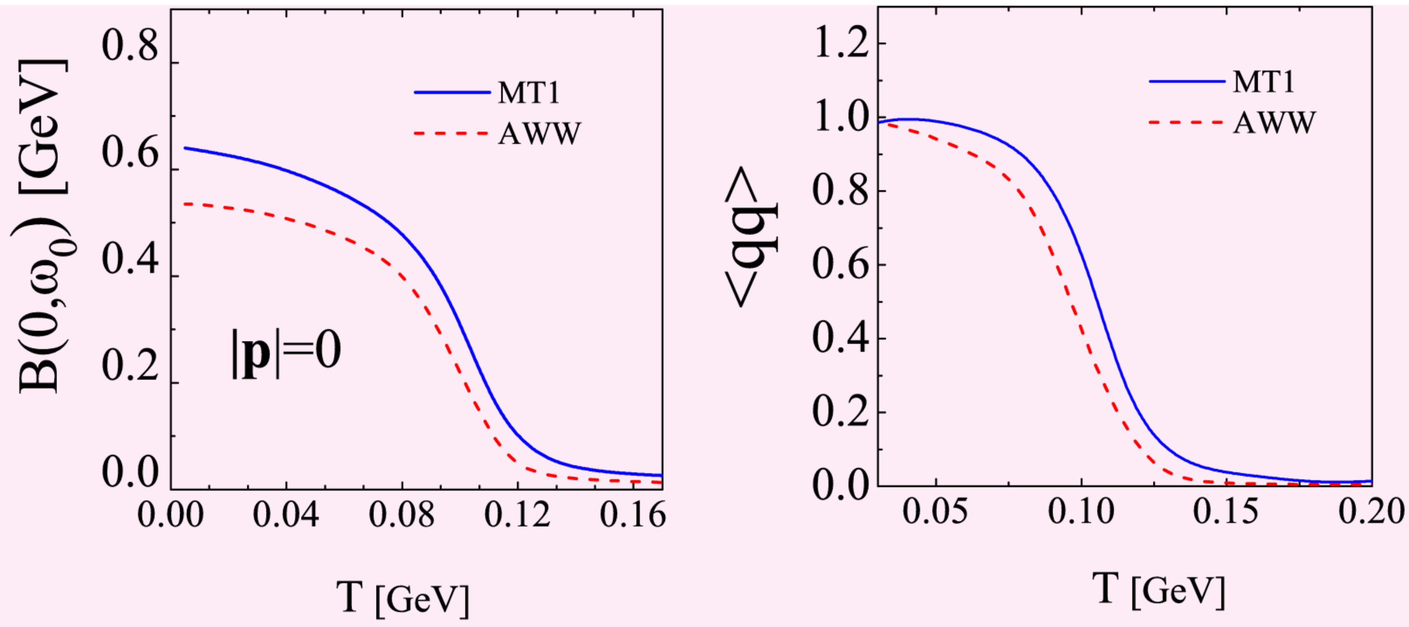


Solutions for $B(\boldsymbol{p}, \omega_0)$ in the chiral limit $m_q = 0$ for the lowest Matsubara frequency and the chiral condensate as functions of T.

$$T_c \approx 115 - 130 MeV$$



The same as above, but in log-scale

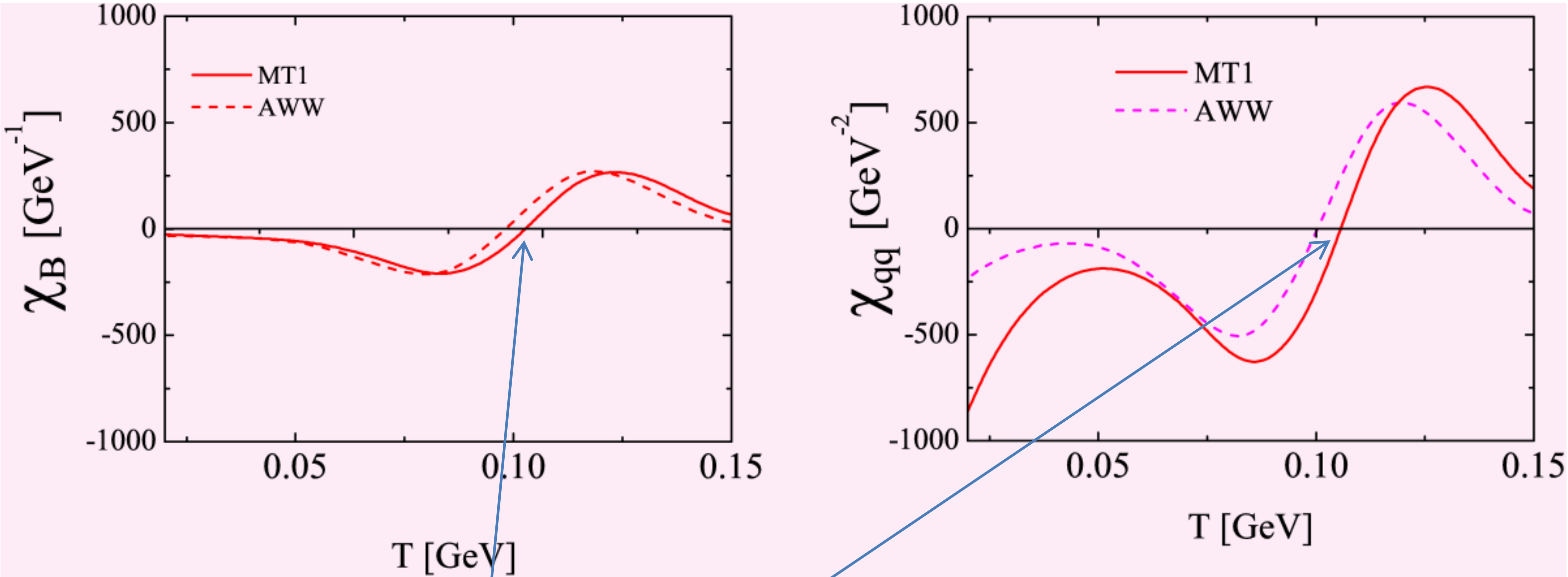


The solutions $B(\mathbf{p}=0,\omega_0)$ (left panel) and the quark condensate (right panel) for the light quark $m_l = 5$ MeV for the lowest Matsubara frequency.

Order parameters? One can use the inflection point of $B(0,\omega_0)$ and of $\langle q\bar{q}\rangle$, i.e. the maximum of the corresponding derivative with respect to the temperature

$$\chi_B(T) = \frac{d^2 B(0,\omega_0)}{dT^2}; \quad \chi_{qq}(T) = \frac{d^2\langle q\bar{q}\rangle}{dT^2}.$$

Then the (pseudo) critical temperature T_c is fixed by the condition that $\chi_B(T)|_{T=T_c} = 0$ and/or $\chi_{qq}(T)|_{T=T_c} = 0$.



The inflection points (second derivative with respect to temperature) for the mass function $B(\mathbf{p} = 0, \omega_0)$ (left panel) and for the normalized quark condensate, (right panel).

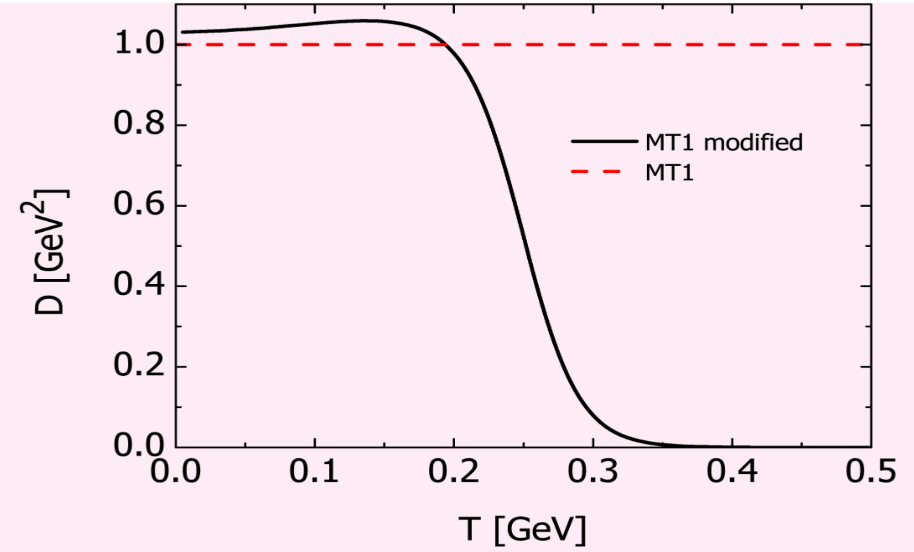
$T_c \sim 100 MeV$; T_c (lattice)$\sim$ 140 – 160 MeV

$$D_{IR}(k^2) = \frac{4\pi^2 D k^2}{\omega^6} e^{-k^2/\omega^2} e^{-m_g^2(T)/\omega^2}$$

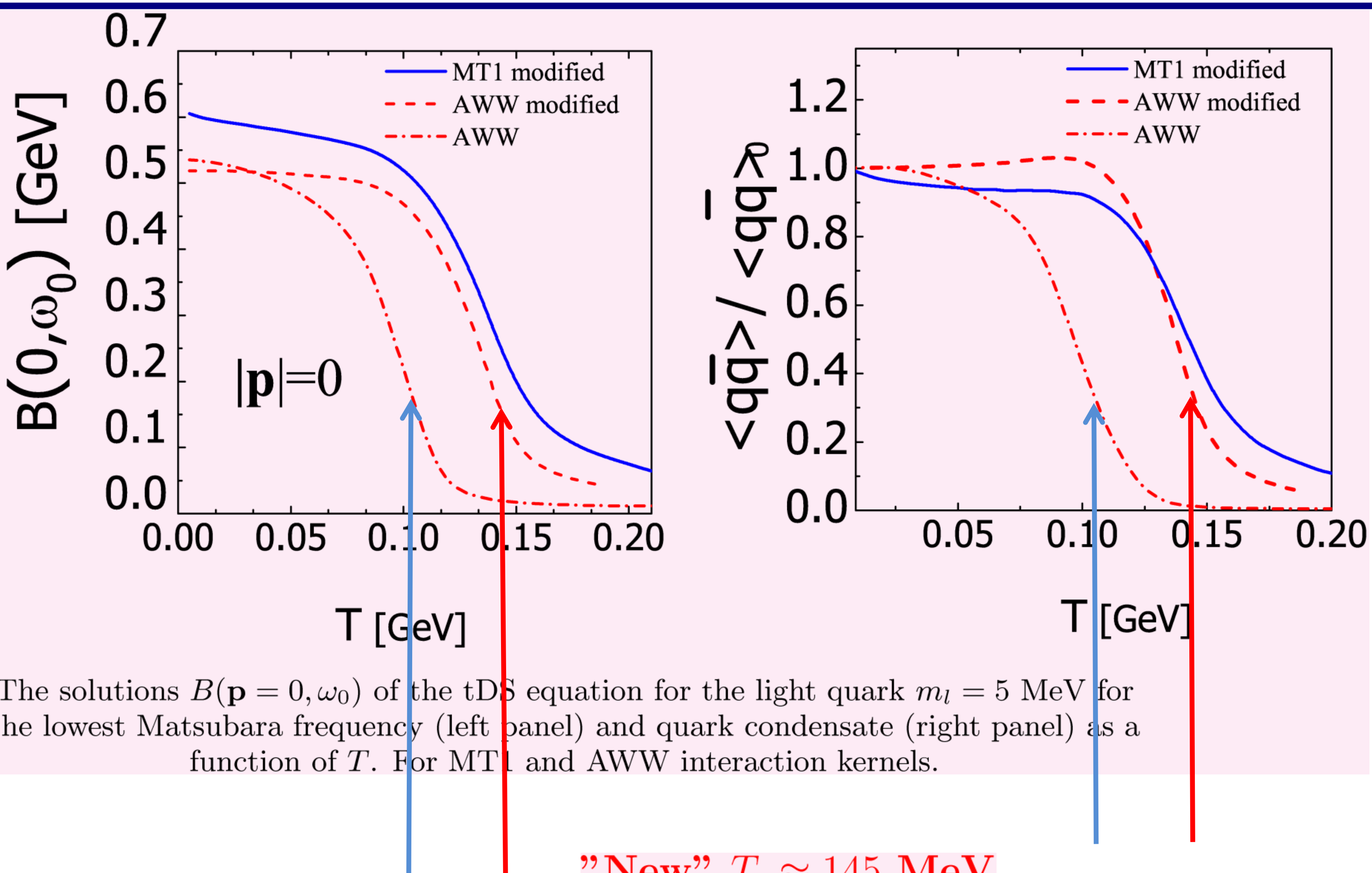
(i) in the IR term the Debye mass is omitted,
(ii) the parameter D receives a T-dependence,
(iii) the UV term, being inspired by perturbative QCD calculations, remains unchanged,

$$D_{IR}(k^2) = \frac{4\pi^2 D(T) k^2}{\omega^6} e^{-k^2/\omega^2}, \qquad D_{UV}(k^2 + {m_g}^2) = \frac{8\pi^2 \gamma_m F(k^2 + {m_g}^2)}{\ln[\tau + (1 + \frac{k^2 + {m_g}^2}{\Lambda_{QCD}^2})^2]}.$$

$$D(T) = D \left[a \left\{ 1 + \tanh\left(-\frac{T - T_p}{\beta} \right) \right\} + b \left\{ 1 - \tanh\left(-\frac{T - T_p}{\beta'} \right) \right\} \exp[-\alpha^2 T^2] \right],$$



An illustration of a possible dependence of the IR term D of the MT1 model on the temperature T.



The solutions $B(\mathbf{p}=0,\omega_0)$ of the tDS equation for the light quark $m_l = 5$ MeV for the lowest Matsubara frequency (left panel) and quark condensate (right panel) as a function of T. For MT1 and AWW interaction kernels.

"New" $T_c \approx 145$ MeV

"Old" $T_c \approx 100 - 110$ MeV

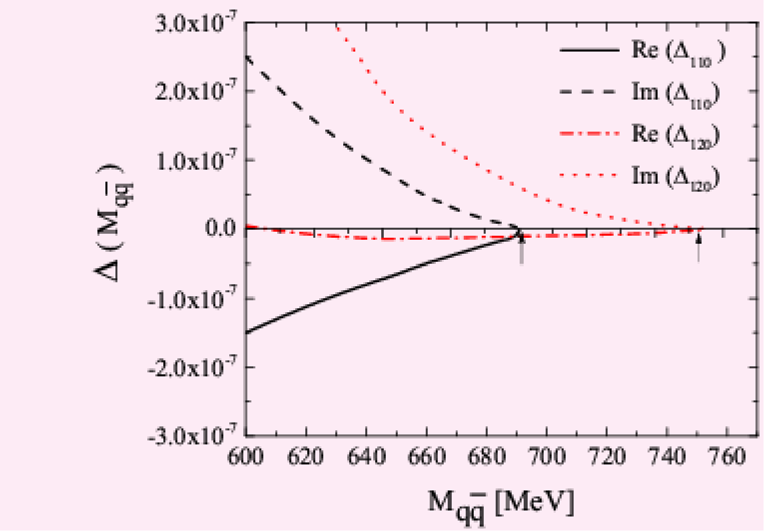
Lattice $T_c = (154 \pm 9)$ MeV (H.-T. Ding et al, Int. J. Mod. Phys. E 24 (2015) 1530007.)

Bethe-Salpeter equation for pseudoscalar particles (non-zero T)

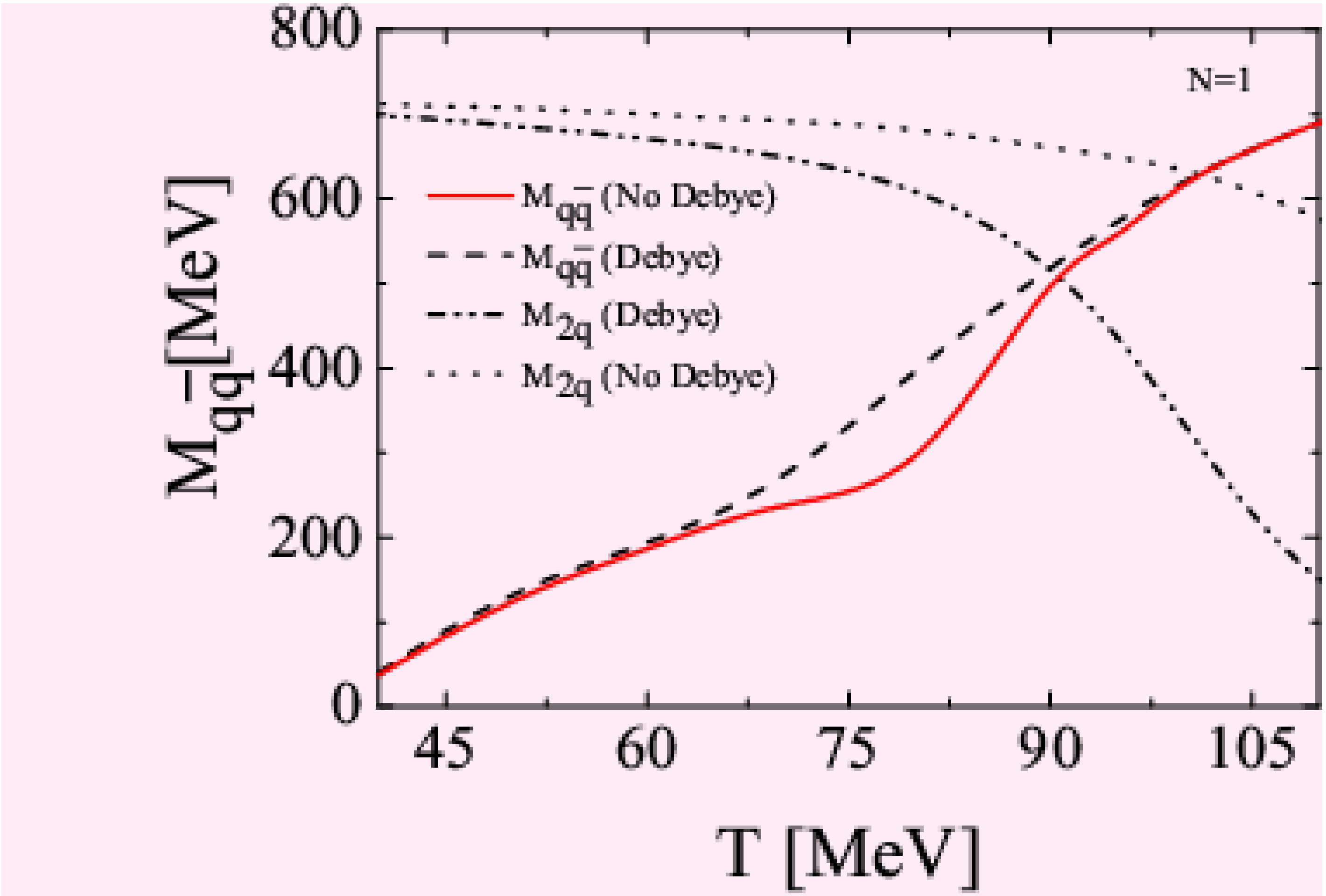
$$\tilde{\Gamma}(P_N, p_n) = \frac{4}{3} T \sum_m \int \frac{d^3q}{(2\pi)^3} \gamma_\mu S^{(+)}(1) \tilde{\Gamma}(P_N, q_m) \tilde{S}^{(-)}(2) \gamma_\nu D_{\mu\nu}(\kappa_{mn}),$$

$$\kappa_{mn} = (\boldsymbol{p} - \boldsymbol{q}, \omega_n - \omega_m)$$

$$\tilde{\Gamma}(P_N, p_n) = (g_1(\boldsymbol{p}, \omega_n) \frac{1}{2} \hat{I} + g_2(\boldsymbol{p}, \omega_n) \frac{1}{2} \gamma_4 + i g_3(\boldsymbol{p}, \omega_n) \frac{1}{2} \frac{\vec{\gamma} \boldsymbol{p}}{|\boldsymbol{p}|} + i g_4(\boldsymbol{p}, \omega_n) \frac{1}{2} \frac{\vec{\gamma} \boldsymbol{P}}{|\boldsymbol{P}|}) \gamma_5$$



Solving of the tBS equation



S.M. Dorkin

SUMMARY

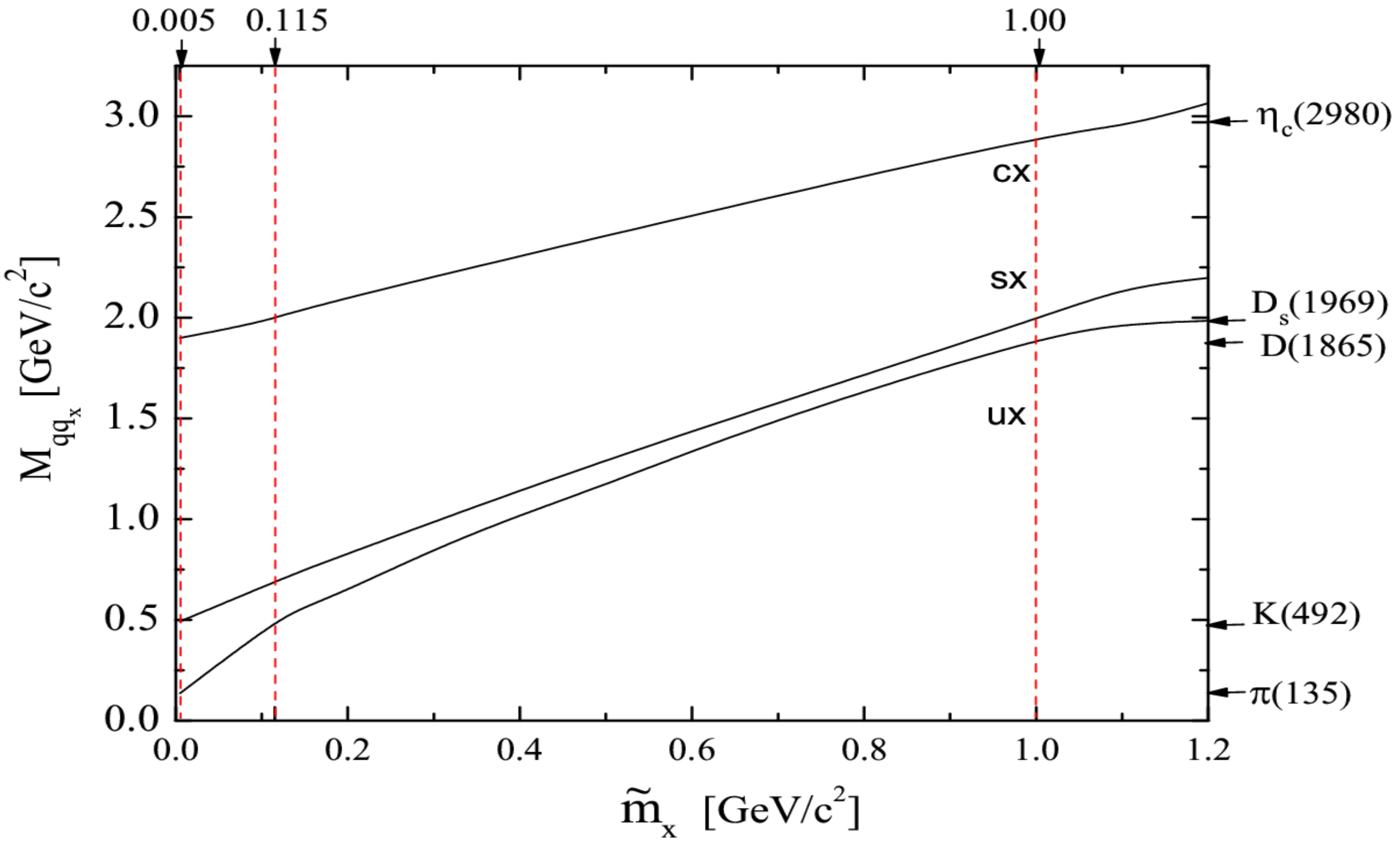
- The considered model, based on the Dyson–Schwinger-Bethe-Salpeter equations with only two-three effective parameters, describes fairly well the vacuum (T=0) properties (masses, electroweak decay constants…) of the scalar, pseudoscalar, vector etc. mesons, and allows for a Poincarè covariant study of processes with mesons

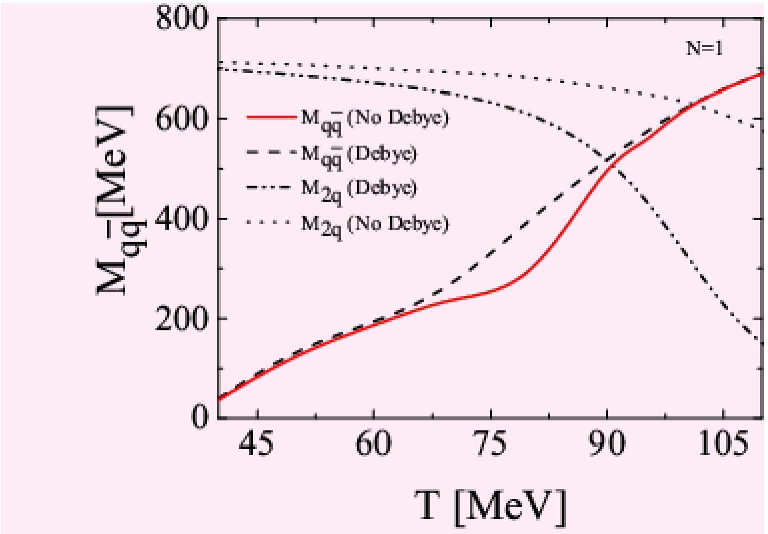
- Within such effective models one can investigate the analytical structure of the quark propagators related to such fundamental characteristics of QCD as confinement and dynamical chiral symmetry breaking phenomena encoded in the chiral condensate

- A direct generalization of the model for **finite** temperatures demonstrates that it still provides qualitatively descriptions of critical phenomena in hot matter, however qiantitavely the critical temperatures Tc relevant to possible signals of **QGP** are underestimate in comparison with lattice calculation results. This is a clear indication that the interaction kernel must receive an additional dependence on temperature.

- We propose a T-dependence of the interaction kernel, which suppresses the IR part at high temperatures and which provides (pseudo-) critical temperatures close to those from lattice calculation. The solutions of the BS-equation at large T (T>100MeV) give dissociation instability against fragmentation into state of two quasi free quarks.

SOLUTION BSE





N=1
$M_{q\bar{q}}$ (No Debye)
$M_{q\bar{q}}$ (Debye)
M_{2q} (Debye)
M_{2q} (No Debye)
$M_{q\bar{q}}$[MeV]
T [MeV]
0
200
400
600
800
45
60
75
90
105