Employment of microscopic model of optical potential for testing the $^{12,14}Be + p$ elastic scattering at 700 MeV

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Motivation

- *First*, in our study of the pA scattering we utilize **the microscopic folding type optical potential (OP)**[1] where one uses the 3-parameter analytic form of the elementary *NN* amplitude of scattering and the respective nuclear density distribution functions of $^{12,14}Be$.
- *Second*, when calculating the *pA* differential cross sections at energies of about 700 MeV we use the relativistic wave equation and thus the relativistic effects are accounted for exactly.
- *The aim* of our study is to explain experimental cross sections of the $^{12,14}Be + p$ elastic scattering[2] and thus to obtain the respective potentials and parameters of the *NN* amplitude bearing in mind to estimate the possible "in-medium" effect when the incident free nucleon interacts with the bounded one in nuclear matter.

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[1] V.Lukyanov *et al.*Phys. Atomic Nuclei,**69**(2006)240

[2] S.Ilieva *et al.* Nuclear Physics A,**875**(2012)8

Microscopic folding OP for *pA* elastic scattering

The microscopic OP is taken in the folding form as done in [1]

$$U(r) = -\frac{(\hbar c)\beta_c}{(2\pi)^2}\,\sigma\,[i+\alpha]\cdot\int j_0(qr)\,\rho(q)\,f(q)\,q^2dq,$$

where $\beta_c = \frac{k}{E}$, $f(q)$ – form factor of the NN-amplitude $F_{NN}(q)$ and $\rho(q)$ – form factor of a nuclear density distribution

$$\rho(q) = \int e^{i\mathbf{qr}}\,\rho(r)\,d^3r.$$

Here we use the charge-independent *NN* high-energy amplitude of scattering

$$F_{NN}(q) = \frac{k}{4\pi}\,\sigma\,[i+\alpha]\cdot f(q), \qquad f(q) = e^{-\beta q^2/2}$$

depended on 3 averaged parameters $\sigma,\ \alpha,\ \beta$ instead of 6 needed for the separate *pp*- and *pn*- amplitudes.

Relativistic wave equation for elastic scattering

The cross sections are calculated by solving the Klein-Gordon equation in its form at conditions $E \gg U$

$$\left(\Delta + k^2\right) \psi(\vec{r}) = 2\bar{\mu} U(r) \left[1 - U/2E\right] \psi(\vec{r}), \qquad U(r) = U^H(r) + U_C(r)$$

Here k is relativistic momentum of a nucleon in c.m. system,

$$k = \frac{M_A k^{\text{lab}}}{\sqrt{(M_A + m)^2 + 2 M_A T^{\text{lab}}}}, \quad k^{\text{lab}} = \sqrt{T^{\text{lab}} \left(T^{\text{lab}} + 2m\right)},$$

and $\bar{\mu} = \dfrac{E M_A}{E + M_A}$ – relativistic reduced mass, $E = \sqrt{k^2 + m^2}$ – total energy, m and M_A – the nucleon and nucleus masses.

Testing the ^{12,14}Be densities in the ^{12,14}Be+p elastic scattering at 700 MeV

In *Ilieva et al.* Nucl. Phys. A875 (2012):

- *Phenomenological* SF-form of the ^{12,14}Be densities are used

$$\rho(r) = \rho_0 \sum_{\epsilon=\pm 1} \frac{\epsilon}{1+\exp(\frac{r-\epsilon R}{a})}$$

- Parameters of radius R and diffuseness a in the SF-dens have been established by fitting to the data on elastic scattering cross sections of ^{12,14}Be+p at 700 MeV by using the Glauber approach.

Our calculation:

- Together with the phenomenological SF-density we apply the VMC and GCM densities of ^{12,14}Be obtained in the framework of the so-called *microscopic* model calculations;
- These densities are used to calculate the folding optical potentials;
- Then we obtain the respective differential cross sections by solving the relativistic wave equation.

Densities of ^{12,14}Be

Density distributions of ^{12}Be and ^{14}Be

1. **Variational Monte Carlo (VMC) model**

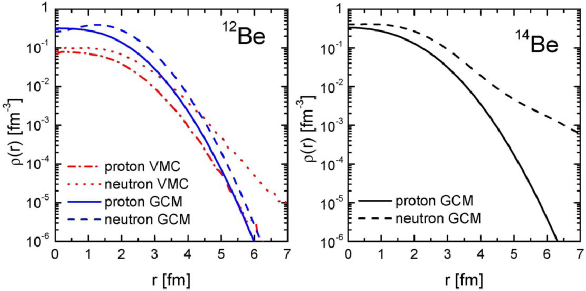
   S. C. Pieper, private communication

   The proton and neutron densities have been computed with the AV18+UX Hamiltonian, in which the Argonne v18 two-nucleon and Urbana X three-nucleon potentials are used.

2. **Generator Coordinate Method (GCM)**

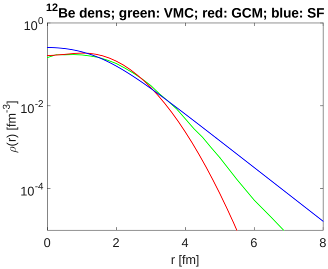
   P. Descouvemont, Phys. Rev. C 52, **704** (1995)

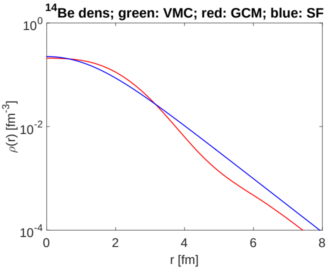
   The ^{14}Be nucleus is investigated in the three-cluster GCM, involving several ^{12}Be+n+n configurations. The ^{12}Be core nucleus is described in the harmonic oscillator model with all possible configurations in the p shell.



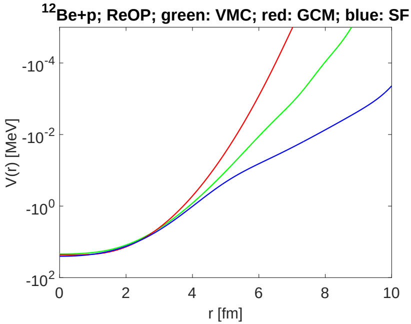
Densities of ^{12,14}Be (total)

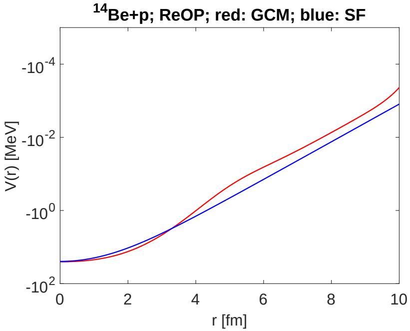
- Generator Coordinate Method (GCM) from Descouvemont, PRC52 (1995)
- Variational Monte Carlo (VMC) model from Pieper, Private Comm. (only ^{12}Be)
- Symmetrized Fermi function (SF); parameters of radius and diffuseness from Ilieva et al. Nucl. Phys. A875 (2012)





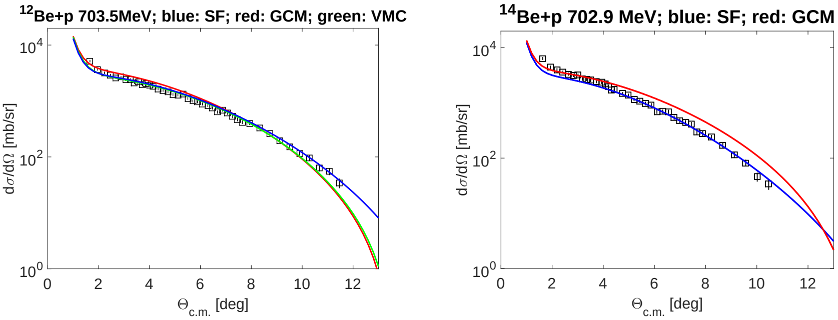
Optical potentials of the ^{12,14}Be+p scattering





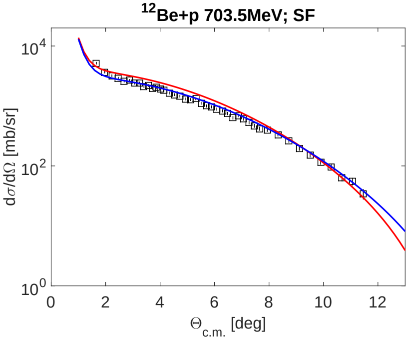
- In the model calculations, forms of the Re and Im parts of OP are the same, and their depths are proportional to $\sigma\alpha$ and σ, respectively.
- For ^{12}Be the all three densities behave differently at r> 4 fm, while for ^{14}Be both densities SF and GSM have here almost the same slope but slightly different absolute values.

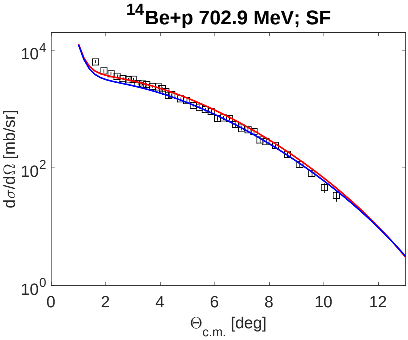
The ^{12,14}Be+p elastic scattering at 700 MeV



- For ^{12}Be the both microscopic densities provide a reasonable agreement with the data at $\Theta < 10^{\circ}$ while SF density is fitting all the data.
- For ^{14}Be one sees acceptable agreements for the GCM density at $\Theta < 5^{\circ}$, but for the better comparison a little change of σ and β parameters are needed.

Comparison of results for two sets of NN-parameters


^{12}Be+p 703.5MeV; SF


^{14}Be+p 702.9 MeV; SF

Blue - for NN-parameters from Ref.[3], Red - from Ref.[2]
The SF parameters are those obtained in Ref.[2]

- ^{12}Be: R=1.37(25), a=0.67(3), R_m=2.70(3)
- ^{14}Be: R=0.99(45), a=0.84(4), R_m=3.21(5)

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[3] Shukla P. 2003 Phys. Rev. C 67 054607

Parameters

nucleus	parameter	Our calc	Nucl Phys A 875 (2012)
^{12}Be	σ (fm^2)	3.870	4.363(pp); 3.758(pn)
	α	0.25	0.092(pp); -0.298(pn)
	β (fm^2)	0.22	0.17
^{14}Be	σ (fm^2)	3.861	4.359(pp); 3.758(pn)
	α	0.25	0.093(pp); -0.298 (pn)
	β (fm^2)	0.22	0.17

Our parameters are calculated via formulas from Shukla P. 2003 Phys. Rev. C 67 054607

Summary

- We use the folding optical potentials which include the nuclear density and NN-amplitude, and thus their forms and parameters are tested when calculated cross sections are compared with existing experimental data.
- In calculations, the relativistic Klein-Gordon wave equation was applied to get the wave functions of scattering and respective differential cross sections of the proton-nucleus scattering.
- Comparisons of calculated cross sections with existing data show good agreements for the phenomenological SF nuclear density having a long tail in the case of ^{14}Be nucleus while the GSM and VMC densities give acceptable agreements for scattering on ^{12}Be for both densities, but some limited agreement for scattering on ^{14}Be.
- Comparisons of our calculations with those basing on Glauber approximation show close agreement while the respective parameters of the NN-amplitude are slightly different.

Thank you
for your attention!