



XXIV International Baldin Seminar
on High Energy Physics Problems
Relativistic Nuclear Physics & Quantum Chromodynamics

September 17 - 22, 2018, Dubna, Russia



Scaling properties of negative particle production in Au+Au collisions from BES-I at RHIC

M. Tokarev*

A. Kechechyan* & I. Zborovský**

*Joint Institute for Nuclear Research, Dubna, Russia

**Nuclear Physics Institute of the CAS, Řež, Czech Republic



- Introduction
- z -Scaling (ideas, definitions,...)
- Properties of data z -presentation
- Self-similarity of negative particle production in **AuAu** collisions at **RHIC**
- Momentum fraction, recoil mass and constituent energy loss vs. \sqrt{s} , centrality, p_T
- Summary



Search for signatures of a phase transition in nuclear matter produced in heavy ion collisions at high energies

Systematic analysis of hadron spectra in pp , pA and AA collisions to search for general features of structure, interaction and fragmentation over a wide scale range

z -Scaling as a tool in high energy physics

Development of z -scaling approach for description of hadron production in AA collisions and verification of self-similarity principle

- New analysis of **STAR** data on negative particle spectra in **AuAu** collisions from **BES-I**
- Verification of scaling properties of spectra in z -presentation
- Development of microscopic scenario of hadron production in AA





"Fundamental symmetry principles dictate the basic laws of physics, control the structure of matter, and define the fundamental forces in Nature."

Leon M. Lederman

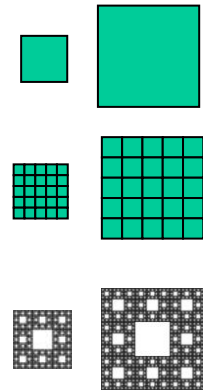
Self-similarity is a property of physical phenomena and the principle to construct theories.

“Scaling” and “Universality” are concepts developed to understanding critical phenomena. Scaling means that systems near the critical points exhibiting self-similar properties are invariant under transformation of a scale. According to universality, quite different systems behave in a remarkably similar fashion near the respective critical points. Critical exponents are defined only by symmetry of interactions and dimension of the space.

H.Stanley, G.Barenblatt,...



- A self-similar object is exactly or approximately similar to a part of itself (i.e. the whole has the same shape as one or more of the parts).
- Self-similarity is a typical property of fractals.
- Scale invariance is an exact form of self-similarity where at any magnification there is a smaller piece of the object that is similar to the whole.



Description of a process in terms of a scaling function and similarity parameter

Reynolds number

$$Re = \rho V D / \eta$$

laminar & turbulent flow



Mach number

$$Ma = v/c$$

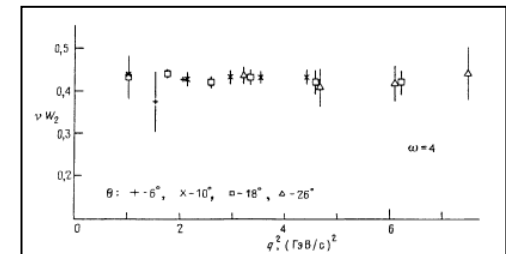
subsonic & supersonic wave



Bjorken variable

$$x = -q^2 / 2(pq)$$

low x & high x

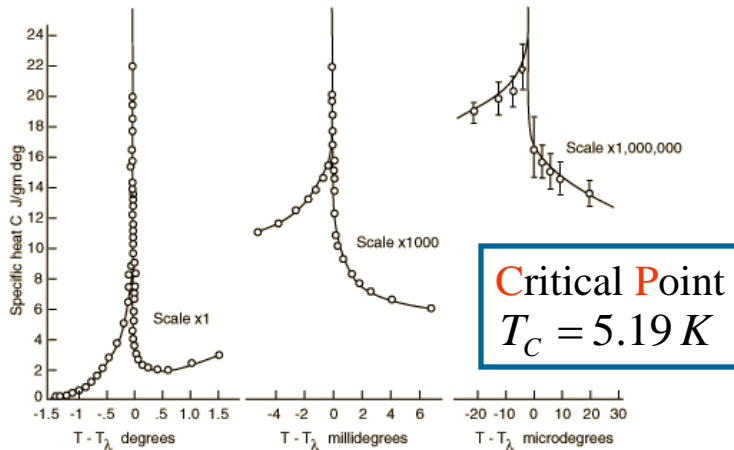


shock wave, explosion, confinement

Violation of a scaling is an indication of new phenomena

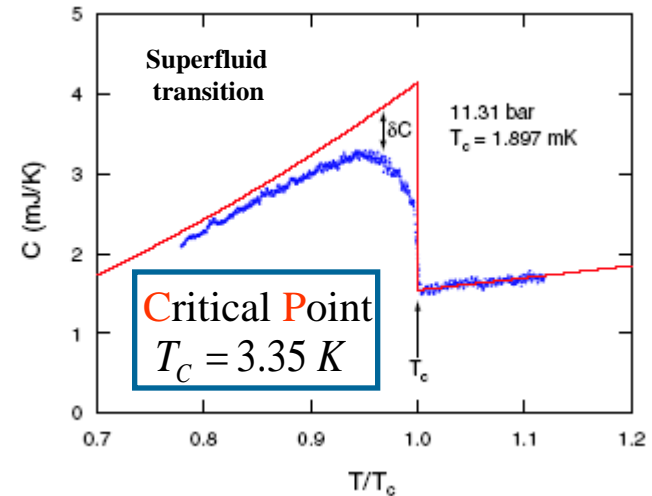


Specific heat of liquid ^4He



H.E. Stanley, 1971

Heat capacity of liquid ^3He



H. Choi et al., PRL 96, 125301 (2006)

- Near a critical point the singular part of thermo-dynamic potentials is a Generalized Homogeneous Function (GHF).
- The Gibbs potential $G(\lambda^{\alpha_\varepsilon} \varepsilon, \lambda^{\alpha_p} p) = \lambda G(\varepsilon, p)$ is GHF of (ε, p) .

$$c_V \sim |\varepsilon|^{-\alpha}$$

$$\varepsilon \equiv (T - T_c) / T_c$$

$$c_V = -T(d^2G / dT^2)$$

Critical exponents define the behavior of thermo-dynamical quantities nearby the Critical Point.

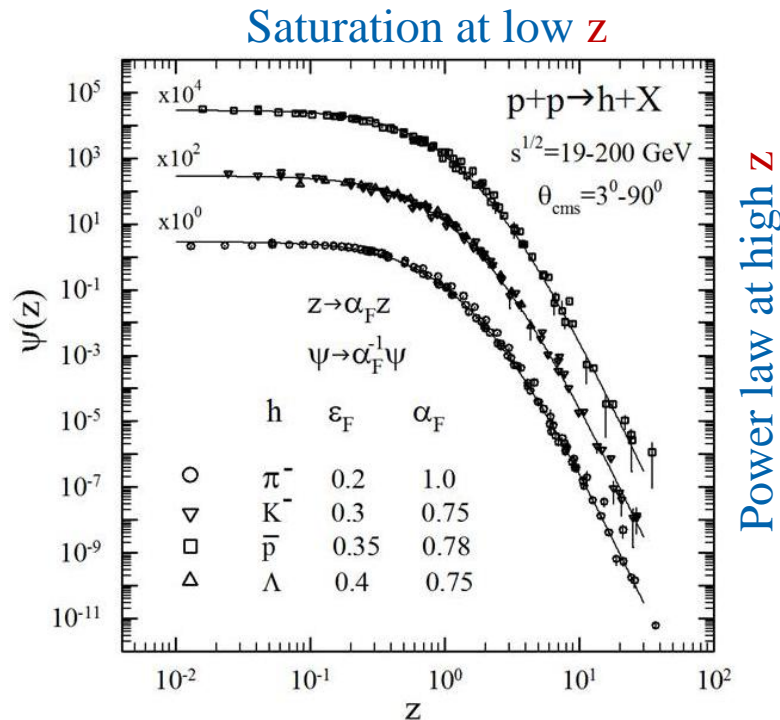


Inclusive cross sections of π^- , K^- , \bar{p} , Λ in pp collisions

FNAL:
PRD 75 (1979) 764

ISR:
NPB 100 (1975) 237
PLB 64 (1976) 111
NPB 116 (1976) 77
(low p_T)
NPB 56 (1973) 333
(small angles)

STAR:
PLB 616 (2005) 8
PLB 637 (2006) 161
PRC 75 (2007) 064901



Energy scan of spectra at U70, ISR, $S\bar{p}pS$, SPS, HERA, FNAL(fixed target), Tevatron, RHIC, LHC

MT & I.Zborovsky
T.Dedovich

Phys.Rev.D75,094008(2007)
Int.J.Mod.Phys.A24,1417(2009)
J. Phys.G: Nucl.Part.Phys.
37,085008(2010)
Int.J.Mod.Phys.A27,1250115(2012)
J.Mod.Phys.3,815(2012)

- Energy & angular independence
- Flavor independence (π , K , \bar{p} , Λ)
- Saturation for $z < 0.1$
- Power law $\Psi(z) \sim z^{-\beta}$ for high $z > 4$

Scaling – “collapse” of data points onto a single curve.
Universality classes – hadron species (ϵ_F , α_F).



Universality: flavor independence of the scaling function

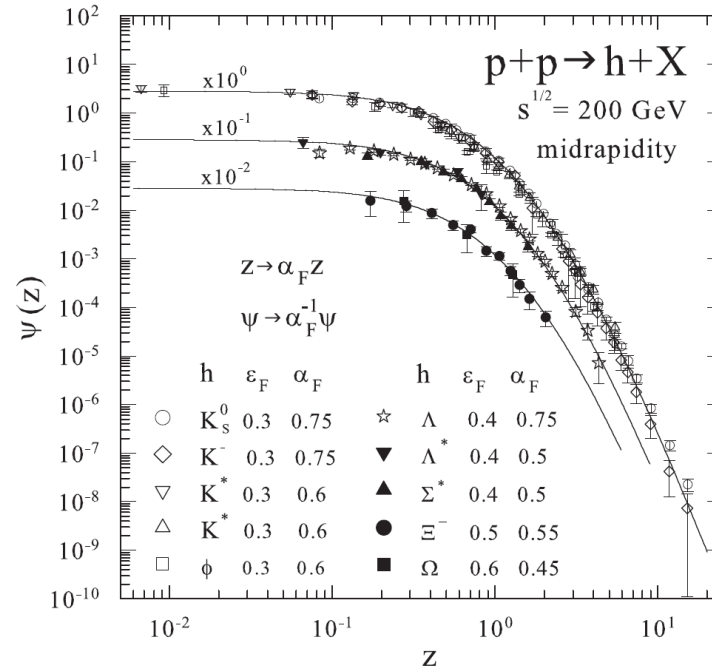
$K_S^0, K^-, K^*, \phi, \Lambda, \Xi, \Omega, \Sigma^*, \Lambda^*$



PRL 92 (2004) 092301
 PRL 97 (2006) 132301
 PLB 612 (2005) 181
 PRC 71 (2005) 064902
 PRC 75 (2007) 064901
 PRL 108 (2012) 072302

M.T. & I. Zborovský
 Int.J.Mod.Phys.
 A32,1750029(2017)

Solid line for π^- meson
 is a reference frame
 $\varepsilon_\pi = 0.2, \alpha_\pi = 1$



PHENIX
 PRC 75 (2007) 051902
 PRD 83 (2011) 052004
 PRC 90 (2014) 054905

- Energy independence
- Angular independence
- Flavor independence
- Saturation for $z < 0.01$
- Power law $\Psi(z) \sim z^{-\beta}$ at large z
- ε_F, α_F independent of $p_T, s^{1/2}$



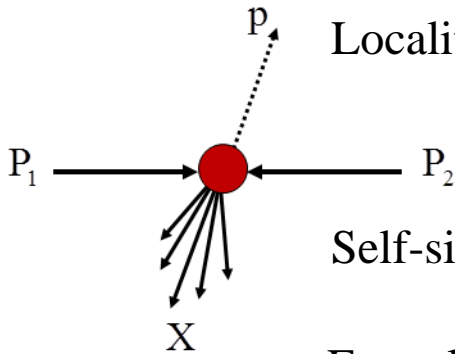
- Energy independence of $\Psi(z)$ ($s^{1/2} > 20 \text{ GeV}$)
- Angular independence of $\Psi(z)$ ($\theta_{\text{cms}} = 3^\circ - 90^\circ$)
- Multiplicity independence of $\Psi(z)$ ($dN_{\text{ch}}/d\eta = 1.5 - 26$)
- Saturation of $\Psi(z)$ at low z ($z < 0.1$)
- Power law, $\Psi(z) \sim z^{-\beta}$, at high z ($z > 4$)
- Flavor independence of $\Psi(z)$ ($\pi, K, \phi, \Lambda, \dots, D, J/\psi, B, Y, \dots, \text{top}$)

These properties reflect **self-similarity**, **locality**, and **fractality** of hadron interactions at a constituent level.

It concerns the **structure** of the colliding objects, constituent interactions and fragmentation process.



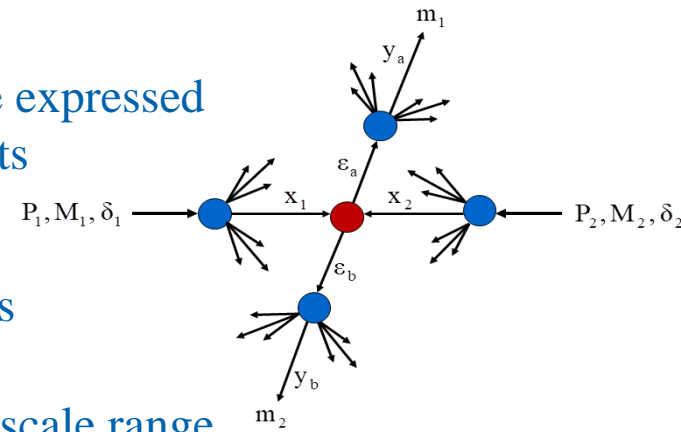
Principles: locality, self-similarity, fractality



Locality: collisions of hadrons and nuclei are expressed via interactions of their constituents (partons, quarks and gluons,...).

Self-similarity: interactions of the constituents are mutually similar.

Fractality: self-similarity is valid over a wide scale range.



Hypothesis of z-scaling :

$$s^{1/2}, p_T, \theta_{cms}$$

Inclusive particle distributions can be described in terms of constituent sub-processes and parameters characterizing bulk properties of the system.

$$X_1, X_2, Y_a, Y_b$$

$$\delta_1, \delta_2, \epsilon_a, \epsilon_b, c$$

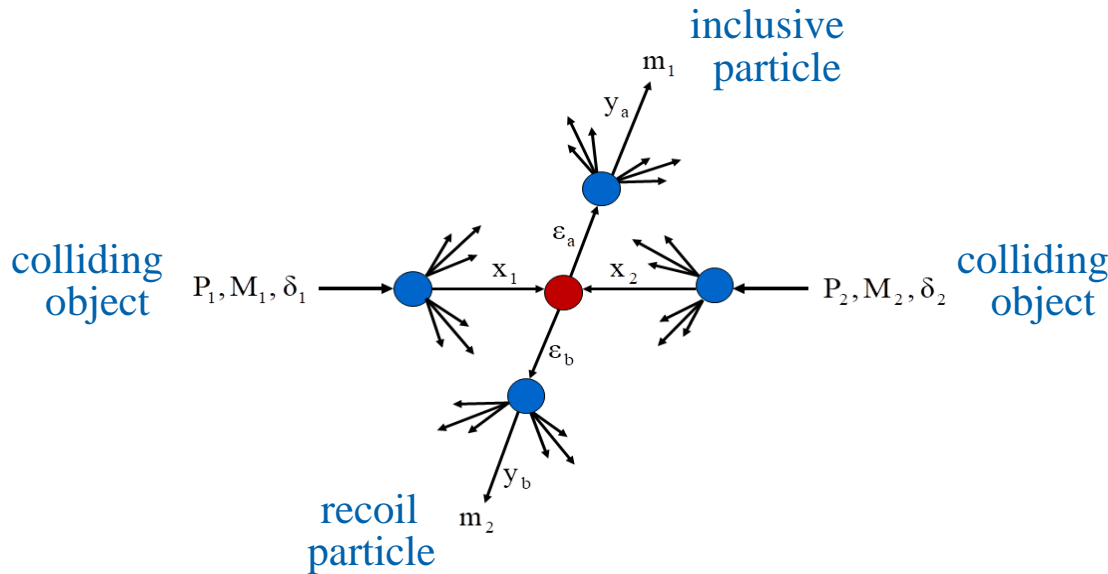
$$Ed^3\sigma/dp^3$$

Scaled inclusive cross section of particles depends in a self-similar way on a single scaling variable z .

$$\Psi(z)$$



Collisions of colliding objects are expressed via interactions of their constituents



P_1, P_2, p – momenta of colliding and produced particles

M_1, M_2, m_1 – masses of colliding and produced particles

x_1, x_2 – momentum fractions of colliding particles carried by constituents

y_a, y_b – momentum fractions of scattered constituents carried by inclusive particle and its recoil

δ_1, δ_2 – fractal dimensions of colliding particles

ϵ_a, ϵ_b – fractal dimensions of scattered constituents (fragmentation dimensions)

m_2 – mass of recoil particle

Elementary sub-process:

$$(x_1 M_1) + (x_2 M_2) \rightarrow (m_1 / y_a) + (x_1 M_1 + x_2 M_2 + m_2 / y_b)$$

Momentum conservation law for sub-process

$$(x_1 P_1 + x_2 P_2 - p / y_a)^2 = M_X^2$$

Mass of recoil system

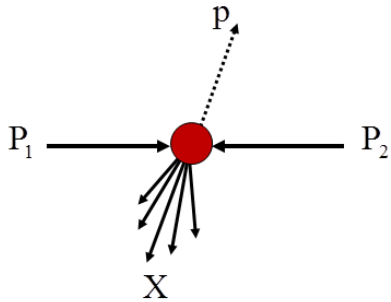
$$M_X = x_1 M_1 + x_2 M_2 + m_2 / y_b$$

M.Tokarev, I.Zborovský
Yu.Panebratsev, G.Skoro
Phys.Rev.D54 5548 (1996)
Int.J.Mod.Phys.A16 1281 (2001)



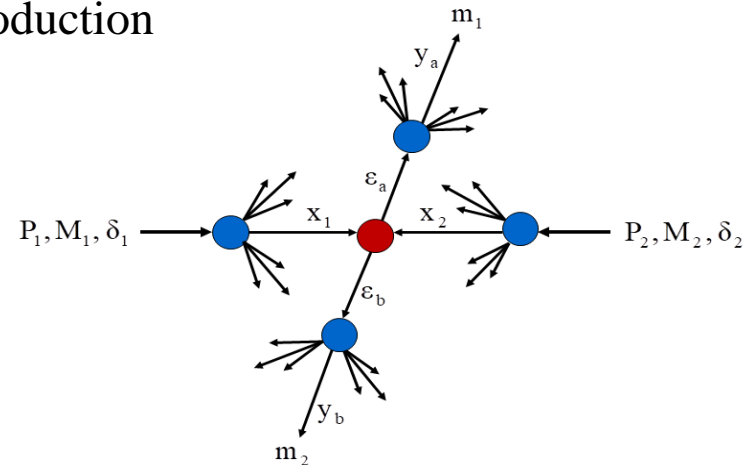
Interactions of constituents are mutually similar

The self-similarity parameter z is a dimensionless quantity, expressed through the dimensional values $P_1, P_2, p, M_1, M_2, m_1, m_2$, characterizing the process of inclusive particle production



$$z = z_0 \cdot \Omega^{-1}$$

$$z_0 = \frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$



- Ω^{-1} is the minimal resolution at which a constituent sub-process can be singled out of the inclusive reaction
- $s_{\perp}^{1/2}$ is the transverse kinetic energy of the sub-process consumed on production of m_1 & m_2
- $dN_{ch}/d\eta|_0$ is the multiplicity density of charged particles at $\eta = 0$
- c is a parameter interpreted as a “specific heat” of created medium
- m_N is an arbitrary constant (fixed at the value of nucleon mass)

Self-similarity over a wide scale range

Fractal measure

$$z = z_0 \cdot \Omega^{-1}$$

$$\Omega = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2} (1 - y_a)^{\varepsilon_a} (1 - y_b)^{\varepsilon_b}$$

$$0 < x_1, x_2 < 1$$

$$0 < y_a, y_b < 1$$

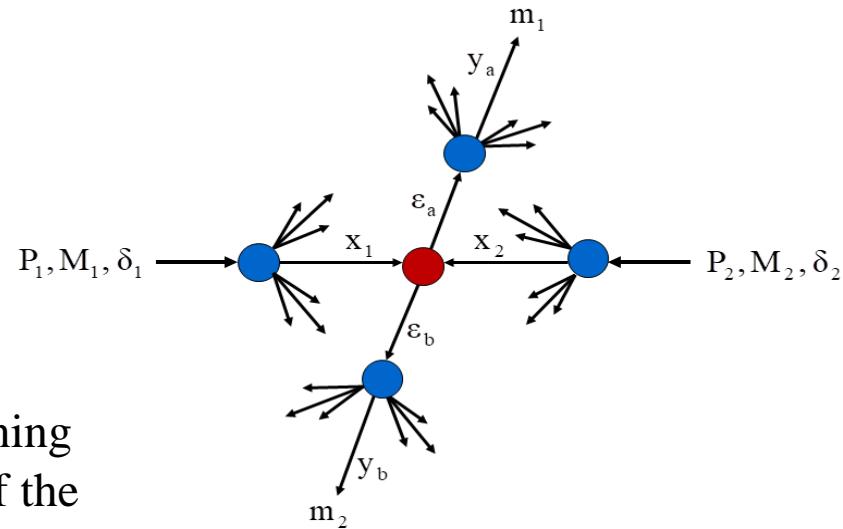
Ω is relative number of configurations containing a sub-process with fractions x_1, x_2, y_a, y_b of the corresponding 4-momenta

$\delta_1, \delta_2, \varepsilon_a, \varepsilon_b$ are parameters characterizing structure of the colliding objects and fragmentation process, respectively

$\Omega^{-1}(x_1, x_2, y_a, y_b)$ characterizes resolution at which a constituent sub-process can be singled out of the inclusive reaction

The fractal measure z diverges as the resolution Ω^{-1} increases.

$$z(\Omega) \Big|_{\Omega^{-1} \rightarrow \infty} \rightarrow \infty$$



Principle of minimal resolution: The momentum fractions x_1, x_2 and y_a, y_b are determined in a way to minimize the resolution Ω^{-1} of the fractal measure z with respect to all constituent sub-processes taking into account 4-momentum conservation law:

Momentum conservation law

$$(x_1 P_1 + x_2 P_2 - p/y_a)^2 = M_X^2$$

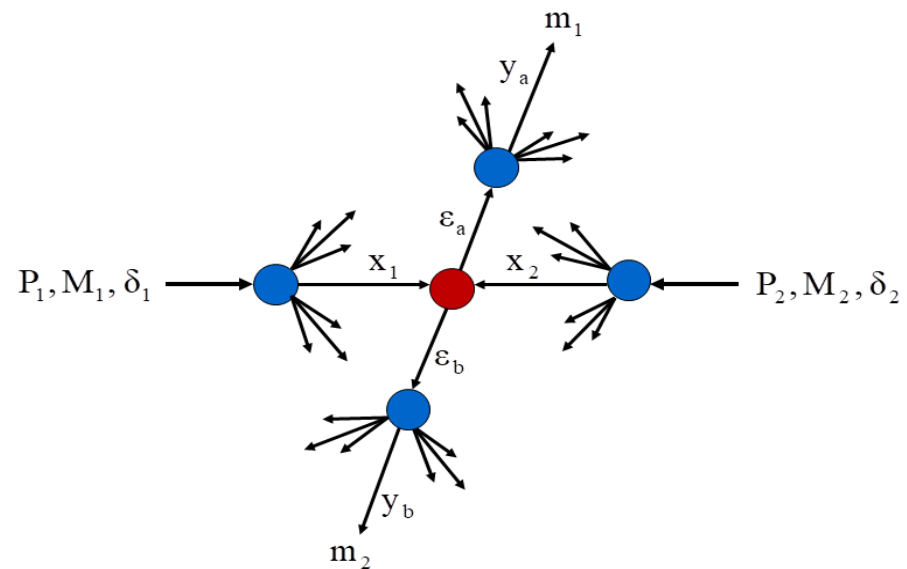
$$\begin{cases} \partial\Omega / \partial x_1 |_{y_a=y_a(x_1, x_2, y_b)} = 0 \\ \partial\Omega / \partial x_2 |_{y_a=y_a(x_1, x_2, y_b)} = 0 \\ \partial\Omega / \partial y_b |_{y_a=y_a(x_1, x_2, y_b)} = 0 \end{cases}$$

Resolution of sub-process

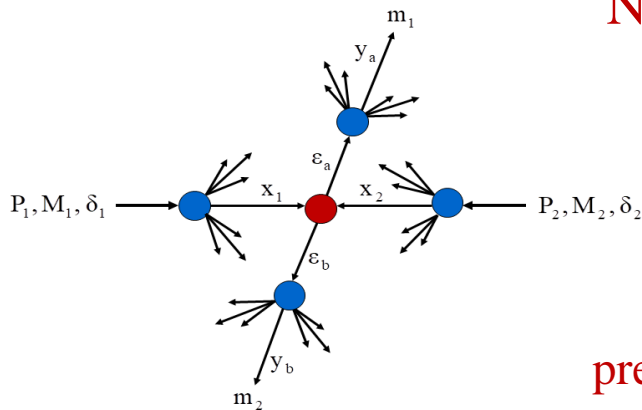
$$\Omega^{-1} = (1 - x_1)^{-\delta_1} (1 - x_2)^{-\delta_2} (1 - y_a)^{-\varepsilon_a} (1 - y_b)^{-\varepsilon_b}$$

Mass of recoil system

$$M_X = x_1 M_1 + x_2 M_2 + m_2 / y_b$$



Fractions x_1, x_2, y_a, y_b are expressed via Lorentz invariants – scalar products of 4-D momenta and particle masses.



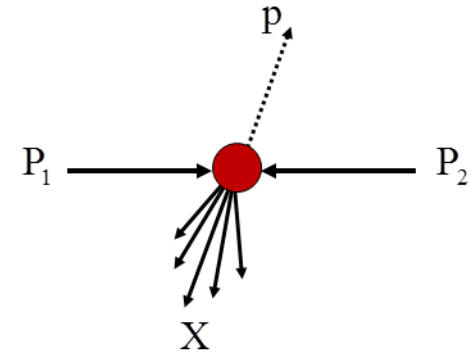
Normalization condition

$$\int_0^{\infty} \Psi(z) dz = 1$$

Scale transformation

$$z \rightarrow \alpha_F z \quad \Psi(z) \rightarrow \alpha_F^{-1} \Psi(z)$$

preserves the normalization condition



$$\Psi(z) = \frac{\pi}{(dN/d\eta) \cdot \sigma_{inel}} \cdot J^{-1} \cdot E \frac{d^3\sigma}{dp^3} \quad \longleftrightarrow \quad \int E \frac{d^3\sigma}{dp^3} dy d^2p_{\perp} = \sigma_{inel} \cdot \langle N \rangle$$

- σ_{in} - the inelastic cross section
- $\langle N \rangle$ - the average multiplicity
- $dN/d\eta$ - the multiplicity density
- $J(z, \eta; p_T^2, y)$ - the Jacobian
- $E d^3\sigma/dp^3$ - the inclusive cross section

The scaling function $\Psi(z)$ is probability density to produce the inclusive particle with the corresponding z .

Self-similarity of h^- hadron production in pp collisions

pp is of interest by itself:

- verification and search for new features
- search for a phase transition with different probes

pp interactions is a reference frame for pA and AA physics

V.V. Abramov et al., Sov. J. Nucl. Phys. 31 (1980) 484

V.V. Abramov et al., JETP Lett. 33 (1981) 289

D. Antreasyan et al., Phys. Rev. D 19 (1979) 764

D. E. Jaffe et al., Phys. Rev. D 40 (1989) 2777

A. Breakstone et al., Z. Phys. C 69 (1995) 55

D. Drijard et al., Nucl. Phys. B 208 (1982) 1

G. Agakishiev et al., Phys. Rev. Lett. 108 (2012) 072302



Self-similarity parameter

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$

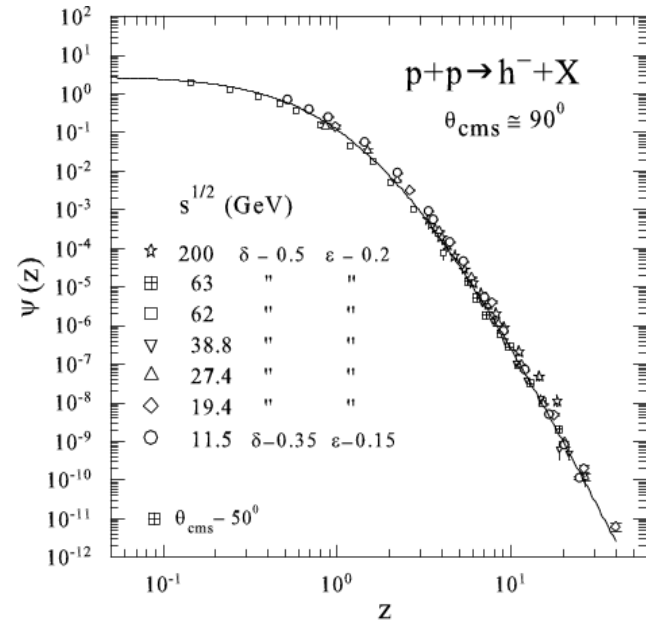
$$\Omega = (1-x_1)^\delta (1-x_2)^\delta (1-y_a)^{\varepsilon_F} (1-y_b)^{\varepsilon_F}$$

- $dN_{ch}/d\eta|_0$ - multiplicity density
- c - “specific heat” of bulk matter
- δ - proton fractal dimension
- ε_F - fragmentation fractal dimension

Scaling function

$$\Psi(z) = \frac{\pi}{(dN/d\eta) \cdot \sigma_{inel}} \cdot J^{-1} \cdot E \frac{d^3\sigma}{dp^3}$$

“Collapse” of data onto a single curve



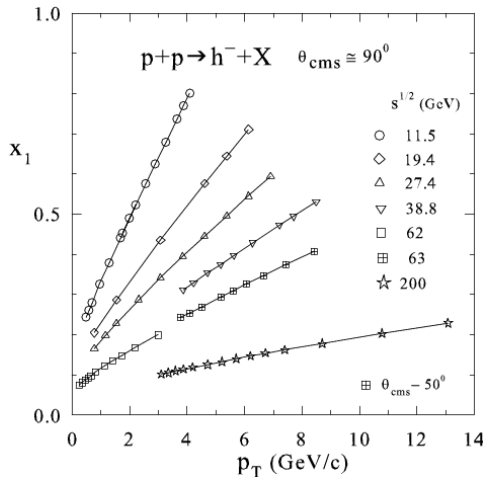
- Energy independence of $\Psi(z)$
- Centrality independence of $\Psi(z)$
- Power law at high z
- Saturation at low z

Universality: the same shape of Ψ vs. \sqrt{s} , p_T



Constituent level of particle production in terms of

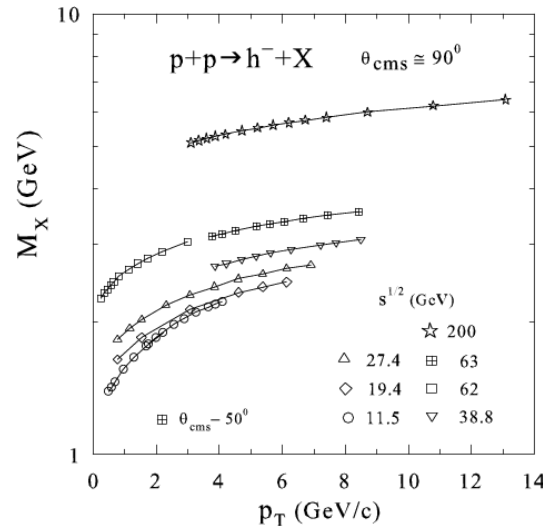
Momentum fraction x_1



Momentum fraction

- increases with p_T
- decreases with \sqrt{s}

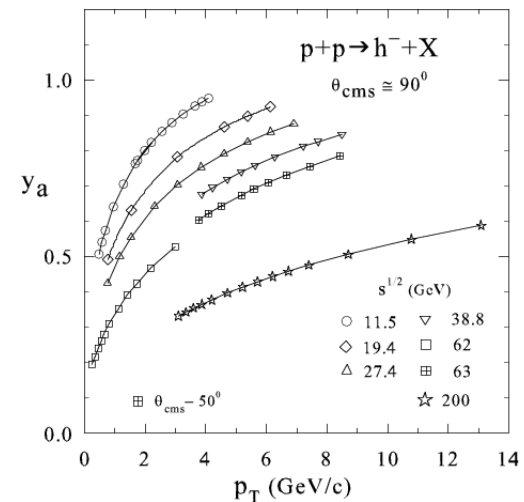
Recoil mass M_X



Recoil mass

- increases with p_T
- increases with \sqrt{s}

Energy loss $\Delta E/E \sim (1-y_a)$

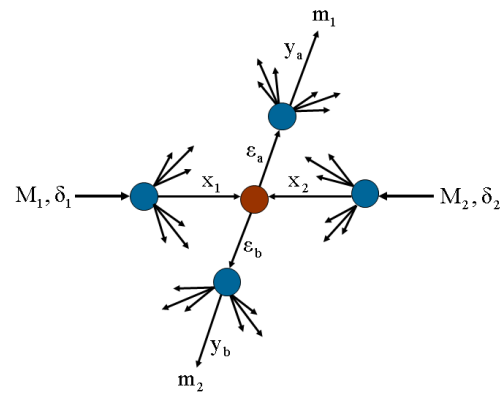


Constituent energy loss

- decreases with p_T
- increases with \sqrt{s}

- pp is a reference frame for pA and AA
- physics at high x_1 and p_T → nearby a kinematic boundary
- asymptotic behavior of $\Psi(z)$ at high z → power law

Energy loss $\Delta E/E \sim (1-y_a)$

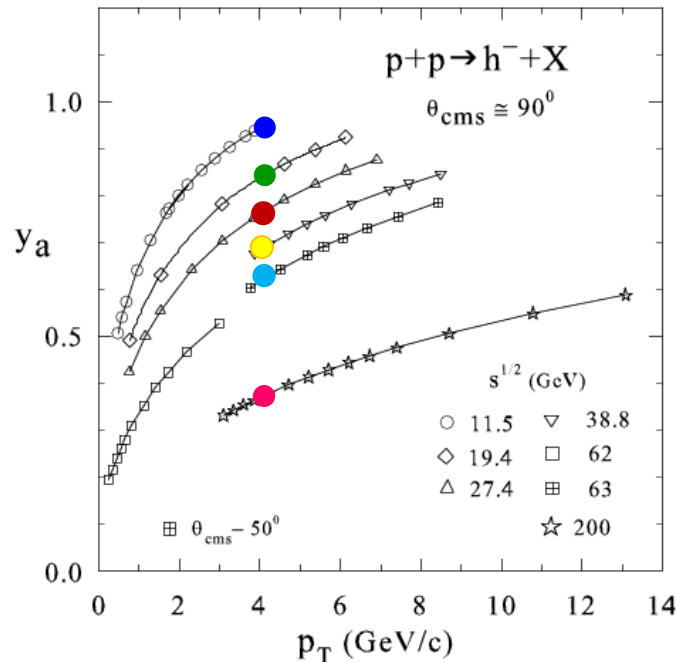


$$(x_1 P_1 + x_2 P_2 - q)^2 = M_X^2$$

$$q = p/y_a$$

p – momentum of produced hadron

q – momentum of scattered constituent



$p_T = 4 \text{ GeV/c}$

5 %
energy loss
 $q \approx 4.2 \text{ GeV/c}$

17 %
energy loss
 $q \approx 4.8 \text{ GeV/c}$

25 %
energy loss
 $q \approx 5.3 \text{ GeV/c}$

32 %
energy loss
 $q \approx 5.9 \text{ GeV/c}$

38 %
energy loss
 $q \approx 6.5 \text{ GeV/c}$

68 %
energy loss
 $q \approx 12.5 \text{ GeV/c}$

Energy loss

- decreases with p_T
- increases with \sqrt{s}

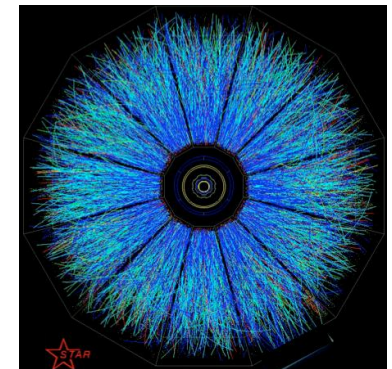
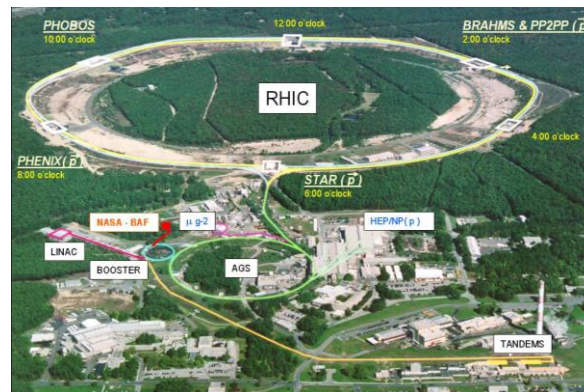
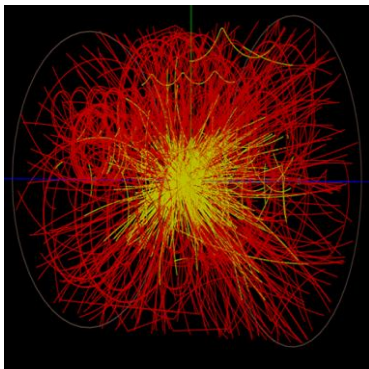
Search for scaling features in AuAu at RHIC

Probing a microscopic structure of a hot and high density nuclear matter at multiple length scales

Self-similarity of hadron production

RHIC beam energy scan with Au+Au:

$$\sqrt{s_{NN}} = 7.7, 11.5, 14.5, 19.6, 27, 39, 62, 130, 200 \text{ GeV}$$



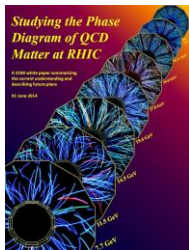
Phase transition and critical phenomena in usual matter (gas, liquid, solid)

“Scaling” and “Universality” are concepts developed to understanding critical phenomena. Scaling means that systems near the critical points exhibiting self-similar properties are invariant under transformation of a scale. According to universality, quite different systems behave in a remarkably similar fashion near the respective critical points. Critical exponents are defined only by symmetry of interactions and dimension of the space.

H.Stanley, G.Barenblatt,...

Phase transition and critical phenomena in nuclear matter

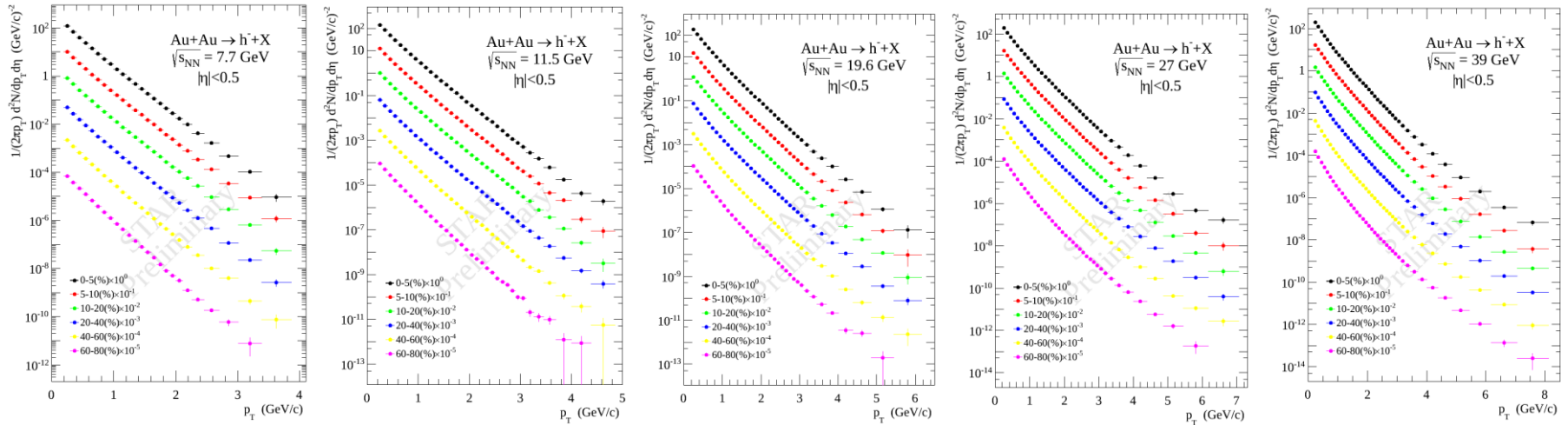
The idea is to vary the collision energy and look for the signatures of QCD phase boundary and QCD critical point i.e. to span the phase diagram from the top RHIC energy (lower μ_B) to the lowest possible energy (higher μ_B). To look for the phase boundary, we would study the established signatures of QGP at 200 GeV as a function of beam energy. Turn-off of these signatures at particular energy would suggest the crossing of phase boundary. Similarly, near critical point, there would be enhanced fluctuations in multiplicity distributions of conserved quantities (net-charge, net-baryon).



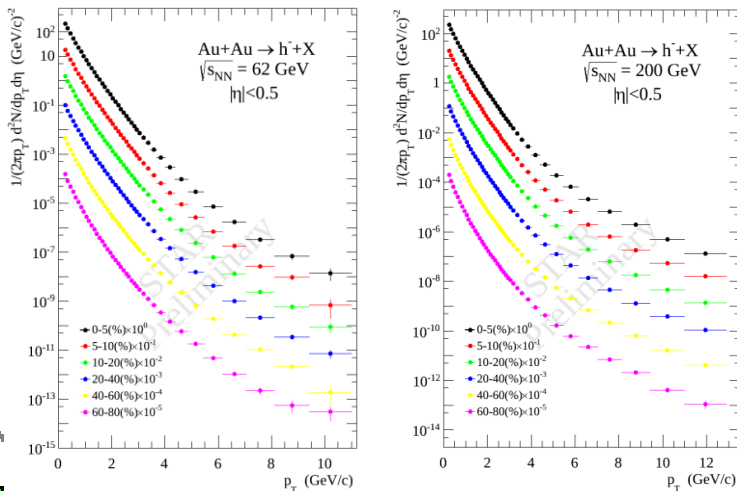
STAR collaboration



BES-I energies



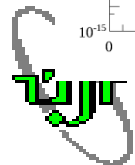
Top RHIC energies



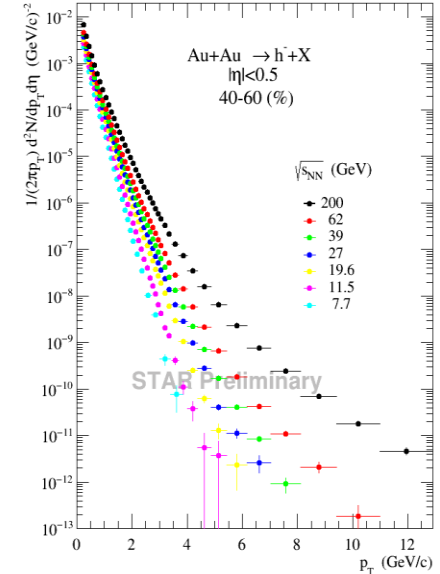
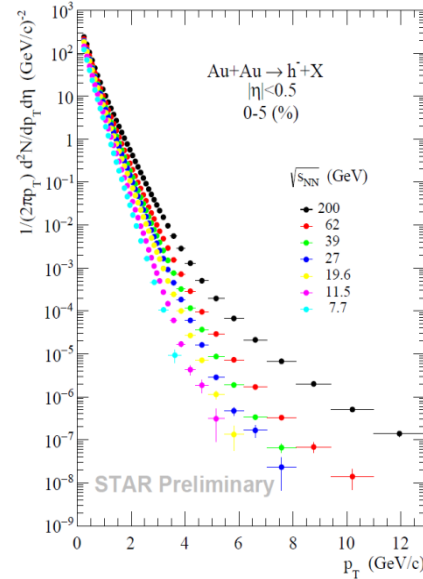
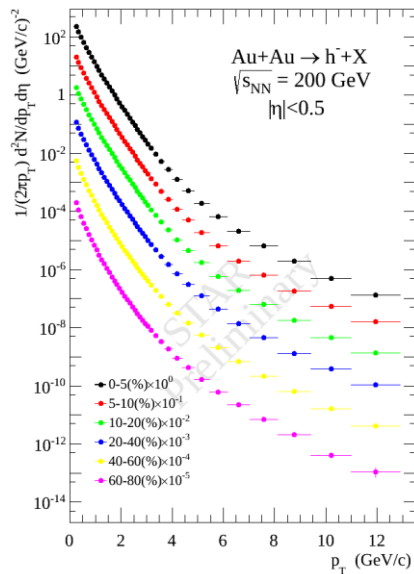
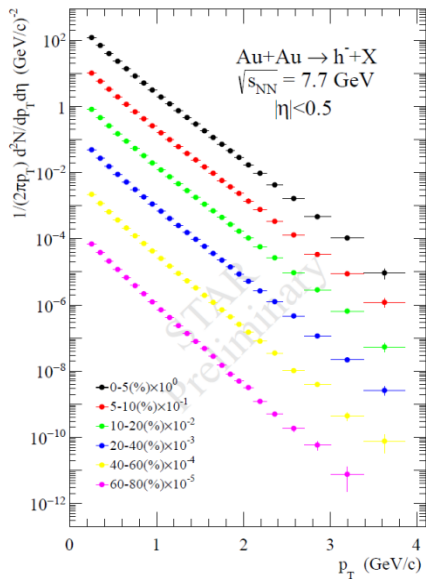
Wide kinematic and dynamical range of particle production:

- Beam energy $\sqrt{s_{NN}} = 7\text{-}200$ GeV
- Centrality 80% - 5% ($dN_{ch}/d\eta|_0 \approx 10\text{-}600$)
- Transverse momentum $p_T = 0.2\text{-}12$ GeV/c

Unprecedented conditions to search for new phenomena in nuclear matter produced in heavy ion collisions.



Ed^3N/dp^3 & $P_{1,2}$, $M_{1,2}$, p , m , $dN/d\eta$



- Centrality dependence of spectra
- Exponential behavior of spectra at low p_T for all energies $\sqrt{s_{NN}}$
- Power behavior of spectra at high p_T and energy $\sqrt{s_{NN}}$

- Energy dependence of spectra
- Difference of yields at various energies strongly increases with p_T for all centralities

The particle yield changes by 7-9 orders of magnitude



Self-similarity parameter

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$

$$\Omega = (1-x_1)^{\delta_{A1}} (1-x_2)^{\delta_{A2}} (1-y_a)^{\varepsilon} (1-y_b)^{\varepsilon}$$

- $dN_{ch}/d\eta|_0$ - multiplicity density
- c_{AA} - “specific heat” of bulk matter
- δ_A - nucleus fractal dimension
- ε_{AA} - fragmentation dimension

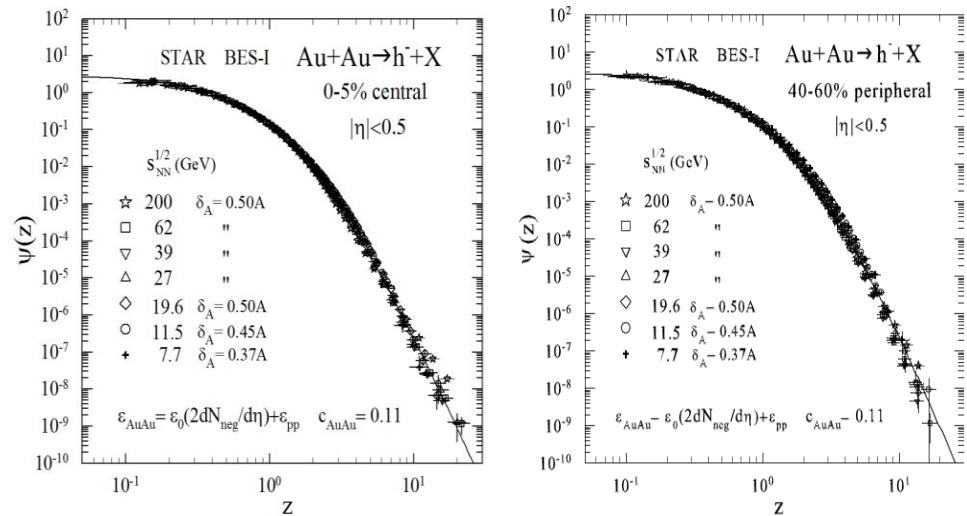
AA collisions:

$$\delta_A = A\delta$$

$$\varepsilon_{AA} = \varepsilon_0 (dN_{AA}/d\eta) + \varepsilon_{pp}$$

$$\Psi(z) = \frac{\pi}{(dN/d\eta) \sigma_{inel}} J^{-1} E \frac{d^3\sigma}{dp^3}$$

“Collapse” of data points onto a single curve



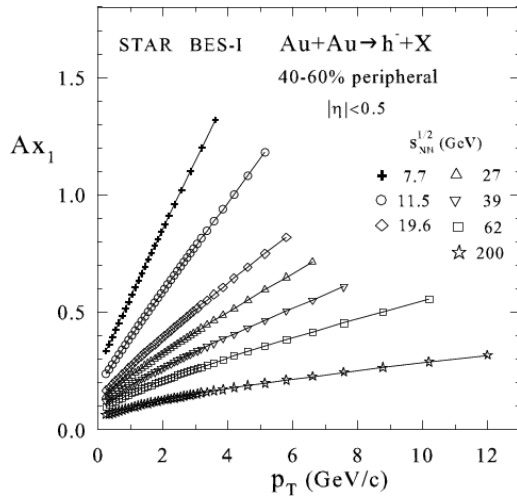
- Energy independence of $\Psi(z)$
- Centrality independence of $\Psi(z)$
- Dependence of ε_{AA} on multiplicity
- Power law at low- and high- z regions

Indication of the decrease of δ for $\sqrt{s_{NN}} < 19.6$ GeV



Constituent level of particle production in terms of

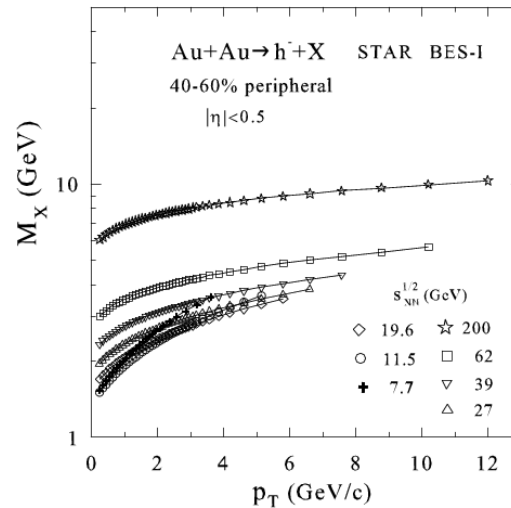
Momentum fraction Ax_1



Momentum fraction

- increases with p_T
- decreases with $\sqrt{s_{NN}}$

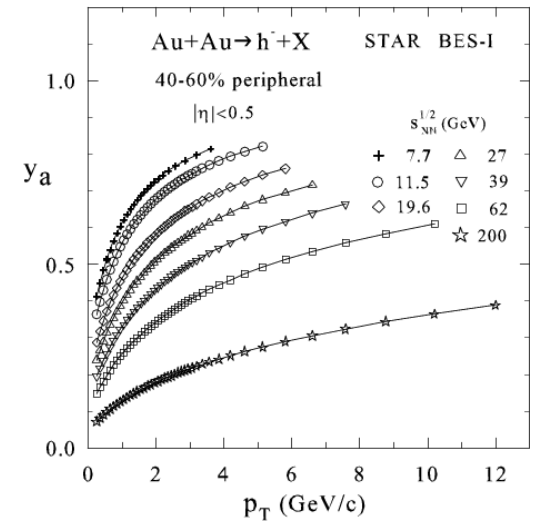
Recoil mass M_X



Recoil mass

- increases with p_T
- increases with $\sqrt{s_{NN}}$

Energy loss $\Delta E/E \sim (1-y_a)$



Energy loss

- decreases with p_T
- increases with $\sqrt{s_{NN}}$

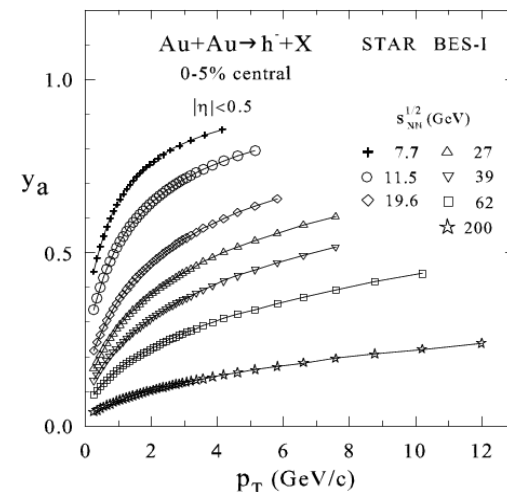
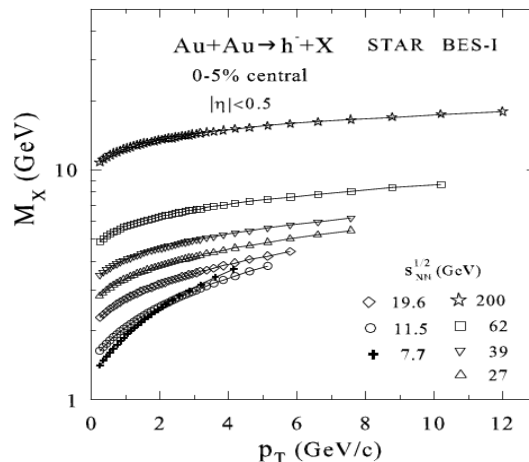
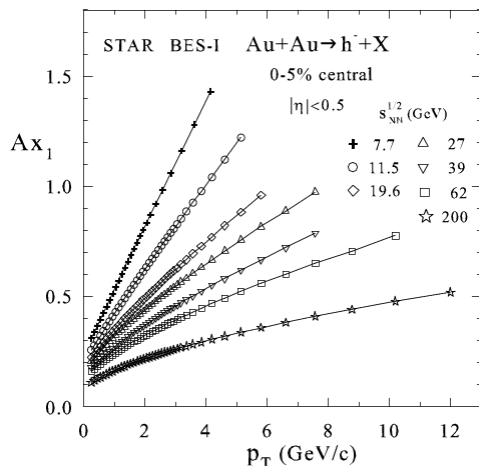
- High x_1 and $p_T \rightarrow$ compressed matter
- High $y_a \rightarrow$ low energy loss, clear PT & CP signatures

Constituent level of particle production in terms of

Momentum fraction Ax_1

Recoil mass M_X

Energy loss $\Delta E/E \sim (1-y_a)$



Momentum fraction

- increases with p_T
- decreases with $\sqrt{s_{NN}}$

Recoil mass

- increases with p_T
- increases with $\sqrt{s_{NN}}$

Energy loss

- decreases with p_T
- increases with $\sqrt{s_{NN}}$

- High x_1 and $p_T \rightarrow$ compressed matter
- Large $M_X \rightarrow$ high density recoil system
- High $y_a \rightarrow$ low energy loss, clear PT & CP signatures



Energy loss $\Delta E/E \sim (1-y_a)$

p_T – transverse momentum of produced hadron

q – momentum of scattered constituent

$$p_T = 4 \text{ GeV}, \quad q = p_T / y_a$$

20 % energy loss
 $q \approx 5 \text{ GeV}/c$

22 % energy loss
 $q \approx 5.1 \text{ GeV}/c$

30 % energy loss
 $q \approx 5.7 \text{ GeV}/c$

35 % energy loss
 $q \approx 6.2 \text{ GeV}/c$

45 % energy loss
 $q \approx 7.3 \text{ GeV}/c$

55 % energy loss
 $q \approx 8.9 \text{ GeV}/c$

75 % energy loss
 $q \approx 16 \text{ GeV}/c$

20 % energy loss
 $q \approx 5 \text{ GeV}/c$

25 % energy loss
 $q \approx 5.3 \text{ GeV}/c$

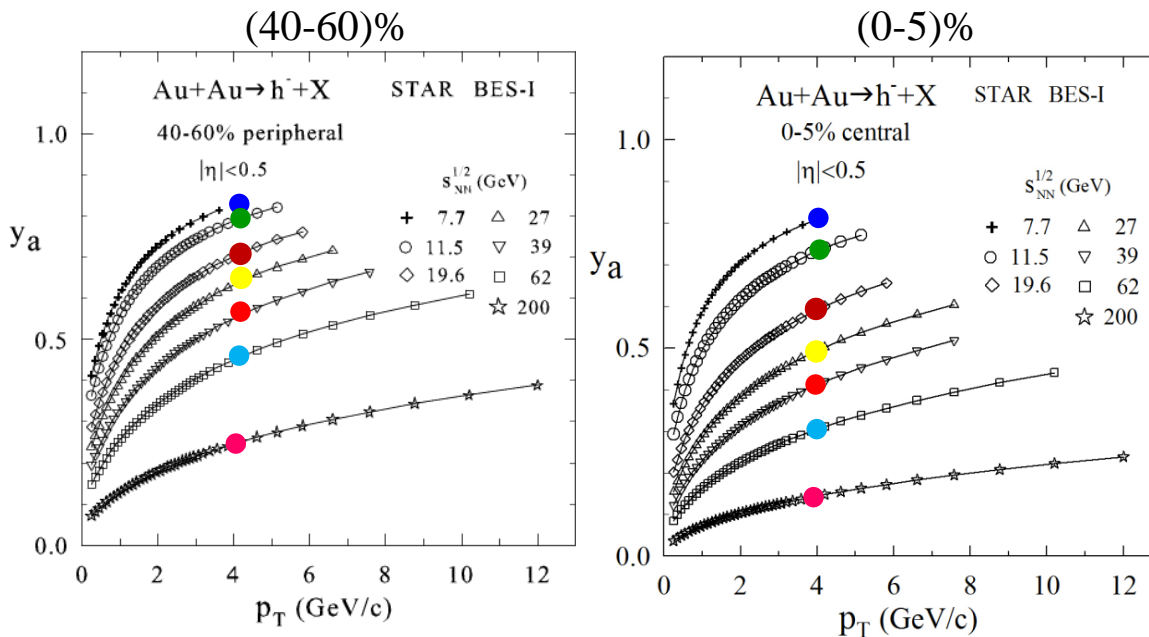
40 % energy loss
 $q \approx 6.7 \text{ GeV}/c$

50 % energy loss
 $q \approx 8 \text{ GeV}/c$

60 % energy loss
 $q \approx 10 \text{ GeV}/c$

70 % energy loss
 $q \approx 13.3 \text{ GeV}/c$

85 % energy loss
 $q \approx 26.6 \text{ GeV}/c$

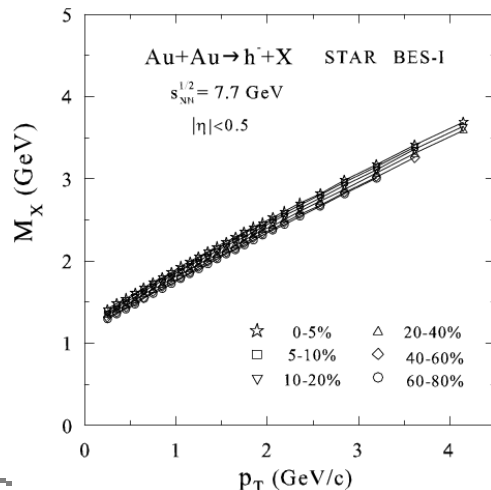
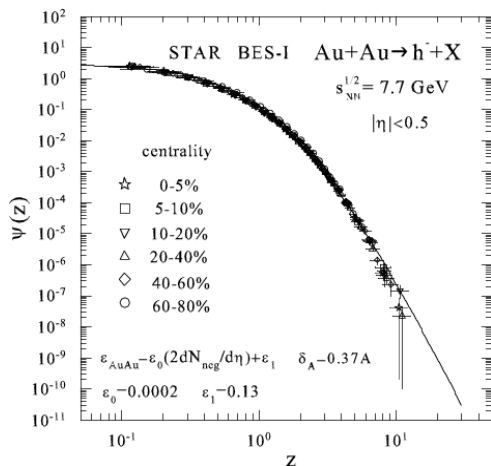


Energy loss

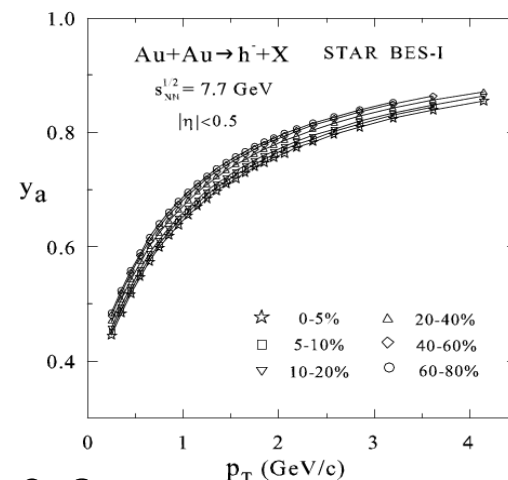
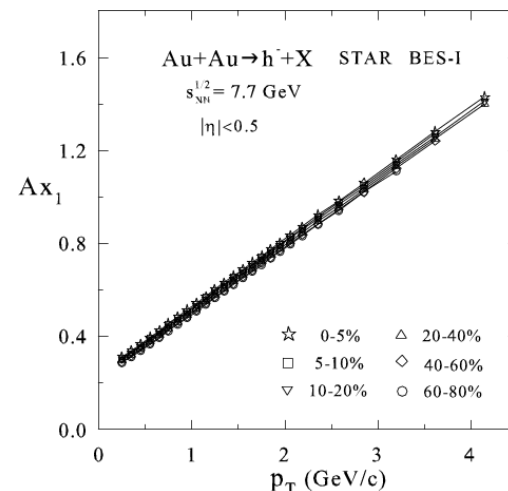
- \triangleright decreases with p_T
- \triangleright increases with $\sqrt{s_{NN}}$
- \triangleright increases with centrality



Au+Au \rightarrow h^- + X



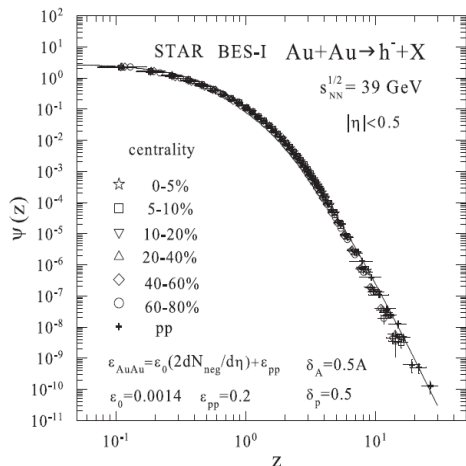
- Scaling behavior of $\Psi(z)$
- Weak dependence of Ax_1 , y_a , M_X on centrality
- Cumulative region is reached
- Smooth dependence vs. variables
- Power behavior of $\Psi(z)$ at $z < 0.4$
- Power behavior of $\Psi(z)$ at $z > 4$
- Linear dependence of M_X and Ax_1 on p_T for all centralities
- Growth and flattening of y_a vs. p_T
- Decrease of $\delta_A = A\delta$ with $\sqrt{s_{NN}}$



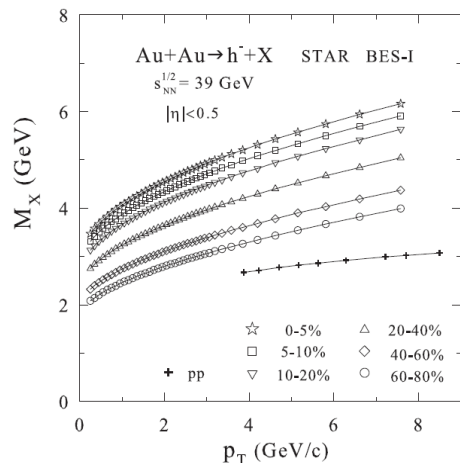
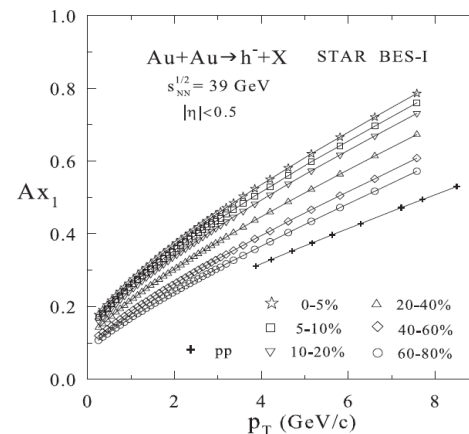
Goal: search for discontinuity and correlation of model parameters δ , c , ε

Scaling function, momentum fractions and recoil mass vs. centrality and p_T at $\sqrt{s_{NN}}=39$ GeV & $|\eta|<0.5$

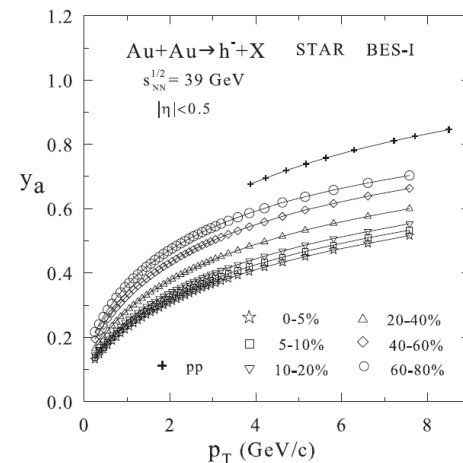
Au+Au \rightarrow h^- + X



- Scaling behavior of $\Psi(z)$
- Strong dependence of Ax_1, y_a, M_X on centrality
- Cumulative region is not reached
- Smooth dependence vs. variables



- Power behavior of $\Psi(z)$ at $z < 0.4$
- Power behavior of $\Psi(z)$ at $z > 4$
- Growth of Ax_1, y_a, M_X on p_T for all centralities
- Independence of $\delta_A = A\delta$ on $\sqrt{s_{NN}}$



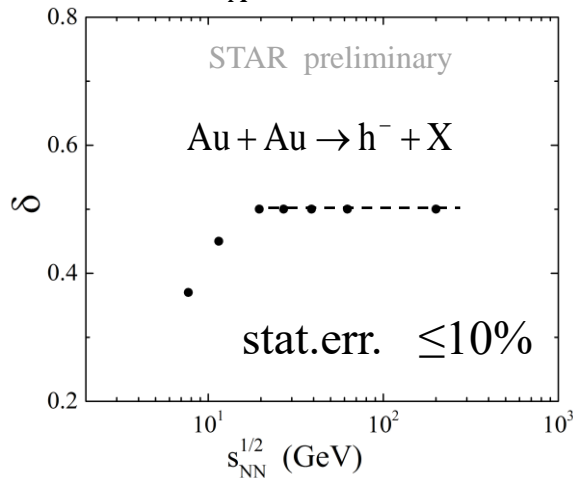
Goal: search for discontinuity and correlation of model parameters δ, c, ϵ



Parameters $\delta_A, \varepsilon_{AA}, c_{AA}$ are determined from the requirement of scaling behavior of Ψ as a function of self-similarity parameter z

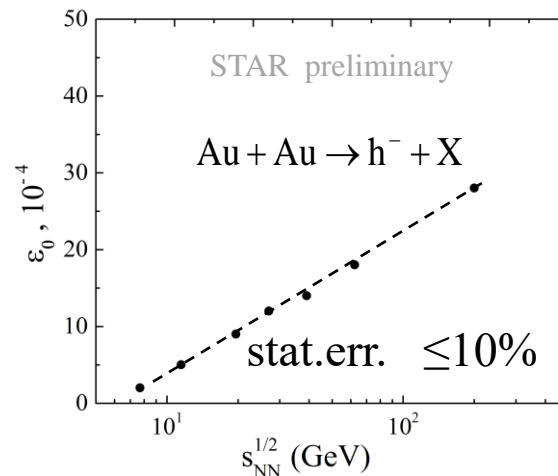
Nucleus fractal dimension

$$\delta_A = A \cdot \delta$$



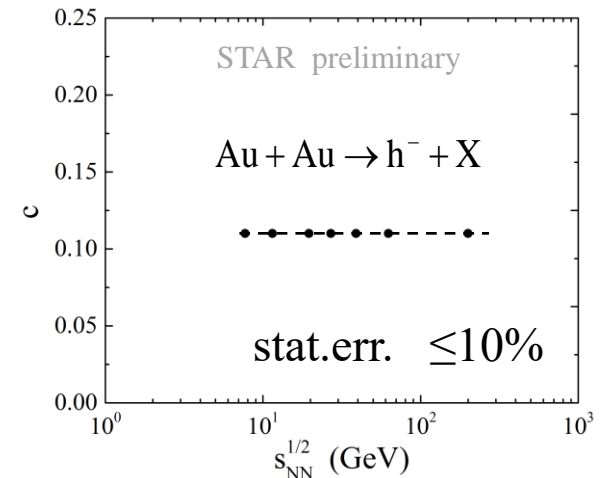
Fragmentation dimension

$$\varepsilon_{AA} = \varepsilon_0 (dN_{AA}/d\eta) + \varepsilon_{pp}$$



“Specific heat”

$$c_{AA}$$



- δ_A decreases with energy for $\sqrt{s_{NN}} \leq 20$ GeV
- δ_A is independent of energy for $\sqrt{s_{NN}} \geq 20$ GeV
- ε_{AA} increases with energy
- c_{AA} is independent of energy

Search for discontinuity and correlations of the model parameters.



- New results of analysis of experimental data on negative hadrons produced in **Au+Au** collisions at **RHIC** from **BES-I** program in the framework of **z**-scaling were presented.
- New confirmation of scaling properties of data **z**-presentation in **AA** collisions were obtained.
- The dependence of momentum fractions, recoil mass vs. collision energy, centrality and transverse momentum was studied.
- The constituent energy loss for a wide kinematic range and centrality of events was estimated.
- STAR **BES-I** data in **z**-presentation demonstrate smooth behavior vs. a collision energy, centrality over a wide range of p_T .
- Discontinuities of model parameters – fractal dimensions δ_A , ε_{AA} and “heat capacity” c_{AA} , are not visible.
- Parameter correlations as signatures of phase transition and **Critical Point** were discussed.

The obtained results can be of interest in searching for a **Critical Point** and signatures of phase transition in nuclear matter produced at **SPS**, **RHIC** and **LHC** in present, and **FAIR** & **NICA** in future.





XXIV International Baldin Seminar on High Energy Physics Problems *Relativistic Nuclear Physics & Quantum Chromodynamics*

September 17 - 22, 2018, Dubna, Russia



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RELATIVISTIC NUCLEAR PHYSICS & QUANTUM CHROMODYNAMICS

SEPTEMBER 17-22, 2018 DUBNA, RUSSIA

Seminar Topics

- Quantum chromodynamics at large distances
- Relativistic heavy ion collisions
- Hadron spectroscopy, multiquarks
- Cumulative processes
- Structure functions of hadrons and nuclei
- Multiparticle dynamics
- Polarization phenomena, spin physics
- Studies of exotic nuclei in relativistic beams
- Applied use of relativistic beams
- Accelerator facilities: status and perspectives
- Project NICA/MPD (Nuclotron-based Ion Collider fAcility/ MultiPurposed Detector) at JINR
- Progress in experimental studies in high energy centers — JINR, CERN, BNL, JLAB, GSI, etc.

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N. M. Piskunov	

Contacts

Programme, registration:
Dr. Elena Bogochayeva
Website and Bulletin Laboratory of High Energy Physics
Joint Institute for Nuclear Research
141980 Dubna, Russia
e: ishepp@nrc.jinr.ru

Visa, accommodation, travel details:
Mrs. Elena Russakovich
International Department
Joint Institute for Nuclear Research
141980 Dubna, Russia
e: russakovich@jinr.ru

XXIV Baldin ISHEPP
+7 (49621) 63990
+7 (49621) 65091
http://relnp.jinr.ru/ishepp/

Thank you for attention!



M. Tokarev

ISHEPP'18, Dubna, Russia



Back-up Slides



$$\sqrt{s}_\perp^2 = \underbrace{y_1 (s_\lambda^{1/2} - M_1 \lambda_1 - M_2 \lambda_2) - m_1}_{\text{energy consumed for the inclusive particle } m_1} + \underbrace{y_2 (s_\chi^{1/2} - M_1 \chi_1 - M_2 \chi_2) - m_2}_{\text{energy consumed for the recoil particle } m_2}$$

energy consumed
for the inclusive particle m_1

energy consumed
for the recoil particle m_2

Fraction decomposition:

$$x_{1,2} = \lambda_{1,2} + \chi_{1,2}$$

$$\lambda_{1,2} = \kappa_{1,2}/y_1 + v_{1,2}/y_2$$

$$\kappa_{1,2} = \frac{(P_{2,1} p)}{(P_2 P_1)}, \quad v_{1,2} = \frac{M_{2,1} m_2}{(P_2 P_1)}$$

$$\chi_{1,2} = (\mu_{1,2}^2 + \omega_{1,2}^2)^{1/2} \mp \omega_{1,2}$$

$$\mu_{1,2}^2 = \alpha^{\pm 1} (\lambda_1 \lambda_2 + \lambda_0) \frac{1 - \lambda_{1,2}}{1 - \lambda_{2,1}}$$

$$\omega_{1,2} = \mu_{1,2} U, \quad U = \frac{\alpha - 1}{2\sqrt{\alpha}} \xi, \quad \alpha = \frac{\delta_2}{\delta_1}$$

$$\lambda_0 = \bar{v}_0/y_2^2 - v_0/y_1^2$$

$$\xi^2 = (\lambda_1 \lambda_2 + \lambda_0) / [(1 - \lambda_1)(1 - \lambda_2)]$$

$$\bar{v}_0 = \frac{0.5 m_2^2}{(P_1 P_2)}, \quad v_0 = \frac{0.5 m_1^2}{(P_1 P_2)}$$

$$s_\lambda = (\lambda_1 P_1 + \lambda_2 P_2)^2$$

$$s_\chi = (\chi_1 P_1 + \chi_2 P_2)^2$$

The scaling function $\Psi(z)$ and self-similarity parameter z
are expressed via Lorentz invariants.

