

How to measure the linear polarization
of gluons in unpolarized proton
using the heavy-quark pair production

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**Novel proposals for heavy quark physics;
Search for polarized gluons in unpolarized proton**

*Baldin ISHEPP XXIV,
Dubna, September 17, 2018*

Outline:

- ❖ Perturbatively stable observables in heavy-quark lepto-production:
 - Azimuthal asymmetries;
 - Callan-Gross ratio $R = F_L / F_T$ in 1PI kinematics
- ❖ Search for linearly polarized gluons in unpolarized proton using the heavy-quark pair production:
 - Maximal values for the $\cos \phi$, $\cos 2\phi$ and R distributions allowed by photon-gluon fusion with unpolarized gluons;
 - Contribution of the linearly polarized gluons, $h_1^{\perp g}$;
 - Recommendations for measurements at EIC and LHeC
- ❖ Outlook for COMPASS and NICA

Our main conclusions:

- $\cos \phi$, $\cos 2\phi$ asymmetries and ratio $R = F_L / F_T$ in heavy-quark pair leptonproduction depend dramatically on the contribution of linearly polarized gluons;
- Future measurements of these quantities at EIC and LHeC seem to be very promising for determination of $h_1^\perp g$;
- $\cos 2\phi$ asymmetry in 1PI charm leptonproduction is predicted to be large ($\sim 15\%$) in the COMPASS kinematics;
- Extraction of the azimuthal asymmetries from available COMPASS data will provide valuable information about the TMD distribution f_1^g

References

Perturbative stability of $\cos 2\phi$ and R quantities:

- N.Ya.Ivanov, A.Capella, A.B.Kaidalov, Nucl. Phys. **B** 586 (2000), 382
- N.Ya.Ivanov, Nucl. Phys. **B** 615 (2001), 266
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Search for linearly polarized gluons in unpolarized proton:

- D.Boer, S.J.Brodsky, P.J.Mulders, C.Pisano, PRL 106 (2011), 132001
- C.Pisano, D.Boer, S.J.Brodsky, P.J.Mulders, JHEP 1310 (2013) 024
- A.V.Efremov, N.Ya.Ivanov, O.V.Teryaev, Phys.Lett. **B** 772 (2017), 283
- A.V.Efremov, N.Ya.Ivanov, O.V.Teryaev, Phys.Lett. **B** 777 (2018), 435
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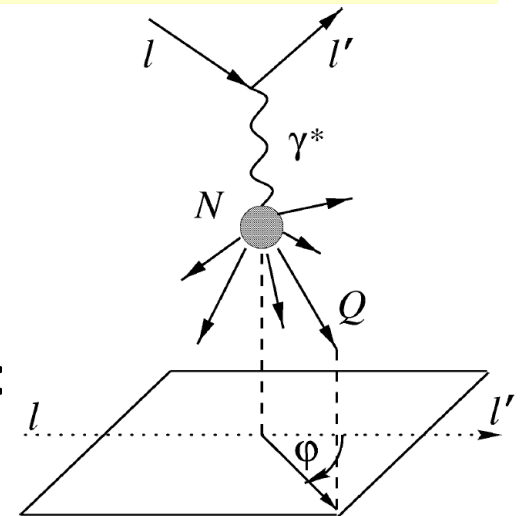
Perturbative stability in charm electroproduction

Definitions and Cross Sections

We consider the Callan-Gross ratio $R = F_L / F_T$ and azimuthal $\cos 2\varphi$ asymmetry, $A = 2xF_A / F_2$, in heavy-quark leptonproduction:

$$l(\ell) + N(p) \rightarrow l(\ell - q) + Q(p_Q) + X[\bar{Q}](p_X)$$

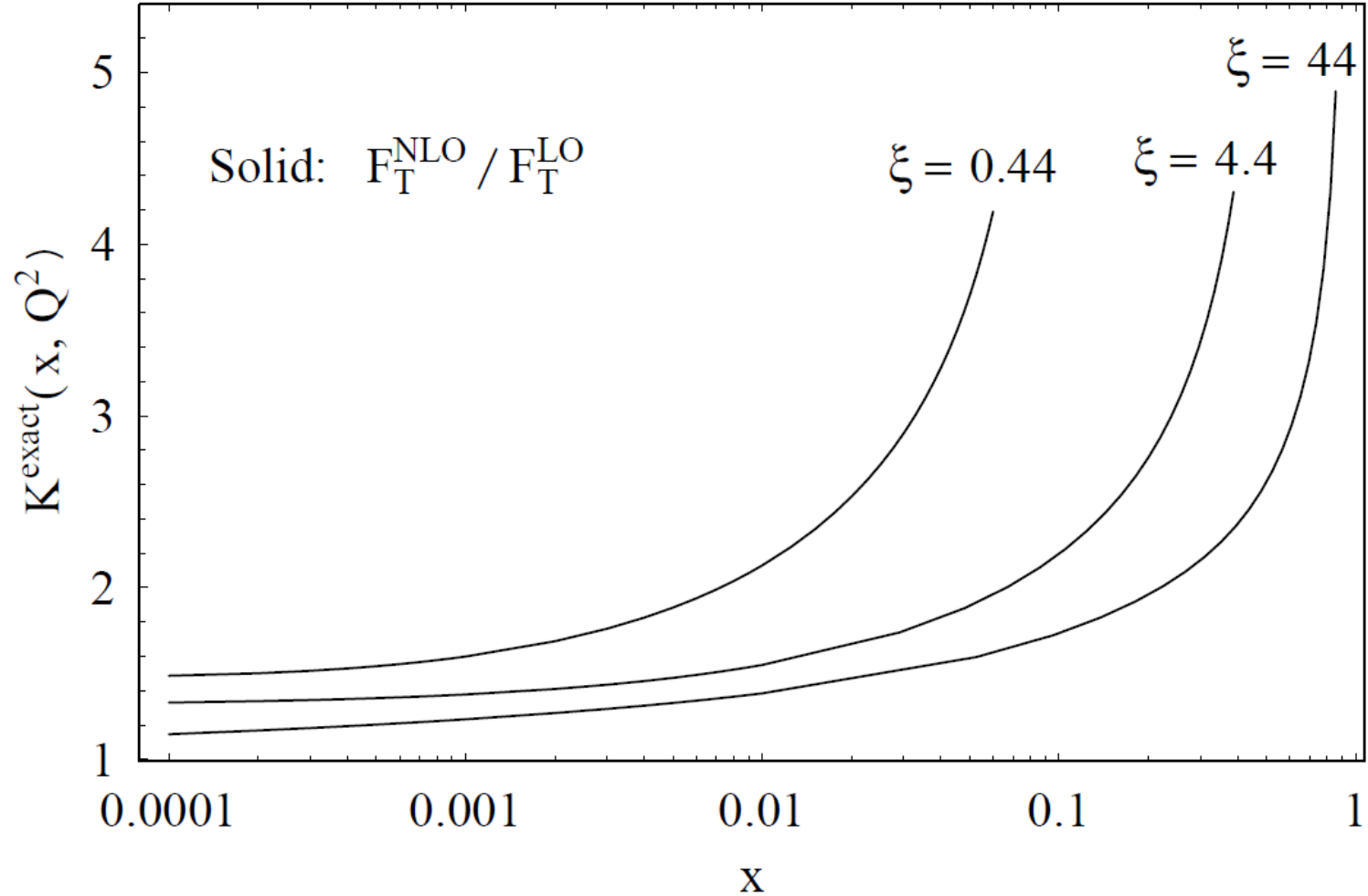
Corresponding cross section in 1PI kinematics is:



$$\frac{d^3\sigma_{lN}}{dx dQ^2 d\varphi} = \frac{\alpha_{em}^2}{xQ^4} \left\{ \left[1 + (1-y)^2 \right] F_2(x, Q^2) - 2xy^2 F_L(x, Q^2) \right. \\ \left. + 4x(1-y) F_A(x, Q^2) \cos 2\varphi + 4x(2-y) \sqrt{2(1-y)} F_I(x, Q^2) \cos \varphi \right\}$$

where $F_2(x, Q^2) = 2x(F_T + F_L)$ and x, y, Q^2 are usual DIS observables

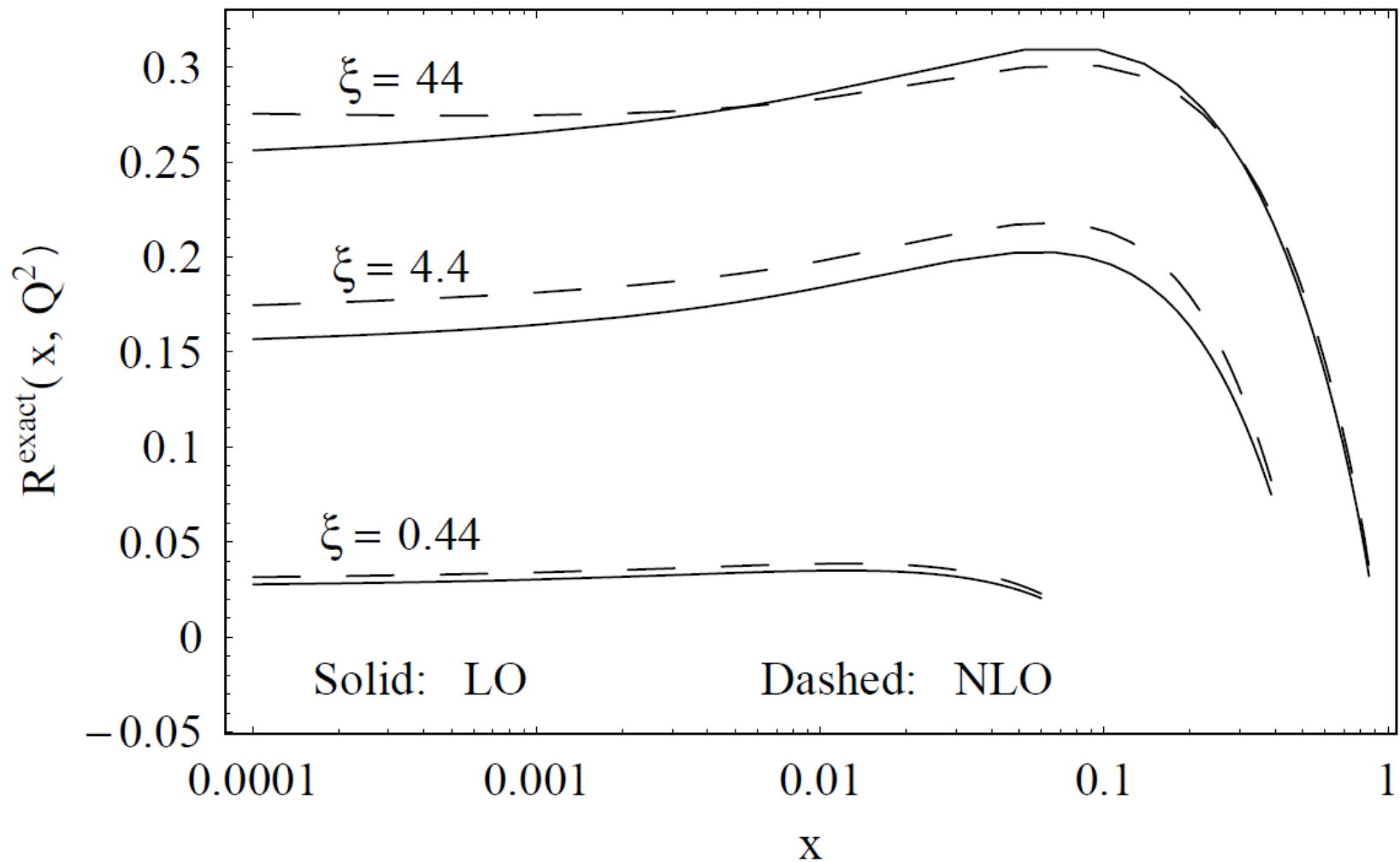
Perturbative **int**ability of the cross section



$$\xi = \frac{Q^2}{m^2}$$

$$K(x, Q^2) = \frac{F_T^{\text{NLO}}}{F_T^{\text{LO}}}$$

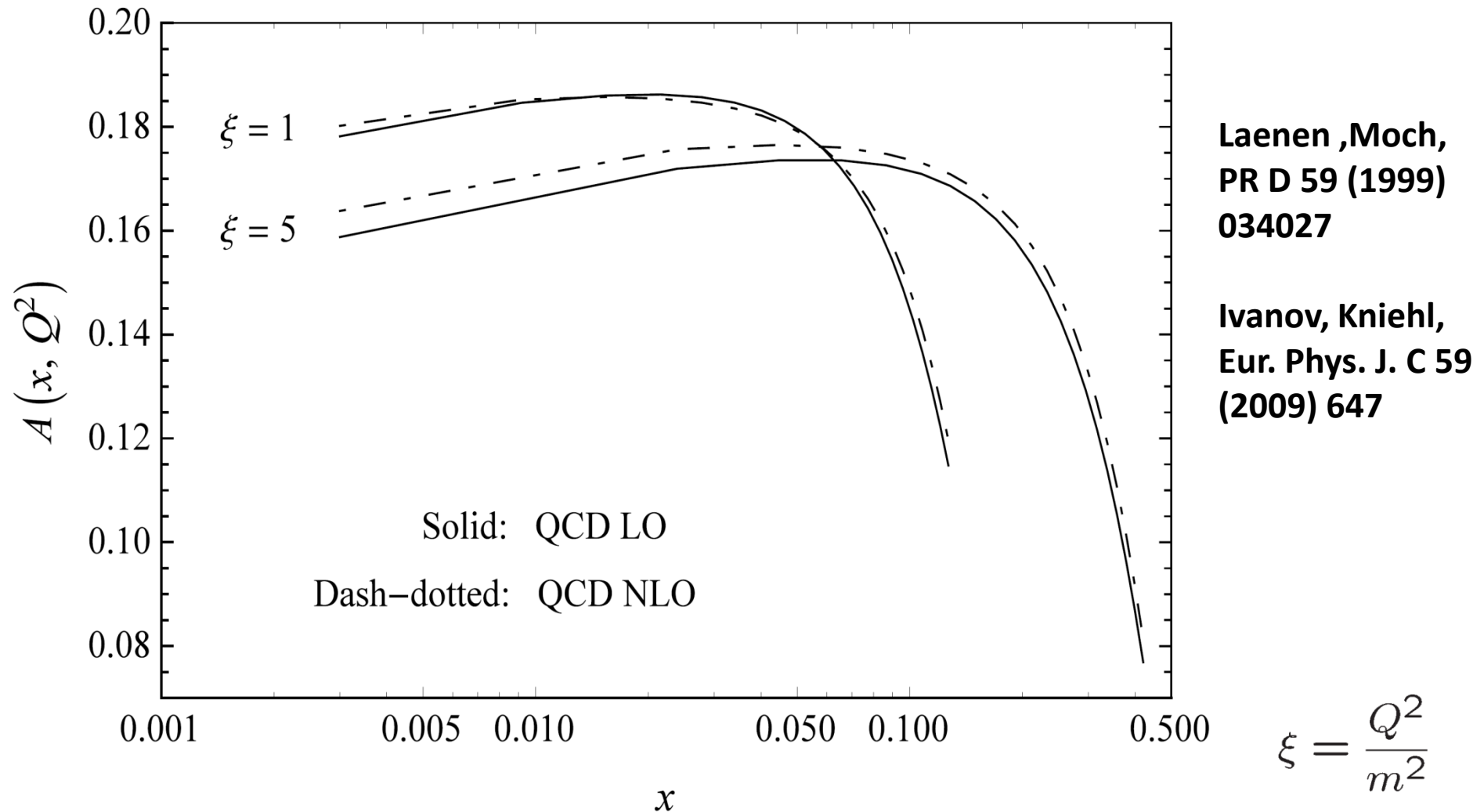
Perturbative stability of $R = F_L / F_T$



$$\xi = \frac{Q^2}{m^2}$$

$$R(x, Q^2) = \frac{F_L}{F_T}$$

Perturbative stability of $A = 2xF_A / F_2$



Laenen, Moch,
PR D 59 (1999)
034027

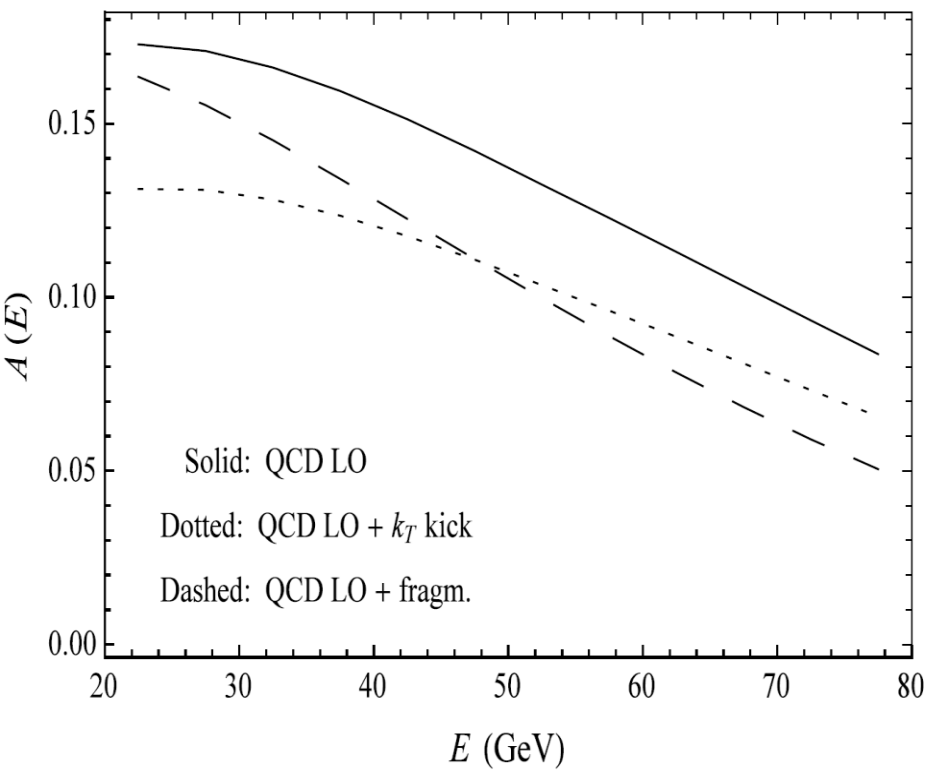
Ivanov, Kniehl,
Eur. Phys. J. C 59
(2009) 647

➤ The soft-gluon NLO NLL corrections are given

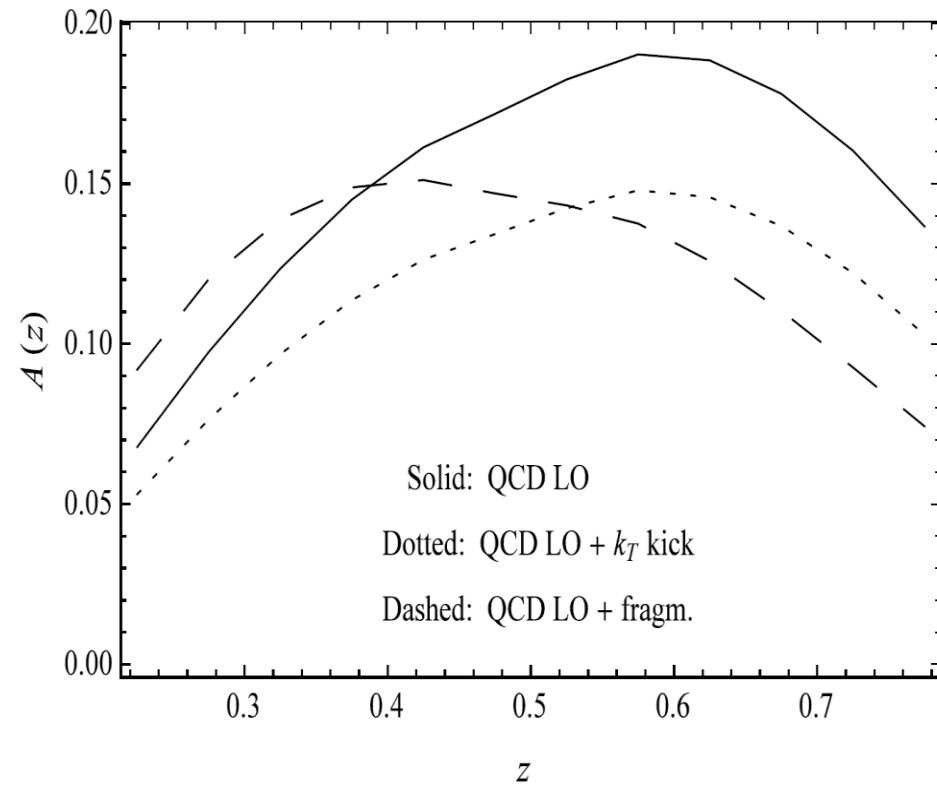
$$A(x, Q^2) = 2x \frac{F_A}{F_2}$$

cos2φ asymmetry in charm electroproduction at COMPASS

cos2φ asymmetry in charm electroproduction can be measured at COMPASS : Efremov, Ivanov, Teryaev, Phys.Lett. B 772 (2017), 283



$$\nu = E_l - E'_l$$

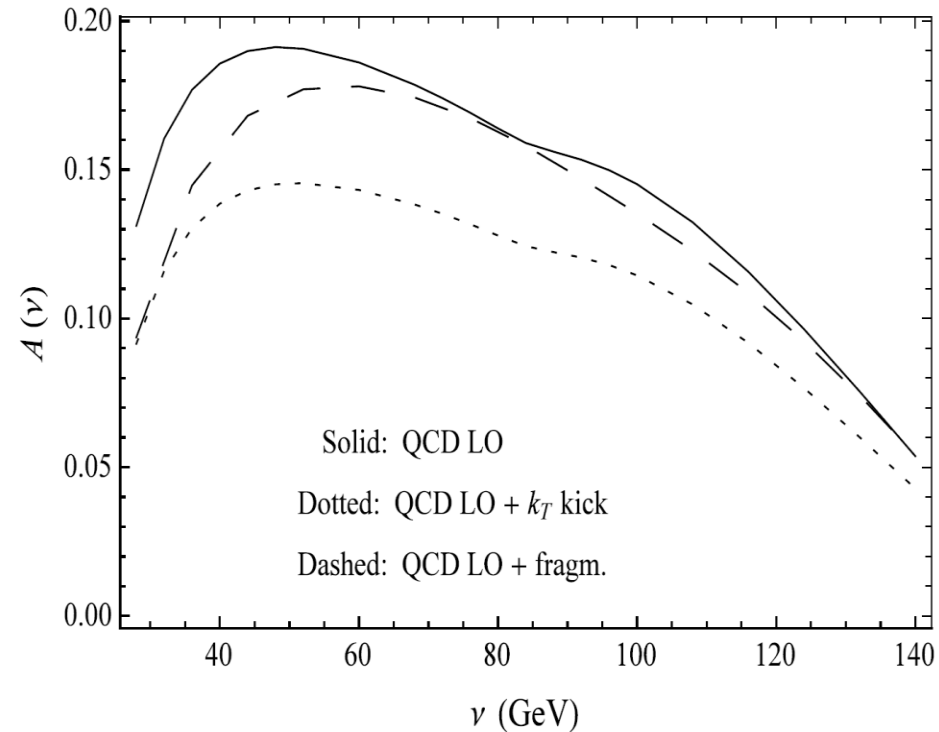
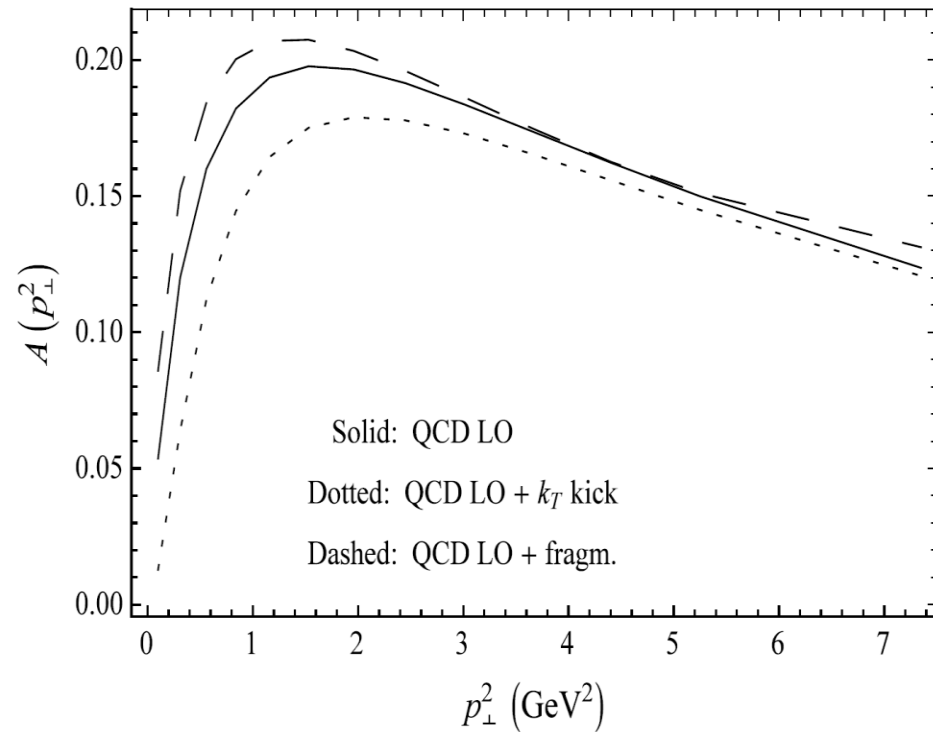


$$z = \frac{E}{\nu}$$

cos2φ asymmetry in charm electroproduction at COMPASS

COMPASS kinematics:

$$0.003 < Q^2 < 10 \text{ GeV}^2, \quad 3 \cdot 10^{-5} < x < 0.1, \quad 20 < E < 80 \text{ GeV}$$



$$\nu = E_l - E_l'$$

Linearly polarized gluons in unpolarized proton

To probe the TMD distribution $h_1^{\perp g}$, the momenta of both heavy quark and anti-quark should be measured (reconstructed) in the reaction:

$$l(\ell) + N(P) \rightarrow l'(\ell - q) + Q(p_Q) + \bar{Q}(p_{\bar{Q}}) + X(p_X)$$

The LO parton-level subprocess is:

$$\gamma^*(q) + g(k_g) \rightarrow Q(p_Q) + \bar{Q}(p_{\bar{Q}}) \quad k_g^\mu \simeq \zeta P^\mu + k_T^\mu$$

Corresponding cross section is:

$$d\sigma \propto L(\ell, q) \otimes \Phi_g(\zeta, k_T) \otimes \left| H_{\gamma^* g \rightarrow Q \bar{Q} X}(q, k_g, p_Q, p_{\bar{Q}}) \right|^2$$

$$\Phi_g^{\mu\nu}(\zeta, k_T) \propto -g_T^{\mu\nu} f_1^g(\zeta, \vec{k}_T^2) + \left(g_T^{\mu\nu} - 2 \frac{k_T^\mu k_T^\nu}{k_T^2} \right) \frac{\vec{k}_T^2}{2m_N^2} h_1^{\perp g}(\zeta, \vec{k}_T^2)$$

The resulting cross section is:

$$d^7\sigma_{lN} \propto A_0 + A_1 \cos \phi_{\perp} + A_2 \cos 2\phi_{\perp} + \bar{q}_T^2 [B_0 \cos 2(\phi_{\perp} - \phi_T) + B_1 \cos(\phi_{\perp} - 2\phi_T) + B'_1 \cos(3\phi_{\perp} - 2\phi_T) + B_2 \cos 2\phi_T + B'_2 \cos 2(2\phi_{\perp} - \phi_T)]$$

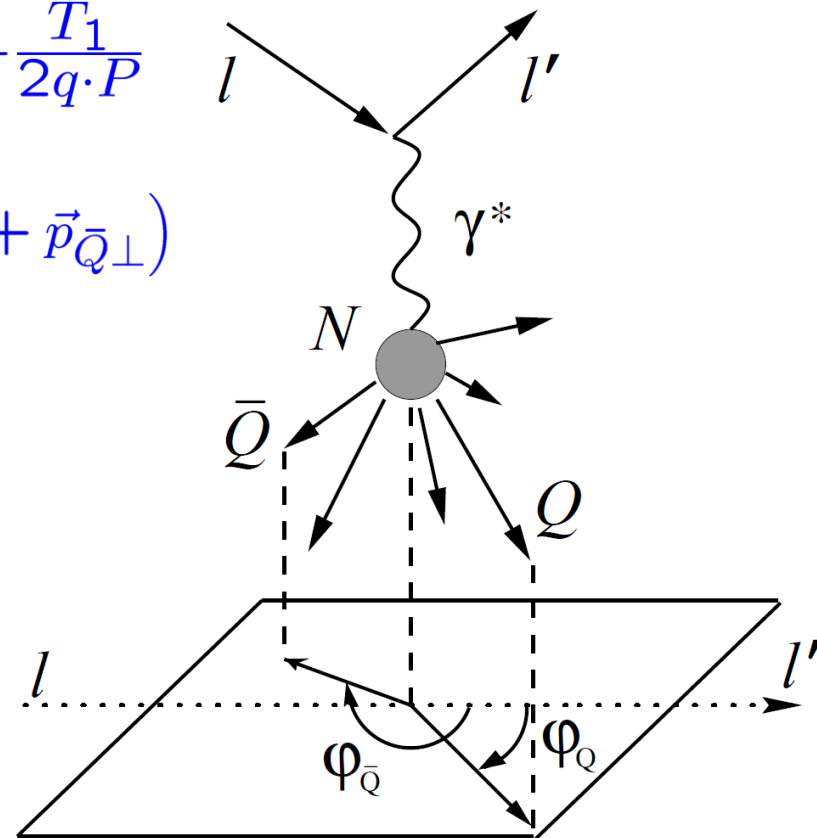
$$\zeta = \frac{-U_1}{y\bar{S} + T_1} = x + \frac{m^2 + \vec{K}_{\perp}^2}{z(1-z)y\bar{S}}, \quad z = -\frac{T_1}{2q \cdot P}$$

$$\vec{K}_{\perp} = \frac{1}{2} (\vec{p}_{Q\perp} - \vec{p}_{\bar{Q}\perp}) \quad \bar{q}_T = \frac{1}{2} (\vec{p}_{Q\perp} + \vec{p}_{\bar{Q}\perp})$$

$$\downarrow \quad \downarrow$$

$$\phi_{\perp} \quad \phi_T$$

$$A_i \sim f_1^g, \quad B_i^{(\prime)} \sim h_1^{\perp g}$$



- Boer, Brodsky, Mulders, Pisano, PRL 106 (2011), 132001
- Pisano, Boer, Brodsky, Mulders, JHEP 1310 (2013) 024

We work in the approximation:

$$|\vec{q}_T| \ll |\vec{K}_\perp|, \quad |\varphi_Q - \varphi_{\bar{Q}}| \sim \pi \quad \Rightarrow \quad \phi_T \simeq \phi_\perp - \frac{\pi}{2}$$

Integration over ϕ_T gives:

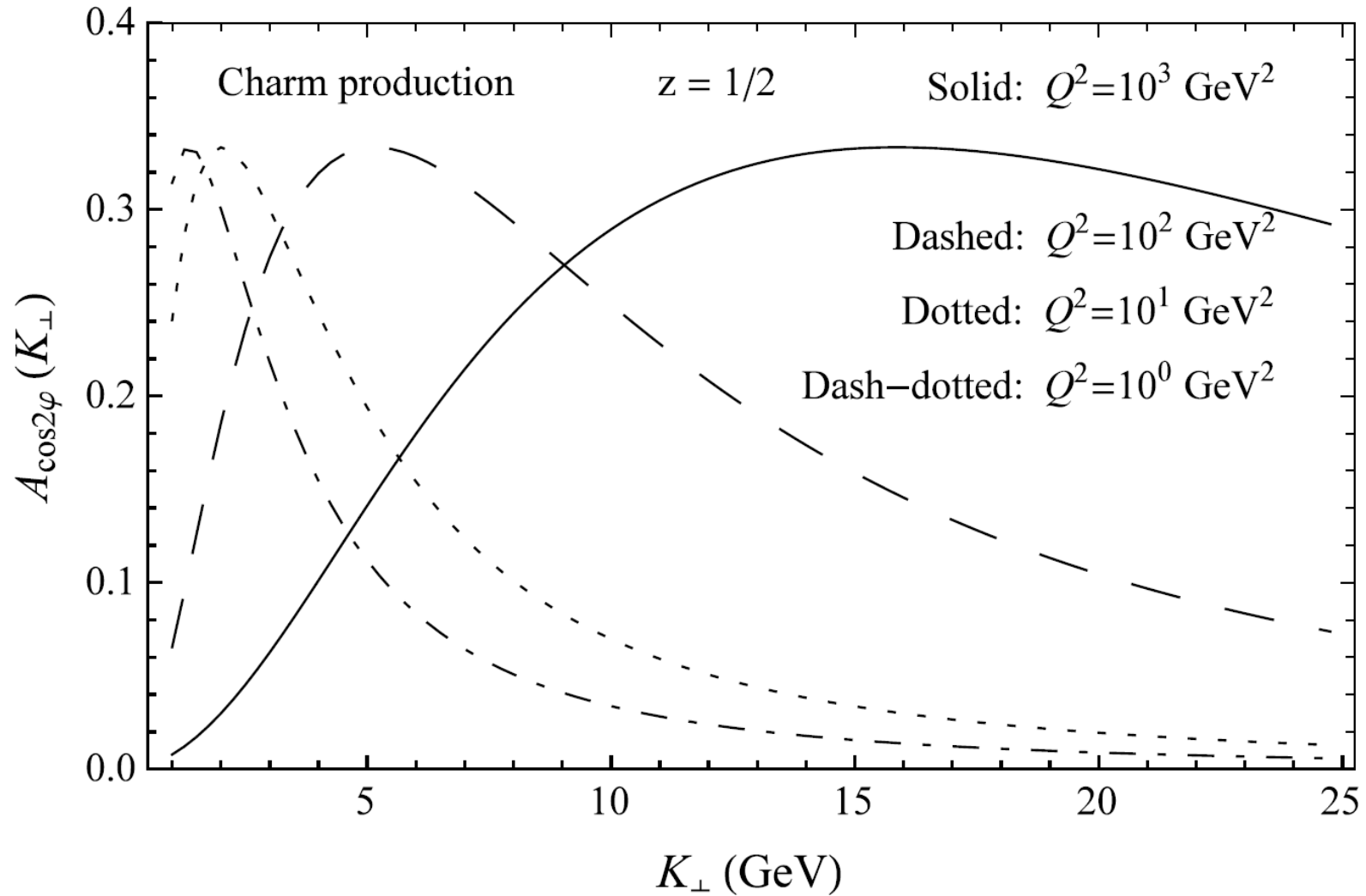
$$\frac{d^6\sigma(\pi)}{dy dx dz d\vec{K}_\perp^2 d\vec{q}_T^2 d\varphi} = \frac{e_Q^2 \alpha_{em}^2 \alpha_s}{8\bar{S}^2} \frac{f_1^g(\zeta, \vec{q}_T^2) \hat{B}_2}{y^3 x \zeta z (1-z)} \left\{ \left[1 + (1-y)^2 \right] \left(1 - 2r \frac{\hat{B}_2^h}{\hat{B}_2} \right) - y^2 \frac{\hat{B}_L}{\hat{B}_2} \left(1 - 2r \frac{\hat{B}_L^h}{\hat{B}_L} \right) \right. \\ \left. + 2(1-y) \frac{\hat{B}_A}{\hat{B}_2} \left(1 - 2r \frac{\hat{B}_A^h}{\hat{B}_A} \right) \cos 2\varphi + (2-y) \sqrt{1-y} \frac{\hat{B}_I}{\hat{B}_2} \left(1 - 2r \frac{\hat{B}_I^h}{\hat{B}_I} \right) \cos \varphi \right\}$$

$$r \equiv r(\zeta, \vec{q}_T^2) = \frac{\vec{q}_T^2}{2m_N^2} \frac{h_1^{\perp g}(\zeta, \vec{q}_T^2)}{f_1(\zeta, \vec{q}_T^2)} \quad \varphi = \varphi_Q$$

$$\hat{B}_i \sim f_1^g, \quad \hat{B}_i^h \sim h_1^{\perp g}$$

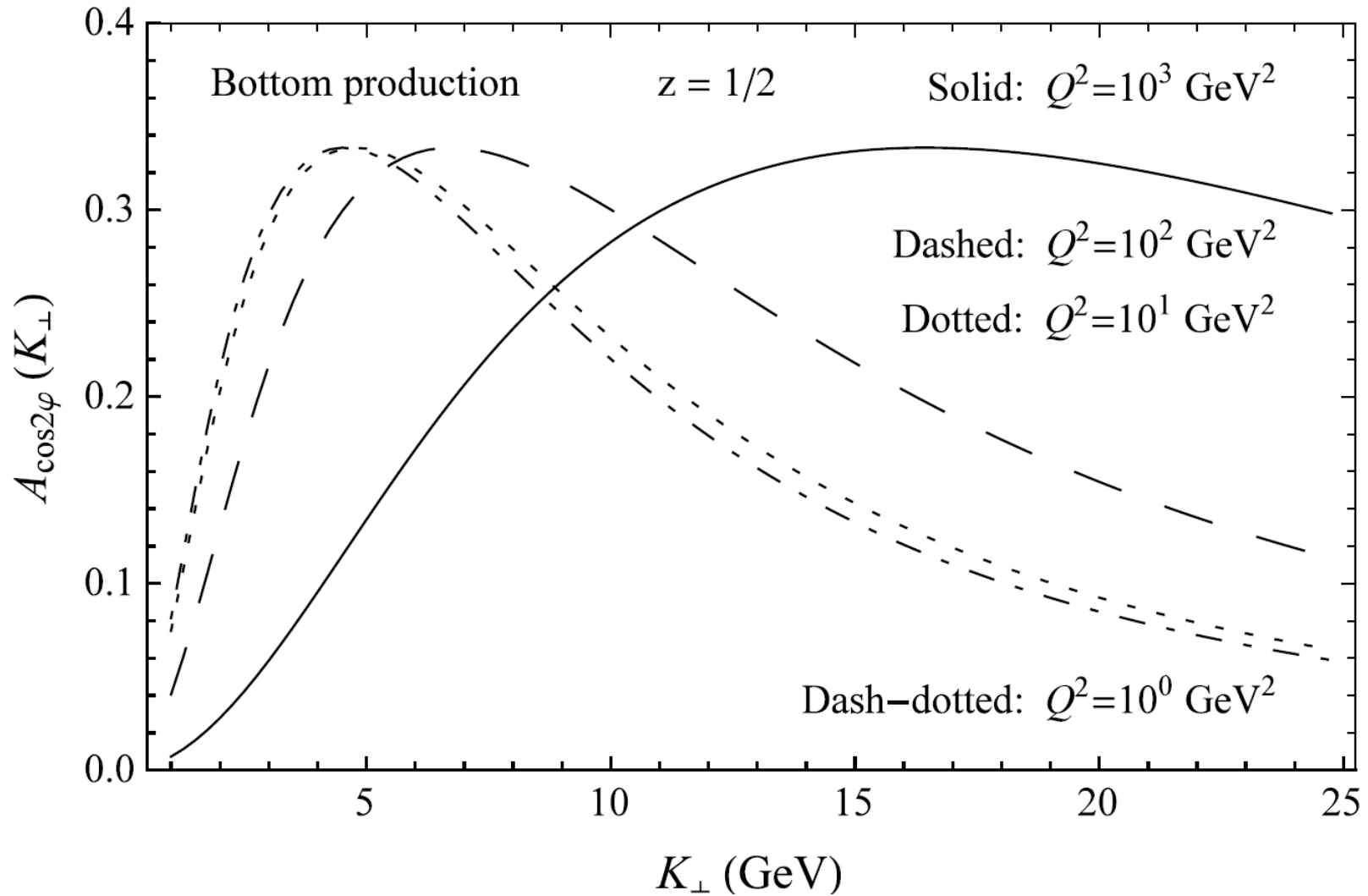
- Efremov, Ivanov, Teryaev, Phys.Lett. B 772 (2017), 283
- Efremov, Ivanov, Teryaev, Phys.Lett. B 777 (2018), 435

pQCD predictions for $\cos 2\varphi$ asymmetry ($r = 0$)



$$A_{\cos 2\varphi}(z, \vec{K}_{\perp}^2) \simeq \frac{\hat{B}_A}{\hat{B}_2}(z, \vec{K}_{\perp}^2) \quad A_{\cos 2\varphi}(z = 1/2, \vec{K}_{\perp}^2 = m^2 + Q^2/4) = \frac{1}{3}$$

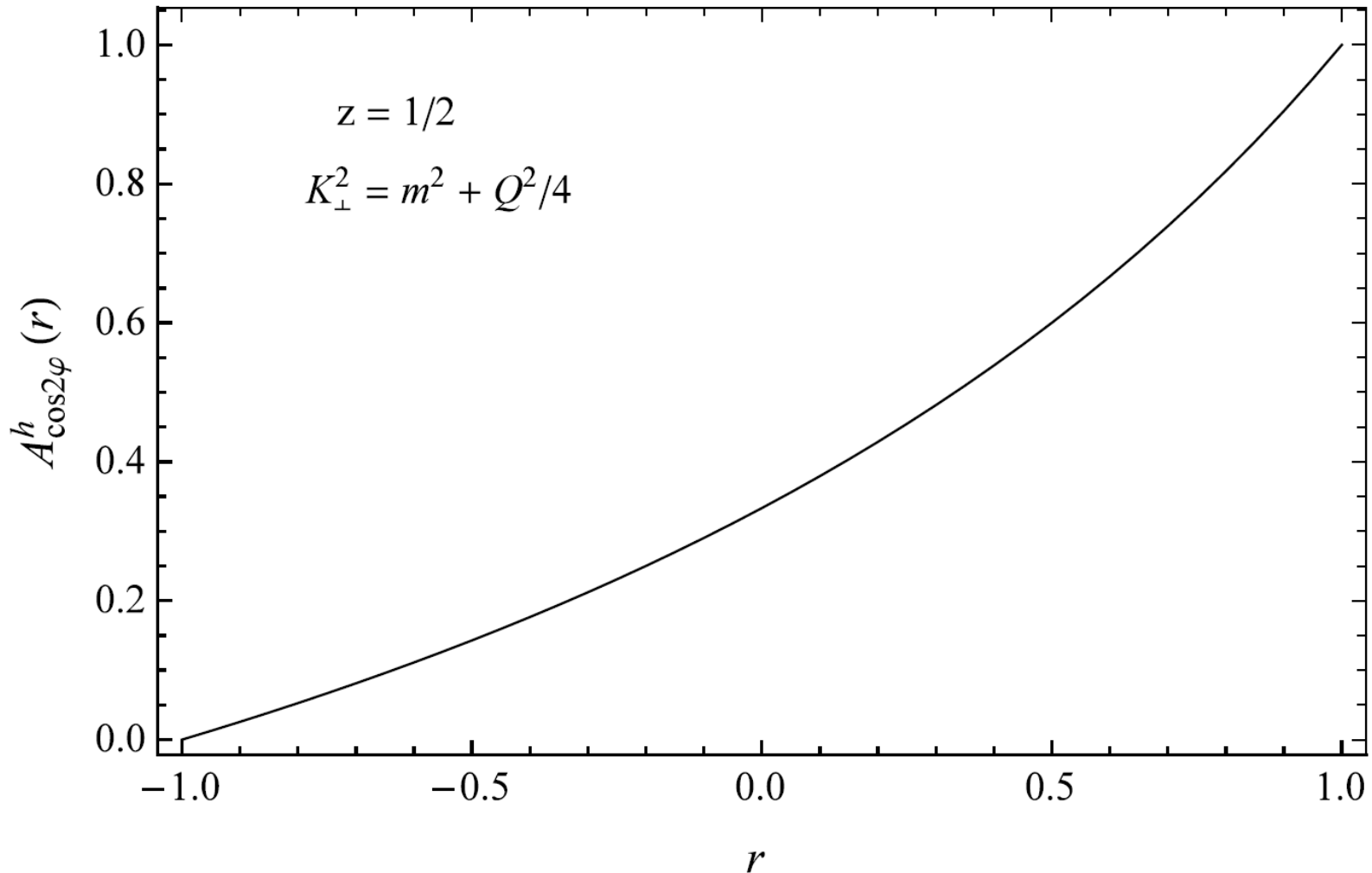
pQCD predictions for $\cos 2\varphi$ asymmetry ($r = 0$)



pQCD predictions for **cos 2φ** asymmetry ($r \neq 0$)

$$A_{\cos 2\varphi}^h(z, \vec{K}_\perp^2, r) \simeq \frac{\hat{B}_A \frac{1-2r\hat{B}_A^h}{\hat{B}_A}}{\hat{B}_2 \frac{1-2r\hat{B}_2^h}{\hat{B}_2}}$$

$$A_{\cos 2\varphi}^h(r) \equiv A_{\cos 2\varphi}^h(z = 1/2, \vec{K}_\perp^2 = m^2 + Q^2/4, r) = \frac{1+r}{3-r}$$



pQCD predictions for **cos φ** asymmetry ($r = 0$)

$$A_{\cos\varphi}(z, \vec{K}_{\perp}^2) \simeq \frac{\hat{B}_1}{\hat{B}_2}(z, \vec{K}_{\perp}^2) \quad \int dz A_{\cos\varphi}(z, \vec{K}_{\perp}^2) = 0$$

$$\begin{cases} A_{\cos\varphi}(z, \vec{K}_{\perp}^2) = -A_{\cos\varphi}(1-z, \vec{K}_{\perp}^2) \\ A_{\cos\varphi}(z, \vec{K}_{\perp}^2) = -A_{\cos\varphi}(z, (z(1-z)Q^2 + m^2)^2/\vec{K}_{\perp}^2) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} A_{\cos\varphi}(z = 1/2, \vec{K}_{\perp}^2) = 0 \\ A_{\cos\varphi}(z, \vec{K}_{\perp}^2 = z(1-z)Q^2 + m^2) = 0 \end{cases}$$

$$\max A_{\cos\varphi}(z, \vec{K}_{\perp}^2) = A_{\cos\varphi}(z = z_{\pm}, \vec{K}_{\perp}^2 = Q^2 \hat{k}_{\pm}^2)$$

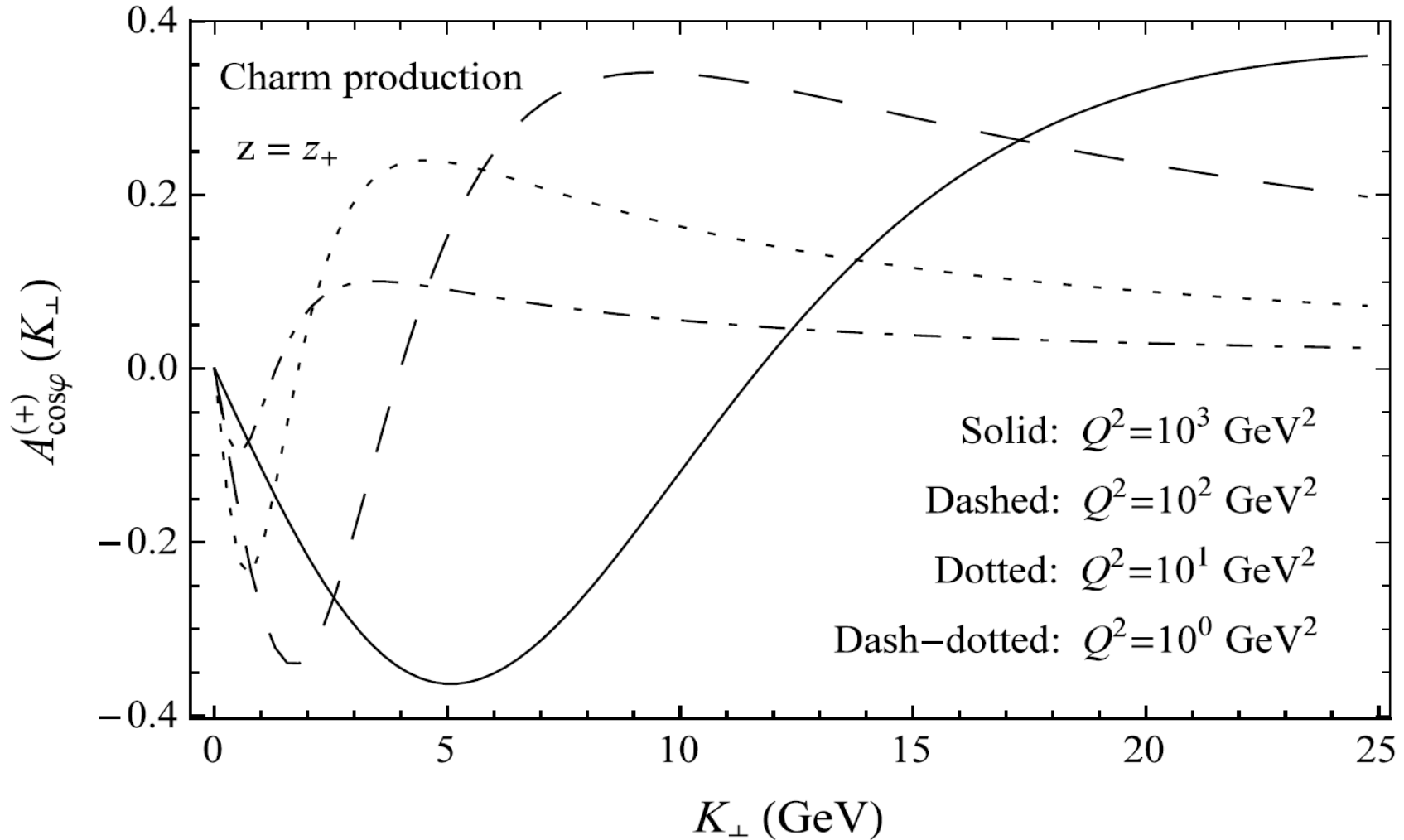
$$\min A_{\cos\varphi}(z, \vec{K}_{\perp}^2) = A_{\cos\varphi}(z = z_{\mp}, \vec{K}_{\perp}^2 = Q^2 \hat{k}_{\pm}^2)$$

$$z_{\pm}(\lambda \rightarrow 0) \simeq \begin{cases} 0.841 \\ 0.159 \end{cases} \quad \hat{k}_{\pm}^2(\lambda \rightarrow 0) \simeq \begin{cases} 0.707 \\ 0.025 \end{cases} \quad \lambda = \frac{m^2}{Q^2}$$

$$A_{\cos\varphi}^{(\pm)}(K_{\perp}) \equiv A_{\cos\varphi}(z = z_{\pm}, K_{\perp}), \quad A_{\cos\varphi}^{(-)} = -A_{\cos\varphi}^{(+)}$$

$$|A_{\cos\varphi}^{(\pm)}|_{\max} \simeq 0.366$$

pQCD predictions for **cos φ** asymmetry ($r = 0$)



$$A_{\cos\varphi}^{(\pm)}(K_{\perp}) \equiv A_{\cos\varphi}(z = z_{\pm}, K_{\perp}), \quad A_{\cos\varphi}^{(-)} = -A_{\cos\varphi}^{(+)}$$

$$|A_{\cos\varphi}^{(\pm)}|_{\max} \simeq 0.366 \quad \text{at} \quad \lambda = \frac{m^2}{Q^2}$$

pQCD predictions for **cos φ** asymmetry ($r \neq 0$)

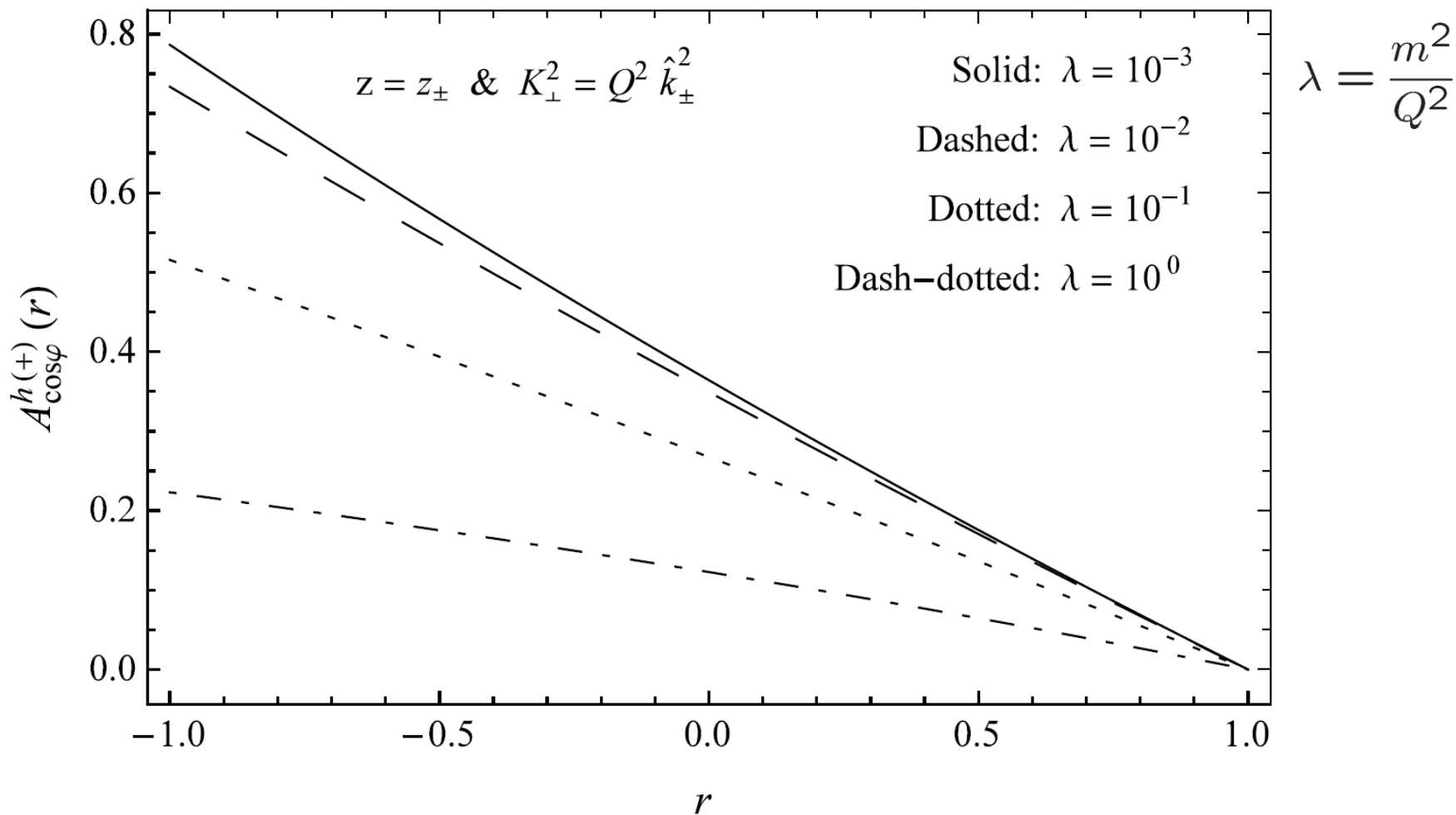
$$A_{\cos\varphi}^h(z, \vec{K}_\perp^2, r) \simeq \frac{\hat{B}_I}{\hat{B}_2} \frac{1 - 2r \hat{B}_I^h / \hat{B}_I}{1 - 2r \hat{B}_2^h / \hat{B}_2}$$

$$|A_{\cos\varphi}^{h(\pm)}|_{\max} \simeq 0.793$$

$$A_{\cos\varphi}^{h(+)}(r) \equiv A_{\cos\varphi}^h(z = z_\pm, \vec{K}_\perp^2 = Q^2 \hat{k}_\pm^2, r)$$

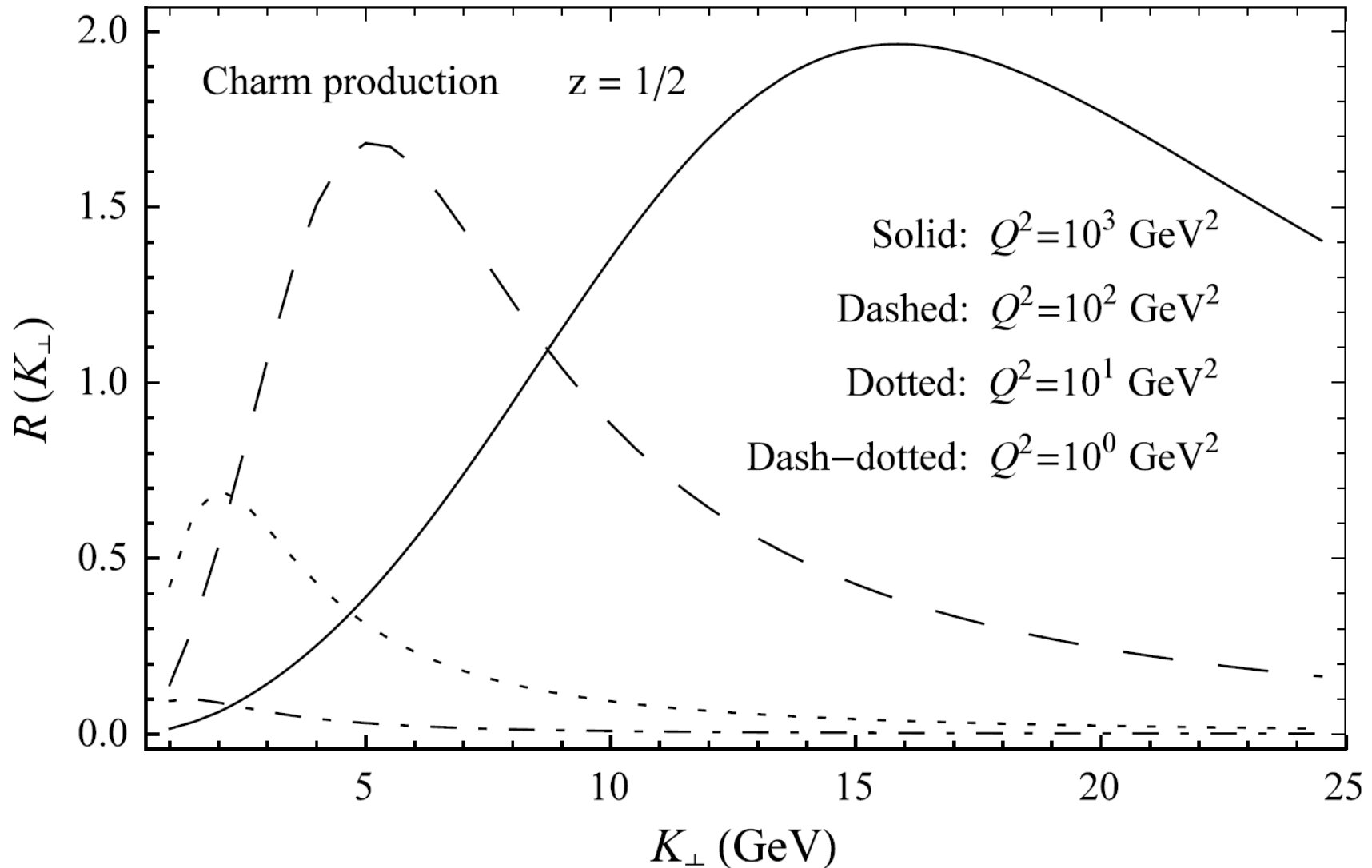
$$A_{\cos\varphi}^{h(-)}(r) \equiv A_{\cos\varphi}^h(z = z_\pm, \vec{K}_\perp^2 = Q^2 \hat{k}_\mp^2, r)$$

$$A_{\cos\varphi}^{h(\pm)}(r) \stackrel{\lambda \rightarrow 0}{\equiv} \pm \frac{(\sqrt{3} - 1)(1 - r)}{2 - r(1 - 2/\sqrt{3})}$$



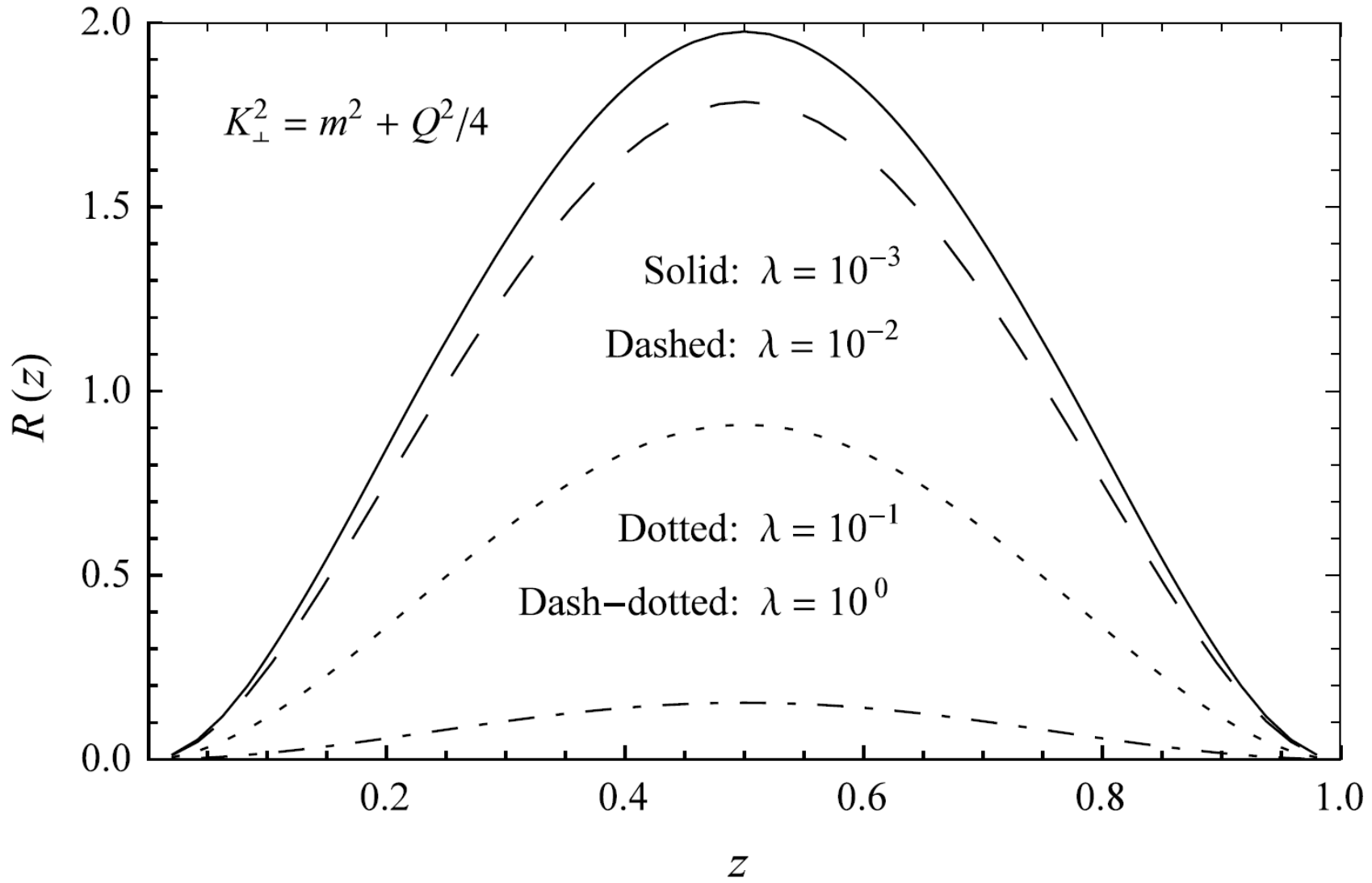
pQCD predictions for $R = F_L / F_T$ ratio ($r = 0$)

$$R(z, \vec{K}_\perp^2) = \frac{d^3\sigma_L}{d^3\sigma_T}(z, \vec{K}_\perp^2, r = 0) = \frac{\hat{B}_L}{\hat{B}_T}(z, \vec{K}_\perp^2)$$



pQCD predictions for $R = F_L / F_T$ ratio ($r = 0$)

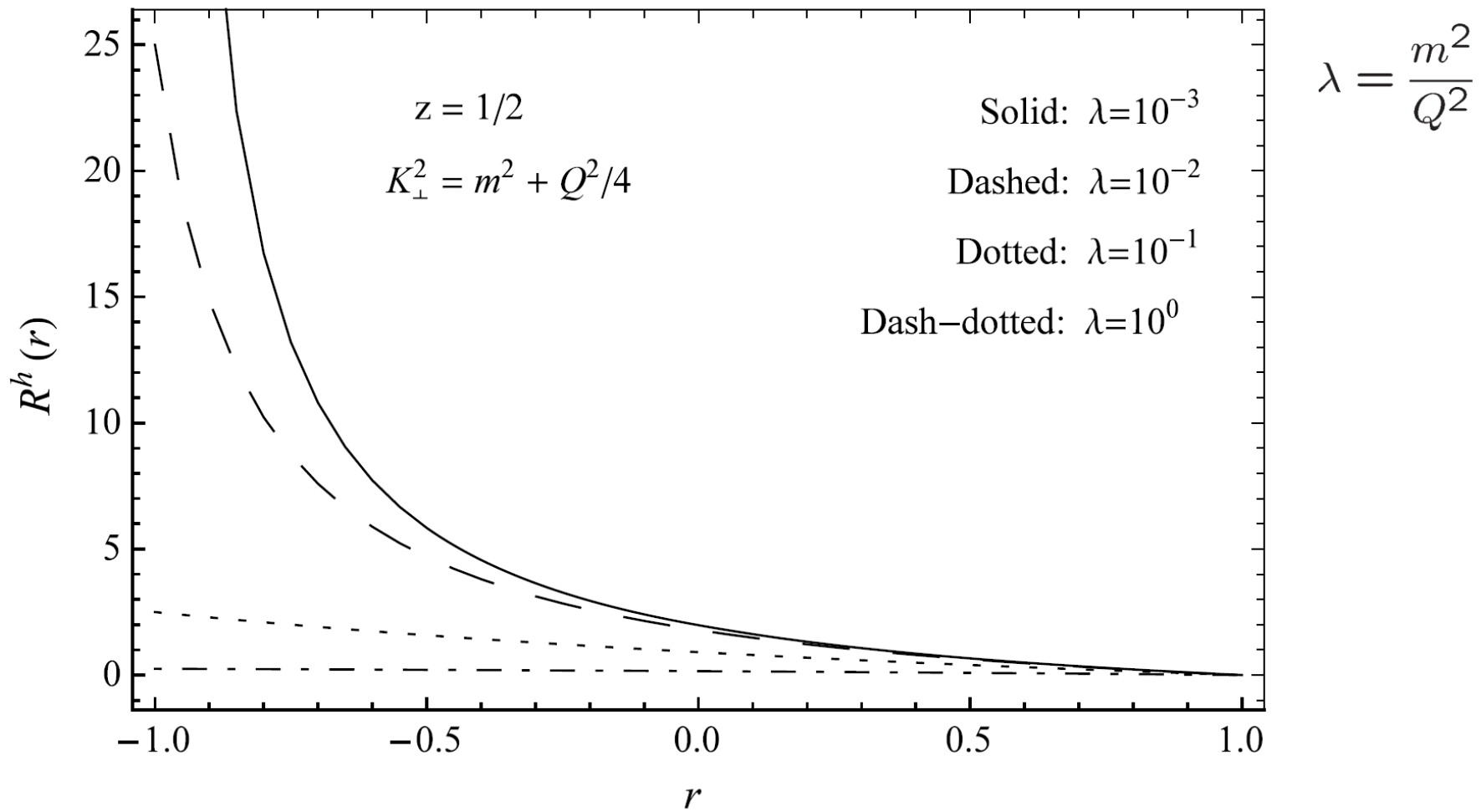
$$R(z = 1/2, \vec{K}_\perp^2 = m^2 + Q^2/4) = \frac{2}{1 + 12\lambda} \quad \lambda = \frac{m^2}{Q^2}$$



pQCD predictions for $R = F_L / F_T$ ratio ($r \neq 0$)

$$R^h(z, \vec{K}_\perp^2, r) = \frac{d^3\sigma_L}{d^3\sigma_T}(z, \vec{K}_\perp^2, r) = \frac{\hat{B}_L}{\hat{B}_T} \frac{1 - 2r\hat{B}_L^h/\hat{B}_L}{1 - 2r\hat{B}_T^h/\hat{B}_T}$$

$$R^h(r) \equiv R^h(z = 1/2, \vec{K}_\perp^2 = m^2 + Q^2/4, r) = \frac{2(1-r)}{1+r+4\lambda(3-r)}$$



Conclusions:

- When a linearly polarized gluon interacts with transverse virtual photon, the heavy-quark production plane is preferably orthogonal to the direction of the gluon polarization. For the longitudinal component, the momenta of emitted quarks and the gluon polarization lie in the same plane;
- The maximal values of the $\cos \varphi$, $\cos 2\varphi$ and $R = F_L / F_T$ quantities allowed by the photon–gluon fusion with unpolarized gluons are large: $(\sqrt{3}-1)/2$, $1/3$ and 2 , respectively;
- These distributions are very sensitive to the linear polarization of gluons: their maximum values vary from 0 to 1 depending on $h_1^\perp g$;
- We conclude that the $\cos \varphi$, $\cos 2\varphi$ and R distributions in heavy-quark pair leptonproduction could be good probes of the linear polarization of gluons inside unpolarized nucleon.

Thank You!

Azimuthal correlations in charm hadroproduction

To probe the TMD distributions in pp - and AA - collisions, the momenta of both heavy quark and anti-quark should be measured,

$$p_1(P_1) + p_2(P_2) \rightarrow Q(p_Q) + \bar{Q}(p_{\bar{Q}}) + X(p_X)$$

Corresponding cross section is:

$$d\sigma \propto \sum_{a,b} \Phi_a(\zeta_a, k_{aT}) \otimes \Phi_b(\zeta_b, k_{bT}) \otimes \left| H_{ab \rightarrow Q\bar{Q}X}(k_a, k_b, p_Q, p_{\bar{Q}}) \right|^2$$

$$k_a^\mu \simeq \zeta_a P_1^\mu + k_{aT}^\mu, \quad k_b^\mu \simeq \zeta_b P_2^\mu + k_{bT}^\mu$$

In this case, both quark and gluon densities do contribute at LO:

$$\Phi_g^{\mu\nu}(\zeta, k_T) \propto -g_T^{\mu\nu} f_1^g(\zeta, \vec{k}_T^2) + \left(g_T^{\mu\nu} - 2 \frac{k_T^\mu k_T^\nu}{k_T^2} \right) \frac{\vec{k}_T^2}{2m_N^2} h_1^{\perp g}(\zeta, \vec{k}_T^2)$$

$$\Phi_q(\zeta, k_T) \propto f_1^q(\zeta, \vec{k}_T^2) \hat{P} + i h_1^{\perp q}(\zeta, \vec{k}_T^2) \frac{[\hat{k}_T, \hat{P}]}{2m_N}$$

The resulting cross section is:

$$\frac{d^6\sigma}{dy_1 dy_2 d^2\vec{K}_\perp d^2\vec{q}_T} = \mathcal{N} \left\{ A + B \vec{q}_T^2 \cos 2(\phi_\perp - \phi_T) + C \vec{q}_T^4 \cos 4(\phi_\perp - \phi_T) \right\}$$

$$\vec{K}_\perp = \frac{1}{2} (\vec{p}_{Q\perp} - \vec{p}_{\bar{Q}\perp}), \quad \vec{q}_T = \vec{p}_{Q\perp} + \vec{p}_{\bar{Q}\perp}$$

Schematically, the functions **A**, **B** and **C** have the following structure:

$$\begin{aligned} A & : f_1^q \otimes f_1^{\bar{q}}, \quad f_1^g \otimes f_1^g, \quad h_1^{\perp g} \otimes h_1^{\perp g} \\ B & : h_1^{\perp q} \otimes h_1^{\perp \bar{q}}, \quad f_1^g \otimes h_1^{\perp g} \\ C & : h_1^{\perp g} \otimes h_1^{\perp g} \end{aligned}$$