On the pair correlations of neutral *K*, *D*, *B* and B_s mesons with close momenta produced in inclusive multiparticle processes

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1. Correlations of pairs of neutral *K* mesons

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- Internal states of the neutral kaon with definite strangeness:

$$|K^0\rangle \quad (S=1), \quad |\overline{K}^0\rangle \quad (S=-1)$$

- Internal states of the neutral kaon with definite *CP* parity (neglecting weak effects of *CP* nonconservation) :

$$|K_S^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$

⟹ *CP* = +1, short-lived neutral kaon decaying into two □ mesons ;

$$|K_L^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$

⟹ *CP* = -1, long-lived neutral kaon decaying into three □ mesons .

In inclusive processes with strangeness conservation, pairs $K^0K^0 \quad (S=2), \quad \overline{K}^0\overline{K}^0 \quad (S=-2)$ are generated incoherently. The internal state of pair $K^0\overline{K}^0 \quad (S=0)$ is non-factorizable at given momenta $\vec{p}_1$, $\vec{p}_2$:

non-diagonal elements of the density matrix between the states

$|K^0\rangle^{(p_1)} \otimes |\bar{K}^0\rangle^{(p_2)}$ and $|\bar{K}^0\rangle^{(p_1)} \otimes |K^0\rangle^{(p_2)}$ are not equal to zero .

As follows from the Bose symmetry with respect to full permutation, *CP* parity of the system $K^0\overline{K}^0$ is always positive ($C = (-1)^L$, $P = (-1)^L$, L is the orbital momentum) .

Symmetric internal state of the pair $K^0\overline{K}^0$, corresponding to even orbital momenta :

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle^{(p_1)} \otimes |\bar{K}^0\rangle^{(p_2)} + |\bar{K}^0\rangle^{(p_1)} \otimes |K^0\rangle^{(p_2)}) =$$
$$= \frac{1}{\sqrt{2}}(|K_S^0\rangle^{(p_1)} \otimes |K_S^0\rangle^{(p_2)} - |K_L^0\rangle^{(p_1)} \otimes |K_L^0\rangle^{(p_2)})$$

(Decomposition into the schemes $K_S^0K_S^0$ and $K_L^0K_L^0$)

Antisymmetric internal state, corresponding to odd orbital momenta :

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle^{(p_1)} \otimes |\bar{K}^0\rangle^{(p_2)} - |\bar{K}^0\rangle^{(p_1)} \otimes |K^0\rangle^{(p_2)}) =$$
$$= \frac{1}{\sqrt{2}}(|K_S^0\rangle^{(p_1)} \otimes |K_L^0\rangle^{(p_2)} - |K_L^0\rangle^{(p_1)} \otimes |K_S^0\rangle^{(p_2)})$$

(Decomposition into the scheme $K_S^0 K_L^0$)

At the selection of the pairs of neutral kaons over decays, the structure functions (double inclusive cross sections) are invariant with respect to the permutation of momenta $\vec{p}_1$ and $\vec{p}_2$ and replacement $K_S^0 \Leftrightarrow K_L^0$.

$$f_{SS}(\mathbf{p}_1, \mathbf{p}_2) = f_{LL}(\mathbf{p}_1, \mathbf{p}_2) = f_{SL}(\mathbf{p}_1, \mathbf{p}_2) + \mathrm{Re}\, \rho_{K^0\bar{K}^0 \to \bar{K}^0 K^0}(\mathbf{p}_1, \mathbf{p}_2)$$

$\rho_{K^0\bar{K}^0 \to \bar{K}^0 K^0} = \rho^*_{\bar{K}^0 K^0 \to K^0 \bar{K}^0}$ -- non-diagonal element of the density matrix of two neutral kaons

- Pair momentum-energy correlations of neutral kaons with small relative momenta

In the framework of the conventional model of one-particle sources, correlation functions R_{SS} and R_{LL}, normalized by 1 at large momentum differences:

$$R_{SS}(\mathbf{k}) = R_{LL}(\mathbf{k}) = \lambda_{K^0 K^0}\left[1 + F_{K^0}(2\mathbf{k}) + 2\, b_{\mathrm{int}}(\mathbf{k})\right] +$$
$$+\lambda_{\bar{K}^0 \bar{K}^0}\left[1 + F_{\bar{K}^0}(2\mathbf{k}) + 2\, \tilde{b}_{\mathrm{int}}(\mathbf{k})\right] +$$
$$+\lambda_{K^0 \bar{K}^0}\left[1 + F_{K^0 \bar{K}^0}(2\mathbf{k}) + 2\, B_{\mathrm{int}}(\mathbf{k})\right]$$

$\vec{k}$ - momentum of one of the kaons in the c.m. frame of the kaon pair ,

$\lambda_{K^0K^0}, \lambda_{\bar{K}^0\bar{K}^0}, \lambda_{K^0\bar{K}^0}$ are relative weights of the pairs $K^0\overline{K}^0$, K^0K^0, $\overline{K}^0\overline{K}^0$

($\lambda_{K^0K^0} + \lambda_{\bar{K}^0\bar{K}^0} + \lambda_{K^0\bar{K}^0} = 1$) .

«Form factors» $F_{K^0}(2\mathbf{k}), F_{\bar{K}^0}(2\mathbf{k}), F_{K^0\bar{K}^0}(2\mathbf{k})$ describe the contribution of Bose statistics without taking into account final-state interaction ;

$b_{\text{int}}(\vec{k})$, $\tilde{b}_{\text{int}}(\vec{k})$ ⟶ S – wave interaction of two K^0 mesons and two $\overline{K}^0$ mesons ; $B_{\text{int}}(\vec{k})$ ⟶ S – wave interaction between the K^0 meson and $\overline{K}^0$ meson .

- If a pair of non-identical neutral kaons $K^0\overline{K}^0$ is generated, but the states $K_S^0K_S^0$ (or $K_L^0K_L^0$) are registered over decays, then the two-particle momentum-energy correlations at small relative momenta have the same character as in the case of ordinary identical bosons (pions) with zero spin .

- For pairs of non-identical states $K_S^0 K_L^0$:

$$R_{SL}(\mathbf{k}) = R_{LS}(\mathbf{k}) = \lambda_{K^0K^0}\left[1 + F_{K^0}(2\mathbf{k}) + 2\,b_{\rm int}(\mathbf{k})\right] +$$
$$+\lambda_{\bar{K}^0\bar{K}^0}\left[1 + F_{\bar{K}^0}(2\mathbf{k}) + 2\,\tilde{b}_{\rm int}(\mathbf{k})\right] +$$
$$+\lambda_{K^0\bar{K}^0}\left[1 - F_{K^0\bar{K}^0}(2\mathbf{k})\right]$$

- At the generation of pairs of non-identical neutral kaons $K^0\overline{K}^0$ and registration of the state $K_S^0 K_L^0$ over decays, pair correlations are analogous to the correlations of identical fermions with equal spin projections (since in this case the pair $K_S^0 K_L^0$ has odd orbital momentum) .

$$R_{SS}(\mathbf{k}) - R_{SL}(\mathbf{k}) = 2\,\lambda_{K^0\bar{K}^0}\left[F_{K^0\bar{K}^0}(2\mathbf{k}) + B_{\rm int}(\mathbf{k})\right]$$

The difference between the correlation functions for pairs of identical neutral kaons $K_S^0 K_S^0$ and pairs of non-identical neutral kaons $K_S^0 K_L^0$ is conditioned exclusively by the generation of $K^0\overline{K}^0$ pairs .

- Form factors $F_{K^0}(2\vec{k})$, $F_{\bar{K}^0}(2\vec{k})$, $F_{K^0\bar{K}^0}(2\vec{k})$ and functions $b_{\text{int}}(\vec{k})$, $\tilde{b}_{\text{int}}(\vec{k})$ and $B_{\text{int}}(\vec{k})$ contain the information on space-time parameters of the generation region of neutral kaons and tend to zero at large relative momenta $q = 2|\vec{k}|$:

$$F_{K^0}(2\mathbf{k}) = \int W_{K^0}(\mathbf{r}) \cos(2\mathbf{kr})\, d^3\mathbf{r}, \qquad F_{\bar{K}^0}(2\mathbf{k}) = \int W_{\bar{K}^0}(\mathbf{r}) \cos(2\mathbf{kr})\, d^3\mathbf{r},$$

$$F_{K^0\bar{K}^0}(2\mathbf{k}) = \int W_{K^0\bar{K}^0}(\mathbf{r}) \cos(2\mathbf{kr})\, d^3\mathbf{r}.$$

$$b_{\text{int}}(\mathbf{k}) = \int W_{K^0}(\mathbf{r})\, b(\mathbf{k},\mathbf{r}) d^3\mathbf{r}, \quad \tilde{b}_{\text{int}}(\mathbf{k}) = \int W_{\bar{K}^0}(\mathbf{r})\, \tilde{b}(\mathbf{k},\mathbf{r}) d^3\mathbf{r}.$$

(due to *CP* invariance $b(\mathbf{k},\mathbf{r}) = \tilde{b}(\mathbf{k},\mathbf{r})$) ;

$$B_{\text{int}}(\mathbf{k}) = \int W_{K^0\bar{K}^0}(\mathbf{r}) B(\mathbf{k},\mathbf{r}) d^3\mathbf{r},$$

$W_{K^0}(\mathbf{r})$, $W_{\bar{K}^0}(\mathbf{r})$, $W_{K^0\bar{K}^0}(\mathbf{r})$ are the distributions of distances between sources of emission of two K^0 mesons, two $\bar{K}^0$ mesons, a K^0 meson and a $\bar{K}^0$ meson, respectively -- in the c.m. frame of the kaon pair .

- Connection of the contribution of final-state interaction into the pair momentum-energy correlations of kaons at small relative momenta with the parameters of S -wave low-energy scattering

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Approximate formula :

$$B(\mathbf{k},\mathbf{r}) = \left(|A_{K^0\bar{K}^0\to K^0\bar{K}^0}(k)|^2 + |A_{K^+K^-\to K^0\bar{K}^0}(k)|^2 \right) \frac{1}{r^2} + $$
$$+2\,\mathrm{Re}\left(A_{K^0\bar{K}^0\to K^0\bar{K}^0}(k)\,\frac{\exp(ikr)\cos\mathbf{kr}}{r} \right),$$

$k = |\vec{k}|$, $r = |\vec{r}|$, $A_{K^0\bar{K}^0\to K^0\bar{K}^0}(k)$, -- amplitude of S -wave elastic

$K^0\overline{K}^0$ - scattering ; $A_{K^+K^-\to K^0\bar{K}^0}(k)$ -- amplitude of the reaction $K^+K^- \to K^0\overline{K}^0$ at the momentum of final K^0 meson equaling k in the c.m.s. of pair $K^0\overline{K}^0$

(cross section of the process $K^+K^- \to K^0\overline{K}^0$:

$$\sigma_{K^+K^-\to K^0\bar{K}^0}(k) = 4\pi\,|A_{K^+K^-\to K^0\bar{K}^0}(k)|^2\,\frac{k}{\tilde{k}},$$

$\tilde{k} = \sqrt{k^2 + (m_0^2 - m_+^2)}$ - momentum of the charged kaon in the c.m. frame .

2. Correlations of pairs of neutral heavy mesons

- Formally, analogous relations are valid also for the neutral heavy mesons D^0 , B^0 and B_s^0 . In doing so, the role of strangeness conservation is played, respectively, by the conservation of charm and beauty in inclusive multiple processes with production of these mesons . In these cases the quasistationary states are also states with definite *CP* parity, neglecting the weak effects of *CP* nonconservation .

For example, $|B_S^0\rangle = \dfrac{|B^0\rangle + |\overline{B}^0\rangle}{\sqrt{2}}$, *CP* parity + 1 ;

$|B_L^0\rangle = \dfrac{|B^0\rangle - |\overline{B}^0\rangle}{\sqrt{2}}$, *CP* parity – 1 .

- In accordance with the mechanism of mixing a particle with the respective antiparticle due to weak interaction through the exchange of two virtual *W* bosons, states with *CP* parity (– 1) have the greater mass and the larger lifetime than states with *CP* parity (+ 1) . The difference of masses is very insignificant in all the cases, ranging from 10^{-12} MeV for K^0 mesons up to 10^{-8} MeV for B_s^0 mesons .

- Concerning the lifetimes, in the case of K^0 mesons they differ by 600 times, but for D^0 , B^0 and B_s^0 mesons the respective difference is very inconsiderable. In connection with this, it is practically impossible to distinguish the states of D^0, B^0 and B_s^0 mesons with definite *CP* parity by the difference in their lifetimes. These states, in principle, can be identified through the purely *CP*-even and purely *CP*-odd decay channels; however, in fact the branching ratio for such decays is very small. For example,

$$Br\,(D^0 \to \pi^+\pi^-) = 1.62 \cdot 10^{-3} \;\; (CP = +1)\;;$$

$$Br\,(D^0 \to K^+K^-) = 4.25 \cdot 10^{-3} \;\; (CP = +1)\;;$$

$$Br\,(B_s^0 \to J/\Psi\;\pi^0) < 1.2 \cdot 10^{-3} \;\; (CP = +1)\;;$$

$$Br\,(B^0 \to J/\Psi\;K_S^0) = 9 \cdot 10^{-4} \;\; (CP = -1)\;;$$

Just as in the case of neutral *K* mesons, the correlation functions for the pairs of states of neutral *D*, *B* and B_s mesons with the same *CP* parity ($R_{SS} = R_{LL}$) and for the pairs of states with different *CP* parity (R_{SL}) do not coincide, and the difference between them is conditioned exclusively by the production of pairs $D^0\overline{D}^0$, $B^0\overline{B}^0$ and $B_s^0\overline{B}_s^0$, respectively. In particular, for B_s^0 mesons the following relation holds:

$$R_{SS}(k) - R_{SL}(k) = 2\,\lambda_{B_s^0\overline{B}_s^0}\,[\,F_{B_s^0\overline{B}_s^0}(2\mathbf{k}) + B_{\text{int}}(\mathbf{k})\,]$$

here $\lambda_{B_s^0\overline{B}_s^0}$ is the relative fraction of generated pairs $B_s^0\overline{B}_s^0$,

$$F_{B_s^0\overline{B}_s^0}(2\mathbf{k}) = \int W_{B_s^0\overline{B}_s^0}(\mathbf{r})\cos(2\mathbf{kr})\,d^3\mathbf{r} \quad ,$$

$$B_{\text{int}}(\mathbf{k}) = \int W_{B_s^0\overline{B}_s^0}(\mathbf{r})\,B(\mathbf{k},\mathbf{r})\,d^3\mathbf{r} \quad ,$$

$$B(\mathbf{k},\mathbf{r}) = |\,A_{B_S^0\overline{B}_S^0}(k)\,|^2\,\frac{1}{r^2} + 2\,\text{Re}\left(A_{B_S^0\overline{B}_S^0}(k)\,\frac{\exp(ikr)\cos\mathbf{kr}}{r}\right) ,$$

where $A_{B_S^0\overline{B}_S^0}(k) \equiv A_{B_S^0\overline{B}_S^0 \to B_s^0\overline{B}_s^0}(k)$ is the amplitude of *S*-wave $B_s^0\overline{B}_s^0$- scattering, $k = |\,\mathbf{k}\,|$, $r = |\,\mathbf{r}\,|$. Let us remark that the B_s^0 and $\overline{B}_s^0$ mesons do not have charged partners (the isotopic spin equals zero) and, on account of that, in the given case the transition similar to $K^+K^- \to K^0\overline{K}^0$ is absent .

Summary

- The phenomenological structure of inclusive cross sections of production of two neutral *K* mesons in collisions of hadrons and nuclei is investigated taking into account strangeness conservation in strong and electromagnetic interactions. As follows directly from strangeness conservation, the double inclusive cross sections of production of two K_L^0 mesons and two K_S^0 mesons coincide.

- Within the model of one-particle sources, the phenomenological formulas for the correlation functions $R_{SS} = R_{LL}$ and $R_{SL} = R_{LS}$, involving the contributions of Bose statistics and *S*- wave strong final-state interaction for two K^0 ($\overline{K}^0$) mesons as well as for K^0 and $\overline{K}^0$, and depending on the relative fractions of generated pairs K^0K^0, $\overline{K}^0\overline{K}^0$ and $K^0\overline{K}^0$, have been derived .

- It is shown that namely the generation of $K^0\overline{K}^0$ pairs with zero strangeness gives rise to the difference between the correlation functions R_{SS} and R_{SL} of two neutral kaons .

- The character of analogous correlations for neutral heavy mesons D^0, B^0, B_s^0 with nonzero charm and beauty is discussed . Contrary to the case of K^0 mesons, here the distinction of respective *CP* -even and *CP* -odd states encounters difficulties, which are connected with the insignificant difference of their lifetimes and the relatively small probability of purely *CP* -even and purely *CP* -odd decay channels . However, one may expect that this will become possible at future colliders .

Thank you !